



Fuzzy Translation and Fuzzy multiplication in BRK-algebras

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Abstract. In this paper, the concepts of fuzzy translation and fuzzy multiplication on a BRK-algebra are introduced. We investigated fuzzy translation and fuzzy multiplication (BRK-subalgebras & BRK-ideals) in BRK-algebras and discussed related properties. Finally, we presented the nation fuzzy magnified- $\alpha\beta$ -translation on BRK-algebra X.

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1. Introduction

The fundamental concept of fuzzy set, popularized by Zadeh [10], was used to generalize several basic concepts of algebra. Fuzzy sets are extremely useful to deal with the many problems in applied mathematics, control engineering, information sciences, expert systems etc. Although there are several generalizations of fuzzy sets, none of them address the issues of members with membership degree 0 who have opposing qualities. Lee [7] handled this problem by introducing the concept of bipolar fuzzy (BF) sets. A BF set is a pair of fuzzy sets, namely a membership and a non-membership function, which represent positive and negative aspects of the given information. Imai and Iseki investigated two classes of abstract algebras: BCI-algebras and BCK-algebras [5]. Recently, Bandaru [1] investigated BRK-algebra which is a generalization of BCK/BCI/BCH/Q/QS/BM-algebras. In [2,3], Elgendy introduced fuzzy BRK-ideal of BRK-algebra and cubic BRK-ideal of BRK-algebra. Some properties of n-dimensional fuzzy subalgebra in BRK-algebras investigated by Zulfiqar [11]. Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras presented in [8]. The contents of the current paper are structured as follows: In Sect. 2, we presented some basic definitions and preliminaries. In Sect. 3, we investigated fuzzy

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translation and fuzzy multiplication of BRK-subalgebras and discussed related properties. In Sect. 4, we introduced fuzzy translation and fuzzy multiplication of BRK-ideals and discussed related results. In Sect. 5, we defined concept of Fuzzy magnified- $\alpha\beta$ -translation of BRK-algebras. At last, some conclusions and future work were presented.

2. Preliminaries

Some elementary aspects that are important for this paper are included in this section.

Definition 1 (1). A BRK-algebra is a non-empty set X with a constant 0 and a binary operation “ $*$ ” satisfying the following conditions :

$$(BRK1) \quad x * 0 = x,$$

$$(BRK2) \quad (x * y) * x = 0 * y, \text{ for all } x, y \in X.$$

A partial ordered relation \leq can be defined by $x \leq y$ if and only if $x * y = 0$. Throughout this paper, X denotes BRK-algebra.

Definition 2 (1). If $(X, *, 0)$ is a BRK-algebra, the following conditions hold:

$$(BRK3) \quad x * x = 0,$$

$$(BRK4) \quad (x * y) = 0 \text{ implies } 0 * x = 0 * y \text{ for all } x, y \in X,$$

$$(BRK5) \quad 0 * (x * y) = (0 * x) * (0 * y) \text{ for all } x, y \in X,$$

.

Definition 3 (1). A subset S of a BRK-algebra X is said to be BRK-subalgebra of X , if $x, y \in S$, implies $x * y \in S$.

Definition 4 (1). A non-empty subset I of a BRK-algebra X is said to be a BRK-ideal of X if it satisfies:

$$(I_1) \quad 0 \in I,$$

$$(I_2) \quad 0 * (x * y) \in I \text{ and } 0 * y \in I \text{ imply } 0 * x \in I, \text{ for all } x, y \in X.$$

Definition 5 (10). A fuzzy subset μ in a non-empty set X is a function $\mu : X \rightarrow [0, 1]$.

Definition 6 (2). A Fuzzy Subset μ in a BRK-algebra X is said to be a Fuzzy BRK-subalgebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\} \forall x, y \in X$.

Definition 7 (2). Let $(X, *, 0)$ be a BRK-algebra. A fuzzy set μ in X is called a fuzzy BRK-ideal of X if it satisfies:

$$(FI_1) \quad \mu(0) \geq \mu(x),$$

$$(FI_2) \quad \mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}, \forall x, y \in X.$$

3. Fuzzy Translation and Fuzzy Multiplication of BRK-subalgebras

This section deals with the notion of Fuzzy translation and Fuzzy multiplication on BRK-algebras. In what follows, X denotes a BRK-algebra, and for any fuzzy set μ of X , we denote $T = 1 - \sup\{\mu(x) | x \in X\}$ unless otherwise specified. We start with,

Definition 8. Let μ be a fuzzy subset of X and $\alpha \in [0, 1]$. A mapping $\mu_\alpha^T: X \rightarrow [0, 1]$ is said to be a fuzzy α -translation of μ if it satisfies: $\mu_\alpha^T(x) = \mu(x) + \alpha, \forall x \in X$.

Definition 9. Let μ be a fuzzy subset of X and $\alpha \in [0, 1]$. A mapping $\mu_\alpha^M: X \rightarrow [0, 1]$ is said to be a fuzzy α -multiplication of μ if it satisfies: $\mu_\alpha^M(x) = \alpha \cdot \mu(x), \forall x \in X$.

Definition 10. A fuzzy α -translation set $\mu_\alpha^T(x)$ of μ is called fuzzy α -translation BRK-subalgebra of X if it satisfies following condition:

$$\mu_\alpha^T(x*y) \geq \min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\},$$

Similarly, we said that $\mu_\alpha^M(x)$ is fuzzy α -multiplication BRK-subalgebra of X if it satisfies:

$$\mu_\alpha^M(x*y) \geq \min\{\mu_\alpha^M(x), \mu_\alpha^M(y)\}.$$

Example 1. Consider a set $X = \{0, a, b, c\}$. We define “*” on X as the following table :

*	0	a	b	c
0	0	a	0	a
a	a	0	a	0
b	b	a	0	a
c	c	b	c	0

Define a fuzzy subset μ of X by $\mu(0) = \mu(a) = 0.6$ and $\mu(b) = \mu(c) = 0.1$, routine calculation gives that μ is fuzzy BRK-subalgebra of X . Here $T = 1 - \sup\{\mu(x) : x \in X\} = 1 - 0.6 = 0.4$. Choose $\alpha = 0.2 \in [0, T]$ and $\beta = 0.3 \in [0, 1]$.

Then the mapping $\mu_{0.2}^T(x): X \rightarrow [0, 1]$ defined by

$$\mu_{0.2}^T(x) = \begin{cases} 0.6 + 0.2 = 0.8 & ; x = 0, a \\ 0.1 + 0.2 = 0.3 & ; x = b, c \end{cases}$$

$\mu_{0.2}^T(x) = \mu(x) + 0.2, \forall x \in X$, is a fuzzy 0.2-translation.

$\mu_{0.3}^M(x): X \rightarrow [0, 1]$ defined by

$$\mu_{0.3}^M(x) = \begin{cases} (0.3)(0.6) = 0.18 & ; x = 0, a \\ (0.3)(0.1) = 0.03 & ; x = b, c \end{cases}$$

$\mu_{0.3}^M(x) = (0.3)\mu(x), \forall x \in X$, is a fuzzy 0.3-multiplication.

Theorem 1. For any fuzzy BRK-subalgebra μ of X and $\alpha \in [0, T]$, the fuzzy α -translation $\mu_\alpha^T(x)$ of μ is a fuzzy BRK-subalgebra of X .

Proof. Let $x, y \in X$ and $\alpha \in [0, T]$. Then $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$.
Now,

$$\begin{aligned} \mu_\alpha^M(x * y) &= \mu(x) + \alpha \\ &\geq \min\{\mu(x), \mu(y)\} + \alpha \\ &= \min\{\mu(x) + \alpha, \mu(y) + \alpha\} \\ &= \min\{\mu_\alpha^M(x), \mu_\alpha^M(y)\} \end{aligned}$$

This completes the proof.

The converse of the above theorem is valid.

Theorem 2. For any fuzzy subset μ of X and $\alpha \in [0, T]$, if the fuzzy α -translation $\mu_\alpha^T(x)$ of μ is a fuzzy BRK-subalgebra of X then so is μ .

Proof. Let $x, y \in X$.

Assume that $\mu_\alpha^T(x)$ of μ is a fuzzy BRK-subalgebra of X for $\alpha \in [0, 1]$.

Then $\mu(x * y) + \alpha = \mu_\alpha^T(x * y) \geq \min\{\mu_\alpha^T(x), \mu_\alpha^T(y)\} = \min\{\mu(x) + \alpha, \mu(y) + \alpha\} = \min\{\mu(x), \mu(y)\} + \alpha$.

Hence, $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$.

Therefore, μ is a fuzzy BRK-subalgebra of X .

Theorem 3. For any fuzzy BRK-subalgebra μ of X and $\alpha \in [0, 1]$, the fuzzy α -multiplication $\mu_\alpha^M(x)$ of μ is a fuzzy BRK-subalgebra of X .

Proof. Let $x, y \in X$ and $\alpha \in [0, 1]$.

Then $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$.

Now,

$\mu_\alpha^M(x * y) = \alpha \cdot \mu(x * y) \geq \alpha \cdot \min\{\mu(x), \mu(y)\} = \min\{\alpha \cdot \mu(x), \alpha \cdot \mu(y)\} = \min\{\mu_\alpha^M(x), \mu_\alpha^M(y)\}$.

This completes the proof.

The following is the converse of the above theorem.

Theorem 4. For any fuzzy subset μ of X and $\alpha \in [0, T]$,if the fuzzy α - multiplication $\mu_\alpha^M(x)$ of μ is a fuzzy BRK-subalgebra of X then so is μ .

Proof. Let $x, y \in X$.

Assume that $\mu_\alpha^M(x)$ of μ is a fuzzy BRK-subalgebra of X for $\alpha \in [0, 1]$.

Then $\alpha \cdot \mu(x * y) = \mu_\alpha^M(x * y) \geq \min\{\mu_\alpha^M(x), \mu_\alpha^M(y)\} = \min\{\alpha \cdot \mu(x), \alpha \cdot \mu(y)\} = \alpha \cdot \min\{\mu(x), \mu(y)\}$

Hence, $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$

Therefore, μ is a fuzzy BRK-subalgebra of X .

4. Fuzzy Translation and Fuzzy Multiplication of BRK-ideals

Definition 11. A fuzzy α -translation set $\mu_\alpha^T(x)$ of μ is called fuzzy α -translation BRK-ideal of X if it satisfies following condition:

(FTI_1) $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$,

$$(FTI_2) \mu_\alpha^T(0 * x) \geq \min\{\mu_\alpha^T(0 * (x * y)), \mu_\alpha^T(0 * y)\}, \forall x, y \in X.$$

Similarly, we said that $\mu_\alpha^T(x)$ is fuzzy α -multiplication BRK-ideal of X if it satisfies:
 (FMI₁) $\mu_\alpha^M(0) \geq \mu_\alpha^M(x)$,
 (FMI₂) $\mu_\alpha^M(0 * x) \geq \min\{\mu_\alpha^M(0 * (x * y)), \mu_\alpha^M(0 * y)\}, \forall x, y \in X.$

Theorem 5. For any fuzzy BRK-ideal μ of X and $\alpha \in [0, T]$, the fuzzy α -translation $\mu_\alpha^T(x)$ of μ is a fuzzy BRK-ideal of X .

Proof. Let $x, y \in X$ and $\alpha \in [0, T]$. Then

$$\mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}$$

Now,

$$\begin{aligned} \mu_\alpha^T(0 * x) &= \mu(0 * x) + \alpha \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\} + \alpha \\ &= \min\{\mu(0 * (x * y)) + \alpha, \mu(0 * y) + \alpha\} \\ &= \min\{\mu_\alpha^M(0 * (x * y)), \mu_\alpha^M(0 * y)\} \end{aligned}$$

Hence, $\mu_\alpha^T(x)$ is a fuzzy BRK-ideal of X .

The following is the converse of the above theorem.

Theorem 6. For any fuzzy subset μ of X and $\alpha \in [0, T]$, if the fuzzy α -translation $\mu_\alpha^T(x)$ of μ is a fuzzy BRK-ideal of X then so is μ .

Proof. Let $x, y \in X$.

Assume that $\mu_\alpha^T(x)$ of μ for $\alpha \in [0, 1]$. Then

$$\begin{aligned} \mu(0 * x) + \alpha &= \mu_\alpha^T(0 * x) \geq \min\{\mu_\alpha^M(0 * (x * y)), \mu_\alpha^M(0 * y)\} = \\ \min\{\mu(0 * (x * y)) + \alpha, \mu(0 * y) + \alpha\} &= \min\{\mu(0 * (x * y)), \mu(0 * y)\} + \alpha \end{aligned}$$

Hence, $\mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}$.

Therefore, μ is a fuzzy BRK-ideal of X .

Theorem 7. For any fuzzy BRK-ideal μ of X and $\alpha \in [0, T]$, the fuzzy α -multiplication $\mu_\alpha^M(x)$ of μ is a fuzzy BRK-ideal of X .

Proof. Let $x, y \in X$ and $\alpha \in [0, T]$. Then

$$\mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}$$

Now,

$$\begin{aligned} \mu_\alpha^M(0 * x) &= \alpha \cdot \mu(0 * x) \geq \alpha \cdot \min\{\mu(0 * (x * y)), \mu(0 * y)\} \\ &= \min\{\alpha \cdot \mu(0 * (x * y)), \alpha \cdot \mu(0 * y)\} \\ &= \min\{\mu_\alpha^M(0 * (x * y)), \mu_\alpha^M(0 * y)\} \end{aligned}$$

Hence, $\mu_\alpha^M(x)$ is a fuzzy BRK-ideal of X.

The following is the converse of the above theorem.

Theorem 8. For any fuzzy subset μ of X and $\alpha \in [0, T]$, if the fuzzy α -multiplication $\mu_\alpha^M(x)$ of μ is a fuzzy BRK-ideal of X then so is μ .

Proof. Let $x, y \in X$.

Suppose that $\mu_\alpha^M(x)$ of μ for $\alpha \in [0, 1]$. Then

$$\begin{aligned} \alpha \cdot \mu(0 * x) &= \mu_\alpha^M(0 * x) \geq \min\{\mu_\alpha^M(0 * (x * y)), \mu_\alpha^M(0 * y)\} = \\ \min\{\alpha \cdot \mu(0 * (x * y)), \alpha \cdot \mu(0 * y)\} &= \alpha \cdot \min\{\mu(0 * (x * y)), \mu(0 * y)\} \end{aligned}$$

Hence, $\mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}$.

Therefore, μ is a fuzzy BRK-ideal of X.

Lemma 1. Let I be a fuzzy α -translation (or fuzzy α -multiplication) BRK-ideal of BRK-algebra X. If $x \leq y$ holds in X, then $\mu_\alpha^T(0 * x) = \mu_\alpha^T(0 * y)$ (or $\mu_\alpha^M(0 * x) = \mu_\alpha^M(0 * y)$).

Proof. Assume that $x \leq y$ holds in X. Then $x * y = 0$. By (BRK4) implies $\mu_\alpha^T(0 * x) = \mu_\alpha^T(0 * y)$

Lemma 2. Let I be a fuzzy α -translation (or fuzzy α -multiplication) BRK-ideal of BRK-algebra X. If $x * y \leq x$ holds in X, then $\mu_\alpha^T(0 * y) \geq \mu_\alpha^T(0 * x)$ (or $\mu_\alpha^M(0 * y) \geq \mu_\alpha^M(0 * x)$).

Proof. Assume that $x * y \leq x$ holds in X. Then $(x * y) * x = 0$. By (BRK2)

$$\mu_\alpha^T(0 * y) = \min\{\mu_\alpha^T(0 * (x * y)), \mu_\alpha^T(0 * x)\}$$

Also,

$$\begin{aligned} \mu_\alpha^T(0 * (x * y)) &\geq \min\{\mu_\alpha^T((0 * (x * y)) * x), \mu_\alpha^T(0 * x)\} \\ &= \min\{\mu_\alpha^T(0), \mu_\alpha^T(x)\} \\ &= \{\mu_\alpha^T(0 * x)\} \end{aligned}$$

Hence, $\mu_\alpha^T(0 * y) \geq \mu_\alpha^T(0 * x)$.

5. Fuzzy magnified- $\alpha\beta$ -translation of BRK-algebras

Definition 12. Let μ be a fuzzy subset of X , $\alpha \in [0, T]$, where $T = 1 - \sup\{\mu(x) : x \in X\}$ and $\beta \in [0, 1]$. A mapping $\mu_{(\alpha\beta)}^{MT} : X \rightarrow [0, 1]$, is said to be a fuzzy magnified- $\alpha\beta$ -translation of μ if it satisfies: $\mu_{(\alpha\beta)}^{MT} = \alpha \cdot x + \beta$.

Example 2. Consider the BRK-algebra $X = 0, a, b, c$ in example 3.4. Define a fuzzy Subset μ of X by

$$\mu(x) = \begin{cases} 0.6 & ; x = 0, a \\ 0.1 & ; x = b, c \end{cases}$$

Then μ is fuzzy BRK-subalgebra of X . Here, $T = 1 - \sup\{\mu(x) : x \in X\} = 1 - 0.6 = 0.4$. Choose $\alpha = 0.5 \in [0, T]$ and $\beta = 0.2 \in [0, 1]$.

Then the mapping $\mu_{(0.5)(0.2)}^{MT} : X \rightarrow [0, 1]$ defined by

$$\mu_{(0.5)(0.2)}^{MT} = \begin{cases} (0.5)(0.6) + 0.2 = 0.5 & ; x = 0, a \\ (0.5)(0.1) + 0.2 = 0.7 & ; x = b, c \end{cases}$$

which satisfies $\mu_{(0.5)(0.2)}^{MT} = \alpha\mu(x) + \beta; \forall x \in X$ is fuzzy magnified-(0.5)(0.2)-translation.

Theorem 9. Let μ be a fuzzy subset of X , $\alpha \in [0, T]$ and $\beta \in [0, 1]$. A mapping $\mu_{(\alpha\beta)}^{MT} : X \rightarrow [0, 1]$ is said to be a fuzzy magnified- $\alpha\beta$ -translation of μ . Then μ is fuzzy BRK-subalgebra of X if and only if $\mu_{(\alpha\beta)}^{MT}$ is fuzzy subalgebra of X .

Proof. Let μ be a fuzzy subset of X , $\alpha \in [0, T]$ and $\beta \in [0, 1]$. A mapping $\mu_{(\alpha\beta)}^{MT} : X \rightarrow [0, 1]$ is said to be a fuzzy magnified- $\alpha\beta$ -translation of μ .

Assume μ is fuzzy BRK-subalgebra of X . Then $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$

Now,

$$\begin{aligned} \mu_{(\alpha\beta)}^{MT}(x * y) &= \alpha \cdot \mu(x * y) + \beta \\ &\geq \alpha \cdot \min\{\mu(x), \mu(y)\} + \beta \\ &= \min\{\alpha \cdot \mu(x) + \beta, \alpha \cdot \mu(y) + \beta\} \\ &= \min\{\mu_{(\alpha\beta)}^{MT}(x), \mu_{(\alpha\beta)}^{MT}(y)\} \end{aligned}$$

Hence, $\mu_{(\alpha\beta)}^{MT}$ is fuzzy subalgebra of X .

Conversely assume, $\mu_{(\alpha\beta)}^{MT}$ is fuzzy subalgebra of X . Then,

$$\begin{aligned} \alpha \cdot \mu(x * y) + \beta &= \mu_{(\alpha\beta)}^{MT}(x * y) \geq \min\{\mu_{(\alpha\beta)}^{MT}(x), \mu_{(\alpha\beta)}^{MT}(y)\} \\ &= \min\{\alpha \cdot \mu(x) + \beta, \alpha \cdot \mu(y) + \beta\} \\ &= \alpha \cdot \min\{\mu(x), \mu(y)\} + \beta \end{aligned}$$

Hence, $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, so μ is fuzzy BRK-subalgebra of X .

Theorem 10. Let μ be a fuzzy subset of X , $\alpha \in [0, T]$ and $\beta \in [0, 1]$. A mapping $\mu_{(\alpha\beta)}^{MT}: X \rightarrow [0, 1]$ is said to be a fuzzy magnified- $\alpha\beta$ -translation of μ . Then μ is fuzzy BRK-ideal of X if and only if $\mu_{(\alpha\beta)}^{MT}$ is fuzzy ideal of X .

Proof. Let μ be a fuzzy subset of X , $\alpha \in [0, T]$ and $\beta \in [0, 1]$. A mapping $\mu_{(\alpha\beta)}^{MT}: X \rightarrow [0, 1]$ is said to be a fuzzy magnified- $\alpha\beta$ -translation of μ . Assume μ is fuzzy BRK-ideal of X . Then

$$\mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}$$

Now,

$$\begin{aligned} \mu_{(\alpha\beta)}^{MT}(0 * x) &= \alpha \cdot \mu(0 * x) + \beta \\ &\geq \alpha \cdot \min\{\mu(0 * (x * y)), \mu(0 * y)\} + \beta \\ &= \min\{\alpha \cdot \mu(0 * (x * y)) + \beta, \alpha \cdot \mu(0 * y) + \beta\} \\ &= \min\{\mu_{(\alpha\beta)}^{MT}(0 * (x * y)), \mu_{(\alpha\beta)}^{MT}(0 * y)\} \end{aligned}$$

Hence, $\mu_{(\alpha\beta)}^{MT}$ is fuzzy BRK-ideal of X .

Conversely, suppose that $\mu_{(\alpha\beta)}^{MT}$ is fuzzy BRK-ideal of X .

$$\begin{aligned} \alpha \cdot \mu(0 * x) &= \mu_{(\alpha\beta)}^{MT}(0 * x) \geq \min\{\mu_{(\alpha\beta)}^{MT}(0 * (x * y)), \mu_{(\alpha\beta)}^{MT}(0 * y)\} \\ &= \min\{\alpha \cdot \mu(0 * (x * y)) + \beta, \alpha \cdot \mu(0 * y) + \beta\} \\ &= \alpha \cdot \min\{\mu(0 * (x * y)), \mu(0 * y)\} + \beta \end{aligned}$$

So, $\mu(0 * x) \geq \min\{\mu(0 * (x * y)), \mu(0 * y)\}$.

Hence, μ is fuzzy BRK-ideal of X .

6. Conclusion

In this article, we investigated the notion of fuzzy translation and fuzzy multiplication BRK-subalgebras and discussed related properties. We introduced fuzzy translation and fuzzy multiplication BRK-ideals and discussed related results. Also, we defined Fuzzy magnified- $\alpha\beta$ -translation of BRK-algebras. As an extension of above results, one could study anti fuzzy translation and fuzzy multiplication on BRK-algebras in other algebraic structures. Method implementations to fix similar concerns in machine learning, decision-making, knowledge science, cognitive science, smart decision-making, etc.

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