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Bi-interior ideal elements in $\wedge e$-semigroups

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Abstract. All the results on semigroups obtained using only sets, can be written in an abstract form in a more general setting. Let us consider a recent paper to justify what we say. The bi-interior ideals of semigroups introduced and studied by M. Murali Krishna Rao in Discuss. Math. Gen. Algebra Appl. in 2018, follow for more general statements about ordered semigroups. The same holds for every result of this sort on semigroups based on right (left) ideals, bi-ideals, quasi-ideals, interior ideals etc. for which we use sets. As a result, we have an abstract formulation of the results on semigroups obtained by sets that is in the same spirit with the abstract formulation of general topology (the so-called topology without points) initiated by Koutský, Nöbeling and, even earlier, by Chittenden, Terasaka, Nakamura, Monteiro and Ribeiro. As a consequence, results on ordered Γ-hypersemigroups and on similar simpler structures can be obtained.

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1. Introduction and prerequisites

The concept of bi-interior ideal of semigroup has been introduced my M.M. Krishna Rao [4] as follows: Let S be a semigroup and A a nonempty subset of S. Then A is called a bi-interior ideal of S if $ASA \cap SAS \subseteq A$. As one can easily see, every bi-ideal A of S is a bi-interior ideal of S and every interior ideal of S is a bi-interior ideal of S. So the concept of bi-interior ideal generalizes the concept of bi-ideal and the concept of interior ideal. As every right (resp. left) ideal and every quasi-ideal of a semigroup S is a bi-ideal of S, the concept of bi-interior ideal generalizes the concepts of right ideal, left ideal, and the concept of quasi-ideal of a semigroup as well. M. Murali Krishna Rao assumes that the bi-ideals and the interior ideals of a semigroup S are subsemigroups of S but this does not make any difference to the investigation.

The results of [4] follows from a more general setting of that of ordered $\wedge e$-semigroups. The same can be said for any similar result based on sets. We casually chose a recent paper by M. Murali Krishna Rao in Discuss. Math. Gen. Algebra Appl. in 2018 as an example to justify what we say. This is in the same spirit with the abstract formulation of general

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topology (the so-called topology without points) initiated by Koutský and Nóbeling [3, 5]. Topology without points has been also studied much earlier by M. Nakamura [Closure in general lattices. Proc. Imp. Acad. Tokyo 17, 5–6 (1941); MR0004225], A. Monteiro and H. Ribeiro [L'operation de fermeture et ses invariants dans les systèmes partiellement ordonnées. Portugal. Math. 3 (1942), 171–184; MR0007973] or, even earlier, by E.W. Chittenden [On general topology and the relation of the properties of the class of all continuous functions to the properties of space. Trans. Amer. Math. Soc. 31, no. 2 (1929), 290–321; MR1501484] and H. Terasaka [Die Theorie der topologischen Verbände. Coll. Papers Fac. Sci. Osaka Univ. Ser. A 8, no. 1 (1940), 33 pp.; MR0032581].

The following definitions are well known: If S is a semigroup, a nonempty subset A of S is called a right (resp. left) ideal of S if $AS \subseteq A$ (resp. $SA \subseteq A$). It is called a bi-ideal of S is $ASA \subseteq A$ (Kehayopulu); and quasi-ideal of S if $AS \cap SA \subseteq A$. S. Lajos considered the bi-ideal of a semigroup S as a subsemigroup of S except in his last publications in which he used the definition given above. It might be mentioned that most of the results hold without the assumption that the bi-ideal is a subsemigroup; as so we do not have to use the term "generalized bi- ideal" so often.

A *poe*-groupoid is a groupoid S at the same time an ordered set having a greatest element "e" (: $e \geq a$ for every $a \in S$) such that $a \leq b$ implies $ac \leq bc$ and $ca \leq cb$ for every $c \in S$. If the multiplication on S is associative, then S is called *poe*-semigroup. An *le*-semigroup is a semigroup S at the same time a lattice having a greatest element e (with respect to the order) such that $a(b \vee c) = ab \vee ac$ and $(a \vee b)c = ac \vee bc$ for every $a, b, c \in S$. Every *le*-semigroup is a *poe*-semigroup. A $\wedge e$-groupoid is a groupoid S at the same time a semilattice under $\wedge$ (: $\wedge$-semilattice) having a greatest element "e" such that $a \leq b$ implies $ac \leq bc$ and $ca \leq cb$ for every $c \in S$; if its multiplication is associative, then it is called $\wedge e$-semigroup. Let S be a *poe*-groupoid. An element a of S is called right (left) ideal element of S if $ae \leq a$ (resp. $ea \leq a$). An element that is both a right and a left ideal element is called ideal element. If S is a $\wedge e$-groupoid, then an element a of S called a quasi-ideal element if $ae \wedge ea \leq a$. An element a of a *poe*-semigroup is called a bi-ideal element if $aea \leq a$ and an interior ideal element if $eae \leq a$. Denote by $r(a)$ (resp. $l(a)$) the right (resp. left) ideal of S generated by a. For an *le*-semigroup S, we have $r(a) = a \vee ae$ and $l(a) = a \vee ea$. An element a of a *poe*-groupoid is called subidempotent if $a^2 \leq a$; it is called idempotent if $a^2 = a$ [1]. An element e' of a *poe*-groupoid S is called an identity (or unity) of S is $ae' = e'a = a$ for every $a \in S$.

The study of *poe*-semigroups plays an essential role in the theory of ordered Γ-hypersemigroups and related simpler structures, like the hypersemigroups, for example.

2. Bi-interior ideal elements in $\wedge e$-semigroups

Proposition 2.1. *If S is a $\wedge e$-groupoid, then every right (resp. left) ideal element of S is a quasi-ideal element of S. If S is a $\wedge e$-semigroup, then every quasi-ideal element of S is a bi-ideal element of S.*

Proof. Let a be a right ideal element of S. Then $ae \wedge ea \leq ae \leq a$ and so a is a quasi-ideal element of S. If a is a quasi-ideal element of S, then $aea \leq ae \wedge ea \leq a$ and so a is a bi-ideal element of S. □

Definition 2.2. An element b of a $\wedge e$-semigroup S is called a *bi-interior ideal element* if $beb \wedge ebe \leq b$.

Proposition 2.3. *Let S be a $\wedge e$-semigroup. Then we have the following:*

(1) *Every right (resp. left) ideal element of S is a bi-interior ideal element of S.*

(2) *Every quasi-ideal element of S is a bi-interior ideal element of S.*

(3) *Every bi-ideal element of S is a bi-interior ideal element of S.*

(4) *Every interior ideal element of S is a bi-interior ideal element of S.*

Proof.

(3) If b be a bi-ideal element of S, then $beb \wedge ebe \leq beb \leq b$, so b is a bi-interior ideal element of S.

(4) If b is an interior ideal element of S, then $beb \wedge ebe \leq ebe \leq b$, so b is a bi-interior ideal element of S.

(2) If q is a quasi-ideal element of S then, by Proposition 2.1, q is a bi-ideal element of S so, by (3), q is a bi-interior ideal element of S.

Independently, if q is a quasi-ideal element of S, then $qeq \wedge eqe \leq qeq \leq qe \wedge eq \leq q$, so q is a bi-interior ideal element of S.

(1) If a is a right (resp. left) ideal element of S then, by Proposition 2.1, a is a quasi-ideal element of S so, by (2), a is a bi-interior ideal element of S. □

Proposition 2.4. *Let S be a $\wedge e$-semigroup. Then we have the following:*

(1) *If a and b are bi-interior ideal elements of S, then $a \wedge b$ is a bi-interior ideal element of S.*

(2) *If a is a right ideal element and b is a left ideal element of S, then $a \wedge b$ is a bi-interior ideal element of S.*

(3) *If b is a bi-interior ideal element and t is an interior ideal element of S, then $b \wedge t$ is a bi-interior ideal element of S.*

Proof.

(1) Let a and b be bi-interior ideal elements of S. Then $aea \wedge eae \leq a$ and $beb \wedge ebe \leq b$, then

$$(a \wedge b)e(a \wedge b) \wedge e(a \wedge b)e \leq aea \wedge eae \leq a$$

and

$$(a \wedge b)e(a \wedge b) \wedge e(a \wedge b)e \leq beb \wedge ebe \leq b.$$

Thus we have

$$(a \wedge b)e(a \wedge b) \wedge e(a \wedge b)e \leq a \wedge b$$

and so $a \wedge b$ is a bi-interior ideal element of S.

(2) If a is a right ideal element of S and b is a left ideal element of S then, by Proposition 2.3(1), a and b are bi-interior ideal elements of S and then, by property (1), $a \wedge b$ is a bi-interior ideal element of S.

(3) If b is a bi-interior ideal element and t is an interior ideal element of S then, by Proposition 2.3(4), a and b are bi-interior ideal elements of S so, by (1), $b \wedge t$ is a bi-interior ideal element of S. □

Proposition 2.5. *Let S be a $\wedge e$-semigroup. Then we have the following:*

(1) *If b is a bi-interior ideal element of S, then the elements be and eb are bi-interior ideal elements of S as well.*

(2) *If b is a bi-interior ideal element of S, $b \leq be$ or $b \leq eb$, then b is a subidempotent element of S.*

(3) *If b is a bi-interior ideal element of S and (the greatest element) e is at the same time the identity of S, then b is a subidempotent element of S.*

Proof.

(1) Let b be a bi-interior ideal element of S. Then

$$(be)e(be) \wedge e(be)e \leq (be)e(be) \leq be$$

and

$$(eb)e(eb) \wedge e(eb)e \leq (eb)e(eb) \leq eb,$$

so be and eb are bi-interior ideal elements of S.

(2) Let b be a bi-interior ideal element of S such that $b \leq be$. Then we have $b^2 \leq (be)b$ and $b^2 \leq b(be) \leq ebe$, thus we have $b^2 \leq beb \wedge ebe \leq b$ and so b is subidempotent. If $b \leq eb$, then $b^2 \leq b(eb)$ and $b^2 \leq (eb)b \leq ebe$, then $b^2 \leq beb \wedge ebe \leq b$ and so b is subidempotent.

(3) Since $b = be = eb$, the proof follows from (2). □

Propositions 2.3, 2.4 and 2.5 generalize the Theorem 3.3 in [4].

Proposition 2.6. *Let S be a $\wedge e$-semigroup. Then we have the following:*

(1) *If a is a right ideal element of S then, for any $b \in S$, the element ab is a bi-interior ideal element of S.*

(2) *If a is a left ideal element of S then, for any $b \in S$, the element ba is a bi-interior ideal element of S.*

(3) *For any $a, b \in S$, the element aeb is a bi-interior ideal element of S.*

Proof. (1) Let a be a right ideal element of S and $b \in S$. Then

$$(ab)e(ab) \wedge e(ab)e \leq (ab)e(ab) \leq (ae)b \leq ab.$$

(2) Let a be a left ideal element of S and $b \in S$. Then

$$(ba)e(ba) \wedge e(ba)e \leq (ba)e(ba) \leq b(ea) \leq ba.$$

(3) Let $a, b \in S$. The element aeb is a bi-ideal element of S. In fact, $(aeb)e(aeb) \leq aeb$. Then, by Proposition 2.3(3), aeb is a bi-interior ideal element of S. □

Proposition 2.7. *The following assertions are satisfied:*

1. *If S is a $\wedge e$-semigroup and $b, t \in S$ such that $tet \wedge ete \leq b \leq t$, then b is a bi-interior ideal element of S.*
2. *If S is an $\wedge e$-semigroup and semilattice under $\vee$ at the same time and $b, t \in S$ such that $tet \vee ete \leq b \leq t$, then b is a bi-interior ideal element of S.*

Proof. (1) We have $beb \wedge ebe \leq tet \wedge ete \leq b$, so b is a bi-interior ideal element of S.

(2) We have $beb \leq tet \leq tet \vee ete \leq b$ and $ebe \leq ete \leq tet \vee ete \leq b$; thus we have $beb \wedge ebe \leq b$ and so b is a bi-interior ideal element of S. □

Proposition 2.8. *Let S be a $\wedge e$-semigroup. If b is a bi-interior ideal element of S and $t \in S$ such that $t \leq b \leq bt$, then bt is a bi-interior ideal element of S.*

Proof. Let b be a bi-interior ideal element of S and $t \leq b \leq bt$. Then we have $(bt)e(bt) \wedge e(bt)e \leq bet \wedge ebe \leq beb \wedge ebe \leq b \leq bt$, thus bt is a bi-interior ideal element of S. □

Theorem 2.9. *Let S be a $\wedge e$-semigroup such that $x \leq xe$ for every $x \in S$. Let a be a minimal right ideal element and b a minimal left ideal element of S. Then ab is a minimal bi-interior ideal element of S.*

Proof. Since a is a right ideal element of S, we have $(ab)e(ab) \leq (ae)b \leq ab$, then ab is a bi-ideal element of S and so ab is a bi-interior ideal element of S (by Prop. 2.3(3)). Let now z be a bi-interior ideal element of S such that $z \leq ab$. Then $ez \leq e(ab) \leq eb \leq b$ and $ze \leq (ab)e \leq ae \leq a$. Since ez is a left ideal element of S, $ez \leq b$, and b is a minimal left ideal element of S, we have $ez = b$. Since ze is a right ideal element of S, $ze \leq a$, and a is a minimal right ideal element of S, we have $ze = a$. Then we have $ab = (ze)(ez) \leq zez$. By hypothesis, we have $ab \leq (ab)e = abe = (ze)(ez)e \leq eze$. Thus we have $ab \leq zez \wedge eze \leq z$. Then we obtain $z = ab$ and the proof is complete. □.

Corollary 2.10. (cf. also [4; Theorem 3.10]) *Let S be a semigroup such that $A \subseteq AS$ for every $A \subseteq S$. If A is a minimal right ideal and B is a minimal left ideal of S, then the product AB is a minimal bi-interior ideal of S.*

3. Bi-interior ideal elements in left simple, simple and bi-interior simple $\wedge e$-semigroups

Definition 3.1. A *poe*-groupoid S is said to be *left* (resp. *right*) *simple* if for every left (resp. right) ideal element a of S we have $a = e$. That is, if e is the only left (resp. right) ideal element of S. It is called *simple* if for every ideal element a of S we have $a = e$; that is, if e is the only ideal element of S.

If S is left (or right) simple, then it is simple.

Proposition 3.2. *If S is a left (resp. right) simple $\wedge e$-semigroup and b is a bi-interior ideal element of S, then b is a right (resp. left) ideal element of S.*

Proof. Let S be left simple and b a bi-interior ideal element of S. Since eb is a left ideal element of S and S is left simple, we have $eb = e$. Since $beb \wedge ebe \leq b$, we get $be \wedge e^2 \leq b$. Since e^2 is a left ideal element of S and S is left simple, we have $e^2 = e$. Then we have $be = be \wedge e \leq b$, then $be \leq b$ and so b is a right ideal element of S. □

Since every bi-ideal element of S is a bi-interior ideal element of S (Prop. 2.3(3)), by Proposition 3.2 we have the following.

Corollary 3.3. *If S is a left (resp. right) simple $\wedge e$-semigroup and b is a bi-ideal element of S, then b is a right (resp. left) ideal element of S.*

Proposition 3.4. *If S is a simple $\wedge e$-semigroup, then every bi-interior ideal element of S is a bi-ideal element of S.*

Proof. Let b be a bi-interior ideal element of S. Then $beb \wedge ebe \leq b$. Since ebe is an ideal element of S and S is simple, we have $ebe = e$. Then $beb \wedge e \leq b$ and so $beb \leq b$. □

Following M. Murali Krishna Rao, we give the following definition.

Definition 3.5. A $\wedge e$-semigroup S is said to be *bi-interior simple* if, for every bi-interior ideal element b of S, we have $b = e$.

Proposition 3.6. *Let S be a $\wedge e$-semigroup. Then S is bi-interior simple if and only if, for every $a \in S$, we have $aea \wedge eae = e$.*

Proof. $\Longrightarrow$. Let $a \in S$. The element $aea \wedge eae$ is a bi-interior ideal element of S. Indeed,

$$\begin{aligned}(aea \wedge eae)e(aea \wedge eae) \wedge e(aea \wedge eae)e &\leq aeaeaea \wedge eaeae \\ &\leq aea \wedge eae.\end{aligned}$$

Since S is bi-interior simple, we have $aea \wedge eae = e$.

$\Longleftarrow$. Let b be a bi-interior ideal element of S. By hypothesis, we have

$$e = beb \wedge ebe \leq b.$$

Then $b = e$ and so S is bi-interior simple. □

Corollary 3.7. [4; Theorem 3.5] *A semigroup M is bi-interior simple if and only if $MaM \cap aMa = M$ for every $a \in M$.*

4. Bi-interior ideal elements in regular $\wedge e$-semigroups

A *poe*-semigroup S is said to be *regular* if, for every $x \in S$, we have $x \le xex$ [1].

Theorem 4.1. *Let S be a $\wedge e$-semigroup. If S is regular then, for every bi-interior ideal element b of S, we have $beb \wedge ebe = b$. In particular, if S is an le-semigroup, then S is regular if and only if for every bi-interior ideal element b of S, we have $beb \wedge ebe = b$.*

Proof. $\Longrightarrow$. Let b be a bi-interior ideal element of S. Then $beb \wedge ebe \le b$. Since S is regular, we have $b \le beb \le (beb)e(beb) \le beb \wedge ebe$. Thus $beb \wedge ebe = b$.
$\Longleftarrow$. Let a be a right ideal element and and b a left ideal element of S. By Proposition 2.4(2), $a \wedge b$ is a bi-interior element of S. By hypothesis, we have $(a\wedge b)e(a\wedge b)\wedge e(a\wedge b)e = a \wedge b$. Then we have

$$a \wedge b \le (a \wedge b)e(a \wedge b) \le (ae)b \le ab \le ae \wedge eb \le a \wedge b.$$

Then $a \wedge b = ab$. Thus, for any $x \in S$, we have

$$x \le r(x) \wedge l(x) = r(x)l(x) = (x \vee xe)(x \vee ex) = x^2 \vee xex,$$

then $x^2 \le x^3 \vee xex^2 \le xex$, then $x \le xex$ and so S is regular. □

Theorem 4.2. *Let S be a $\wedge e$-semigroup. If S is regular, then every bi-interior ideal element of S is subidempotent. "Conversely", if S is an le-semigroup, then S is regular if and only if every bi-interior ideal element of S is idempotent.*

Proof. $\Longrightarrow$. Let b be a bi-interior ideal element of S. Since S is regular, we have $b \le beb$. Then we have $b^2 \le (beb)b \le beb \wedge ebe = b$, thus b is subidempotent.
$\Longleftarrow$. Let a be a right ideal element and b a left ideal element of S. By Proposition 2.4(2), $a \wedge b$ is a bi-interior ideal element of S. By hypothesis, we have

$$a \wedge b = (a \wedge b)^2 = (a \wedge b)(a \wedge b) \le ab \le ae \wedge eb \le a \wedge b,$$

thus $a \wedge b = ab$, and S is regular (see the proof of Theorem 4.1). □

Proposition 4.3. *Let S be a regular $\wedge e$-semigroup. Then b is a bi-interior ideal element of S if and only if b is a bi-ideal element of S.*

Proof. $\Longrightarrow$. Let b be a bi-interior element of S. Since S is regular, we have

$$b \quad \le \quad beb \le (beb)e(beb) \le (beb) \wedge (ebe) \le b.$$

Thus we have $b = beb$, and b is a bi-ideal element of S.
The $\Leftarrow$-part follows from Proposition 2.3(3), and it holds for $\wedge e$-semigroups in general. □

Theorem 4.4. *Let S be a regular $\wedge e$-semigroup. Then b is a bi-interior ideal element of S if and only if there exists a right ideal element r and a left ideal element l of S such that $b = rl$.*

Proof. $\Longrightarrow$. Let b be a bi-interior ideal element of S. Since S is regular, by Theorem 4.1, we have $beb \wedge ebe = b$. The element be and eb are right and left ideal elements of S, respectively. It is enough to prove that $b = (be)(eb)$.
Since S is regular, we have $b \le beb$. Thus we have

$$(be)(eb) \le (beb)e(beb) \le (beb) \wedge (ebe) = b.$$

We also have $b \le beb \le (beb)e(beb) \le (be)(eb)$ and so $b = (be)(eb)$.
The "$\Leftarrow$-part follows by Proposition 2.6(1) (or 2.6(2)) and it holds for $\wedge e$-semigroups in general. □

Corollary 4.5. (cf. also [4; Theorem 3.28]) *Let M be a regular semigroup. Then B is a bi-interior ideal of M if and only if there exists a right ideal R and a left ideal L of M such that $B = RL$.*

Proposition 4.6. *Let S be a poe-semigroup, b a subidempotent bi-ideal element of S and $a \in S$ such that $a \le b$ and $a = aba$. Then a is a bi-interior ideal element of S.*

Proof. Since $a \le b$, we have $ba \le b^2 \le b$ and $ab \le b^2 \le b$. Then $a = a(ba) \le ab$ and $a = (ab)a \le ba$. Then $aea \le (ab)e(ba) = a(beb)a \le aba = a$ and so a is a bi-ideal element of S. Then, by Proposition 2.3(3), it is a bi-interior ideal element of S as well. □

Theorem 4.7. *A $\wedge e$-semigroup S is regular if and only if for every bi-interior ideal element b, every ideal element i and every left ideal element l of S, we have $b \wedge i \wedge l \le bil$.*

Proof. $\Longrightarrow$. Let b be a bi-interior ideal element, i an ideal element and l a left ideal element of S. Since S is regular, we have

$$\begin{aligned} b \wedge i \wedge l &\le (b \wedge i \wedge l)e(b \wedge i \wedge l) \\ &\le \Big((b \wedge i \wedge l)e(b \wedge i \wedge l)e(b \wedge i \wedge l)\Big)\Big(e(b \wedge i \wedge l)e(b \wedge i \wedge l)\Big)e(b \wedge i \wedge l). \end{aligned}$$

We have
$b \wedge i \wedge l \le b,\ e(b \wedge i \wedge l)e \le e,\ (b \wedge i \wedge l) \le b$ and so
$(b \wedge i \wedge l)e(b \wedge i \wedge l)e(b \wedge i \wedge l) \le beb.$
$(b \wedge i \wedge l)e \le e,\ b \wedge i \wedge l \le b,\ e(b \wedge i \wedge l) \le e$ and so
$(b \wedge i \wedge l)e(b \wedge i \wedge l)e(b \wedge i \wedge l) \le ebe.$
Thus we have
$(b \wedge i \wedge l)e(b \wedge i \wedge l)e(b \wedge i \wedge l) \le beb \wedge ebe \le b.$
Moreover,
$e(b \wedge i \wedge l)e(b \wedge i \wedge l) \le eiei \le i$ and $e(b \wedge i \wedge l) \le el \le l.$
Hence we obtain $b \wedge i \wedge l \le bil$.

$\Longleftarrow$. Let a be a right ideal element and b a left ideal element of S. Since a is a bi-interior ideal element, e an ideal element and b a left ideal element of S, by hypothesis, we have
$a \wedge b = a \wedge e \wedge b \le aeb \le ab \le ae \wedge eb \le ab$, then $a \wedge b = ab$
and so S is regular. □

Example 4.8. We consider the $\wedge e$-semigroup $S = \{a, b, c, d, e\}$ given by Table 1 and Figure 1. This is an le-semigroup at the same time.

$\cdot$	a	b	c	d	e
a	e	b	a	d	e
b	b	b	b	b	b
c	a	b	c	d	e
d	d	b	d	d	d
e	e	b	e	d	e

Table 1

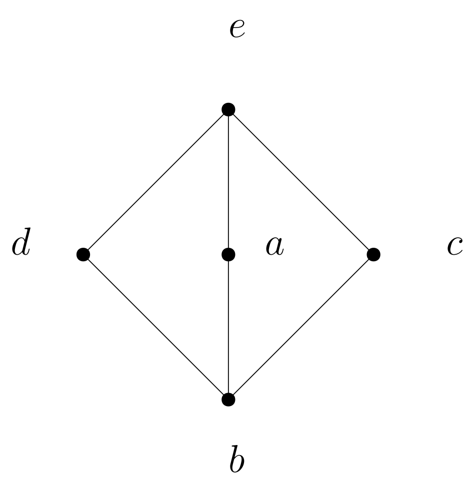

Figure 1

This is regular as $x \leq xex$ for every $x \in S$.
The bi-interior ideal elements of S are the sets b, d and e. The results of sections 2 and 4 can be applied.
This is not left simple, right simple, simple or bi-interior simple.

Note. We do not have to assume that all semigroups in [4] have unity. In case we need it, the assumption $A \subseteq AM$ and $A \subseteq MA$ for every nonempty subset A of S provides a more general situation. It is not known if the Theorem 3.33 in [4] holds since its proof is wrong. The proof of the "$\Rightarrow$"-part of Theorem 3.14 in [4] is wrong; however the above Theorem 4.7 shows that it can be proved and the Theorem 3.14 in [4] holds.

5. Conclusion

The results of the present paper generalize corresponding results by M.M. Krishna Rao in Discuss. Math. Gen. Algebra Appl. In a similar way all the results on semigroups based on sets can be written in an abstract form using elements (instead of sets).

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