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# On the solutions of the Diophantine equation $p^x + (p+4k)^y = z^2$ for prime pairs $p$ and $p+4k$

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**Abstract.** In this paper, we solve the Diophantine equation $p^x + (p+4k)^y = z^2$ in $\mathbb{N}_0$ for prime pairs $(p, p+4k)$. First, we consider cousin primes $p$ and $p+4$. Then we extend the study to solving $p^x + (p+4)^y = z^{2n}$, where $n \in \mathbb{N}\backslash\{1\}$. Furthermore, we solve the equation $p^x + (p+4k)^y = z^2$ for $k \geq 2$. As a result, we show that this equation has a unique solution $(p, p+4k, x, y, z) = (3, 11, 5, 2, 122)$ whenever $x > 1$ and $y > 1$. Finally, we show the finiteness of number of solutions in $\mathbb{N}$.

**2020 Mathematics Subject Classifications**: 11A07, 11A41, 11D61, 11D72

**Key Words and Phrases**: Diophantine equation, exponential Diophantine equation, nonlinear Diophantine equation, cousin primes, Legendre symbol

## 1. Introduction

Diophantine equations of type

$$p^x + q^y = z^2 \tag{1}$$

have been widely studied for various fixed values of $p$ and $q$. Some of these can be seen in [1, 2, 11, 14–17, 19] and [20]. In 2015, Bacani and Rabago [3] provided the solutions of the Diophantine equation $p^x + q^y = z^2$, where $p$ and $q$ are twin primes; that is, $p$ and $q$ differ by 2. It was shown that this equation has infinitely many solutions in the set $\mathbb{N}_0$ of nonnegative integers, assuming that the twin prime conjecture, also known as Polignac's conjecture, holds. In 2018, Burshtein [4] made a study on the Diophantine equation $p^x + (p+4)^y = z^2$, where $p$ and $p+4$ are primes, and $x+y = 2, 3, 4$. Also in 2018, Neres [13] investigated the solvability of the Diophantine equation $p^x + (p+8) = z^2$ for prime pairs $p > 3$ and $p+8$. Most recently, Dockan and Pakapongpun [6] published a paper on the Diophantine equation $p^x + (p+20)^y = z^2$ for prime pairs $p$ and $p+20$.

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Motivated by the papers mentioned above, this work will deal with the Diophantine equations of the form $p^x + (p+4k)^y = z^2, k \in \mathbb{N}$, in the set of nonnegative integers. We first deal with the case where $k = 1$. That is, we consider the equation

$$p^x + (p+4)^y = z^2, \tag{2}$$

where $p$ and $p+4$ are primes. These primes which differ by 4 are called cousin primes. We then extend the work to solving the Diophantine equation $p^x + (p+4)^y = z^{2n}$ for all $n > 1$. Lastly, we study the solutions of Diophantine equations of the form

$$p^x + (p+4k)^y = z^2, \qquad k \geq 2 \tag{3}$$

where $p$ and $p+4k$ are both primes.

## 2. Preliminaries

Diophantine equations usually refer to any equations in one or more unknowns that are to be solved in the set $\mathbb{Z}$ of integers [5]. The simplest equation known is the *linear Diophantine equation* in two unknowns, written as $ax + by = c$, where $a, b$ and $c$ are integers. Indian mathematician *Brahmagupta* is believed to be the first person to describe the general solution of such equations. Now, the general solution is already well-established (cf. [18], p.134). Euclidean Algorithm is one of the ways to solve linear Diophantine equations.

If there are linear Diophantine equations, there are also *nonlinear Diophantine equations*. Quadratic equations in two unknowns $x$ and $y$ ( $ex^2 + fxy + gy^2 + hx + jy = k$) for fixed integers $e, f, g, h, j$ and $k$) are considered nonlinear. The famous quadratic Diophantine equation is the Pythagoras equation:

$$x^2 + y^2 = z^2. \tag{4}$$

The triples $(3, 4, 5), (5, 12, 13)$, and $(7, 24, 25)$ are just a few of the infinitely many solutions of (4).

Another famous nonlinear Diophantine equation is given in the Fermat's Last Theorem, which states that the equation $x^n + y^n = z^n$ has no solutions in $\mathbb{Z}$ for $n > 2$.

In our time, many mathematicians study various nonlinear Diophantine equations. Keskin [7] investigated positive integer solutions of $x^2 - kxy \mp y^2 \mp x = 0$ and $x^2 - kxy - y^2 \mp y = 0$, Abu Muriefah and AL-Rashed [12] studied the Diophantine equation $x^2 - 4p^m = \pm y^n$, and Luca and Soydan [10] considered Diophantine equation $2^m + nx^2 = y^n$. Acu, Burshtein, Dockan, Neres, Rabago, Sroysang, and Suvarnamani are among the mathematicians who studied Diophantine equations of the form $a^x + b^y = z^2$ (cf. [1, 2, 4, 6, 13–17, 19, 20]).

## 3. Main Results

### 3.1. On the Diophantine equation $p^x + (p+4)^y = z^2$

We begin the discussion by considering the following two lemmas.

**Lemma 1.** *The Diophantine equation* (2)*, where* $p$ *and* $p+4$ *are cousin primes, has no solutions in* $\mathbb{N}_0$ *if* $x$ *and* $y$ *are of the same parity.*

*Proof.* Suppose that $x$ and $y$ are both even. Taking equation (2) modulo 4, by first replacing $p+4$ by $q$, we get $p^x + q^y \equiv 2 \pmod 4$ whenever $p \equiv 1 \pmod 4$ or $p \equiv -1 \pmod 4$. On the other hand, since $p^x + q^y$ is even for any cousin primes $p$ and $q$, then $z^2$ is even and $z^2 \equiv 0 \pmod 4$. Now, suppose that $x$ and $y$ are both odd. If $p \equiv 1 \pmod 4$, then $p^x + q^y \equiv 1 + 1 \equiv 2 \pmod 4$. If $p \equiv -1 \pmod 4$, then $p^x + q^y \equiv (-1) + (-1) \equiv 2 \pmod 4$. Therefore, in any case, $p^x + q^y \not\equiv z^2 \pmod 4$. □

We now consider the case where $x$ and $y$ are of different parity.

**Lemma 2.** *The Diophantine equation* (2)*, where* $p$ *and* $q$ *are cousin primes, and* $x$ *and* $y$ *are of different parity, has exactly two solutions in* $\mathbb{N}_0$*, namely,* $(p, q, x, y, z) = (3, 7, 1, 0, 2)$ *and* $(3, 7, 2, 1, 4)$.

*Proof.* **Case 1**: $x$ is odd and $y$ is even, i.e. $y = 2l$ for some $l \in \mathbb{N}_0$. Write $p^x$ as $p^x = z^2 - q^{2l} = (z + q^l)(z - q^l)$. Since $p$ is prime, there exist integers $\alpha$ and $\beta$ with $\alpha < \beta$ s.t. $\alpha + \beta = x$ and $p^\alpha(p^{\beta-\alpha} - 1) = (z + q^l) - (z - q^l) = 2q^l$. Since $p \neq 2, q$, we have $\alpha = 0$ and will imply that $p^x - 1 = 2q^l$. By factoring, we get

$$(p-1)(p^{x-1} + p^{x-2} + \cdots + 1) = 2q^l. \tag{5}$$

Note that $q$ is prime. So, $p - 1 = 2q^j$ for some $j \leq l$. If $j \geq 1$, then $p - 1 < 2(p+4)^j$. Hence this can only have a solution if $j = 0$. If $j = 0$, then $p = 3$ and we have from (5) that

$$3^{x-1} + 3^{x-2} + \cdots + 1 = 7^l. \tag{6}$$

Substituting $x = 1$ to (6), we get $l = 0$, and $(p, q, x, y, z) = (3, 7, 1, 0, 2)$ is a solution to (2). If $x > 1$ then $l > 1$. Take modulo 7 to (6) to obtain $3^{x-1} + 3^{x-2} + \cdots + 1 \equiv 0 \pmod 7$. Multiply this congruence by $3 - 1$ to get $3^x - 1 \equiv 0 \pmod 7$, or equivalently $3^x \equiv 1 \pmod 7$. This can only happen if $x \equiv 0 \pmod 6$, which will yield no solutions since we assumed $x$ to be odd.

**Case 2**: $x$ is even and $y$ is odd, i.e. $x = 2m$ for some $m \in \mathbb{N}_0$. Using the same argument as in Case 1, we arrive at an analogous equation $q^y - 1 = 2p^m$. By factoring, we get

$$(q-1)(q^{y-1} + q^{y-2} + \cdots + 1) = 2p^m \tag{7}$$

which implies that $q - 1 = 2p^j$ for some $j \leq m$. If $j = 0$, then $q = 3$ which is not possible. If $j > 1$, then $q - 1 = p + 3 < 2p^j$ for any odd prime $p$. Hence, this can only have a solution if $j = 1$. Using (7), we have

$$7^{y-1} + 7^{y-2} + \cdots + 1 = 3^{m-j} = 3^{m-1}. \tag{8}$$

Substituting $y = 1$ to (8), we get $m = 1$, and $(p, q, x, y, z) = (3, 7, 2, 1, 4)$ becomes a solution to (2). If $j > 1$ then $m > 1$. Taking modulo 3 to equation (8), we get $y - 1 + 1 \equiv 0 \pmod 3$. Thus, $y \equiv 0 \pmod 3$. By letting $y = 3y_1$ and substituting this to (8), we arrive at $7^{3y_1-1} + 7^{3y_1-2} + \cdots + 1 = 3^{m-1}$. Multiply this equation by $7-1$ to get $7^{3y_1} - 1 = 6 \cdot 3^{m-1} = 2 \cdot 3^m$. We then have

$$(7^3 - 1) \cdot \sum_{i=0}^{y_1-1} (7^3)^i = 2 \cdot 3^m,$$

which implies that

$$2 \cdot 3^2 \cdot 19 \cdot \sum_{i=0}^{y_1-1} (7^3)^i = 2 \cdot 3^m,$$

which is not possible since 19 does not divide $2 \cdot 3^m$. We have proven Lemma 2. □

By using the above two lemmas, we now have our main theorem.

**Theorem 1.** *The Diophantine equation* (2)*, where* $p$ *and* $q$ *are cousin primes, has exactly two solutions* $(p, q, x, y, z)$ *in* $\mathbb{N}_0$*, namely,* $(3, 7, 1, 0, 2)$ *and* $(3, 7, 2, 1, 4)$.

Corollary to the theorem is the following result.

**Corollary 1.** *The Diophantine equation* $p^x + q^y = z^{2n}$*, where* $p$ *and* $q$ *are cousin primes, and* $z$ *is not a perfect square, has exactly two solutions* $(p, q, x, y, z, n)$ *in* $\mathbb{N}_0$*, namely,* $(3, 7, 1, 0, 2, 1)$*, and* $(3, 7, 2, 1, 2, 2)$.

*Proof.* Here, we are considering the Diophantine equation $p^x + q^y = (z^n)^2$. By Theorem 1, this has only two solutions in $\mathbb{N}_0$ and we get those solutions when $z^n = 2$ or $z^n = 4$. The first equality gives us that $z = 2$ and $n = 1$, hence the solution $(p, q, x, y, z, n) = (3, 7, 1, 0, 2, 1)$. The second one has a solution $(z, n) = (4, 1)$ or $(2, 2)$. Since $z$ is assumed to be not a perfect square, then we get $(z, n) = (2, 2)$. This gives the solution $(3, 7, 2, 1, 2, 2)$. □

**Remark 1.** *In the corollary, if* $z$ *happens to be a perfect square, then just put its power to* $n$.

## 3.2. The Diophantine equation $p^x + (p + 4k)^y = z^2$

We now extend the work to solving the Diophantine equation (3), where $p$ and $p + 4k$ are both primes such that $k \geq 2$.

We first consider the generalization of Lemma 1.

**Theorem 2.** *The Diophantine equation* (3)*, where* $p$ *and* $p + 4k$ *are both primes, has no solution in* $\mathbb{N}_0$ *if* $x$ *and* $y$ *are of the same parity.*

*Proof.* Suppose that $x$ and $y$ are both odd. If $p \equiv 1 \pmod 4$ then $p + 4k \equiv 1 \pmod 4$. It follows that $p^x + (p + 4k)^y \equiv 2 \pmod 4$. Also, if $p \equiv -1 \pmod 4$ then $p + 4k \equiv -1 \pmod 4$ and $p^x + (p + 4k)^y \equiv -1 + (-1) \equiv 2 \pmod 4$. On the other hand, since $p^x + (p + 4k)^y$ is even, it follows that $z^2$ is even. Hence, $z^2 \equiv 0 \pmod 4$. Thus, $p^x + (p + 4k)^y \not\equiv z^2 \pmod 4$.

Now, suppose that $x$ and $y$ are both even. Then, $p^x + (p+4k)^y \equiv 1 + 1 \equiv 2 \pmod 4$. Hence, $p^x + (p+4k)^y \not\equiv z^2 \pmod 4$.

Therefore, the Diophantine equation (3) has no solutions in $\mathbb{N}_0$ for both cases. □

Because of the previous theorem, from now on, we will only be considering the case where $x \geq 1$ and $y \geq 1$ because the case $x = 0$ or $y = 0$ can easily be handled. We divide the proof into three cases: (1) $x = 1$ or $y = 1$, (2) $x > 1$ even and $y > 1$ odd; and (3) $x > 1$ odd and $y > 1$ even. We have the first case.

**Theorem 3.** *Consider the Diophantine equation* (3)*, where* $p$ *and* $p + 4k$ *are primes and* $x = 1$ *or* $y = 1$*. Then* (3) *has a solution if and only if* $p \equiv 3 \pmod 4$*,* $x = 2m$ *for some positive integer* $m$*,* $y = 1$ *and* $k = (2p^m - p + 1)/4$.

*Proof.* Firstly, take $x = 1$ and $y$ even, then take $y = 1$ and $x$ even. So we get $p + (p+4k)^y = z^2$ with $y$ even and $p^x + (p+4k) = z^2$ with $x$ even, respectively.

Assume that $p + (p+4k)^y = z^2$, where $y$ is even. Then $z$ is even and therefore $p + (p+4k)^y \equiv 0 \pmod 4$. Since $y$ is even we get $p \equiv 3 \pmod 4$. Let $y = 2m$. Then $z - (p+4k)^m = 1$ and $z + (p+4k)^m = p$. It follows that $2(p+4k)^m + 1 = p$, which is impossible because $2(p+4k)^m + 1 > p$.

Now, consider the equation $p^x + (p+4k) = z^2$, where $x$ is even. Then, $p + 1 \equiv 0 \pmod 4$ since $z$ is even. Therefore, $p \equiv -1 \pmod 4$. Let $x = 2m$. Then $z - p^m = 1$ and $z + p^m = p + 4k$. Therefore, $2p^m + 1 = p + 4k$ and so $k = \dfrac{2p^m - p + 1}{4}$. Since $p \equiv -1 \pmod 4$, it follows that $k$ is an integer for any positive integer $m$. Therefore taking $x = 2m$, $p \equiv -1 \pmod 4$, and $k = \dfrac{2p^m - p + 1}{4}$, it can be seen that $p^x + (p+4k) = (p^m + 1)^2$. Therefore, $p^x + (p+4k) = z^2$ has a solution if and only if $p \equiv 3 \pmod 4$, $x = 2m$, and $k = \dfrac{2p^m - p + 1}{4}$. □

From this theorem, some solutions to (3) are given by $(p, p+4k, x, y, z) = (11, 23, 2, 1, 12)$, $(3, 19, 4, 1, 10)$ and $(3, 163, 8, 1, 82)$. We also have the following result.

**Theorem 4.** *Consider the Diophantine equation* (3)*, where* $p$ *and* $p + 4k$ *are primes. Let* $x > 1$ *and* $y > 1$ *be odd and even integers, respectively. Then,* (3) *has only the solution* $(p, p+4k, x, y, z) = (3, 11, 5, 4, 122)$.

*Proof.*

Let $y = 2l$ with $l \geq 1$ and $q = p + 4k$. Then we get

$$p^x = z^2 - q^{2l} = (z - q^l)(z + q^l).$$

It can be shown that $\gcd(z - q^l, z + q^l) = 1$. So it follows that $z - q^l = 1$ and $z + q^l = p^x$. From here, we get $2q^l = p^x - 1$. Thus, $2q^l = (p-1)(1 + p + p^2 + ... + p^{x-1})$. Assume that $p > 3$. Then $p - 1 = 2q^j = 2(p+4k)^j$ for some $j \geq 1$. This is impossible since $p - 1 < 2(p+4k)^j$ for $j \geq 1$. Therefore, $p = 3$. So we get $3^x - 1 = 2(3+4k)^l$. Having

$3^x - 1 = 2(3+4k)^l$, it is seen that $3 \nmid k$ and the Legendre symbol $\left(\frac{3}{3+4k}\right)^x$ is equal to 1. Thus, $\left(\frac{3}{3+4k}\right) = 1$ since $x$ is odd. This implies that

$$\left(\frac{k}{3}\right) = \left(\frac{3+4k}{3}\right) = (-1)^{\frac{3-1}{2}\cdot\frac{3+4k-1}{2}}\left(\frac{3}{3+4k}\right) = -1.$$

Therefore $k \equiv 2 \pmod 3$. Let $k = 3a+2$ with $a \geq 0$. Then,

$$3^x - 1 = 2(3+4k)^l = 2(11+12a)^l.$$

This shows that $-1 \equiv 2(-1)^l \pmod 3$, hence $l$ is even. Suppose $l = 2r$ for some positive integer $r$. So we get

$$3^x - 1 = 2[(11+12a)^r]^2.$$

By Theorem 2.3 given in [8], the equation $2x^2 + 1 = 3^n$ has only three positive solutions $(x,n) = (1,1),(2,2),(11,5)$. Therefore, we get $x = 5$ and $(11+12a)^r = 11$; that is, $11 + 12a = 11$ and $r = 1$. Thus, $y = 2l = 4r = 4$. This shows that the equation

$$p^x + (p+4k)^y = z^2, \quad x > 1, y > 1 \qquad \text{with } x \text{ odd and } y \text{ even}$$

has only the solution $(p, p+4k, x, y, z) = (3, 11, 5, 4, 122)$.

□

Here is an analog of Theorem 4 for the case where $x$ is even and $y$ is odd.

**Theorem 5.** *Consider the Diophantine equation*(3) *where $p$ and $p+4k$ are primes. Let $x > 1$ and $y > 1$ be positive even and odd integers, respectively. Then,* (3) *has no solutions in* $\mathbb{N}$.

*Proof.* Let $q = p+4k$. Then $q^y - 1 = 2p^m$. Hence, $q - 1 = 2p^\alpha$ and $1 + q + q^2 + ... + q^{y-1} = p^\beta$ for some positive integers $\alpha$ and $\beta$. Since $q - 1 = 2p^\alpha$, it follows that $q \equiv 3 \pmod 4$. Consequently, $p \equiv 3 \pmod 4$ since $p = q - 4k$.

Using the fact that $q - 1 = 2p^\alpha$, it is seen that the Legendre symbol $\left(\frac{q}{p}\right)$ is equal to 1. Since $p \equiv 3 \pmod 4$ and $q \equiv 3 \pmod 4$, then by using the quadratic reciprocity law, we get $\left(\frac{q}{p}\right) = -1$. On the other hand, the equality $1 + q + q^2 + ... + q^{y-1} = p^\beta$ implies $p^\beta \equiv 1 \pmod q$, so we have $(-1)^\beta = \left(\frac{p}{q}\right)^\beta = 1$, which shows that $\beta$ is even. Let $\beta = 2r$. Then, we get $1 + q + q^2 + ... + q^{y-1} = (p^r)^2$. That is,

$$\frac{1 - q^y}{1 - q} = (p^r)^2.$$

By the result given in [9], the equation

$$\frac{x^n - 1}{x - 1} = y^2, x > 1, y > 1, n > 2$$

has only solutions $(x,n) = (7,4),(3,5)$. Since $q = p+4k > 7$, and $y$ is odd with $y > 2$, we see that

$$\frac{1-q^y}{1-q} = (p^r)^2$$

has no solutions. Therefore, equation (3) has no solutions if $x > 1, y > 1$, $y$ is odd, and $x$ is even. □

From Lemma 2 and Theorems 4 and 5, we get the following corollary.

**Corollary 2.** *Let $p$ and $p+4k$ be prime numbers. Then, the equation $p^x + (p+4k)^y = z^2, x > 1, y > 1$ has only the solution*

$$(p, p+4k, x, y, z) = (3, 11, 5, 4, 122).$$

### 3.3. Finiteness of number of solutions

In this section, we prove that (3) has a finite number of solutions for any given value of $p$ and $k$.

**Theorem 6.** *Let $k$ be a fixed positive integer such that $p$ and $p+4k$ are primes. Then, the Diophantine equation (3) has at most two solutions $(x,y,z)$ in $\mathbb{N}$.*

*Proof.* Fix a value for $p$ and $k$. We have the following cases. If $x = 1$, then by Theorem 3, there are no solutions. If $y = 1$, then there is only one solution for a fixed value of $p$ and $k$. Finally, if $x > 1$ and $y > 1$, then by Corollary 2, there is only one solution. Hence, in any case, there are at most two solutions. □

**Remark 2.** *By applying the results in Theorem 6 and Corollary 1, it follows that for each $k \in \mathbb{N}$, the Diophantine equation $p^x + (p+4k)^y = z^{2n}$ has only a finite number of solutions $(x,y,z,n)$.*

## 4. Conclusion

In this paper, we have shown that there are only two solutions to the Diophantine equation (2) in $\mathbb{N}_0$ when $p$ and $p+4$ are primes. There are also two solutions for the equation $p^x + (p+4)^y = z^{2n}$, whenever $z$ is not a perfect square. Lastly, we studied the more general form $p^x + (p+4k)^y = z^2$ and have shown that there's a unique solution if $x$ and $y$ are both greater than 1. In general, there's a finite number of solutions in the set $\mathbb{N}$.

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