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On ψgs-closed Sets in Bitopological Spaces

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Abstract. In this paper, the properties of ψgs-closed sets in bitopological spaces are investigated. The relationships between ψgs-closed set and other closed sets in bitopological spaces are established and some properties of ψgs-closure and ψgs-interior are provided.

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1. Introduction

Over the years, many researchers have introduced different types of sets in topological spaces. One of these sets is the semi-open set, which was introduced and studied by Levine [12]. Thereafter, the notion of generalized closed sets (briefly, g-closed set) in topological spaces was introduced and investigated in [13]. In 2000, the concepts between closed sets and g-closed sets in topological spaces were studied in [10] and a few years later, the same author [11] studied ψ- closed sets in topological spaces. Ramya and Parvathi introduced a new concept of ψ- generalized closed (briefly, ψg-closed) sets in topological spaces. Recently, a new class of sets namely ψ generalized semi-closed (briefly, ψgs-closed) sets were introduced in topological spaces and some of their basic properties were investigated.

Nowadays, a new concept coined from topological spaces is the so-called bitopological spaces (briefly, BTS). Bose [2], studied semi continuity and semi open mappings in BTS. Thereafter, in [7] and [8], the concepts on generalized closed and semi open sets in bitopological spaces were investigated.

The concepts of bitopological spaces have been widely investigated up to other types of spaces, like soft bitopological spaces. The researcher of this present study was inspired by the work of Şenel and Cagman where in [5] they studied soft closed sets on soft bitopological spaces. Thereafter in [6] they investigated soft topological subspaces. In addition, in [3] a new approach to Hausdorff space theory via the soft sets was investigated by Şenel and further studied soft topology generated by L-soft sets in [4]. With all these concepts in

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mind, we are motivated to define and introduce ψgs-closed sets in bitopological spaces and will intend to further study in other spaces such as soft bitopological spaces.

Moreover, we are interested to find the properties of ψgs-closed sets in BTS and their relationship to other existing sets and will intend to investigate the properties of ψgs-interior and ψgs-closure of a set. In general, this study establishes some properties of ψgs-closed set in bitopological spaces. Specifically, this study investigates some properties of ψgs-closed set in BTS; establishes the relationships between ψgs-closed set and other closed sets in BTS; and provides some properties of ψgs-closure and ψgs-interior in BTS.

The major contributions of this study are the original results on ψgs-closed set in BTS. The findings reveal that every (i,j)-ψ-closed set, τ_i closed set, regular-closed set, semi-closed set, α-closed set, ψ-closed set, αgs-closed set in (X,τ_i) is (i,j)-ψgs-closed where $i,j \in \{1,2\}$. Hence, (i,j)-ψgs-closed set is bigger than those of the mentioned sets. Also, it was found out that the intersection of (i,j)-ψgs-closed sets is (i,j)-ψgs-closed. Furthermore, the results on ψgs-closure and ψgs-interior in BTS are analogous to that in other spaces.

We are motivated to have the results or theorems since these results could also be applied in other spaces to come up with analogous results or theorems. This study could serve as a resource material for future researches and possible applications. This may encourage other mathematics enthusiasts to come up with more results and to establish possible research directions for further study.

2. Preliminaries

In this section, some basic definitions and some known results are provided. Examples are also given for a clearer understanding of several terms defined.

A collection τ of subsets of a nonempty set X is a *topology* on X if $\varnothing, X \in \tau$, $\{M_\omega : \omega \in \Omega\} \subseteq \tau$ implies $\cup_{\omega\in\Omega} M_\omega \in \tau$, and $A, B \in \tau$ implies $A \cap B \in \tau$. If τ is a topology on X, then (X,τ) is called a *topological space*, and the elements of τ are called *τ-open* (or simply open) sets. A subset F of X is said to be *τ-closed* (or simply closed) if its complement $X \setminus F$ is open.

The *interior* of A, denoted by $int(A)$, is the union of all open sets contained in A. That is,

$$int(A) = \bigcup \{O \in \tau : O \subseteq A\}.$$

The *closure* of A, denoted by $cl(A)$, is the intersection of all closed sets containing A. That is,

$$cl(A) = \bigcap \{F \subseteq X : F \text{ is closed and } F \supseteq A\}.$$

Now, if τ_1 and τ_2 are arbitrary topologies on X then (X,τ_1,τ_2) is called a *bitopological space*. The interior of A and the closure of A with respect to τ_i are denoted by $int_i(A)$ and $cl_i(A)$, respectively. Note that through out this context $i,j \in \{1,2\}$ such that $i \neq j$.

Definition 1. Let (X,τ) be a topological space. A subset A of X is called

(i) *semi-open set* [12] if $A \subseteq cl(int(A))$;

(ii) *regular-open set* [19] if $A = int(cl(A))$;

(iii) *α-open set* [16] if $A \subseteq int(cl(int(A)))$;

(iv) *semi-generalized closed* (briefly, *sg-closed*) *set* [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ);

(v) *αgs-closed set* [18] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ);

(vi) *ψ-closed set* [11] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ); and

(vii) *ψ generalized semi-closed* (briefly, *ψgs-closed*) *set* [9] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).

The complement of semi-open (resp. regular-open, α-open, gs-closed, αgs-closed, ψ-closed, and ψgs-closed) set is called semi-closed (resp. regular-closed, α-closed, gs-open, αgs-open, ψ-open, and ψgs-open) set.

Definition 2. Let (X, τ_1, τ_2) be a bitopological space. A subset A of X is called

(i) *(i, j)-semi open* set [15] if $A \subseteq cl_j(int_i(A))$;

(ii) *(i, j)-semi generalized closed* (briefly, *(i, j)-sg closed*) set [17] if (i, j)-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is (i, j)-semi open; and

(iii) *(i, j)-ψ-closed* set [20] if (i, j)-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is (i, j)-sg open.

The complement of (i, j)-semi open (resp. (i, j)-sg closed and (i, j)-ψ-closed) set is called (i, j)-semi closed (resp. (i, j)-sg open and (i, j)-ψ-open) set.

The next result was proven in [14].

Lemma 1. *If a subset A of X is semi-open (respectively, semi-closed, sg-closed, gs-closed, g-closed, ψ-closed) set in (X, τ_i) for $i \in \{1, 2\}$, then it is semi-open (respectively, semi-closed, sg-closed, gs-closed, g-closed, ψ-closed) set in (X, τ_1, τ_2).*

The next Theorem is a composition of several results from Gowsalya, S. and Balamani, N. in [9].

Theorem 1. *Let (X, τ_1, τ_2) be bitopological space. Then*

(i) Every semi-closed set in (X, τ) is ψgs-closed in (X, τ).

(ii) Every closed set in (X, τ) is ψgs-closed set in (X, τ).

(iii) Every regular-closed set in (X, τ) is ψgs-closed in (X, τ).

(iv) Every α-closed set in (X, τ) is ψgs-closed in (X, τ).

(v) Every ψ-closed set in (X, τ) is ψgs-closed in (X, τ).

(vi) Every αgs-closed set in (X, τ) is ψgs-closed in (X, τ).

3. ψgs-closed sets and its relationship to other closed sets in BTS

In this section, some properties of ψgs-closed sets in BTS are investigated. Moreover, the relationship to some other existing closed sets in BTS is established.

Definition 3. A subset A of a bitopological space (X, τ_1, τ_2) is called (i, j)-ψ *generalized semi-closed* (briefly, (i, j)-ψ*gs-closed*) set if (i, j)-$\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is (i, j)-semi-open in $(X, \tau_1, \tau_2), i, j \in \{1, 2\}$ where $i \neq j$. The complement of (i, j)-ψgs-closed set is called (i, j)-ψ*gs-open* set.

Example 1. Let (X, τ_1, τ_2) be a bitopological space such that $X = \{a, b, c\}$, $\tau_1 = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$, and $\tau_2 = \{\varnothing, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $A = \{b, c\}$. Note that X is the only $(1, 2)$-semi open set containing $A = \{b, c\}$. Since $A = \{b, c\}$ is a ψ closed set, it follows that (i, j)-$\psi cl(\{b, c\}) = \{b, c\} \subseteq X$. Thus $A = \{b, c\}$ is a $(1, 2)$-ψ generalized semi closed set. Similarly, $\varnothing, \{b\}, \{c\}$ and X are $(1, 2)$-ψ generalized semi closed sets.

Throughout this context, the open (resp., closed) set in (X, τ_1, τ_2) is denoted by (i, j)-open (resp., (i, j)-closed) set.

Proposition 1. *Every (i, j)-ψ-closed set is (i, j)-ψgs-closed.*

Proof. Let A be (i, j)-ψ-closed set and U be (i, j)-semi-open in (X, τ_1, τ_2) such that $A \subseteq U$. Then (i, j)-$\psi cl(A) = A \subseteq U$. Hence A is (i, j)-ψgs-closed in (X, τ_1, τ_2). □

Theorem 2. *Every ψgs-closed set in (X, τ_i) is ψgs-closed set in (X, τ_1, τ_2).*

Proof. Let A be ψgs-closed set in (X, τ_i). Then $\psi cl(A) \subseteq U$ where U is semi-open in τ_i such that $A \subseteq U$. Since, for each $i \in \{1, 2\}$, every semi-open in (X, τ_i) is semi-open in (X, τ_1, τ_2) by Lemma 1, it follows that U is semi-open in (X, τ_1, τ_2). Moreover, by Lemma 1, every ψ-closed in τ_i is ψ-closed in (X, τ_1, τ_2). Thus (i, j)-$\psi cl(A) \subseteq \psi cl(A) \subseteq U$. Hence A is a ψgs-closed set in (X, τ_1, τ_2). □

The following corollary follows from Theorem 1 and Theorem 2.

Corollary 1. *Let (X, τ_i) be a topological space and (X, τ_1, τ_2) be bitopological space. Then the following statements hold.*

(i) Every τ_i closed set is (i, j)-ψgs-closed set.

(ii) Every regular-closed set in (X, τ_i) is (i, j)-ψgs-closed.

(iii) Every semi-closed set in (X, τ_i) is (i, j)-ψgs-closed.

(iv) Every α-closed set in (X, τ_i) is (i, j)-ψgs-closed.

(v) Every ψ-closed set in (X, τ_i) is (i, j)-ψgs-closed.

(vi) Every αgs-closed set in (X, τ_i) is (i, j)-ψgs-closed.

Theorem 3. *Let A be a (i,j)-ψ-closed set and $A \subseteq B \subseteq (i,j)$-$\psi cl(A)$. Then B is also a (i,j)-ψgs-closed set.*

Proof. Let A be a (i,j)-ψ-closed set and $A \subseteq (i,j)$-$\psi cl(A)$. Suppose U is (i,j)-semi-open such that $B \subseteq U$. We want to show (i,j)-$\psi cl(B) \subseteq U$. Since $A \subseteq B$ and $B \subseteq U$, it follows that $A \subseteq U$. Also, by Proposition 1, A is a (i,j)-ψgs-closed set since A is a (i,j)-ψ-closed set; and so (i,j)-$\psi cl(A) \subseteq U$. Note that $B \subseteq (i,j)$-$\psi cl(A)$ implies

$$(i,j)\text{-}\psi cl(B) \subseteq (i,j)\text{-}\psi cl((i,j)\text{-}\psi cl(A)) = (i,j)\text{-}\psi cl(A) \subseteq U.$$

□

Theorem 4. *Let $\{A_k | A_k$ is (i,j)-ψgs-closed set, $k \in \mathbb{N}\}$. Then $\bigcap_{k=1}^{\infty} A_k$ is (i,j)-ψgs-closed set.*

Proof. Suppose $U = \bigcap_{k=1}^{\infty} U_k$ is (i,j)-semi-open such that $\bigcap_{k=1}^{\infty} A_k \subseteq U$. We want to show that (i,j)-$\psi cl(\bigcap_{k=1}^{\infty} A_k) \subseteq U$. From our assumption, A_k is (i,j)-ψgs-closed set for each $k \in \mathbb{N}$. It follows that (i,j)-$\psi cl(A_k) \subseteq U_k$ for each k such that $A_k \subseteq U_k$ where U_k is (i,j)-semi-open. Now,

$$(i,j)\text{-}\psi cl(\bigcap_{k=1}^{\infty} A_k) \subseteq \bigcap_{k=1}^{\infty} (i,j)\text{-}\psi cl(A_k) \subseteq \bigcap_{k=1}^{\infty} U_k = U.$$

□

Note that if A_k is (i,j)-ψgs-closed set, then by Definition 3, $X \setminus A_k$ is (i,j)-ψgs-open set. Moreover, by De Morganś law, $X \setminus (\bigcap_{k=1}^{\infty} A_k) = \bigcup_{k=1}^{\infty} (X \setminus A_k)$. Hence the following corollary follows.

Corollary 2. *If $\{B_k | B_k$ is (i,j)-ψgs-open set, $k \in \mathbb{N}\}$, then $\bigcup_{k=1}^{\infty} B_k$ is (i,j)-ψgs-open set.*

4. ψgs-closure and ψgs-interior in BTS

In this section, the ψgs-closure and ψgs-interior in bitopological spaces are introduced and some of their properties are explored.

Definition 4. Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. An element $x \in A$ is called (i,j)-ψgs-interior point of A if there exists a (i,j)-ψgs-open set O such that $x \in O \subseteq A$. The set of all (i,j)-ψgs-interior points of A is called the (i,j)-ψgs-interior of A and is denoted by (i,j)-ψgs-$Int(A)$.

Theorem 5. *The ψgs-interior of a subset A of X is the countable union of ψgs-open sets contained in A, that is,*

$$(i,j)\text{-}\psi gs\text{-}Int(A) = \cup\{O : O \text{ is } (i,j)\text{-}\psi gs\text{-open and } O \subseteq A\}.$$

Proof. Let $x \in (i,j)$-ψgs-$Int(A)$. Then there exists a (i,j)-ψgs-open set O such that $x \in O \subseteq A$ implying $x \in \cup\{O : O$ is (i,j)-ψgs-open and $O \subseteq A\}$. Next, suppose $y \in \cup\{O : O$ is (i,j)-ψgs-open and $O \subseteq A\}$. Then there exists (i,j)-ψgs-open set $O_0 \subseteq A$ such that $y \in O_0$. Thus $y \in (i,j)$-ψgs-$Int(A)$. □

The previous theorem implies that (i,j)-ψgs-$Int(A)$ is contained in A, where A is any subset of X, being the union of all (i,j)-ψgs-open sets contained in A. Now Corollary 2 entails that the arbitrary union of (i,j)-ψgs-open sets is also (i,j)-ψgs-open, hence we can say that (i,j)-ψgs-$Int(A)$ is (i,j)-ψgs-open. Consequently, (i,j)-ψgs-$Int(A)$ is the largest (i,j)-ψgs-open set contained in A, as stated in the following remark.

Remark 1. *Let (X, τ_1, τ_2) be a bitopological space and $A, B \subseteq X$. Then the following hold:*

(i) (i,j)-ψgs-$Int(A) \subseteq A$;

(ii) (i,j)-ψgs-$Int(A)$ *is* (i,j)-ψgs*-open set; and*

(iii) If $B \subseteq A$ *such that* B *is* (i,j)-ψgs*-open set, then* $B \subseteq (i,j)$-ψgs-$Int(A)$.

Theorem 6. *Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. A is (i,j)-ψgs-open set, if and only if (i,j)-ψgs-$Int(A) = A$.*

Proof. Let A be (i,j)-ψgs-open set and $x \in A$. Note that by Remark 1 (i), (i,j)-ψgs-$Int(A) \subseteq A$. Hence it suffices to show $A \subseteq (i,j)$-ψgs-$Int(A)$. Suppose $x \notin (i,j)$-ψgs-$Int(A)$. Then $x \notin O$ for all (i,j)-ψgs-open sets O such that $A \subseteq O$. It follows that $x \in X \setminus O$ such that $A \cap (X \setminus O) = \varnothing$. This is a contradiction since $x \in A \cap X \setminus O$. Thus $x \in (i,j)$-ψgs-$Int(A)$. Consequently, (i,j)-ψgs-$Int(A) = A$. Conversely, suppose (i,j)-ψgs-$Int(A) = A$. By Remark 1 (ii), (i,j)-ψgs-$Int(A)$ is (i,j)-ψgs-open set, and so A is (i,j)-ψgs-open set. □

Definition 5. Let $A \subseteq X$. Then $x \in X$ is (i,j)-*ψgs-adherent* to A if $V \cap A \neq \varnothing$ for every (i,j)-ψgs-open set V containing x. The set of all (i,j)-ψgs-adherent points of A is called the (i,j)-*ψgs-closure* of A and is denoted by (i,j)-ψgs-Cl(A).

Theorem 7. *The (i,j)-ψgs-closure of a subset A of X is the countable intersection of (i,j)-ψgs-closed sets containing A, that is,*

$$(i,j)\text{-}\psi gs\text{-}Cl(A) = \bigcap\{F : F \textit{ is } (i,j)\text{-}\psi gs\textit{-closed and } A \subseteq F\}.$$

Proof. Let $x \in (i,j)$-ψgs-$Cl(A)$. Then $O \cap A \neq \varnothing$ for every (i,j)-ψgs-open set O containing x. Suppose $x \notin \cap\{F : F$ is (i,j)-ψgs-closed and $A \subseteq F\}$. It follows that there exists (i,j)-ψgs-closed F_0 such that $A \subseteq F_0$ and $x \in X \setminus F_0$. Note that $X \setminus F_0$ is (i,j)-ψgs-open set containing x such that $(X \setminus F_0) \cap A = \varnothing$, a contradiction. Thus

$$x \in \cap\{F : F \text{ is } (i,j)\text{-}\psi\text{gs-closed and } A \subseteq F\}.$$

Next, let $y \in \cap\{F : F$ is (i,j)-ψgs-closed and $A \subseteq F\}$. Then $y \in F$ for all (i,j)-ψgs-closed such that $A \subseteq F$. Suppose on the contrary, $y \notin (i,j)$-ψgs-$Cl(A)$. It implies that $U \cap A = \varnothing$ for some (i,j)-ψgs-closed set U containing y. Hence there exists (i,j)-ψgs-closed set $X \setminus U$ such that $y \notin X \setminus U$ and $A \subseteq X \setminus U$, a contradiction. Consequently, $y \in (i,j)$-ψgs-$Cl(A)$. □

Theorem 7 indicates that (i,j)-ψgs-Cl(A) contains A since the intersection of all (i,j)-ψgs-closed sets contains A. Now Theorem 4 implies that the arbitrary intersection of (i,j)-ψgs-closed sets is also (i,j)-ψgs-closed, thus it follows that (i,j)-ψgs-Cl(A) is (i,j)-ψgs-closed. Hence, (i,j)-ψgs-Cl(A) is the smallest (i,j)-ψgs-closed set that contains A, as stated in the following remark.

Remark 2. *Let (X, τ_1, τ_2) be a bitopological space and $A, B \subseteq X$. Then the following hold:*

(i) $A \subseteq (i,j)$-ψgs-$Cl(A)$;

(ii) (i,j)-ψgs-$Cl(A)$ is (i,j)-ψgs-closed set; and

(iii) If $A \subseteq B$ such that B is (i,j)-ψgs-closed set, then (i,j)-ψgs-$Cl(A) \subseteq B$.

Theorem 8. *A is (i,j)-ψgs-closed set, if and only if (i,j)-ψgs-$Cl(A) = A$.*

Proof. Let A be (i,j)-ψgs-closed set and $x \in (i,j)$-ψgs-$Cl(A)$. Then for all (i,j)-ψgs-open set U containing x, we have $U \cap A \neq \varnothing$. Suppose on the contrary, $x \notin A$. Then $x \in X \setminus A$ where $X \setminus A$ is (i,j)-ψgs-open and $(X \setminus A) \cap A = \varnothing$, a contradiction since $x \in (i,j)$-ψgs-$Cl(A)$. Thus $x \in A$, and so (i,j)-ψgs-$Cl(A) \subseteq A$. Note that by Remark 2 (i), $A \subseteq (i,j)$-ψgs-$Cl(A)$, and hence (i,j)-ψgs-$Cl(A) = A$. Conversely, suppose (i,j)-ψgs-$Cl(A) = A$. By Remark 2 (ii), (i,j)-ψgs-$Cl(A)$ is (i,j)-ψgs-closed set, and so A is (i,j)-ψgs-closed set. □

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