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Left and Right Magnifying Elements in the Semigroup of all Binary Relations

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Abstract. An element a of a semigroup S is called left [right] magnifying if there exists a proper subset M of S such that $S = aM$ [$S = Ma$]. Let X be a nonempty set and $\mathcal{B}_X$ the semigroup of binary relations on X. In this paper, we give necessary and sufficient conditions for elements in $\mathcal{B}_X$ to be left or right magnifying.

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1. Introduction and Preliminaries

An element a of a semigroup S is called *left [right] magnifying* if there exists a proper subset M of S such that $S = aM$ [$S = Ma$] in which the concept of such definition was first introduced by Ljapin [4]. In 1971, Migliorini [6] studies week left [right] and strong left [right] magnifying element of semigroups which specifically gives a definition of a proper subset M of S as a subsemigroup. In the following years, there are several studies on other properties of magnifying elements (see [1–3, 5, 7]). The semigroup of all binary relations are widely known and there are many research in the area of this type of semigroup ([8, 11, 12]). In 2018, Chinram, Petchkaew and Baupradist [9] give necessary and sufficient conditions for elements in some generalized linear transformation semigroups. Then in the next year, Baupradist, Panityakul and Chinram [10] give necessary and sufficient conditions for elements in semigroups of linear transformations with restricted range to be left or right magnifying. Our research is motivated by these studies. In this paper, we give necessary and sufficient conditions for elements in the semigroups of binary relations to be left and right magnifying.

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Throughout this paper, we let X be a nonempty set and $|X|$ be the cardinality of X. It is well-known that the set of all binary relations on X form a semigroup, denoted by $\mathcal{B}_X$, under composition:

$$\alpha \circ \beta = \{(x, y) \mid \exists z \in X \text{ such that } (x, z) \in \alpha \text{ and } (z, y) \in \beta\}$$
$$\text{for all } \alpha, \beta \in \mathcal{B}_X.$$

Let $a, b \in X$, $A \subseteq X$ and $\alpha \in \mathcal{B}_X$. The *domain* and *range* of α are denoted by $\mathrm{dom}\alpha = \{x \in X \mid (x, y) \in \alpha\}$ and $\mathrm{ran}\alpha = \{y \in X \mid (x, y) \in \alpha\}$, respectively. We use the following notions:

$$\begin{aligned}
\alpha^{-1} &= \{(a, b) \mid (a, b) \in \alpha\},\\
(a)\alpha &= \{y \in X \mid (a, y) \in \alpha\},\\
(a)\alpha^{-1} &= \{x \in X \mid (x, a) \in \alpha\},\\
(A)\alpha &= \{y \in X \mid y \in (a)\alpha \text{ for some } a \in A\},\\
(A)\alpha^{-1} &= \{x \in X \mid x \in (a)\alpha^{-1} \text{ for some } a \in A\},\\
\alpha|_A &= \{(a, b) \mid a \in A \text{ and } (a, b) \in \alpha\}
\end{aligned}$$

Note that we can write $(a)\alpha$ instead of $(\{a\})\alpha$. The universal relation $X \times X$ is denoted by ω and the identity relation on X is denoted by i_X. Here, we will write a relation from the right, $(a)\alpha$ rather than $\alpha(a)$ and compose from the left to the right, $(a)(\alpha\beta)$ rather than $(\beta \circ \alpha)(a)$, for $\alpha, \beta \in \mathcal{B}_X$.

2. Left Magnifying Elements of $\mathcal{B}_X$

Lemma 1. *Let $\alpha \in \mathcal{B}_X$. $|(y)\alpha^{-1}| = 1$ for all $y \in \mathrm{ran}\alpha$ if and only if for every $a, b \in \mathrm{dom}\alpha$, $(a)\alpha \cap (b)\alpha \neq \emptyset$ implies $a = b$.*

Proof. Assume that $|(y)\alpha^{-1}| = 1$ for all $y \in \mathrm{ran}\alpha$. Let $a, b \in \mathrm{dom}\alpha$. Suppose that $(a)\alpha \cap (b)\alpha \neq \emptyset$. Let $c \in (a)\alpha \cap (b)\alpha$. Thus $a \in (c)\alpha^{-1}$ and $b \in (c)\alpha^{-1}$. Since $a, b \in \mathrm{ran}\alpha$, we have $|(c)\alpha^{-1}| = 1$, then $a = b$. Conversely, assume that for every $a, b \in \mathrm{dom}\alpha$, $(a)\alpha \cap (b)\alpha \neq \emptyset$ implies $a = b$. Let $y \in \mathrm{ran}\alpha$. Suppose that $|(y)\alpha^{-1}| > 1$. Then there exist two distinct elements $a, b \in \mathrm{dom}\alpha$ such that $a, b \in (y)\alpha^{-1}$. Thus $y \in (a)\alpha$ and $y \in (b)\alpha$. That is, $(a)\alpha \cap (b)\alpha \neq \emptyset$. As a result, $a = b$ in which we obtain a contradiction. Therefore $|(y)\alpha^{-1}| = 1$.

Lemma 2. *Let $\alpha \in \mathcal{B}_X$. $|(y)\alpha^{-1}| = 1$ for all $y \in \mathrm{ran}\alpha$ if and only if $(A\alpha)\alpha^{-1} = A$ for every $A \subseteq \mathrm{dom}\alpha$.*

Proof. Assume that $|(y)\alpha^{-1}| = 1$ for all $y \in \mathrm{ran}\alpha$. It is clear that $A \subseteq (A\alpha)\alpha^{-1}$. We want to show that $(A\alpha)\alpha^{-1} \subseteq A$. Let $x \in (A\alpha)\alpha^{-1}$. Then there exists $b \in A\alpha$ such that $x \in (b)\alpha^{-1}$. We have that $b \in (x)\alpha$ and therefore there exists $a \in A$ such that $b \in (a)\alpha$. That is, $b \in (x)\alpha \cap (a)\alpha$. As a result, $(x)\alpha \cap (a)\alpha \neq \emptyset$. By Lemma 1, $x = a \in A$.

Conversely, assume that $(A\alpha)\alpha^{-1} = A$ for all $A \subseteq \mathrm{dom}\alpha$. Let $y \in \mathrm{ran}\alpha$. We will show that $|(y)\alpha^{-1}| = 1$. Since $y \in \mathrm{ran}\alpha$, there exist $a \in \mathrm{dom}\alpha$ such that $y \in (a)\alpha$. Clearly, $a \in (y)\alpha^{-1}$. Suppose that $|(y)\alpha^{-1}| > 1$. Then there exist $b \in (y)\alpha^{-1}$ such that $b \neq a$. Since $\{a\} \subseteq \mathrm{dom}\alpha$ and by assumption, we have $((\{a\})\alpha)\,\alpha^{-1} = \{a\}$. Therefore

$$b \in (y)\alpha^{-1} \subseteq ((a)\alpha)\alpha^{-1} = \{a\}.$$

Thus $b = a$, a contradiction. As a result, $|(y)\alpha^{-1}| = 1$.

Combining Lemmas 1 and 2 we have the following Lemma.

Lemma 3. *Let $\alpha \in \mathcal{B}_X$. Then the following are equivalent:*

(i) for every $a, b \in dom\alpha$, $(a)\alpha \cap (b)\alpha \neq \emptyset$ implies $a = b$;

(ii) $|(y)\alpha^{-1}| = 1$ for all $y \in ran\alpha$;

(iii) $(A\alpha)\alpha^{-1} = A$ for every $A \subseteq dom\alpha$.

Lemma 4. *Let $\alpha \in \mathcal{B}_X$. If α is a left magnifying element of $\mathcal{B}_X$, then*

(i) $dom(\alpha) = X$,

(ii) for any $x, y \in dom(\alpha)$, $(a)\alpha \cap (b)\alpha \neq \emptyset$ implies $a = b$.

Proof. Assume that α is a left magnifying element of $\mathcal{B}_X$. Then there exists a proper subset M of $\mathcal{B}_X$ such that $\alpha M = \mathcal{B}_X$. Since $i_X \in \mathcal{B}_X$, there exists a relation $\beta \in M$ such that $\alpha\beta = i_X$ and since $\mathrm{dom}(i_X) = X$, we obtain $\mathrm{dom}\alpha = X$. Let $x, y \in \mathrm{dom}\alpha$. Suppose that $(a)\alpha \cap (b)\alpha \neq \emptyset$. Then there exists an element $z \in (x)\alpha \cap (y)\alpha$. We get

$$(z)\beta \subseteq ((x)\alpha)\beta = (x)(\alpha\beta) = (x)i_X = \{x\}, \text{ and}$$

$$(z)\beta \subseteq ((y)\alpha)\beta = (y)(\alpha\beta) = (y)i_X = \{y\}.$$

Thus $(z)\beta \subseteq \{x\}$ and $(z)\beta \subseteq \{y\}$. We have that $x = y$. Therefore $(x)\alpha \cap (y)\alpha = \emptyset$.

Lemma 5. *Let $\alpha \in \mathcal{B}_X$. If for any $x, y \in dom(\alpha)$ such that $(x)\alpha \cap (y)\alpha \neq \emptyset$, $x = y$ and α is a function, then α is a one-to-one function.*

The proof of Lemma 5 is obvious and immediately obtained.

Lemma 6. *Let $\alpha \in \mathcal{B}_X$. If α is a bijective function on X, then α is not left magnifying element of $\mathcal{B}_X$.*

Proof. We are going to proof this Lemma similarly to [9, Lemma 2.]. Suppose that α is a left magnifying element of $\mathcal{B}_X$. Then there exists a proper subset M of $\mathcal{B}_X$ such that $\alpha M = \mathcal{B}_X$. Since α is a bijective function,

$$\alpha M = \alpha\alpha^{-1}\alpha M = \alpha\alpha^{-1}\mathcal{B}_X \subseteq \alpha\mathcal{B}_X \subseteq \mathcal{B}_X = \alpha M.$$

Then $\alpha M = \alpha\mathcal{B}_X$. Hence $M = \alpha^{-1}\alpha M = \alpha^{-1}\alpha\mathcal{B}_X = \mathcal{B}_X$; we achieve a contradiction. Therefore α is not a left magnifying element of $\mathcal{B}_X$.

Lemma 7. *Let $\alpha \in \mathcal{B}_X$ and $dom\alpha = X$. If α is not a bijective function on X and for every $a, b \in dom\alpha$, $(a)\alpha \cap (b)\alpha \neq \emptyset$ implies $a = b$, then α is a left magnifying element of $\mathcal{B}_X$.*

Proof. Let $\alpha \in \mathcal{B}_X$ and $\mathrm{dom}\alpha = X$. Assume that α is not a bijective function on X and for every $a, b \in \mathrm{dom}\alpha$, $(a)\alpha \cap (b)\alpha \neq \emptyset$ implies $a = b$. Let $M = \{h \in \mathcal{B}_X \mid \mathrm{dom}h \neq X$ or h is not a one-to-one function$\}$. Then M is a proper subset of $\mathcal{B}_X$. We will show that $\alpha M = \mathcal{B}_X$. Let β be a relation in $\mathcal{B}_X$. For each $x \in (\mathrm{dom}\beta)\alpha$. Then there exists a unique $x' \in \mathrm{dom}\beta$ such that $x \in (x')\alpha$. Define a relation γ in $\mathcal{B}_X$ by

$$(x)\gamma = (x')\beta$$

where $x' \in \mathrm{dom}\beta$ such that $x \in (x')\alpha$. If $(\mathrm{dom}\beta)\alpha \neq X$, then $\mathrm{dom}\gamma \neq X$. So $\gamma \in M$. If β is not a function, by the definition of γ, clearly, γ is not a function. Consequently $\gamma \in M$.

Suppose that $(\mathrm{dom}\beta)\alpha = X$ and β is a function. We will show that γ is not a one-to-one function. Let $x \in X$ and $c \in (x)\alpha$. Thus we have $c \in X = (\mathrm{dom}\beta)\alpha$. Then there exists $x' \in \mathrm{dom}\beta$ such that $c \in (x')\alpha$. That is, $(x)\alpha \cap (x')\alpha \neq \emptyset$. Consequently, $x = x' \in \mathrm{dom}\beta$. Therefore $\mathrm{dom}\beta = X$. Next, we will show that α is not a function from $\mathrm{dom}\beta = X$ onto X. Suppose the contrary that α is a function from X onto X. By our assumption and Lemma 5, α is a one-to-one function. Hence α is a bijective function on X. This is a contradiction. Therefore α is not a function from $\mathrm{dom}\beta = X$ onto X. Finally, we will show that γ is not a one-to-one function. Since α is not a function from $\mathrm{dom}\beta$ onto X, there exists $x_0 \in \mathrm{dom}\beta$ such that $|(x_0)\alpha| > 1$ and that there exist two distinct elements u, v in $(x_0)\alpha$. By the definition of γ, we have $(u)\gamma = (v)\gamma = (x_0)\beta$. Therefore γ can not be a one-to-one function. Hence $\gamma \in M$.
By Lemma 3(iii), we obtain

$$\begin{aligned}\mathrm{dom}(\alpha\gamma) &= (\mathrm{ran}\alpha \cap \mathrm{dom}\gamma)\alpha^{-1} \\ &= (\mathrm{ran}\alpha \cap (\mathrm{dom}\beta)\alpha)\,\alpha^{-1} \\ &= ((\mathrm{dom}\beta)\alpha)\alpha^{-1} \\ &= \mathrm{dom}\beta.\end{aligned}$$

Therefore $\mathrm{dom}(\alpha\gamma) = \mathrm{dom}\beta$. For each $x \in \mathrm{dom}(\alpha\gamma)$, we have

$$(x)(\alpha\gamma) = ((x)\alpha)\gamma = (x)\beta.$$

Then $\alpha\gamma = \beta$, and as a result, $\alpha M = \mathcal{B}_X$. Hence the proof is complete.

Theorem 1. *Let $\alpha \in \mathcal{B}_X$. Then α is a left magnifying element of $\mathcal{B}_X$ if and only if*

(i) *$dom\alpha = X$,*

(ii) *for every $x, y \in dom\alpha$, $(x)\alpha \cap (y)\alpha \neq \emptyset$ implies $x = y$, and*

(iii) *α is not a bijective function on X.*

Proof. It follows from Lemma 4, Lemma 6 and Lemma 7.

Example 1. *Let* $X = \mathbb{N}$ *and* $\alpha \in \mathcal{B}_X$ *defined by*

$$(x)\alpha = \{2^x, 3^x\} \text{ for all } x \in X.$$

Following the assumption we have $dom\alpha = X$*, for every* $x, y \in X$ *such that* $x \neq y$*,* $(x)\alpha \cap (y)\alpha = \emptyset$ *and* α *is not a function. So that* α *is not a bijective on* X*. Let* $M = \{h \in \mathcal{B}_X \mid domh \neq X \text{ or } h \text{ is not a one-to-one function}\}$*. Let* β *be any relation in* $\mathcal{B}_X$*. By Lemma 7, we can define a relation* $\gamma \in \mathcal{B}_X$ *such that* $\gamma \in M$ *and* $\gamma\alpha = \beta$*. For example, if* β *is a relation in* $\mathcal{B}_X$ *such that* $(x)\beta = \{2x\}$ *for all odd integer* x*. Thus* $dom\beta = \{x \in \mathbb{N} \mid x \text{ is odd}\}$*. So* $(dom\beta)\alpha = \{2^x \mid x \text{ is odd}\} \cup \{3^x \mid x \text{ is odd}\}$*. Define a relation* γ *in* $\mathcal{B}_X$ *by* $(x)\gamma = (y)\beta$ *if* $x = 2^y$ *or* $x = 3^y$ *for some odd integer* y*. Thus* $dom\gamma = \{2^x \mid x \text{ is odd}\} \cup \{3^x \mid x \text{ is odd}\} \neq X$ *and* $dom\gamma = \{2^x \mid x \text{ is odd}\} \cup \{3^x \mid x \text{ is odd}\} \subseteq ran\alpha$*. We have*

$$\begin{aligned} dom(\alpha\gamma) &= (ran\alpha \cap dom\gamma)\alpha^{-1} \\ &= (dom\gamma)\,\alpha^{-1} \\ &= (\{2^x \mid x \text{ is odd}\} \cup \{3^x \mid x \text{ is odd}\})\,\alpha^{-1} \\ &= \{x \in \mathbb{N} \mid x \text{ is odd}\} \\ &= dom\beta. \end{aligned}$$

That is, $dom(\alpha\gamma) = dom\beta$*. For each an odd integer* x*,*

$$(x)(\alpha\gamma) = ((x)\alpha)\gamma = (\{2^x, 3^x\})\,\gamma = (x)\beta \cup (x)\beta = (x)\beta.$$

Therefore $\alpha\gamma = \beta$*.*

3. Right Magnifying Elements of $\mathcal{B}_X$

Lemma 8. *If* α *is a right magnifying element of* $\mathcal{B}_X$*, then there exists a subset* A *of* X *such that* $\alpha|_A$ *is an onto function from* A *to* X*.*

Proof. Assume that α is a right magnifying element of $\mathcal{B}_X$. Then there exists a proper subset M of $\mathcal{B}_X$ such that $M\alpha = B_X$. Since $i_X \in \mathcal{B}_X$, then there exists $\gamma \in M$ such that $\gamma\alpha = i_X$. It is clear that $\text{ran}\gamma \cap \text{dom}\alpha \neq \emptyset$. We put $A := \text{ran}\gamma \cap \text{dom}\alpha$. First, we will show that $\alpha|_A$ is a function from A to X. Let $x \in A$. Then there exists $u \in \text{dom}\gamma$ such that $x \in (u)\gamma$. Therefore

$$(x)\alpha \subseteq ((u)\gamma)\alpha = (u)(\gamma\alpha) = (u)i_X = \{u\}.$$

Thus $|(x)\alpha| = 1$. As a result, $\alpha|_A$ is a function from A to X. Next, we will show that $\alpha|_A$ is onto. Let $y \in X$. Then $y \in \text{ran}(i_X) = \text{ran}(\gamma\alpha)$, giving that there exists $v \in \text{dom}(\gamma\alpha)$ such that $y \in (v)(\gamma\alpha) = ((v)\gamma)\,\alpha$. We have $y \in ((v)\gamma)\,\alpha$ when $(v)\gamma \in (\text{ran}\gamma \cap \text{dom}\alpha)$. This leads us to a conclusion that $\alpha|_A$ is onto.

The proof of the following Lemma is done similarly to [9, Lemma 5.].

Lemma 9. *If $\alpha \in \mathcal{B}_X$ is bijective, then α is not a right magnifying element.*

Proof. Assume that α is a right magnifying element of $\mathcal{B}_X$. Then there exists a proper subset M of $\mathcal{B}_X$ such that $M\alpha = \mathcal{B}_X$. Since α is a bijective function,

$$M\alpha = M\alpha\alpha^{-1}\alpha = \mathcal{B}_X\alpha^{-1}\alpha \subseteq \mathcal{B}_X\alpha \subseteq \mathcal{B}_X = M\alpha.$$

Then $M\alpha = \mathcal{B}_X\alpha$. Hence $M = M\alpha\alpha^{-1} = \mathcal{B}_X\alpha\alpha^{-1} = \mathcal{B}_X$. This is a contradiction. Therefore α is not a right magnifying element of $\mathcal{B}_X$.

Lemma 10. *Let $\alpha \in \mathcal{B}_X$. If there exists a subset A of X such that $\alpha|_A$ is an onto function from A to X and α is not a bijective function on X, then α is a right magnifying element of $\mathcal{B}_X$.*

Proof. Assume that there exists a subset A of X such that $\alpha|_A$ is an onto function from A to X and α is not a bijective function on X. Let $M = \{h \in \mathcal{B}_X \mid \text{ran}h \neq X\}$. Then M is a proper subset of $\mathcal{B}_X$.

We will show that there exists a proper subset C of X such that $\alpha|_C$ is an onto function from C to X. If $A \neq X$, then it is immediate by setting $C := A$. If $A = X$, then α is an onto function on X, but as α is not a bijective function on X, α is not a one-to-one function on X. Then there exists $u \in X$ which $\alpha|_{X\setminus\{u\}}$ is an onto function from $X \setminus \{u\}$ to X. By put $C := X \setminus \{u\}$, the proof is immediately obtained.

Let $\beta \in \mathcal{B}_X$ and $x \in \text{dom}\beta$. Then $(x)\beta \subseteq X$. Since $\alpha|_C$ is onto, there exists a nonempty subset C_x of C such that $(C_x)\alpha = (x)\beta$. From these we defined a relation $\gamma \in \mathcal{B}_X$ by

$$(x)\gamma = C_x \text{ for each } x \in \text{dom}\beta.$$

It is clearly that $\text{ran}\gamma \subseteq C \neq X$, then $\gamma \in M$. Consider

$$(x)\beta = (C_x)\alpha = ((x)\gamma)\alpha = (x)(\gamma\alpha),$$

where $x \in \text{dom}\beta$, we have $\gamma\alpha = \beta$. Therefore $\mathcal{B}_X = M\alpha$.

Theorem 2. *Let $\alpha \in \mathcal{B}_X$. Then α is a right magnifying element of $\mathcal{B}_X$ if and only if there exists a subset A of X such that α is an onto function from A to X and α is not a bijective function on X.*

Proof. Trivial from Lemma 8, Lemma 9 and Lemma 10.

Example 2. *Let $X = \mathbb{N}$ and α be a relation in $\mathcal{B}_X$ defined by*

$$(x)\alpha = \begin{cases} \{\frac{x}{2}\} & \textit{for all even } x, \\ \{x, x+1\} & \textit{for all odd } x. \end{cases}$$

Then $\alpha|_A$ is an onto function from A to $\mathbb{N}$, where $A = \{x \in \mathbb{N} \mid x \text{ is even}\}$, and α is not a bijective function on $\mathbb{N}$. We put $M := \{h \in \mathcal{B}_X \mid \text{ran}h \neq X\}$. Let $\beta \in \mathcal{B}_X$. By Lemma 3, there exists a relation γ in $\mathcal{B}_X$ such that $\gamma\alpha = \beta$.

For example, (a) if $\beta \in \mathcal{B}_X$ such that $(x)\beta = \{x+3\}$ for each x where x is odd. Then $dom\beta = \{x \in \mathbb{N} \mid x \text{ is odd}\}$. Define $\gamma \in \mathcal{B}_X$ by

$$(x)\gamma = 2(x+3) \text{ for all } x \text{ that is odd.}$$

Thus $dom\gamma = dom\beta$ and $ran\gamma \subseteq dom\alpha \neq X$. We have $\gamma \in M$. For each odd integer x, then

$$\begin{aligned}(x)(\gamma\alpha) &= ((x)\gamma)\alpha \\ &= (2(x+3))\,\alpha \\ &= \left\{\frac{2(x+3)}{2}\right\} \\ &= \{x+3\} \\ &= (x)\beta.\end{aligned}$$

Therefore $\gamma\alpha = \beta$.
(b) If $\beta = \{(1,2),(1,3),(2,3)\}$, then $\beta \in \mathcal{B}_X$.
Define $\gamma \in \mathcal{B}_X$ by $\gamma = \{(1,2),(1,6),(2,6)\}$. Thus

$$(1)(\gamma\alpha) = ((1)\gamma)\,\alpha = (\{2,6\})\,\alpha = \{1,3\}, \text{ and}$$

$$(2)\gamma\alpha = ((1)\gamma)\,\alpha = (\{6\})\,\alpha = \{3\}.$$

Therefore, $\gamma\alpha = \beta$.
(c) If $\beta \in \mathcal{B}_X$ such that $(x)\beta = \{x, x+2\}$ for all $x \in \mathbb{N}$. Define $\gamma \in \mathcal{B}_X$ by

$$(x)\gamma = \{2x, 2(x+2)\} \text{ for all } x \text{ is an integer.}$$

Clearly that $dom(\gamma\alpha) = dom\beta$. Let $x \in \mathbb{N}$. Then

$$\begin{aligned}(x)(\gamma\alpha) &= ((x)\gamma)\alpha \\ &= (\{2x, 2(x+2)\})\,\alpha \\ &= \left\{\frac{2x}{2}, \frac{2(x+2)}{2}\right\} \\ &= \{x, x+2\} \\ &= (x)\beta.\end{aligned}$$

Therefore $(x)(\gamma\alpha) = (x)\beta$ for all $x \in \mathbb{N}$.

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