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# Some New Results of Fixed Point in Fuzzy Soft G-Metric Spaces

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**Abstract.** The main goal of the present paper is to study and prove some results of fixed points for mappings satisfying different conditions in fuzzy soft $G$-metric spaces.

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**Key Words and Phrases**: Fuzzy set, Fuzzy soft set, Fuzzy soft $G$-metric space, Fixed point

## 1. Introduction

Most of the real life problems have various uncertainties and that classical mathematics may not be able to model properly. Among these uncertainties, there are two types of mathematical tools for dealing with such problems: fuzzy set theory [32] and theory of soft sets due to Molodstov [19], both of which aid in the solution of problems in various fields. Maji et al. [23] introduced the fuzzy soft set, which is a hybrid of fuzzy and soft sets. Roy and Maji [22] presented some results on the use of fuzzy soft sets in decision-making problems. The applications of fuzzy soft sets have been extensively studied (see, e.g., [1, 3, 4, 6, 12, 15, 16, 18, 31]). Beaulaa and Gunaseeli defined the fuzzy soft metric space in [7], and Sayed and Alahmari [25] defined the notions of some mappings and proved several fixed point theorems in fuzzy soft metric spaces. Mustafa and Sims [20] introduced the concept of G-metric space to extend and generalize the concept of metric space. Since then, a number of authors have investigated a variety of well-known results in G-metric space (see, e.g., [5, 13, 21, 24, 26–30]). Güler et al. [11] proposed the idea of a soft G-metric space based on a soft element and found some of its properties. Then they came up with the concepts ”soft G-convergence” and ”soft continuity”. They also established that fixed points in soft G-metric spaces exist and are unique. Güler and Yildirim presented

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soft G-Cauchy sequences and soft G-complete metric spaces in [10], and Shrivastava et al. established fixed point results of mapping defined on soft G-metric spaces in [17]. Using fuzzy soft elements, Sayed et al. proposed the concept of fuzzy soft G-metric space in [2]. They also investigated on fuzzy soft continuity and convergence in fuzzy soft G-metric spaces. The main goal of the present paper is to study and prove some results of fixed points for mappings satisfying different conditions in fuzzy soft $G$-metric spaces.

## 2. Preliminaries

Basic definitions of fuzzy soft sets and fuzzy soft G-metric spaces are presented in this section.
Throughout this study, we will use the terms $X$ to refer to an initial universe, $E$ to refer to the set of all parameters for $X$, and $I = [0, 1]$ to refer to the initial universe.

**Definition 1.** *[32] The membership function $\mu_A(x)$ of a fuzzy set $A$ is a function $\mu_A : X \to [0, 1]$ So, every element $x$ in $X$ has membership degree: $\mu_A(x) \in [0, 1]$.
$A$ is completely determined by the set of tuples: $A = \{(x, \mu_A(x)) : x \in X\}$.*

An empty fuzzy set, denoted by $\tilde{0}$ is one whose membership value is $0, \forall x \in X$. In contrast to the above, when $\mu_A(x) = 1$, then the fuzzy set $A$ is the universal fuzzy set, denoted by $\tilde{1}$.

**Definition 2.** *[8] A fuzzy soft set $f_A$ over $X$ is a pair $(f, A)$, where $f$ is a mapping $f : A \to I^X$ defined by*

$$f_A(e) = \begin{cases} \tilde{0}, & if\, e \notin A; \\ otherwise, & if\, e \in A. \end{cases}$$

**Definition 3.** *[9] A fuzzy soft set $f_A$ over $X$ is said to be:*

*(a) null fuzzy soft set, denoted by $\tilde{\phi}$, if for all $e \in A, f_A(e) = \tilde{0}$,*

*(b) absolute fuzzy soft set, denoted by $\tilde{E}$, if for all $e \in A, f_A(e) = \tilde{1}$.*

The fuzzy soft real numbers were defined in [14], denoted by $\tilde{\bar{r}}, \tilde{\bar{s}}, \tilde{\bar{t}}, ...$ etc, and $\bar{\bar{r}}, \bar{\bar{s}}, \bar{\bar{t}}$ will be denoted in particular type of fuzzy soft real numbers such that $\bar{\bar{r}}(e)$ is a fuzzy number for all $e \in E$.
Let $A \subseteq E$. $\mathcal{R}(A)^*$ be a set of all nonnegative fuzzy soft real numbers and $FSC(f_A)$ denotes a collection of all fuzzy soft points of a fuzzy soft set $f_A$ over $X$.

**Definition 4.** *[2] Let $E$ be a nonempty set of parameters, $A \subseteq E$ and $\tilde{E}$ be the absolute fuzzy soft. A mapping $\tilde{G} : FSC(\tilde{E}) \times FSC(\tilde{E}) \times FSC(\tilde{E}) \to \mathcal{R}(A)^*$ is said to be a fuzzy soft G-metric on $\tilde{E}$ if $\tilde{G}$ satisfies the following conditions:
$(FS\tilde{G}_1) : \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{0}$ if $f_{e_1} = f_{e_2} = f_{e_3}$;
$(FS\tilde{G}_2) : \tilde{0} \tilde{<} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$ for all $f_{e_1}, f_{e_2} \tilde{\in} FSC(\tilde{E})$ with $f_{e_1} \neq f_{e_2}$;
$(FS\tilde{G}_3) : \tilde{G}(f_{e_1}, f_{e_1}, f_{e_2}) \tilde{\leq} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})$ for all $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$ with $f_{e_2} \neq f_{e_3}$;*

$(FS\tilde{G}_4) : \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \tilde{G}(f_{e_1}, f_{e_3}, f_{e_2}) = \tilde{G}(f_{e_2}, f_{e_3}, f_{e_1}) = ...;$
$(FS\tilde{G}_5) : \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tilde{\leq} \tilde{G}(f_{e_1}, f_e, f_e) + \tilde{G}(f_e, f_{e_2}, f_{e_3})$ *for all* $f_{e_1}, f_{e_2}, f_{e_3}, f_e \tilde{\in} FSC(\tilde{E})$.
*The fuzzy soft set* $\tilde{E}$ *with a fuzzy soft G-metric* $\tilde{G}$ *on* $\tilde{E}$ *is said to be a fuzzy soft G-metric space and is denoted by* $(\tilde{E}, \tilde{G})$.

**Definition 5.** *[2] Let* $(\tilde{E}, \tilde{G})$ *be a fuzzy soft G-metric space and* $\{f_{e_n}\}$ *a sequence of fuzzy soft elements in* $\tilde{E}$. *The sequence* $\{f_{e_n}\}$ *is said to be fuzzy soft G-convergent to* $f_e$ *in* $\tilde{E}$ *if for every* $\tilde{\epsilon} \tilde{\geq} \tilde{0}$, *chosen arbitrary, there exists a natural number* $N = N(\tilde{\epsilon})$ *such that* $\tilde{0} \tilde{\leq} \tilde{G}(f_{e_n}, f_{e_n}, f_e) \tilde{<} \tilde{\epsilon}$ *whenever* $n \geq N$ *i.e* $n \geq N \Rightarrow \{f_{e_n}\} \tilde{\in} \tilde{\tilde{B}}(f_e, \tilde{\tilde{t}}, \tilde{\epsilon})$. *We denote this by* $f_{e_n} \to f_e$ *as* $n \to \infty$ *or* $Lim_{n\to\infty}\{f_{e_n}\} = f_e$.

**Definition 6.** *[2] Let* $(\tilde{E}, \tilde{G})$ *be a fuzzy soft G-metric space and* $\{f_{e_n}\}$ *be sequence of fuzzy soft elements in* $\tilde{E}$. *Then the sequence* $\{f_{e_n}\}$ *is said to be fuzzy soft G-Cauchy if for every* $\tilde{\epsilon} \tilde{\geq} \tilde{0}$, *there exist* $\tilde{\delta} \tilde{>} \tilde{0}$ *and a positive integer* $N = N(\tilde{\epsilon})$ *such that* $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l}) \tilde{<} \tilde{\epsilon}$ *for all* $n, m, l \geq N$; *that is* $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l}) \to \tilde{0}$ *as* $n, m, l \to \infty$.

**Definition 7.** *[2] A fuzzy soft G-metric space* $(\tilde{E}, \tilde{G})$ *is said to be fuzzy soft G-complete if every fuzzy soft G-Cauchy sequence in* $(\tilde{E}, \tilde{G})$ *is fuzzy soft G-convergent in* $(\tilde{E}, \tilde{G})$.

**Definition 8.** *[2] Let* $(\tilde{E}, \tilde{G}), (\tilde{\acute{E}}, \tilde{\acute{G}})$ *be two fuzzy soft G-metric spaces. Then a function* $T : \tilde{E} \to \tilde{\acute{E}}$ *is fuzzy soft G-continuous at a fuzzy soft element* $f_e \tilde{\in} FSC(\tilde{E})$ *if and only if for every* $\tilde{\epsilon} \tilde{>} \tilde{0}$, *there exists* $\tilde{\delta} \tilde{\geq} \tilde{0}$ *such that* $f_{e_1}, f_{e_2} \tilde{\in} FSC(\tilde{E})$ *and* $\tilde{G}(f_e, f_{e_1}, f_{e_2}) \tilde{<} \tilde{\delta}$ *implies that* $\tilde{\acute{G}}(Tf_e, Tf_{e_1}, Tf_{e_2}) \tilde{<} \tilde{\epsilon}$.
*A function* $T$ *is fuzzy soft G-continuous if and only if it is fuzzy soft G-continuous at all fuzzy soft elements* $f_e \tilde{\in} FSC(\tilde{E})$

## 3. Main Results

We start our main results in this paper with the following theorem (Theorem 4.2 in [2])

**Theorem 1.** *Let* $(\tilde{E}, \tilde{G})$ *be a fuzzy soft G-complete and* $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ *be a mapping that satisfies the following condition for all* $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$,

$$\begin{aligned} \tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}) \tilde{\leq} \bar{\bar{a}} \tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}) + \bar{\bar{b}} \tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}) + \\ \bar{\bar{c}} \tilde{G}(f_{e_3}, Tf_{e_3}, Tf_{e_3}) + \bar{\bar{d}} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}), \end{aligned} \tag{1}$$

*where* $\tilde{0} \tilde{\leq} \bar{\bar{a}} + \bar{\bar{b}} + \bar{\bar{c}} + \bar{\bar{d}} \tilde{<} \tilde{1}$.
*Then* $T$ *has a unique fixed point, say* $f_e$, *and* $T$ *is a fuzzy soft G-continuous at* $f_e$.

We will show that this theorem fails to verify if the set of parameter is not finite. For this, we will provide the following two examples:

**Example 1.** *Let* $X = E = \{\frac{1}{n} : n \in \mathbb{N}\}$. *Consider the fuzzy soft G-metric space* $(\tilde{E}, \tilde{G})$, *where*

$$\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \frac{\tilde{\tilde{1}}}{3} \{|f_{e_1} - f_{e_2}| + |f_{e_2} - f_{e_3}| + |f_{e_1} - f_{e_3}|\}, \forall f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E}).$$

*Note that,* $(\tilde{E}, \tilde{G})$ *is a symmetric fuzzy soft G-metric(Proposition 3.3, [2]).*
*Now, we show that* $(\tilde{E}, \tilde{G})$ *is fuzzy soft G-complete. For this, suppose that* $\{f_{e_n}\}_{n\in\mathbb{N}}$ *is a fuzzy soft G-Cauchy sequence of fuzzy soft elements in* $(\tilde{E}, \tilde{G})$. *Take the fuzzy soft real number* $\tilde{\epsilon}$ *such that* $\tilde{\epsilon}(\lambda) = \lambda, \forall \lambda \in E$, *that is* $\tilde{\epsilon}(\frac{1}{h}) = \frac{1}{h}, \forall h \in \mathbb{N}$. *Then,* $\exists k \in \mathbb{N}$ *such that* $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l}) \tilde{<} \tilde{\epsilon}, \forall n, m, l \geq k$, *which implies to* $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l})(\frac{1}{h}) < \tilde{\epsilon}(\frac{1}{h}), \forall h \in \mathbb{N}$.
*Hence*

$$\frac{\tilde{\tilde{1}}}{3} (|f_{e_n} - f_{e_m}| + |f_{e_m} - f_{e_l}| + |f_{e_n} - f_{e_l}|) (\frac{1}{h}) < \frac{1}{h'}, \forall n, m, l \geq k, h \in \mathbb{N}$$

*Which implies to*

$$\frac{\tilde{\tilde{1}}}{3} (|f_{e_n} - f_{e_m}| + |f_{e_m} - f_{e_l}| + |f_{e_n} - f_{e_l}|) \leq \frac{1}{h'}, \forall n, m, l \geq k, h \in \mathbb{N}$$

*So, we obtain that the sequence* $\{f_{e_n}\}_{n\in\mathbb{N}}$ *is eventually constant, and hence fuzzy soft G-convergent. Hence* $(\tilde{E}, \tilde{G})$ *is fuzzy soft G-complete.*

*Now suppose* $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ *is defined as:*

$$T(f_e) = \frac{\tilde{\tilde{1}}}{8}(f_e), \forall f_e \in FSC(\tilde{E})$$

*Note that* $T$ *satisfies the condition (1). For any* $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$ *and* $\eta \in E$, *we have*

$$\begin{aligned}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3})(\eta) &= \left(\frac{\tilde{\tilde{1}}}{3} \left\{\frac{\tilde{\tilde{1}}}{8}|f_{e_1} - f_{e_2}| + \frac{\tilde{\tilde{1}}}{8}|f_{e_2} - f_{e_3}| + \frac{\tilde{\tilde{1}}}{8}|f_{e_1} - f_{e_3}|\right\}\right)(\eta) \\ &= \frac{\tilde{\tilde{1}}}{24} \{|f_{e_1} - f_{e_2}| + |f_{e_2} - f_{e_3}| + |f_{e_1} - f_{e_3}|\} \\ &= \frac{\tilde{\tilde{1}}}{8} \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})(\eta) \\ &\tilde{\leq} \frac{\tilde{\tilde{1}}}{8} \left\{\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}) + \tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}) + \tilde{G}(f_{e_3}, Tf_{e_3}, Tf_{e_3})\right\}(\eta).\end{aligned}$$

*Hence, we obtain*

$$\begin{aligned}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}) \tilde{\leq} & \frac{\tilde{\tilde{1}}}{8}\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}) + \frac{\tilde{\tilde{1}}}{8}\tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}) + \frac{\tilde{\tilde{1}}}{8}\tilde{G}(f_{e_3}, Tf_{e_3}, Tf_{e_3}) + \\ & \frac{\tilde{\tilde{1}}}{8}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}).\end{aligned}$$

*All conditions of Theorem 1 are satisfied but* $T$ *is a fixed point free map.*

**Example 2.** *Let* $X = E = \{\frac{1}{n} : n \in \mathbb{N}\}$. *Consider the fuzzy soft G-metric space* $(\tilde{E}, \tilde{G})$, *where the fuzzy soft G-metric is defined as:*

$$\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) = \begin{cases} f_{e_1} + f_{e_2} + f_{e_3} + \tilde{1}, & \text{if } f_{e_1} \neq f_{e_2} \neq f_{e_3} \neq \tilde{0}; \\ f_{e_1} + f_{e_3} + \tilde{1}, & \text{if } f_{e_1} = f_{e_2} \neq f_{e_3} \neq \tilde{0}; \\ f_{e_2} + f_{e_3} + \tilde{\tilde{2}}, & \text{if } f_{e_1} = \tilde{0}, f_{e_2} \neq f_{e_3} \neq \tilde{0}; \\ f_{e_2} + \tilde{\tilde{3}}, & \text{if } f_{e_1} = \tilde{0}, f_{e_2} = f_{e_3} \neq \tilde{0}; \\ f_{e_3} + \tilde{\tilde{2}}, & \text{if } f_{e_1} = f_{e_2} = \tilde{0}, f_{e_3} \neq \tilde{0}; \\ \tilde{0}, & \text{if } f_{e_1} = f_{e_2} = f_{e_3}, \end{cases}$$

*and extend the definition by symmetry in its arguments.*
*It is easy to show that* $(\tilde{E}, \tilde{G})$ *is a fuzzy soft G-metric which is not symmetric.*
*Now, we show that* $(\tilde{E}, \tilde{G})$ *is fuzzy soft G-complete. For this, suppose that* $\{f_{e_n}\}_{n\in\mathbb{N}}$ *is a fuzzy soft G-Cauchy sequence of fuzzy soft elements in* $(\tilde{E}, \tilde{G})$.
*Take the fuzzy soft real number* $\tilde{\epsilon}$ *such that* $\tilde{\epsilon}(\lambda) = \lambda, \forall \lambda \in E$, *that is* $\tilde{\epsilon}(\frac{1}{h}) = \frac{1}{h}, \forall h \in \mathbb{N}$.
*Then,* $\exists k \in \mathbb{N}$ *such that* $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l}) \tilde{<} \tilde{\epsilon}, \forall n, m, l \geq k$, *Which implies to*
$\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l})(\frac{1}{h}) \tilde{<} \tilde{\epsilon}(\frac{1}{h}), \forall h \in \mathbb{N}$, *that is* $\tilde{G}(f_{e_n}, f_{e_m}, f_{e_l})(\frac{1}{h}) \tilde{<} \frac{1}{h'}, \forall n, m, l \geq k, \forall h \in \mathbb{N}$.
*This is possible only if the sequence* $\{f_{e_n}\}_{n\in\mathbb{N}}$ *is constant, and therefore it is fuzzy soft G-convergent. We conclude that* $(\tilde{E}, \tilde{G})$ *is fuzzy soft G-complete.*

*Now suppose* $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ *is defined as:*

$$T(f_e) = \tilde{\bar{\frac{1}{8}}}(f_e), \forall f_e \in FSC(\tilde{E})$$

*Note that* $T$ *satisfies the condition (1). For any* $f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$ *and* $\eta \in E$, *we have*

$$\begin{aligned}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3})(\eta) &= \tilde{\bar{\tfrac{1}{8}}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})(\eta) \\ &\tilde{\leq} \tilde{\bar{\tfrac{1}{8}}}\{\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}) + \tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}) + \tilde{G}(f_{e_3}, Tf_{e_3}, Tf_{e_3}) + \\ &\quad \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})\}(\eta).\end{aligned}$$

*Hence, we obtain*

$$\begin{aligned}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}) \tilde{\leq} &\tilde{\bar{\tfrac{1}{8}}}\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}) + \tilde{\bar{\tfrac{1}{8}}}\tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}) + \tilde{\bar{\tfrac{1}{8}}}\tilde{G}(f_{e_3}, Tf_{e_3}, Tf_{e_3}) + \\ &\tilde{\bar{\tfrac{1}{8}}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}).\end{aligned}$$

*All conditions of Theorem 1 are satisfied but* $T$ *can be seen it has no fixed point.*

Next, we see that keeping the set of parameters finite, one can obtain the following new results.

**Theorem 2.** *Suppose* $(\tilde{E}, \tilde{G})$ *is a fuzzy soft G-metric space and* $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ *is a mapping satisfying the following condition:*

$$\bar{\bar{a}}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}) \tilde{\leq} \bar{\bar{b}} \left\{ \begin{array}{l} \tilde{G}(f_{e_1}, Tf_{e_2}, Tf_{e_2}) \\ +\tilde{G}(f_{e_1}, Tf_{e_3}, Tf_{e_3}) \\ +\tilde{G}(f_{e_3}, Tf_{e_1}, Tf_{e_1}) \end{array} \right\} + \bar{\bar{c}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tag{2}$$

$\forall f_{e_1}, f_{e_2}, f_{e_3} \tilde{\in} FSC(\tilde{E})$ *and* $\tilde{0} \tilde{\leq} \bar{\bar{a}}, \bar{\bar{b}}, \bar{\bar{c}} \tilde{<} \tilde{1}$ *with* $\tilde{\bar{3}}\bar{\bar{b}} + \bar{\bar{c}} \tilde{<} \bar{\bar{a}}$.
*Then* $T$ *has an unique fixed point, say* $f_e$, *and at* $f_e$, $T$ *is fuzzy soft G-continuous.*

*Proof.* Assume that $f_{e_0} \tilde{\in} FSC(\tilde{E})$ is an arbitrary fuzzy soft element and define the sequence $\{g_{e_n}\}_{n\in\mathbb{N}}$ as follows: $Tg_{e_0} = g_{e_1}, Tg_{e_1} = g_{e_2}, Tg_{e_2} = g_{e_3}, ..., Tg_{e_n} = g_{e_{n+1}}$.
Consider that $g_{e_n} \neq g_{e_{n+1}}$.
Substituting $f_{e_1} = g_{e_n}, f_{e_2} = g_{e_{n+1}}$ and $f_{e_3} = g_{e_{n+1}}$ in (2), we obtain

$$\bar{\bar{a}}\tilde{G}(Tg_{e_n}, Tg_{e_{n+1}}, Tg_{e_{n+1}}) \tilde{\leq} \bar{\bar{b}} \left\{ \begin{array}{l} \tilde{G}(g_{e_n}, Tg_{e_{n+1}}, Tg_{e_{n+1}}) \\ +\tilde{G}(g_{e_{n+1}}, Tg_{e_{n+1}}, Tg_{e_{n+1}}) \\ +\tilde{G}(g_{e_{n+1}}, Tg_{e_n}, Tg_{e_n}) \end{array} \right\} + \bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{b}}\left\{\begin{array}{l}\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\\+\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\\+\tilde{G}(g_{e_{n+1}},g_{e_{n+1}},g_{e_{n+1}})\end{array}\right\}+\bar{\bar{c}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\frac{\bar{\bar{b}}+\bar{\bar{c}}}{(\bar{\bar{a}}-\tilde{2}\bar{\bar{b}})}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{k}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\text{ where }\bar{\bar{k}}=\frac{\bar{\bar{b}}+\bar{\bar{c}}}{(\bar{\bar{a}}-\tilde{2}\bar{\bar{b}})}\tilde{<}\tilde{1}.$$

On continuing this process $(n+1)$ times; we obtain

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{k}}^{(n+1)}\tilde{G}(g_{e_0},g_{e_1},g_{e_1}).$$

Similarly, we will conclude that

$$\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\tilde{\leq}\bar{\bar{k}}^{n}\tilde{G}(g_{e_0},g_{e_1},g_{e_1}).$$

Next, we show that $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Then for all $n,m\in\mathbb{N}$, $n<m$, we have

$$\begin{aligned}\tilde{G}(g_{e_n},g_{e_m},g_{e_m})&\tilde{\leq}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+...+\tilde{G}(g_{e_{m-1}},g_{e_m},g_{e_m})\\&\tilde{\leq}(\bar{\bar{k}}^{n}+\bar{\bar{k}}^{(n+1)}+...+\bar{\bar{k}}^{m}\tilde{G}(g_{e_0},g_{e_1},g_{e_1})\\&\tilde{\leq}\frac{\bar{\bar{k}}^{n}}{\tilde{1}-\bar{\bar{k}}}\tilde{G}(g_{e_0},g_{e_1},g_{e_1}).\end{aligned}$$

Hence, $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Since $(\tilde{E},\tilde{G})$ is fuzzy soft $G$-complete, there exists $f_e\tilde{\in}FSC(\tilde{E})$ such that $\{g_{e_n}\}_{n\in\mathbb{N}}$ fuzzy soft G-converges to $f_e$.
Next, we will show that $f_e$ is a fixed point of $T$.
For this, we take $f_{e_1}=g_{e_n}$ and $f_{e_2}=f_{e_3}=f_e$ in (3.1), then

$$\bar{\bar{a}}\tilde{G}(Tg_{e_n},Tf_e,Tf_e)\tilde{\leq}\bar{\bar{b}}\left\{\begin{array}{l}\tilde{G}(g_{e_n},Tf_e,Tf_e)\\+\tilde{G}(f_e,Tf_e,Tf_e)\\+\tilde{G}(f_e,Tg_{e_n},Tg_{e_n})\end{array}\right\}+\bar{\bar{c}}\tilde{G}(g_{e_n},f_e,f_e)$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e,Tf_e,Tf_e)\tilde{\leq}\bar{\bar{b}}\left\{\begin{array}{l}\tilde{G}(f_e,Tf_e,Tf_e)\\+\tilde{G}(f_e,Tf_e,Tf_e)\\+\tilde{G}(f_e,f_e,f_e)\end{array}\right\}+\bar{\bar{c}}\tilde{G}(f_e,f_e,f_e).$$

$$\Rightarrow\ \tilde{G}(f_e,Tf_e,Tf_e)\tilde{\leq}\frac{\tilde{2}\bar{\bar{b}}}{\bar{\bar{a}}}\tilde{G}((f_e,Tf_e,Tf_e).$$

This is a contradiction, so $Tf_e=f_e$ i.e. $f_e$ is a fixed point of $T$
Now, to prove uniqueness, assume that $f_e$ and $g_e$ are two fixed points of $T$. Then by inequality (2), we have

$$\bar{\bar{a}}\tilde{G}(Tf_e,Tg_e,Tg_e)\tilde{\leq}\bar{\bar{b}}\left\{\begin{array}{l}\tilde{G}(f_e,Tg_e,Tg_e)\\+\tilde{G}(g_e,Tg_e,Tg_e)\\+\tilde{G}(g_e,Tf_e,Tf_e)\end{array}\right\}+\bar{\bar{c}}\tilde{G}(f_e,g_e,g_e)$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e,g_e,g_e)\tilde{\leq}\bar{\bar{b}}\left\{\begin{array}{l}\tilde{G}(f_e,g_e,g_e)\\+\tilde{G}(g_e,g_e,g_e)\\+\tilde{G}(g_e,f_e,f_e)\end{array}\right\}+\bar{\bar{c}}\tilde{G}(f_e,g_e,g_e).$$

So, we deduct that $(\bar{\bar{a}}-\bar{\bar{b}}-\bar{\bar{c}})\tilde{G}(f_e,g_e,g_e)\tilde{\leq}\bar{\bar{b}}\tilde{G}(g_e,f_e,f_e)$.
$\Rightarrow\ \tilde{G}(f_e,g_e,g_e)\tilde{\leq}\frac{\bar{\bar{b}}}{(\bar{\bar{a}}-\bar{\bar{b}}-\bar{\bar{c}})}\tilde{G}(g_e,f_e,f_e)$ and by use of the same argument, we will find

$$\tilde{G}(g_e,f_e,f_e)\tilde{\leq}\frac{\bar{\bar{b}}}{(\bar{\bar{a}}-\bar{\bar{b}}-\bar{\bar{c}})}\tilde{G}(f_e,g_e,g_e).$$

Therefore, we get $\tilde{G}(f_e, g_e, g_e)\tilde{\leq}\left(\frac{\bar{\bar{b}}}{(\bar{\bar{a}}-\bar{\bar{b}}-\bar{\bar{c}})}\right)^2\tilde{G}(f_e, g_e, g_e)$, since $\tilde{\bar{3}}\bar{\bar{b}}+\bar{\bar{c}}\tilde{<}\bar{\bar{a}}$, this is a contradiction implies that $f_e = g_e$.
To show that $T$ is fuzzy soft G-continuous at $f_e$.
Let $\{h_{e_n}\}_{n\in\mathbb{N}}$ be a sequence of fuzzy soft elements in $\tilde{E}$ such that $\{h_{e_n}\}_{n\in\mathbb{N}} \to f_e$, then we can deduce that using inequality (2), we have

$$\bar{\bar{a}}\tilde{G}(Tf_e, Th_{e_n}, Th_{e_n})\tilde{\leq}\bar{\bar{b}}\left\{\begin{array}{l}\tilde{G}(f_e, Th_{e_n}, Th_{e_n})\\ +\tilde{G}(h_{e_n}, Th_{e_n}, Th_{e_n})\\ +\tilde{G}(h_{e_n}, Tf_e, Tf_e)\end{array}\right\}+\bar{\bar{c}}\tilde{G}(f_e, h_{e_n}, h_{e_n})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e, Th_{e_n}, Th_{e_n})\tilde{\leq}\bar{\bar{b}}\left\{\begin{array}{l}\tilde{G}(f_e, Th_{e_n}, Th_{e_n})\\ +\tilde{G}(h_{e_n}, Th_{e_n}, Th_{e_n})\\ +\tilde{G}(h_{e_n}, f_e, f_e)\end{array}\right\}+\bar{\bar{c}}\tilde{G}(f_e, h_{e_n}, h_{e_n})$$

Taking the limit as $n\to\infty$ from which, we see that
$(\bar{a}-\tilde{\bar{2}}\bar{\bar{b}})\tilde{G}(f_e, Th_{e_n}, Th_{e_n})\to\tilde{0}$ and so, by proposition (4.1) in [2] we have that the sequence $\{Th_{e_n}\}_{n\in\mathbb{N}}$ is fuzzy soft G-convergent to $Tf_e = f_e$, therefore proposition (4.4) in [2] implies that $T$ is fuzzy soft G-continuous at $f_e$.

**Theorem 3.** *Suppose $(\tilde{E},\tilde{G})$ is a fuzzy soft G-metric space and $T:(\tilde{E},\tilde{G})\to(\tilde{E},\tilde{G})$ is a mapping satisfying the following condition:*

$$\bar{\bar{a}}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3})+\bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}),\\ \tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}),\\ \tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}),\\ \tilde{G}(f_{e_3}, Tf_{e_3}, Tf_{e_3})\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \qquad (3)$$

*$\forall f_{e_1}, f_{e_2}, f_{e_3}\tilde{\in}FSC(\tilde{E})$ and $\tilde{0}\tilde{\leq}\bar{\bar{a}},\bar{\bar{b}},\bar{\bar{c}}\tilde{<}\tilde{1}$ with $\bar{\bar{c}}-\bar{\bar{b}}\tilde{<}\bar{\bar{a}}$.*
*Then $T$ has an unique fixed point, say $f_e$, and at $f_e$, $T$ is fuzzy soft G-continuous.*

*Proof.* Assume that $f_{e_0}\tilde{\in}FSC(\tilde{E})$ is an arbitrary fuzzy soft element and define the sequence $\{g_{e_n}\}_{n\in\mathbb{N}}$ as follows: $Tg_{e_0} = g_{e_1}, Tg_{e_1} = g_{e_2}, Tg_{e_2} = g_{e_3}, ..., Tg_{e_n} = g_{e_{n+1}}$.
Consider that $g_{e_n}\neq g_{e_{n+1}}$.
Substituting $f_{e_1} = g_{e_n}$, $f_{e_2} = g_{e_{n+1}}$ and $f_{e_3} = g_{e_{n+1}}$ in (3), we obtain

$$\bar{\bar{a}}\tilde{G}(Tg_{e_n}, Tg_{e_{n+1}}, Tg_{e_{n+1}})+\bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(Tg_{e_n}, Tg_{e_{n+1}}, Tg_{e_{n+1}}),\\ \tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}),\\ \tilde{G}(g_{e_{n+1}}, Tg_{e_{n+1}}, Tg_{e_{n+1}}),\\ \tilde{G}(g_{e_{n+1}}, Tg_{e_{n+1}}, Tg_{e_{n+1}})\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}g_{e_{n+2}}),$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})+\bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}),\\ \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}),\\ \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, Tg_{e_{n+2}}),\\ \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})+\bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}),\\ \tilde{G}(g_{e_n}, g_{e_{n+1}}, Tg_{e_{n+1}})\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

We now have two cases:

Case (1): If $\min\left\{\begin{array}{l}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, , g_{e_{n+2}}),\\ \tilde{G}(g_{e_n}, g_{e_{n+1}}, Tg_{e_{n+1}})\end{array}\right\} = \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})$, then

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) + \bar{\bar{b}}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

$\Rightarrow\ \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})\tilde{\leq}\frac{\bar{\bar{c}}}{\bar{\bar{a}}+\bar{\bar{b}}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$

Case (2): If $\min\left\{\begin{array}{l}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, , g_{e_{n+2}}),\\ \tilde{G}(g_{e_n}, g_{e_{n+1}}, Tg_{e_{n+1}})\end{array}\right\} = \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$, then

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) + \bar{\bar{b}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

$\Rightarrow\ \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})\tilde{\leq}\frac{\bar{\bar{c}}-\bar{\bar{b}}}{\bar{\bar{a}}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$

From (1) and (2), we have

$$\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})\tilde{\leq}\bar{\bar{k}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

Similarly, we will conclude that

$$\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})\tilde{\leq}\bar{\bar{k}}\tilde{G}(g_{e_{n-1}}, g_{e_n}, g_{e_n}) \text{ and so}$$
$$\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})\tilde{\leq}\bar{\bar{k}}^n\tilde{G}(g_{e_0}, g_{e_1}, g_{e_1})$$

Next, we show that $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.

Then for all $n, m \in \mathbb{N}$, $n < m$, we have

$$\begin{aligned}\bar{\bar{a}}\tilde{G}(g_{e_n}, g_{e_m}, g_{e_m})&\tilde{\leq}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}) + \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) + ... + \tilde{G}(g_{e_{m-1}}, g_{e_m}, g_{e_m})\\ &\tilde{\leq}(\bar{\bar{k}}^n + \bar{\bar{k}}^{(n+1)} + ... + \bar{\bar{k}}^{(m-1)})\tilde{G}(g_{e_0}, g_{e_1}, g_{e_1})\\ &\tilde{\leq}\frac{\bar{\bar{k}}^n}{\bar{\bar{1}}-\bar{\bar{k}}}\tilde{G}(g_{e_0}, g_{e_1}, g_{e_2}).\end{aligned}$$

Hence, $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.

Since $(\tilde{E}, \tilde{G})$ is fuzzy soft $G$-complete, there exists $f_e\tilde{\in}FSC(\tilde{E})$ such that $\{g_{e_n}\}_{n\in\mathbb{N}}$ fuzzy soft G-converges to $f_e$.

Next, we will show that $f_e$ is a fixed point of $T$.

For this, we take $f_{e_1} = g_{e_n}$ and $f_{e_2} = f_{e_3} = f_e$ in (3), then

$$\bar{\bar{a}}\tilde{G}(Tg_{e_n}, Tf_e, Tf_e) + \bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(Tg_{e_n}, Tf_e, Tf_e),\\ \tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}),\\ \tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n}, f_e, f_e)$$

As $n \to \infty$, we have

$$\bar{\bar{a}}\tilde{G}(f_e, Tf_e, Tf_e) + \bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(f_e, f_e, f_e)$$

$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e, Tf_e, Tf_e)\tilde{\leq}\tilde{0}$, since $\tilde{0}\tilde{\leq}\bar{\bar{a}}\tilde{<}\tilde{1}$

This is a contradiction, so $Tf_e = f_e$ i.e. $f_e$ is a fixed point of $T$.

Now, to prove uniqueness, assume that $f_e$ and $g_e$ are two fixed points of $T$. Then by inequality (3), we have

$$\bar{\bar{a}}\tilde{G}(Tf_e, Tg_e, Tg_e) + \bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(Tf_e, Tg_e, Tg_e),\\ \tilde{G}(f_e, Tf_e, Tg_e),\\ \tilde{G}(g_e, Tg_e, Tg_e),\\ \tilde{G}(g_e, Tg_e, Tg_e)\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(f_e, g_e, g_e)$$

$\Rightarrow\ \tilde{G}(f_e, g_e, g_e)\tilde{\leq}\frac{\bar{\bar{c}}}{\bar{\bar{a}}}\tilde{G}(f_e, g_e, g_e)$, this is a contradiction implies that $f_e = g_e$. To show that $T$ is fuzzy soft G-continuous at $f_e$.
Let $\{h_{e_n}\}_{n\in\mathbb{N}}$ be a sequence of fuzzy soft elements in $\tilde{E}$ such that $\{h_{e_n}\}_{n\in\mathbb{N}} \to f_e$, then we can deduce that using inequality (3), we have

$$\bar{\bar{a}}\tilde{G}(Tf_e, Th_{e_n}, Th_{e_n}) + \bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(Tf_e, Th_{e_n}, Th_{e_n}),\\ \tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(h_{e_n}, Th_{e_n}, Th_{e_n}),\\ \tilde{G}(h_{e_n}, Th_{e_n}, Th_{e_n})\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(f_e, h_{e_n}, h_{e_n})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e, Th_{e_n}, Th_{e_n}) + \bar{\bar{b}}\ \ min\left\{\begin{array}{l}\tilde{G}(f_e, Th_{e_n}, Th_{e_n}),\\ \tilde{G}(f_e, f_e, f_e),\\ \tilde{G}(h_{e_n}, Th_{e_n}, Th_{e_n}),\\ \tilde{G}(h_{e_n}, Th_{e_n}, Th_{e_n})\end{array}\right\}\tilde{\leq}\bar{\bar{c}}\tilde{G}(f_e, h_{e_n}, h_{e_n})$$

Taking the limit as $n \to \infty$ from which, we see that
$\tilde{G}(f_e, Th_{e_n}, Th_{e_n}) \to \tilde{0}$ and so, by proposition (4.1) in [2] we have that the sequence $\{Th_{e_n}\}_{n\in\mathbb{N}}$ is fuzzy soft G-convergent to $Tf_e = f_e$, therefore proposition (4.4) in [2] implies that $T$ is fuzzy soft G-continuous at $f_e$.

**Theorem 4.** *Suppose $(\tilde{E}, \tilde{G})$ is a fuzzy soft G-metric space and $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ is a mapping satisfying the following condition:*

$$\bar{\bar{a}}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}) + \bar{\bar{b}}\left[\frac{min\left\{\begin{array}{l}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_2})\cdot\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}),\\ \tilde{G}(f_{e_1}, f_{e_2}, Tf_{e_3})\cdot\tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2})\end{array}\right\}}{min\left\{\begin{array}{l}\tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3})\cdot\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}),\\ \tilde{G}(f_{e_1}, f_{e_2}, f_{e_3})\cdot\tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2})\end{array}\right\}}\right] \tilde{\leq}\bar{\bar{c}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_2}) \quad (4)$$

*$\forall f_{e_1}, f_{e_2}, f_{e_3}\tilde{\in}FSC(\tilde{E})$ and $\tilde{0}\tilde{\leq}\bar{\bar{a}}, \bar{\bar{b}}, \bar{\bar{c}}\tilde{<}\tilde{1}$ with $\bar{\bar{c}} - \bar{\bar{b}}\tilde{<}\bar{\bar{a}}$.*
*Then $T$ has an unique fixed point, say $f_e$, and at $f_e$, $T$ is fuzzy soft G-continuous.*

*Proof.* Assume that $f_{e_0}\tilde{\in}FSC(\tilde{E})$ is an arbitrary fuzzy soft element and define the sequence $\{g_{e_n}\}_{n\in\mathbb{N}}$ as follows: $Tg_{e_0} = g_{e_1}, Tg_{e_1} = g_{e_2}, Tg_{e_2} = g_{e_3}, ..., Tg_{e_n} = g_{e_{n+1}}$.
Consider that $g_{e_n} \neq g_{e_{n+1}}$.
Substituting $f_{e_1} = g_{e_n}, f_{e_2} = g_{e_{n+1}}$ and $f_{e_3} = g_{e_{n+1}}$ in (4), we obtain

$$\bar{\bar{a}}\tilde{G}(Tg_{e_n}, Tg_{e_{n+1}}, Tg_{e_{n+1}}) + \bar{\bar{b}}\left[\frac{min\left\{\begin{array}{l}\tilde{G}(Tg_{e_n}, Tg_{e_{n+1}}, Tg_{e_{n+1}})\cdot\tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}),\\ \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})\cdot\tilde{G}(g_{e_{n+1}}, Tg_{e_{n+1}}, Tg_{e_{n+1}})\end{array}\right\}}{min\left\{\begin{array}{l}\tilde{G}(Tg_{e_n}, Tg_{e_{n+1}}, Tg_{e_{n+1}}), \tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}),\\ \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}), \tilde{G}(g_{e_{n+1}}, Tg_{e_{n+1}}, Tg_{e_{n+1}})\end{array}\right\}}\right] \tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+\bar{\bar{b}}\left[\frac{min\left\{\begin{array}{c}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\cdot\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\cdot\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}}{min\left\{\begin{array}{c}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}}),\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}}\right] \tag{5}$$
$$\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+\bar{\bar{b}}\left[\frac{min\left\{\begin{array}{c}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\cdot\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\cdot\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}}{min\left\{\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}}),\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\right\}}\right]$$
$$\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

We now have four cases:

Case (1): If $\left[\frac{min\left\{\begin{array}{c}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\cdot\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\cdot\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}}{min\left\{\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}}),\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\right\}}\right]=$

$$\left[\frac{\left\{\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\cdot\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\right\}}{\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})}\right]=\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})$$

Then, we have

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+\bar{\bar{b}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$
$$\Rightarrow\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\frac{\bar{\bar{c}}}{\bar{\bar{a}}+\bar{\bar{b}}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

Case (2): If $\left[\frac{min\left\{\begin{array}{c}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\cdot\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\cdot\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}}{min\left\{\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}}),\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\right\}}\right]=$

$$\left[\frac{\left\{\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\cdot\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\right\}}{\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})}\right]=\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

Then, we have

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+\bar{\bar{b}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$
$$\Rightarrow\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\frac{\bar{\bar{c}}-\bar{\bar{b}}}{\bar{\bar{a}}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

Case (3): If $\left[\frac{min\left\{\begin{array}{c}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\cdot\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\cdot\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}}{min\left\{\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}}),\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\right\}}\right]=$

$$\left[\frac{\left\{\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\cdot\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\right\}}{\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})}\right]=\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})$$

Then, we have

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+\bar{\bar{b}}\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{c}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$
$$\Rightarrow\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\frac{\bar{\bar{c}}}{\bar{\bar{a}}+\bar{\bar{b}}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

Case (4): If $\left[\dfrac{min\left\{\begin{array}{l}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) \cdot \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}),\\ \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}) \cdot \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})\end{array}\right\}}{min\left\{\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}), \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})\right\}}\right] =$

$\left[\dfrac{\left\{\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}) \cdot \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})\right\}}{\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}})}\right] = \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$

Then, we have

$$\bar{\bar{a}}\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) + \bar{\bar{b}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}) \tilde{\leq} \bar{\bar{c}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

$\Rightarrow\ \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) \tilde{\leq} \frac{\bar{\bar{c}}-\bar{\bar{b}}}{\bar{\bar{a}}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$

From Cases (1), (2), (3) and (4), we have

$$\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) \tilde{\leq} \bar{\bar{k}}\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

On continuing this process $(n+1)$ times, we have

$$\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) \tilde{\leq} \bar{\bar{k}}^{n+1}\tilde{G}(g_{e_0}, g_{e_1}, g_{e_1})$$

Similarly, we will conclude that

$$\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}) \tilde{\leq} \bar{\bar{k}}^{n}\tilde{G}(g_{e_0}, g_{e_1}, g_{e_1})$$

Next, we show that $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Then for all $n, m \in \mathbb{N}$, $n < m$, we have

$$\begin{aligned}\bar{\bar{a}}\tilde{G}(g_{e_n}, g_{e_m}, g_{e_m}) &\tilde{\leq} \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}) + \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) + ... + \tilde{G}(g_{e_{m-1}}, g_{e_m}, g_{e_m})\\ &\tilde{\leq} (\bar{\bar{k}}^n + \bar{\bar{k}}^{(n+1)} + ... + \bar{\bar{k}}^{(m-1)})\tilde{G}(g_{e_0}, g_{e_1}, g_{e_1})\\ &\tilde{\leq} \frac{\bar{\bar{k}}^n}{\tilde{1}-\bar{\bar{k}}}\tilde{G}(g_{e_0}, g_{e_1}, g_{e_2}).\end{aligned}$$

Hence, $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Since $(\tilde{E}, \tilde{G})$ is fuzzy soft $G$-complete, there exists $f_e \tilde{\in} FSC(\tilde{E})$ such that $\{g_{e_n}\}_{n\in\mathbb{N}}$ fuzzy soft G-converges to $f_e$.
Next, we will show that $f_e$ is a fixed point of $T$.
For this, we take $f_{e_1} = g_{e_n}$ and $f_{e_2} = f_{e_3} = f_e$ in the inequality (4), then from the inequality (5), we have

$$\bar{\bar{a}}\tilde{G}(Tg_{e_n}, Tf_e, Tf_e) + \bar{\bar{b}}\left[\dfrac{min\left\{\begin{array}{l}\tilde{G}(Tg_{e_n}, Tf_e, Tf_e) \cdot \tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}),\\ \tilde{G}(g_{e_n}, f_e, f_e) \cdot \tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}}{min\left\{\begin{array}{l}\tilde{G}(Tg_{e_n}, Tf_e, Tf_e), \tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}),\\ \tilde{G}(g_{e_n}, f_e, f_e), \tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}}\right] \tilde{\leq} \bar{\bar{c}}\tilde{G}(g_{e_n}, f_e, f_e)$$

As $n \to \infty$, we have

$$\bar{\bar{a}}\tilde{G}(f_e, Tf_e, Tf_e) + \bar{\bar{b}}\left[\dfrac{min\left\{\begin{array}{l}\tilde{G}(f_e, Tf_e, Tf_e) \cdot \tilde{G}(f_e, f_e, f_e),\\ \tilde{G}(f_e, f_e, f_e) \cdot \tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}}{min\left\{\begin{array}{l}\tilde{G}(f_e, Tf_e, Tf_e), \tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, f_e, f_e), \tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}}\right] \tilde{\leq} \bar{\bar{c}}\tilde{G}(f_e, f_e, f_e)$$

$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e, Tf_e, Tf_e) \tilde{\leq} \tilde{0}$, since $\tilde{0} \tilde{\leq} \bar{\bar{a}} \tilde{<} \tilde{1}$.
This is a contradiction, so $Tf_e = f_e$ i.e. $f_e$ is a fixed point of $T$.
Now, to prove uniqueness, assume that $f_e$ and $g_e$ are two fixed points of $T$. Then by inequality (5), we have

$$\bar{\bar{a}}\tilde{G}(Tf_e, Tg_e, Tg_e) + \bar{\bar{b}}\left[\frac{min\left\{\begin{array}{c}\tilde{G}(Tf_e, Tg_e, Tg_e)\cdot\tilde{G}(g_e, Tg_e, Tg_e),\\ \tilde{G}(f_e, g_e, g_e)\cdot\tilde{G}(g_e, Tg_e, Tg_e)\end{array}\right\}}{min\left\{\begin{array}{c}\tilde{G}(Tf_e, Tg_e, Tg_e)\cdot\tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, g_e, g_e)\cdot\tilde{G}(g_e, Tg_e, Tg_e)\end{array}\right\}}\right]\tilde{\le}\bar{\bar{c}}\tilde{G}(f_e, g_e, g_e)$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e, g_e, g_e) + \bar{\bar{b}}\left[\frac{min\left\{\begin{array}{c}\tilde{G}(f_e, g_e, g_e)\cdot\tilde{G}(g_e, g_e, g_e),\\ \tilde{G}(f_e, g_e, g_e)\cdot\tilde{G}(g_e, g_e, g_e)\end{array}\right\}}{min\left\{\begin{array}{c}\tilde{G}(f_e, g_e, g_e)\cdot\tilde{G}(f_e, f_e, f_e),\\ \tilde{G}(f_e, g_e, g_e)\cdot\tilde{G}(g_e, g_e, g_e)\end{array}\right\}}\right]\tilde{\le}\bar{\bar{c}}\tilde{G}(f_e, g_e, g_e)$$

$\Rightarrow\ \tilde{G}(f_e, g_e, g_e)\tilde{\le}\frac{\bar{\bar{c}}}{\bar{\bar{a}}}\tilde{G}(f_e, g_e, g_e)$, since $\tilde{0}\tilde{\le}\frac{\bar{\bar{c}}}{\bar{\bar{a}}}\tilde{<}\tilde{1}$.

This is a contradiction, implies that $f_e = g_e$.

To show that $T$ is fuzzy soft G-continuous at $f_e$.

Let $\{h_{e_n}\}_{n\in\mathbb{N}}$ be a sequence of fuzzy soft elements in $\tilde{E}$ such that $\{h_{e_n}\}_{n\in\mathbb{N}} \to f_e$, then we can deduce that using inequality (5), we have

$$\bar{\bar{a}}\tilde{G}(Tf_e, Th_{e_n}, Th_{e_n}) + \bar{\bar{b}}\left[\frac{min\left\{\begin{array}{c}\tilde{G}(Tf_e, Th_{e_n}, Th_{e_n})\cdot\tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, h_{e_n}, h_{e_n})\cdot\tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}}{min\left\{\begin{array}{c}\tilde{G}(Tf_e, Th_{e_n}, Th_{e_n})\cdot\tilde{G}(f_e, Tf_e, Tf_e),\\ \tilde{G}(f_e, h_{e_n}, h_{e_n})\cdot\tilde{G}(f_e, Tf_e, Tf_e)\end{array}\right\}}\right]\tilde{\le}$$
$$\bar{\bar{c}}\tilde{G}(f_e, h_{e_n}, h_{e_n})$$

$$\Rightarrow\ \bar{\bar{a}}\tilde{G}(f_e, Th_{e_n}, Th_{e_n}) + \bar{\bar{b}}\left[\frac{min\left\{\begin{array}{c}\tilde{G}(f_e, Th_{e_n}, Th_{e_n})\cdot\tilde{G}(f_e, f_e, f_e),\\ \tilde{G}(f_e, h_{e_n}, h_{e_n})\cdot\tilde{G}(f_e, f_e, f_e)\end{array}\right\}}{min\left\{\begin{array}{c}\tilde{G}(f_e, Th_{e_n}, Th_{e_n})\cdot\tilde{G}(f_e, f_e, f_e),\\ \tilde{G}(f_e, h_{e_n}, h_{e_n})\cdot\tilde{G}(f_e, f_e, f_e)\end{array}\right\}}\right]\tilde{\le}\bar{\bar{c}}\tilde{G}(f_e, h_{e_n}, h_{e_n})$$

Taking the limit as $n \to \infty$ from which, we see that $\tilde{G}(f_e, Th_{e_n}, Th_{e_n}) \to \tilde{0}$ and so, by proposition (4.1) in [2] we have that the sequence $\{Th_{e_n}\}_{n\in\mathbb{N}}$ is fuzzy soft G-convergent to $Tf_e = f_e$, therefore proposition (4.4) in [2] implies that $T$ is fuzzy soft G-continuous at $f_e$.

**Theorem 5.** *Suppose $(\tilde{E}, \tilde{G})$ is a fuzzy soft G-metric space and $T : (\tilde{E}, \tilde{G}) \to (\tilde{E}, \tilde{G})$ is a mapping satisfying the following condition:*

$$min\left\{\begin{array}{c}\left[\tilde{G}(f_{e_1}, Tf_{e_1}, Tf_{e_1}) + \tilde{G}(f_{e_1}, Tf_{e_2}, Tf_{e_2})\right],\\ \tilde{G}(Tf_{e_1}, Tf_{e_2}, Tf_{e_3}),\\ \left[\tilde{G}(f_{e_2}, Tf_{e_2}, Tf_{e_2}) + \tilde{G}(f_{e_2}, Tf_{e_1}, Tf_{e_1})\right]\end{array}\right\}\tilde{\le}\bar{\bar{a}}\tilde{G}(f_{e_1}, f_{e_2}, f_{e_3}) \tag{6}$$

*$\forall f_{e_1}, f_{e_2}, f_{e_3}\tilde{\in}FSC(\tilde{E})$ and $\tilde{0}\tilde{\le}\bar{\bar{a}}\tilde{<}\tilde{1}$.*

*Then $T$ has an unique fixed point, say $f_e$, and at $f_e$, $T$ is fuzzy soft G-continuous.*

*Proof.* Assume that $f_{e_0}\tilde{\in}FSC(\tilde{E})$ is an arbitrary fuzzy soft element and define the sequence $\{g_{e_n}\}_{n\in\mathbb{N}}$ as follows: $Tg_{e_0} = g_{e_1}, Tg_{e_1} = g_{e_2}, Tg_{e_2} = g_{e_3}, ..., Tg_{e_n} = g_{e_{n+1}}$.

Consider that $g_{e_n} \neq g_{e_{n+1}}$.

Substituting $f_{e_1} = g_{e_n}, f_{e_2} = g_{e_{n+1}}$ and $f_{e_3} = g_{e_{n+1}}$ in (6), we obtain

$$min\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},Tg_{e_n},Tg_{e_n})+\tilde{G}(g_{e_n},Tg_{e_{n+1}},Tg_{e_{n+1}})\right],\\ \tilde{G}(Tg_{e_n},Tg_{e_{n+1}},Tg_{e_{n+1}}),\\ \left[\tilde{G}(g_{e_{n+1}},Tg_{e_{n+1}},Tg_{e_{n+1}})+\tilde{G}(g_{e_{n+1}},Tg_{e_n},Tg_{e_n})\right]\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

$$\Rightarrow\ min\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}}),\\ \left[\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+\tilde{G}(g_{e_{n+1}},g_{e_{n+1}},g_{e_{n+1}})\right]\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

$$\Rightarrow\ min\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}) \qquad (7)$$

We now have two cases:
Case (1): If $\min\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}=\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})$
Then, (7) is reduced to

$$\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

Case (2): If $\min\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}=$
$\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right]$
Then, (7) is reduced to

$$\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

$$\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\left[\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})-\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\right]\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

$$\Rightarrow\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

On continuing this process $(n+1)$ times; we obtain

$$\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}^{n+1}\tilde{G}(g_{e_0},g_{e_1},g_{e_1})$$

Similarly, we will conclude that

$$\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})\tilde{\leq}\bar{\bar{a}}^{n}\tilde{G}(g_{e_0},g_{e_1},g_{e_1})$$

Next, we show that $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Then for all $n,m\in\mathbb{N}$, $n<m$, we have
$\tilde{G}(g_{e_n},g_{e_m},g_{e_m})\tilde{\leq}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+...+\tilde{G}(g_{e_{m-1}},g_{e_m},g_{e_m})$
$\tilde{\leq}(\bar{\bar{a}}^{n}+\bar{\bar{a}}^{(n+1)}+...+\bar{\bar{a}}^{m}\tilde{G}(g_{e_0},g_{e_1},g_{e_1})$
$\tilde{\leq}\frac{\bar{\bar{a}}^{n}}{1-\bar{\bar{a}}}\tilde{G}(g_{e_0},g_{e_1},g_{e_1})$.

Hence, $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Since $(\tilde{E},\tilde{G})$ is fuzzy soft $G$-complete, there exists $f_e\tilde{\in}FSC(\tilde{E})$ such that $\{g_{e_n}\}_{n\in\mathbb{N}}$ fuzzy soft G-converges to $f_e$.
Next, we will show that $f_e$ is a fixed point of $T$.
For this, we take $f_{e_1}=g_{e_n}$ and $f_{e_2}=f_{e_3}=f_e$ in (6), then

$$min\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},Tg_{e_n},Tg_{e_n})+\tilde{G}(g_{e_n},Tf_e,Tf_e)\right],\\ \tilde{G}(Tg_{e_n},Tf_e,Tf_e),\\ \left[\tilde{G}(f_e,Tf_e,Tf_e)+\tilde{G}(f_e,Tg_{e_n},Tg_{e_n})\right]\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},f_e,f_e)$$

$$\Rightarrow\quad min\left\{\begin{array}{c}\left[\tilde{G}(f_e,f_e,f_e)+\tilde{G}(f_e,Tf_e,Tf_e)\right],\\ \tilde{G}(f_e,Tf_e,Tf_e),\\ \left[\tilde{G}(f_e,Tf_e,Tf_e)+\tilde{G}(f_e,f_e,f_e)\right]\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(f_e,f_e,f_e)$$

$\Rightarrow\ \tilde{G}(f_e,Tf_e,Tf_e)\tilde{\leq}\tilde{0}$
This is a contradiction, so $Tf_e=f_e$ i.e. $f_e$ is a fixed point of $T$.
Now, to prove uniqueness, assume that $f_e$ and $g_e$ are two fixed points of $T$. Then by inequality (6), we have

$$min\left\{\begin{array}{c}\left[\tilde{G}(f_e,f_e,f_e)+\tilde{G}(f_e,g_e,g_e)\right],\\ \tilde{G}(f_e,g_e,g_e),\\ \left[\tilde{G}(g_e,g_e,g_e)+\tilde{G}(g_e,f_e,f_e)\right]\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(f_e,f_e,f_e)$$

$\Rightarrow\ \tilde{G}(f_e,g_e,g_e)\tilde{\leq}\bar{\bar{a}}\tilde{G}(f_e,g_e,g_e)$, this is a contradiction implies that $f_e=g_e$.
To show that $T$ is fuzzy soft G-continuous at $f_e$.
Let $\{h_{e_n}\}_{n\in\mathbb{N}}$ be a sequence of fuzzy soft elements in $\tilde{E}$ such that $\{h_{e_n}\}_{n\in\mathbb{N}}\to f_e$, then we can deduce that using inequality (6), we have

$$min\left\{\begin{array}{c}\left[\tilde{G}(f_e,Tf_e,Tf_e)+\tilde{G}(f_e,Th_{e_n},Th_{e_n})\right],\\ \tilde{G}(Tf_e,Th_{e_n},Th_{e_n}),\\ \left[\tilde{G}(h_{e_n},Th_{e_n},Th_{e_n})+\tilde{G}(h_{e_n},Tf_e,Tf_e)\right]\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(f_e,h_{e_n},h_{e_n})$$

$$\Rightarrow\quad min\left\{\begin{array}{c}\left[\tilde{G}(f_e,f_e,f_e)+\tilde{G}(f_e,Th_{e_n},Th_{e_n})\right],\\ \tilde{G}(f_e,Th_{e_n},Th_{e_n}),\\ \left[\tilde{G}(h_{e_n},Th_{e_n},Th_{e_n})+\tilde{G}(h_{e_n},f_e,f_e)\right]\end{array}\right\}\tilde{\leq}\bar{\bar{a}}\tilde{G}(f_e,h_{e_n},h_{e_n})$$

Taking the limit as $n\to\infty$ from which, we see that
$\tilde{G}(f_e,Th_{e_n},Th_{e_n})\to\tilde{0}$ and so, by proposition (4.1) in [2] we have that the sequence $\{Th_{e_n}\}_{n\in\mathbb{N}}$ is fuzzy soft G-convergent to $Tf_e=f_e$, therefore proposition (4.4) in [2] implies that $T$ is fuzzy soft G-continuous at $f_e$.

**Theorem 6.** *Suppose $(\tilde{E},\tilde{G})$ is a fuzzy soft G-metric space and $T:(\tilde{E},\tilde{G})\to(\tilde{E},\tilde{G})$ is a mapping satisfying the following condition:*

$$\tilde{G}(Tf_{e_1},Tf_{e_2},Tf_{e_3})\tilde{\leq}\bar{\bar{a}}\ max\left\{\begin{array}{c}\left[\tilde{G}(f_{e_1},Tf_{e_1},Tf_{e_1})+\tilde{G}(f_{e_1},Tf_{e_2},Tf_{e_2})\right],\\ \tilde{G}(f_{e_1},f_{e_2},f_{e_3}),\\ \left[\tilde{G}(f_{e_2},Tf_{e_2},Tf_{e_2})+\tilde{G}(f_{e_2},Tf_{e_1},Tf_{e_1})\right]\end{array}\right\} \tag{8}$$

$\forall f_{e_1},f_{e_2},f_{e_3}\tilde{\in}FSC(\tilde{E})$ *and* $\tilde{0}\tilde{\leq}\bar{\bar{a}}\tilde{<}\tilde{1}$.
*Then $T$ has an unique fixed point, say $f_e$, and at $f_e$, $T$ is fuzzy soft G-continuous.*

*Proof.* Assume that $f_{e_0}\tilde{\in}FSC(\tilde{E})$ is an arbitrary fuzzy soft element and define the sequence $\{g_{e_n}\}_{n\in\mathbb{N}}$ as follows: $Tg_{e_0}=g_{e_1},Tg_{e_1}=g_{e_2},Tg_{e_2}=g_{e_3},...,Tg_{e_n}=g_{e_{n+1}}$.
Consider that $g_{e_n}\neq g_{e_{n+1}}$.
Substituting $f_{e_1}=g_{e_n},f_{e_2}=g_{e_{n+1}}$ and $f_{e_3}=g_{e_{n+1}}$ in (8), we obtain

$$\tilde{G}(Tg_{e_n},Tg_{e_{n+1}},Tg_{e_{n+1}})\tilde{\leq}\bar{\bar{a}}\ max\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},Tg_{e_n},Tg_{e_n})+\tilde{G}(g_{e_n},Tg_{e_{n+1}},Tg_{e_{n+1}})\right],\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \left[\tilde{G}(g_{e_{n+1}},Tg_{e_{n+1}},Tg_{e_{n+1}})+\tilde{G}(g_{e_{n+1}},Tg_{e_n},Tg_{e_n})\right]\end{array}\right\}$$

$$\Rightarrow\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}\ max\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\\ \left[\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})+\tilde{G}(g_{e_{n+1}},g_{e_{n+1}},g_{e_{n+1}})\right]\end{array}\right\}$$

$$\Rightarrow\ \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}\ max\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\} \tag{9}$$

We now have three cases:

Case (1): If $max\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}=\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right]$

Then, (9) is reduced to

$$\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right]$$

$$\Rightarrow\qquad \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\frac{\bar{\bar{a}}}{\tilde{1}-\tilde{\bar{a}}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

Case (2): If $max\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}=\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$

Then, (9) is reduced to

$$\Rightarrow\qquad \tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\tilde{\leq}\bar{\bar{a}}\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})$$

Case (3): If $max\left\{\begin{array}{c}\left[\tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}})+\tilde{G}(g_{e_n},g_{e_{n+2}},g_{e_{n+2}})\right],\\ \tilde{G}(g_{e_n},g_{e_{n+1}},g_{e_{n+1}}),\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})\end{array}\right\}=\tilde{G}(g_{e_{n+1}},g_{e_{n+2}},g_{e_{n+2}})$

Then, (9) is reduced to

$$\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) \tilde{\leq} \bar{\bar{a}} \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}), \text{ which is a contradiction.}$$

From cases $(1), (2)$ and $(3)$; we have

$$\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) \tilde{\leq} \bar{\bar{a}} \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

On continuing this process $(n+1)$ times; we have

$$\tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) \tilde{\leq} \bar{\bar{a}}^{n+1} \tilde{G}(g_{e_0}, g_{e_1}, g_{e_1})$$

Similarly, we will conclude that

$$\tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_n}) \tilde{\leq} \bar{\bar{a}} \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}})$$

Next, we show that $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Then for all $n, m \in \mathbb{N}$, $n < m$, we have

$$\begin{aligned}\tilde{G}(g_{e_n}, g_{e_m}, g_{e_m}) &\tilde{\leq} \tilde{G}(g_{e_n}, g_{e_{n+1}}, g_{e_{n+1}}) + \tilde{G}(g_{e_{n+1}}, g_{e_{n+2}}, g_{e_{n+2}}) + ... + \tilde{G}(g_{e_{m-1}}, g_{e_m}, g_{e_m}) \\ &\tilde{\leq} (\bar{\bar{a}}^n + \bar{\bar{a}}^{(n+1)} + ... + \bar{\bar{a}}^m \tilde{G}(g_{e_0}, g_{e_1}, g_{e_1}) \\ &\tilde{\leq} \frac{\bar{\bar{a}}^n}{\tilde{1} - \bar{\bar{a}}} \tilde{G}(g_{e_0}, g_{e_1}, g_{e_1}).\end{aligned}$$

Hence, $\{g_{e_n}\}_{n\in\mathbb{N}}$ is a fuzzy soft G-Cauchy sequence.
Since $(\tilde{E}, \tilde{G})$ is fuzzy soft $G$-complete, there exists $f_e \tilde{\in} FSC(\tilde{E})$ such that $\{g_{e_n}\}_{n\in\mathbb{N}}$ fuzzy soft G-converges to $f_e$.
Next, we will show that $f_e$ is a fixed point of $T$.
For this, we take $f_{e_1} = g_{e_n}$ and $f_{e_2} = f_{e_3} = f_e$ in (8), then

$$\tilde{G}(Tg_{e_n}, Tf_e, Tf_e) \tilde{\leq} \bar{\bar{a}}\ max \left\{ \begin{array}{c} \left[\tilde{G}(g_{e_n}, Tg_{e_n}, Tg_{e_n}) + \tilde{G}(g_{e_n}, Tf_e, Tf_e)\right], \\ \tilde{G}(g_{e_n}, f_e, f_e), \\ \left[\tilde{G}(f_e, Tf_e, Tf_e) + \tilde{G}(f_e, Tg_{e_n}, Tg_{e_n})\right] \end{array} \right\}$$

$$\Rightarrow \quad \tilde{G}(f_e, Tf_e, Tf_e) \tilde{\leq} \bar{\bar{a}}\ max \left\{ \begin{array}{c} \left[\tilde{G}(f_e, f_e, f_e) + \tilde{G}(f_e, Tf_e, Tf_e)\right], \\ \tilde{G}(f_e, f_e, f_e), \\ \left[\tilde{G}(f_e, Tf_e, Tf_e) + \tilde{G}(f_e, f_e, f_e)\right] \end{array} \right\}$$

$$\Rightarrow \quad \tilde{G}(f_e, Tf_e, Tf_e) \tilde{\leq} \bar{\bar{a}} \tilde{G}(f_e, Tf_e, Tf_e)$$

$$\Rightarrow \quad (\tilde{1} - \bar{\bar{a}}) \tilde{G}(f_e, Tf_e, Tf_e) \tilde{\leq} \tilde{0}$$

This is a contradiction, so $Tf_e = f_e$ i.e. $f_e$ is a fixed point of $T$.
Now, to prove uniqueness, assume that $f_e$ and $g_e$ are two fixed points of $T$. Then by inequality (8), we have

$$\tilde{G}(Tf_e, Tg_e, Tg_e) \tilde{\leq} \bar{\bar{a}}\ max \left\{ \begin{array}{c} \left[\tilde{G}(f_e, Tf_e, Tf_e) + \tilde{G}(f_e, Tg_e, Tg_e)\right], \\ \tilde{G}(f_e, g_e, g_e), \\ \left[\tilde{G}(g_e, Tg_e, Tg_e) + \tilde{G}(g_e, Tf_e, Tf_e)\right] \end{array} \right\}$$

$$\Rightarrow \quad \tilde{G}(f_e, g_e, g_e) \tilde{\leq} \bar{\bar{a}}\ max \left\{ \begin{array}{c} \left[\tilde{G}(f_e, f_e, f_e) + \tilde{G}(f_e, g_e, g_e)\right], \\ \tilde{G}(f_e, g_e, g_e), \\ \left[\tilde{G}(g_e, g_e, g_e) + \tilde{G}(g_e, f_e, f_e)\right] \end{array} \right\}$$

$$\Rightarrow \quad \tilde{G}(f_e, g_e, g_e) \tilde{\leq} \bar{\bar{a}}\ max \left\{ \ \tilde{G}(f_e, g_e, g_e), \tilde{G}(g_e, f_e, f_e) \ \right\}$$

We now have two cases:
Case (I): If $max \left\{ \ \tilde{G}(f_e, g_e, g_e), \tilde{G}(g_e, f_e, f_e) \ \right\} = \tilde{G}(f_e, g_e, g_e)$, then we get

$$\tilde{G}(f_e, g_e, g_e) \tilde{\leq} \bar{\bar{a}} \tilde{G}(f_e, g_e, g_e)$$

This is a contradiction implies that $f_e = g_e$

Case (II): If $max\left\{ \tilde{G}(f_e, g_e, g_e), \tilde{G}(g_e, f_e, f_e) \right\} = \tilde{G}(g_e, f_e, f_e)$, then we get

$$\tilde{G}(f_e, g_e, g_e) \tilde{\leq} \bar{\bar{a}} \tilde{G}(g_e, f_e, f_e)$$

So, we deduct that $\tilde{G}(f_e, g_e, g_e) \tilde{\leq} \bar{\bar{a}} \tilde{G}(g_e, f_e, f_e)$.

By repeated use of the same argument, we will find $\tilde{G}(g_e, f_e, f_e) \tilde{\leq} \bar{\bar{a}} \tilde{G}(f_e, g_e, g_e)$.

Therefore, we get $\tilde{G}(f_e, g_e, g_e) \tilde{\leq} \bar{\bar{a}}^2 \tilde{G}(g_e, f_e, f_e)$.

Since $\bar{\bar{a}} \tilde{<} (\tilde{\tilde{1/2}})$, this is a contradiction implies that $f_e = g_e$

To show that $T$ is fuzzy soft G-continuous at $f_e$.

Let $\{h_{e_n}\}_{n \in \mathbb{N}}$ be a sequence of fuzzy soft elements in $\tilde{E}$ such that $\{h_{e_n}\}_{n \in \mathbb{N}} \to f_e$, then by using inequality (8), we can deduce that

$$\tilde{G}(f_e, Th_{e_n}, Th_{e_n}) = \tilde{G}(Tf_e, Th_{e_n}, Th_{e_n}) \tilde{\leq} \bar{\bar{a}}\ max \left\{ \begin{array}{c} \left[\tilde{G}(f_e, Tf_e, Tf_e) + \tilde{G}(f_e, Th_{e_n}, Th_{e_n})\right], \\ \tilde{G}(f_e, h_{e_n}, h_{e_n}), \\ \left[\tilde{G}(h_{e_n}, Th_{e_n}, Th_{e_n}) + \tilde{G}(h_{e_n}, Tf_e, Tf_e)\right] \end{array} \right\}$$

$$\Rightarrow \quad \tilde{G}(f_e, Th_{e_n}, Th_{e_n}) \tilde{\leq} \bar{\bar{a}} \tilde{G}(f_e, Th_{e_n}, Th_{e_n})$$

$$\Rightarrow \quad (\tilde{1} - \bar{\bar{a}}) \tilde{G}(f_e, Th_{e_n}, Th_{e_n}) \tilde{\leq} \tilde{0}$$

Taking the limit as $n \to \infty$ from which, we see that $\tilde{G}(f_e, Th_{e_n}, Th_{e_n}) \to \tilde{0}$ and so, by proposition (4.1) in [2] we have that the sequence $\{Th_{e_n}\}_{n \in \mathbb{N}}$ is fuzzy soft G-convergent to $Tf_e = f_e$, therefore proposition (4.4) in [2] implies that $T$ is fuzzy soft G-continuous at $f_e$.

## 4. Conclusion

In this paper, some new results of fixed points for mappings satisfying different conditions in fuzzy soft $G$-metric spaces were presented and proved. We'll hope to improve the search performance even more in the future for some more important results in this space.


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