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On β-local Functions in ideal topological spaces

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Abstract. In this paper, by using β-open sets in [1] we introduce and investigate the concepts of the β-local function, I_{s^*g}-β-closed sets and I_g-β-closed sets in an ideal topological space. In addition to the properties, an operation cl^*_β is defined and the properties are obtained similarly with the local function in [8].

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1. Introduction

Kuratowski [11] has introduced the concept of an ideal topological space in 1930. Further, Jankovic and Hamlet [8] have studied ideal topological spaces and obtained their significant properties. They introduced the concept of I-open sets and studied topologies via ideals quite extensively. Abd-El-Monsef et al.[2] further explored the ideas of I-open sets. The concept of I_g-closed sets has been given by Dontchev et al. [6] in 1999 and the idea of I_{s^*g}-closed sets was first introduced by Khan and Hamza [9]. The concepts of the s-local function was first introduced by Abd. El-Monsef et al. [3] and further investigated by Khan and Noiri [10].

Recently, Al-Omari and Noiri [5] have introduced and investigated the notion of local function Γ^* in an ideal topological space and showed that Γ^* is equivalent to the δ-local function due to Hatir et al. [7]. In this paper, by using β-open sets in [1] we introduce and investigate the concepts of the β-local function, I_{s^*g}-β-closed sets and I_g-β-closed sets in an ideal topological space. And also, an operation cl^*_β is defined and the properties are obtained similarly with the local function in [8].

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2. Preliminaries

Throughout this paper (X, τ) and (X, τ, I) denote a topological space and an ideal topological space, respectively. The collection of closed sets in X is denoted by τ_F. For any subset A of X the closure and the interior of A are denoted by $cl(A)$ and $Int(A)$, respectively.

We now recall certain definitions, which would be required for our study.

Definition 1. *[8] An **ideal** I on a topological spaces (X, τ) is a nonempty collection of subsets of X, which satisfies the following conditions:*

- *$A \in I$ and $B \in I$ **implies** $A \cup B \in I$,*
- *$A \in I$ and $B \subset A$ **implies** $B \in I$.*

Then the triplet (X, τ, I) is called an ideal topological space.

Definition 2. *[8] Let (X, τ, I) be an ideal topological space. For a set $A \subset X$, $A^*(X, \tau) = \{x \in X : A \cap U \notin I$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau : x \in U\}$, is called the **local-function** of A with respect to I and τ. $A^*(X, \tau)$ is simply denoted by A^*.*

Definition 3. *[5] Let (X, τ, I) be an ideal topological space. For a set $A \subset X$, $\Gamma^*(A)(I, \tau) = \{x \in X : A \cap U \notin I$ for every regular open set U containing $x\}$ is called the **local function** Γ^* of A with respect to I and τ.*

Definition 4. *[3],[10] Let (X, τ, I) be an ideal topological space and A be a subset of X. Then $(A)^{*^s}(I, \tau) = \{x \in X : A \cap U \notin I$ for every $U \in SO(X, x)\}$ is called the **semi-local function** of A with respect to I and τ, where $SO(X, x) = \{U \in SO(X) | x \in U\}$. When there is no ambiguity we write A^{*^s} for $(A)^{*^s}(I, \tau)$.*

Definition 5. *Let (X, τ) be a topological space. A subset A of X is said to be:*

(i) β-open [1] if $A \subset cl(Int(cl(A)))$,

(ii) semi-open [12] if $A \subset cl(Int(A))$,

(iii) regular-open [13] if $A = Int(cl(A))$.

The family of all β-open (resp. semi-open, regular open) sets in X is denoted by $\beta O(X)$ (resp. $SO(X), RO(X)$).

Definition 6. *[1] Let (X, τ) be a topological space. A subset A of X is said to be **β-closed** if its complement is β-open.*

Definition 7. *[4] Let (X, τ) be a topological space and A be a subset of X. The **β-closure** of A is defined by the intersection of all β-closed sets containing the set A and it is denoted by $\beta cl(A)$.*

3. β-local functions

In order to define the generalized version of the local function [8], we now introduce the concept of the β-local function.

Definition 8. *Let (X, τ, I) be an ideal topological space. For a set $A \subset X$, $A^*_\beta(I, \beta O(X)) =\{x \in X : A \cap U \notin I$ for every $U \in \beta O(x)\}$, where $\beta O(x) = \{U \in \beta O(X) : x \in U\}$, is called the **$\beta$-local function** of A with respect to I and $\beta O(X)$. $A^*_\beta(I, \beta O(X))$ is simply denoted by A^*_β.*

Example 1. *Let $X = \{a, b, c, d\}$ be a nonempty set with the topology $\tau = \{\phi, X, \{a\}, \{a, b, c\}\}$. Then the collection of closed sets is $\tau_F = \{X, \phi, \{b, c, d\}, \{d\}\}$. Applying Definition 5, we compute the collection $\beta O(X)= \{\phi, X, \{a\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{c, d, a\}, \{d, a, b\}, \{a, c\}\}$. Next, we consider $I = \{\phi, \{b\}\}$. If $A = \{c, d\} \subset X$ then it may be easily verified that $A^*_\beta = \{c, d\}$ and $A^* = \{b, c, d\}$.*

We need the following lemma for our analysis.

Lemma 1. *[4] Let A be a subset of a topological space (X, τ). Then $x \in \beta cl(A)$ if and only if $A \cap U \neq \phi$ for every U in $\beta O(x)$.*

Theorem 1. *Let (X, τ, I) be an ideal topological space and A, B be subsets of X. Then the following properties hold:*

(1). If $A \subset B$ then $A^*_\beta \subset B^*_\beta$,

(2). $(A \cup B)^*_\beta = A^*_\beta \cup B^*_\beta$,

(3). $(A \cap B)^*_\beta \subset A^*_\beta \cap B^*_\beta$,

(4). $(A^*_\beta)^*_\beta \subset A^*_\beta$,

(5). $A^*_\beta = \beta cl(A^*_\beta) \subset \beta cl(A)$.

Proof. (1). If $x \notin B^*_\beta$, then there exists $U \in \beta O(x)$ such that $U \cap B \in I$. Since $A \subset B$, $U \cap A \in I$ and hence $x \notin A^*_\beta$. This shows that $A^*_\beta \subset B^*_\beta$.
(2). Let $x \in (A \cup B)^*_\beta$, then using Definition 8, we have $U \cap (A \cup B) \notin I$ for every $U \in \beta O(x)$ and $(U \cap A) \cup (U \cap B) \notin I$. Now, since I is an ideal, three cases can be possible:
case I $(U \cap A) \notin I$ and $(U \cap B) \notin I$,
case II $(U \cap A) \in I$ and $(U \cap B) \notin I$,
case III $(U \cap A) \notin I$ and $(U \cap B)$ in I.
It may be seen easily that for all three cases $x \in A^*_\beta \cup B^*_\beta$. Therefore, we have $(A \cup B)^*_\beta \subset A^*_\beta \cup B^*_\beta$. By (1), $A^*_\beta \subset (A \cup B)^*_\beta$ and $B^*_\beta \subset (A \cup B)^*_\beta$. Hence, $A^*_\beta \cup B^*_\beta \subset (A \cup B)^*_\beta$ and we obtain

$$A^*_\beta \cup B^*_\beta = (A \cup B)^*_\beta.$$

(3). Since, $A \cap B \subset A$ and $A \cap B \subset B$, by (1), $(A \cap B)^*_\beta \subset (A)^*_\beta$ and $(A \cap B)^*_\beta \subset (B)^*_\beta$ and hence,

$$(A \cap B)^*_\beta \subset (A)^*_\beta \cap (B)^*_\beta.$$

(4). Let $x \in (A^*_\beta)^*_\beta$, then by Definition 8, we have, $A^*_\beta \cap U \notin I$ for every $U \in \beta O(x)$ and $A^*_\beta \cap U \neq \phi$. Now, let $y \in A^*_\beta \cap U$, then $y \in U$ and $U \in \beta O(y)$. Since $y \in A^*_\beta$, $A \cap U \notin I$. Hence, $x \in A^*_\beta$. Therefore, $(A^*_\beta)^*_\beta \subset A^*_\beta$.
(5). We know that $A^*_\beta \subset \beta cl(A^*_\beta)$.
We show that $\beta cl(A^*_\beta) \subset A^*_\beta$.
Let $x \in \beta cl(A^*_\beta)$. Then by Lemma 1, we have, $A^*_\beta \cap U \neq \phi$ for every $U \in \beta O(x)$. Now, let $y \in A^*_\beta \cap U$, then we have $y \in A^*_\beta$ and $y \in U \in \beta O(X)$. Therefore, we have, $A \cap U \notin I$ and hence, $x \in A^*_\beta$. Therefore, $\beta cl(A^*_\beta) \subset A^*_\beta$ and hence, $A^*_\beta = \beta cl(A^*_\beta)$. This implies A^*_β is β-closed.
Next, we show that $A^*_\beta \subset \beta cl(A)$. If $x \notin \beta cl(A)$, then there exists $U \in \beta O(x)$ such that $U \cap A = \phi \in I$ and $x \notin A^*_\beta$. Therefore, $A^*_\beta \subset \beta cl(A)$.

Remark 1. *Let (X, τ, I) be an ideal topological space and $A \subset X$. Then the following holds:*

(1). If $I = \{\phi\}$, then $A^*_\beta = \beta cl(A)$,

(2). If $K \in I$, then $K^*_\beta = \phi$ and hence, $\{\phi\}^*_\beta = \phi$,

*(3). It is not necessary that **3(a).** $A \subset A^*_\beta$ or **3(b).** $A^*_\beta \subset A$,*

(4). $A^*_\beta(I, \beta O(X)) = (A)^{*^s}(I, \tau)$ if $SO(X) = \beta O(X)$,

(5). $A^*_\beta(I, \beta O(X)) = A^*(I, \tau)$ if $\beta O(X) = \tau(X)$.

In order to verify **3(a).** and **3(b).** of Remark 1, we explore the following example:

Example 2. Let $X = \{a, b, c, d\}$ be a nonempty set with the topology $\tau = \{\phi, X, \{a\}, \{a, b, c\}\}$ and the collection of closed sets is $\tau_F = \{\phi, X, \{b, c, d\}, \{d\}\}$. Next, by applying Definition 5, we compute the collection βO(X)= $\{\phi, X, \{a\}, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}, \{c, d, a\}, \{d, a, b\}\}$. Considering $I = \{\phi, \{b\}\}$ and $A, B \subset X$ where, $A = \{b, c, d\}$ and $B = \{a, b, c\}$ then by applying Definition 8, $A^*_\beta = \{c, d\}$ and $B^*_\beta = X$. In view of above assertions 3(a) and 3(b) have been verified.

Lemma 2. *Let (X, τ, I) be an ideal topological space. Then the following properties hold:*
(1). *$RO(X) \subset \tau \subset SO(X) \subset \beta O(X)$,*
(2). *$A^*_\beta \subset A^{*^s} \subset A^* \subset \Gamma^*(A)$ for every subset A of X.*

Proof. **(1).** The proof is obvious by the Definition 5.
(2). First, we show that $A^*_\beta \subset A^{*^s}$. Let $x \in A^*_\beta$. Then, $A \cap U \notin I$ for every $U \in \beta O(x)$. Since $SO(X) \subset \beta O(X)$, $A \cap U \notin I$ for every $U \in SO(x)$ and $x \in A^{*^s}$. Hence, we have $A^*_\beta \subset A^{*^s}$. Similarly, by using the fact that $RO(X) \subset \tau \subset SO(X)$, we may establish $A^{*^s} \subset A^*$ and $A^* \subset \Gamma^*(A)$.

Definition 9. *Let (X, τ, I) be an ideal topological space. We define $cl^*_\beta(A) = A \cup A^*_\beta$ for every subset A of X.*

Theorem 2. *Let (X, τ, I) be an ideal topological space. Then the following properties hold:*
(1). $A \subset cl^*_\beta(A)$,
(2). $cl^*_\beta(\phi) = \phi$ *and* $cl^*_\beta(X) = X$,
(3). $A \subset B$ *implies* $cl^*_\beta(A) \subset cl^*_\beta(B)$,
(4). $cl^*_\beta(A) \cup cl^*_\beta(B) = cl^*_\beta(A \cup B)$,
(5). $(cl^*_\beta(A))^*_\beta \subset cl^*_\beta(A) = cl^*_\beta(cl^*_\beta(A))$.

Proof. **(1).** This follows directly by the Definition 9.
(2). We have $cl^*_\beta(\phi) = \{\phi\} \cup \{\phi\}^*_\beta = \phi$. Similarly, it may be verified that $cl^*_\beta(X) = X$.
(3). Given, $A \subset B$. By Definition 9, $cl^*_\beta(A) = A \cup A^*_\beta$ and $cl^*_\beta(B) = B \cup B^*_\beta$. Next, by Theorem 1 (1), we have $A^*_\beta \subset B^*_\beta$. Therefore, we obtain $A \cup A^*_\beta \subset B \cup B^*_\beta$ and hence $cl^*_\beta(A) \subset cl^*_\beta(B)$.
(4). By Theorem 1 (2),
$$cl^*_\beta(A \cup B) = (A \cup B) \cup (A^*_\beta \cup B^*_\beta) = cl^*_\beta(A) \cup cl^*_\beta(B).$$
Hence,
$$cl^*_\beta(A \cup B) = cl^*_\beta(A) \cup cl^*_\beta(B).$$
(5). First, we show that $(cl^*_\beta(A))^*_\beta \subset cl^*_\beta(A)$.

Let if possible $x \notin cl^*_\beta(A)$. This implies $x \notin A^*_\beta$ and there exists $U \in \beta O(x)$ such that $U \cap A \in I$ and we conclude that $U \cap A^*_\beta = \phi$ and $\phi \in I$. For if $A^*_\beta \cap U \neq \phi$ then there exists $y \in A^*_\beta \cap U$ and $U \in \beta O(y)$. Then $y \in A^*_\beta$ implies $U \cap A \notin I$, which is a contradiction as $U \cap A \in I$. Hence, $U \cap A^*_\beta = \phi$. Now, we obtain $(A \cup A^*_\beta) \cap U = (A \cap U) \cup (A^*_\beta \cap U) \in I$. This implies that $(cl^*_\beta(A)) \cap U \in I$. By Definition 8, we obtain, $x \notin (cl^*_\beta(A))^*_\beta$. Hence, we obtain $(cl^*_\beta(A))^*_\beta \subset cl^*_\beta(A)$.
Next, we show that $cl^*_\beta(cl^*_\beta(A)) = cl^*_\beta(A)$. Now, we have $cl^*_\beta(cl^*_\beta(A)) = cl^*_\beta(A) \cup (cl^*_\beta(A))^*_\beta$. Since $(cl^*_\beta(A))^*_\beta \subset cl^*_\beta(A)$, we obtain $cl^*_\beta(cl^*_\beta(A)) \subset cl^*_\beta(A)$. It is obvious that $cl^*_\beta(A) \subset cl^*_\beta(cl^*_\beta(A))$. Therefore, $cl^*_\beta(A) = cl^*_\beta(cl^*_\beta(A))$.

Theorem 3. Let (X, τ, I) be an ideal topological space. Let $\tau^*_\beta = \{U \subset X : cl^*_\beta(X \setminus U) = X \setminus U\}$. Then τ^*_β is a topology for X such that $\tau^* \subset \tau^*_\beta$ and $\beta O(X) \subset \tau^*_\beta$.

Proof. By Theorem 2, we obtain that $cl^*_\beta(A) = A \cup A^*_\beta$ is a Kuratowski Closure Operator. Therefore, τ^*_β is the topology for X generated by cl^*_β.
First, we show that $\tau^* \subset \tau^*_\beta$. By Lemma 2(2), for every subset A of X, $cl^*_\beta(A) = A \cup A^*_\beta \subset A \cup A^* = cl^*(A)$. Let A be a τ^*-closed set, then $cl^*(A) = A$ and $cl^*_\beta(A) \subset A$. Hence $cl^*_\beta(A) = A$ and A is τ^*_β-closed.
Secondly, we show that $\beta O(X) \subset \tau^*_\beta$. Suppose that A is β-closed. If $x \notin A$, then by Lemma 1, there exists U in $\beta O(x)$ such that $U \cap A = \phi \in I$. Hence $x \notin A^*_\beta$. This shows $A^*_\beta \subset A$. Therefore, $cl^*_\beta(A) = A \cup A^*_\beta = A$ and A is τ^*_β-closed. We obtain that $\beta O(X) \subset \tau^*_\beta$.

Definition 10. *Let (X, τ, I) be an ideal topological space. A subset A of X is said to be I_g-β-closed if $A^*_\beta \subset U$ whenever $A \subset U$ and U in $\beta O(X)$.*

Theorem 4. *For a subset A of an ideal topological space (X, τ, I), the following properties are equivalent:*

(1). *A is I_g-β-closed;*
(2). *$cl^*_\beta(A) \subset U$ whenever $A \subset U$ and U is β-open;*
(3). *For every $x \in cl^*_\beta(A)$, $\beta cl(\{x\}) \cap A \neq \phi$;*
(4). *$cl^*_\beta(A) - A$ contains no nonempty β-closed set;*
(5). *$A^*_\beta - A$ contains no nonempty β-closed set.*

Proof. **(1).** $\Rightarrow$ **(2).** By hypothesis, A is I_g-β-closed. Therefore, $A^*_\beta \subset U$ whenever $A \subset U$ and U in $\beta O(X)$. This implies $A^*_\beta \cup A \subset U$ and hence, $cl^*_\beta(A) \subset U$ whenever $A \subset U$ and $U \in \beta O(X)$.
(2). $\Rightarrow$ **(3).** Suppose $x \in cl^*_\beta(A)$. If $\beta cl(\{x\}) \cap A = \phi$, then $A \subset (X - \beta cl(\{x\}))$, where $(X - \beta cl(\{x\}))$ is β-open and by hypothesis, $cl^*_\beta(A) \subset (X - \beta cl(\{x\}))$. Therefore, $cl^*_\beta(A) \cap \beta cl(\{x\}) = \phi$, which is a contradiction, since $x \in cl^*_\beta(A)$. Hence, for every $x \in cl^*_\beta(A)$, $\beta cl(\{x\}) \cap A \neq \phi$
(3). $\Rightarrow$ **(4).** Let if possible, $F \subset cl^*_\beta(A) - A$, where F is a nonempty β-closed set and $x \in F$. This implies $F \subset X - A$ and $F \cap A = \phi$. Therefore, $\beta cl(\{x\}) \cap A = \phi$, which is a contradiction to our hypothesis as $\beta cl(\{x\}) \cap A \neq \phi$. Hence, $cl^*_\beta(A) - A$ contains no nonempty β-closed set.
(4). $\Rightarrow$ **(5).** The proof is obvious, since $A^*_\beta \subset cl^*_\beta(A)$.
(5). $\Rightarrow$ **(1).** Let $A \subset U$ and U is any β-open set of X. By Theorem 1(5), A^*_β is β-closed and $A^*_\beta \cap (X - U) \subset A^*_\beta - A$, where, $A^*_\beta \cap (X - U)$ is β-closed. By (5), $A^*_\beta \cap (X - U) = \phi$. Therefore, $A^*_\beta \subset U$ and hence A is I_g-β-closed.

4. I_{s^*g}-β-closed sets

In this section, the notion of I_{s^*g}-β-closed sets is defined with an illustrative example. Moreover, some properties of these closed sets has been also explored.

Definition 11. *Let (X, τ, I) be an ideal topological space. A subset A of X is said to be I_{s^*g}-β-**closed** (resp. I_{s^*g}-closed [9]) if $A^*_\beta \subset U$ (resp. $A^* \subset U$) whenever $A \subset U$ and U is semi-open. The complement of an **I_{s^*g}-β-closed** set is said to be **I_{s^*g}-β-open**. The family of I_{s^*g}-β-closed (resp. I_{s^*g}-closed) sets is denoted by $I_{s^*g}\ \beta C(X)$ (resp. $I_{s^*g}C(X)$).*

Theorem 5. *Let (X, τ, I) be an ideal topological space and A a subset of X. If A is I_{s^*g}-closed, then it is I_{s^*g}-β-closed. But the converse is not always true.*

Proof. Suppose that A is I_{s^*g}-closed. For every $U \in SO(X)$ containing A, we have $A^* \subset U$ and by Lemma 2(2), $A^*_\beta \subset A^* \subset U$. This shows that A is I_{s^*g}-β-closed.

Example 3. *Let $X = \{a, b, c, d\}$ be a nonempty set with the topology $\tau = \{\phi, X, \{b, c\}, \{a, b, c\}, \{b\}, \{a, b\}\}$ and the collection of closed sets is $\tau_F = \{X, \phi, \{a, c, d\}, \{c, d\}, \{a, d\}, \{d\}\}$. Applying the Definition 5, we compute the collection $\beta O(X)$= $\{\phi, X, \{a, b\}, \{b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{d, a, b\}\}$. Considering $I = \{\phi, \{a\}\}$ and applying Definition 11, we compute the collection of $I_{s^*g}\beta C(X)$= $\{\phi, X, \{a, c, d\}, \{c, d\}, \{a, d\}, \{a, c\}, \{d\}, \{a\}, \{c\}\}$ and $I_{s^*g}C(X) = \{\ \phi,\ X, \{a, c, d\}, \{c, d\}, \{a, d\}, \{d\}, \{a\}\}$. It can be verified that the subsets $\{a, c\}$ and $\{c\}$ of X are $I_{s^*g}\beta$-closed but not I_{s^*g}-closed.*

Theorem 6. *Let (X, τ, I) be an ideal topological space and A, B be subsets of X.*
(1). *If A and B are I_{s^*g}-β-closed, then $A \cup B$ is I_{s^*g}-β-closed.*
(2). *If A is closed in X, then A is I_{s^*g}-β-closed.*
(3). *If U is open in X and A is I_{s^*g}-β-open, then $U \cap A$ is I_{s^*g}-β-open.*

Proof. **(1).** Let $A \cup B \subset U$ and $U \in SO(X)$. Then, we know that $A \subset U$ and $B \subset U$. Since A and B both are I_{s^*g}-β-closed, we have $A^*_\beta \subset U$ and $B^*_\beta \subset U$. Hence, $A^*_\beta \cup B^*_\beta \subset U$. Now by Theorem 1(2), $(A \cup B)^*_\beta = A^*_\beta \cup B^*_\beta \subset U$. Hence, we obtain $A \cup B$ is I_{s^*g}-β-closed.
(2). Let $A \subset U$ and $U \in SO(X)$. By Lemma 2, $A^*_\beta \subset A^* \subset Cl(A) = A \subset U$. This shows that A is I_{s^*g}-β-closed.
(3). The proof is a direct consequence of (1) and (2).

5. Conclusion

The concept of the β-local function, the operation cl^*_β and I_g-β-closed sets have been introduced with illustrative examples. Moreover, certain properties have been also studied and explored. It may be concluded that the concept of the topology τ^*_β is more generalized version of τ^* and β-open sets, which may be further useful to enrich the class of continuous functions.

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