Investigation of the Infinite Discrete Levels from Finite discrete Levels in Position Dependent Mass Quantum Systems

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Abstract. We find quantum systems having finite distinct discrete energy levels can reflect infinite distinct energy levels under suitable form of position dependent mass systems. A model example has been investigated considering the fractional Harmonic Oscillator [6].In addition to this,we also notice the same behaviour in other quantum models. In all the cases, we find finite distinct discrete levels becoming infinite distinct discrete levels under position dependent mass quantum systems without the change in the potential energy.

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1. Introduction

Bounded quantum structures are associated with actual strength levels [2]—. These systems can have infinite or finite distinct discrete levels. A model infinite quantum system is [6]:

$$h = p^2 + \lambda x^2 + \frac{x^2}{(1 + gx^2)} \tag{1}$$

This systems reflect infinite levels as long as $\lambda \gg 0$. However a drastic change in energy levels is noticed in the limit $\lambda = 0$. Under this condition model Hamiltonian becomes

$$H = p^2 + \frac{x^2}{(1 + x^2)} \tag{2}$$

The model potential satisfies the condition $V(x) = \frac{x^2}{(1+x^2)} \ll 1$,which is basically the reason for "distinct discrete levels"(DDL). This mannequin has been studied detail by Rath and Kaabar [11]. However, we consider here a slightly modified DDL as:

$$H^1 = p^2 + \frac{x^2}{(1 + 2x^2)} \tag{3}$$

which is recast as

$$= [p^2 + M(x)x^2] \tag{4}$$

where $M(x) = \frac{1}{(1+2x^2)}$. Energy levels of this oscillator has been reflected in Fig. 1. Below, we focus our attention on new models without the change in potential energy as follows.
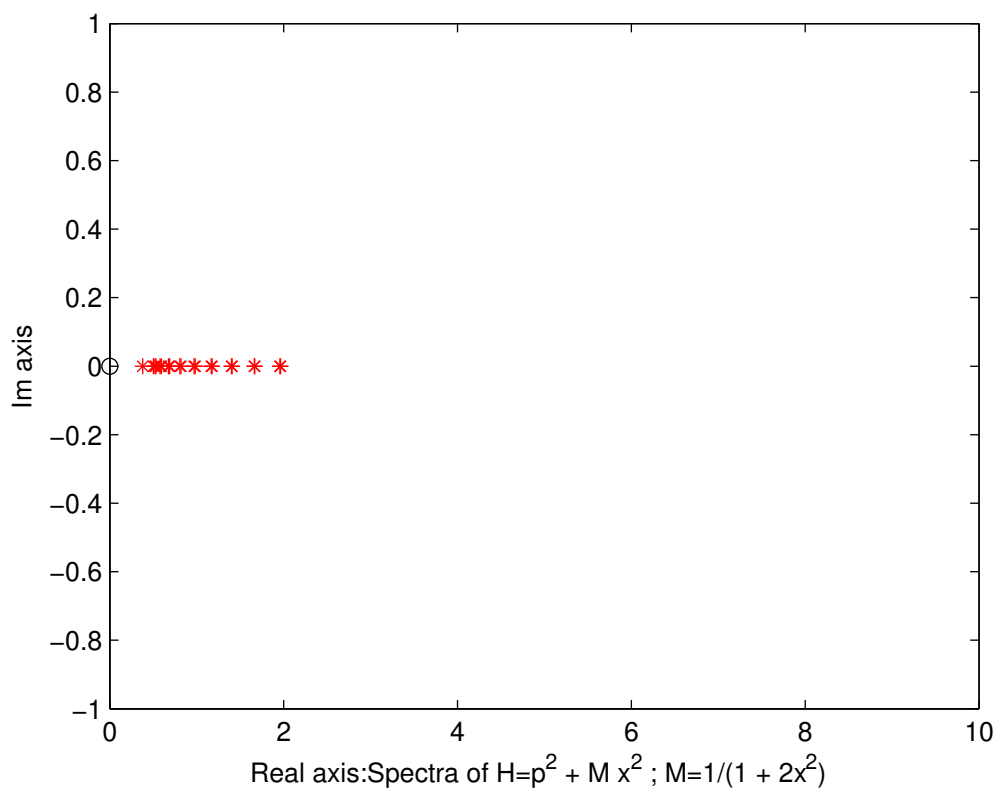

Figure 1: Energy levels of $H = [p^2 + Mx^2]$; $with M = \frac{1}{(1+2x^2)}$

2. Position dependent mass system without potential change

At the outset, we would like to state that all our equations are valid subject to validity of commutation relation

$$[x, p] = i \tag{5}$$

$$H_{PDM} = p^2/M(x) + M(x)x^{2N} = \frac{p^2}{M} + V(x) \tag{6}$$

Hence, the corresponding M(x) can be written as

$$M(x) = M = \frac{V(x)}{x^{2N}} \qquad\qquad N = 1, 2, 3... \tag{7}$$

and so on. In fact, for N=1 model Hamiltonian was also previously proposed by Mathews and Lakshnman for the study of nonlinear analysis in view of its potential applications in quantum field theory [4]. As such the Hamiltonian is not self-adjoint($H_{PDM} \neq H_{PDM}^{\dagger}$) [1, 3, 7, 9, 13–15]. In its self adjoint form, it is expressed as [15]:

$$H_{PDMS} = H^{(2)} = [-\frac{1}{M^{1/4}}\partial_x \frac{1}{M^{1/2}}\partial_x \frac{1}{M^{1/4}} + M(x)x^2] \tag{8}$$

Hence the resultant Hamiltonian becomes von Roos model Hamiltonian [13, 14]. Energy levels of this Hamiltonian are reflected in Fig. 2.

In subsequent applications of von Roos model we use

$$p^2/M = T = [-\frac{1}{M^{1/4}}\partial_x \frac{1}{M^{1/2}}\partial_x \frac{1}{M^{1/4}}] \tag{9}$$

Hence in all the PDM calculations, the corresponding Hamiltonian is self-adjoint in nature($T = T^{\dagger}$). Apart from this we present a few models as

Case-I: Trivial exponential model

$$H^{(3)} = p^2 + 10(1 - exp(-x^2))) \tag{10}$$

Hence, the corresponding PDM operator can be written as

$$H_{PDM}^{(4)} = H^{(4)} = \frac{p^2}{M} + 10(1 - exp(-x^2)); \qquad with M = 10\frac{[1 - exp(-x^2)]}{x^2} \tag{11}$$

The respective energy levels are reflected in Fig. 3 and Fig. 4, respectively.

Similarly, we select a model PDM Hamiltonian as

$$H^{(5)} = \frac{p^2}{M} + 10(1 - exp(-x^4)); \qquad with M = \frac{10[1 - exp(-x^4))]}{x^4} \tag{12}$$

The respective energy levels are reflected in Fig. 5. One can also extend this approach to

$$H^{(6)} = \frac{p^2}{M} + 10(1 - exp(-x^6)); \qquad with M = \frac{10[1 - exp(-x^6))]}{x^6} \tag{13}$$
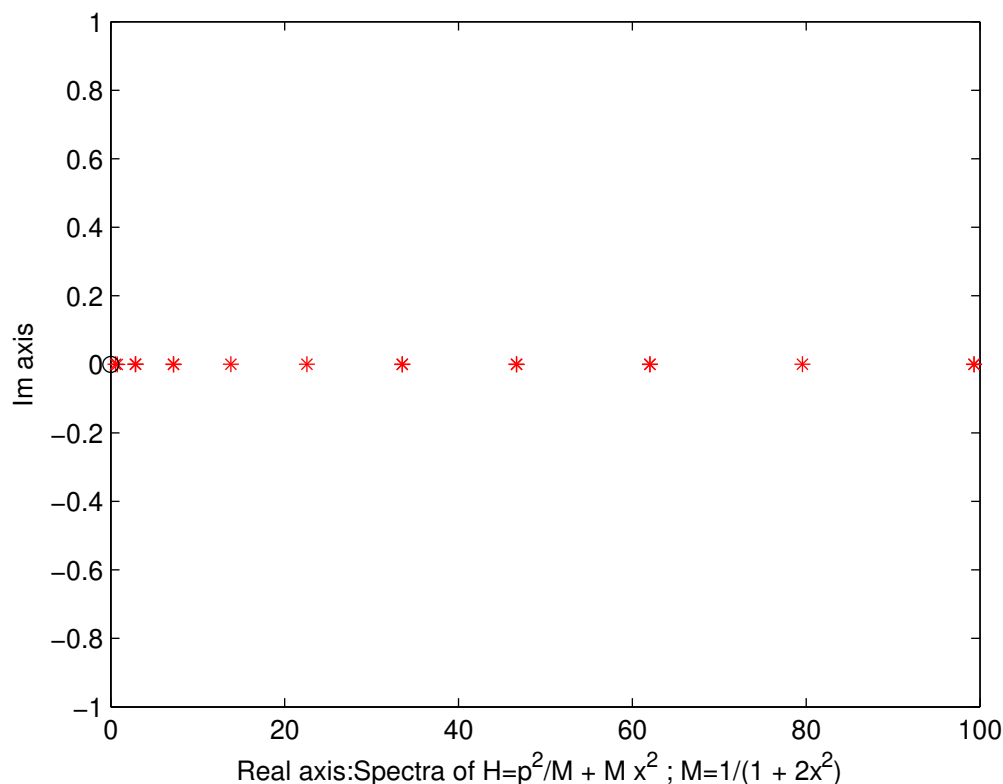
Figure 2: Energy levels of $H = [p^2/M + Mx^2]$; $with M = \frac{1}{(1+2x^2)}$

The respective energy levels are reflected in Fig. 6.

Case-II:Non-trivial exponential model

Here, we consider the model Hamiltonian as

$$H^{(7)} = p^2 + \frac{(1 - exp(-x^2))}{(1 + exp(-x^2))} \tag{14}$$

whose energy levels has finite distinct discrete levels as seen in Fig. 7.

The corresponding PDM operators as

$$H^{(8)} = \frac{p^2}{M} + \frac{(1 - \exp(-x^2))}{(1 + \exp(-x^2))}; \qquad with M \frac{(1 - \exp(-x^2))}{x^2(1 + \exp(-x^2))} \tag{15}$$

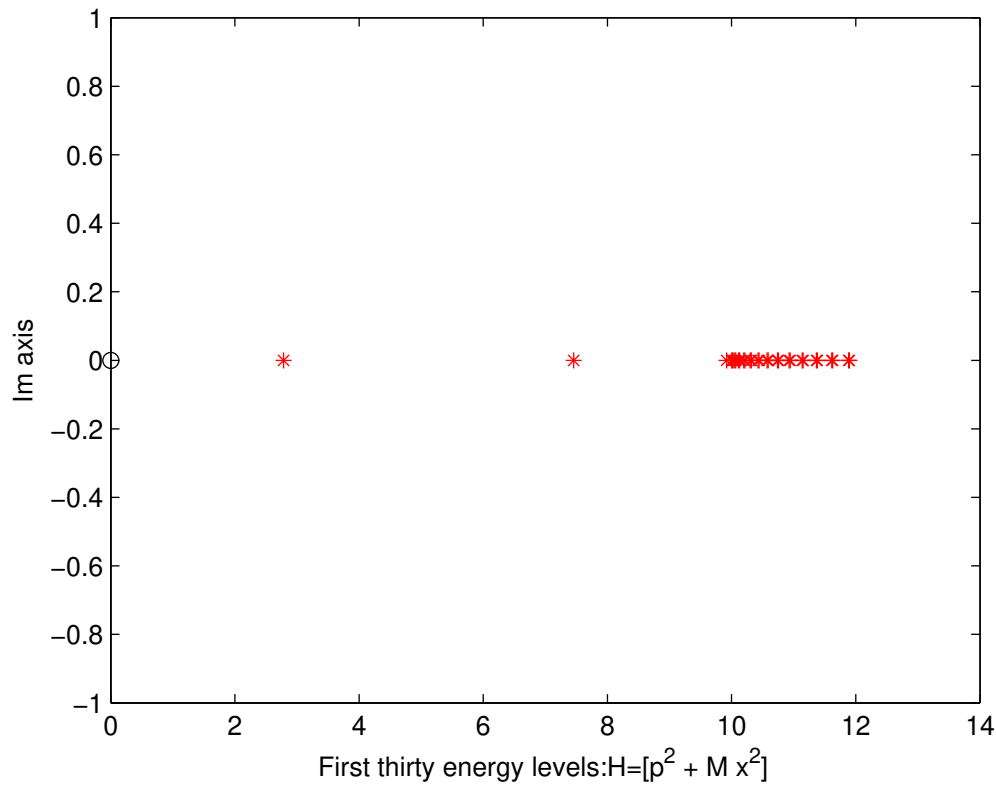whose energy levels are reflected in Fig. 8. Apart from this we also consider similar cases as

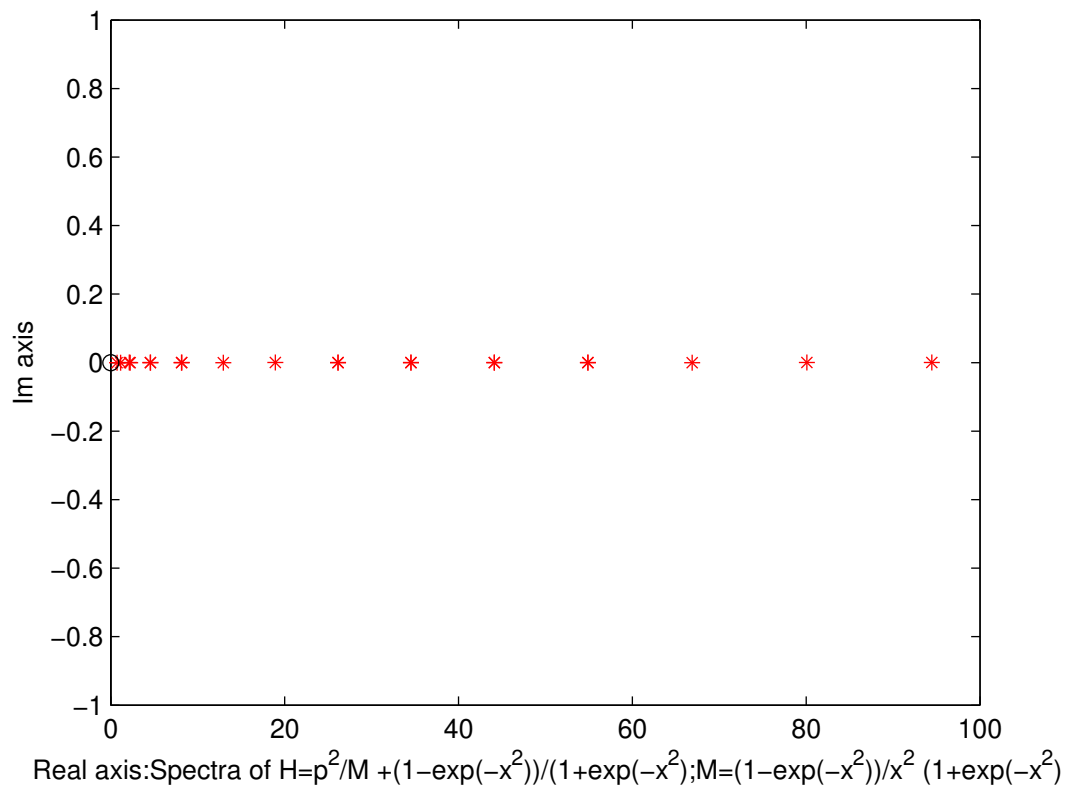Figure 3: Energy levels of $H = p^2 + Mx^2$; $\quad with M = 10\frac{(1-exp(-x^2))}{x^2}$

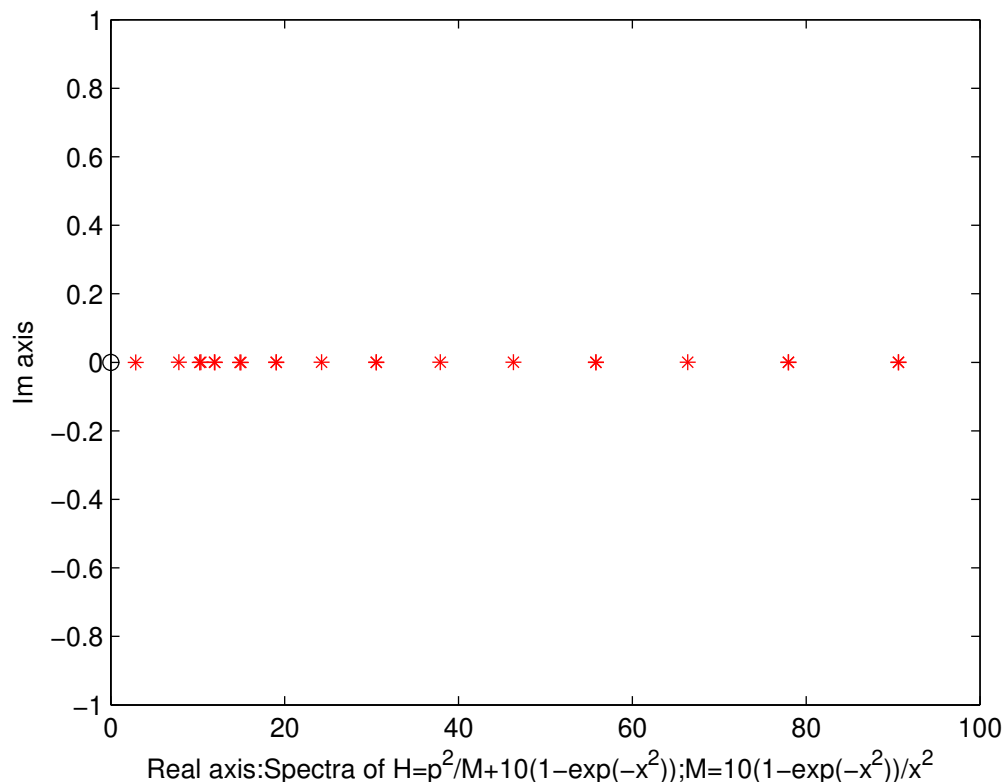Figure 8: Energy levels of $H = p^2/M + Mx^2$; $\quad M = \frac{(1-exp(-x^2))}{x^2(1+\exp -x^2)}$

Figure 4: Energy levels of $H = p^2/M + Mx^2$; $\quad with M = 10\frac{(1-exp(-x^2))}{x^2}$

$$H^{(9)} = \frac{p^2}{M} + \frac{(1 - \exp(-x^2))}{(1 + \exp(-x^2))}; \qquad with M \frac{(1 - \exp(-x^2))}{x^4(1 + \exp(-x^2))} \tag{16}$$

and

$$H^{(10)} = \frac{p^2}{M} + \frac{(1 - \exp(-x^2))}{(1 + \exp(-x^2))}; \qquad with M \frac{(1 - \exp(-x^2))}{x^6(1 + \exp(-x^2))} \tag{17}$$

Here, the energy levels are reflected in Figs. 9,10, respectively. These models are nontrivial because of the potential nature. The finite levels are due to the validity of condition $V(x) \ll 1$.
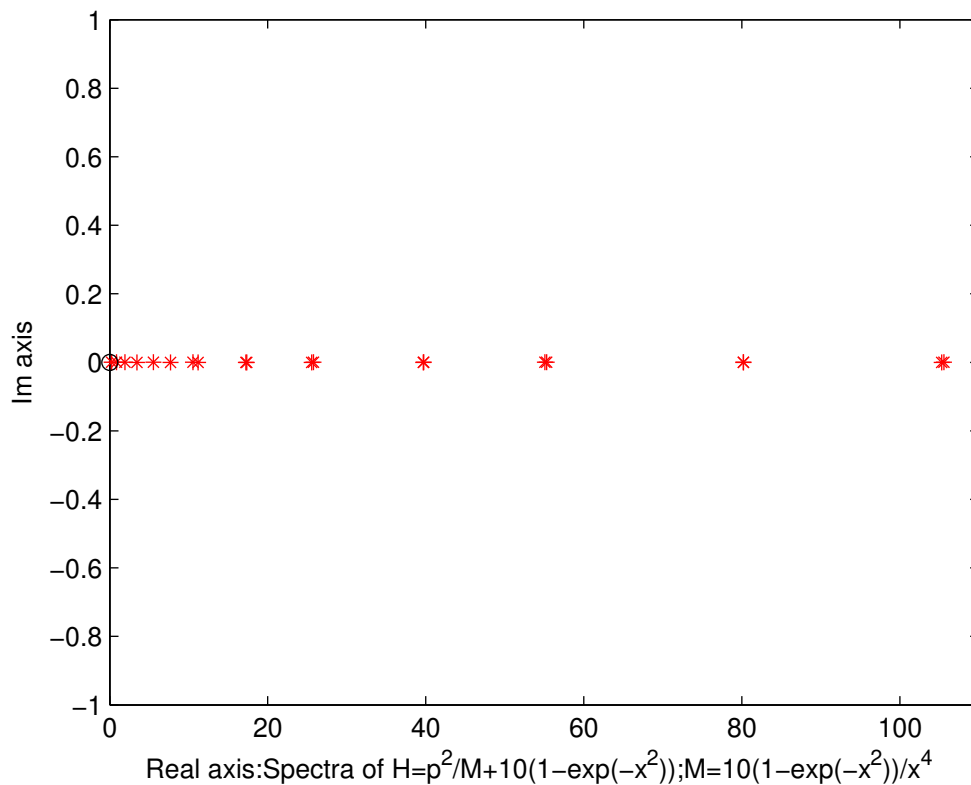
Figure 5: Energy levels of $H = p^2/M + Mx^4$; $\qquad with M = 10\frac{(1-exp(-x^2))}{x^4}$
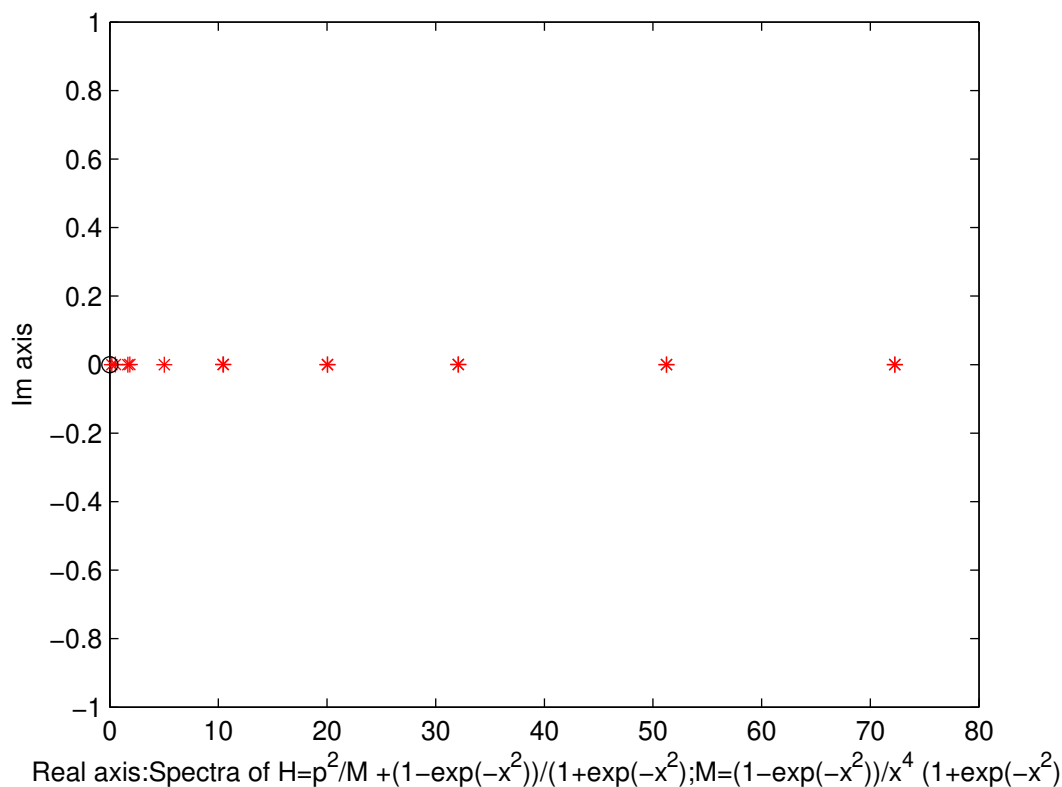

Figure 9: Energy levels of $H = p^2/M + Mx^4$; $\qquad M = \frac{(1-exp(-x^2))}{x^4(1+\exp -x^2)}$

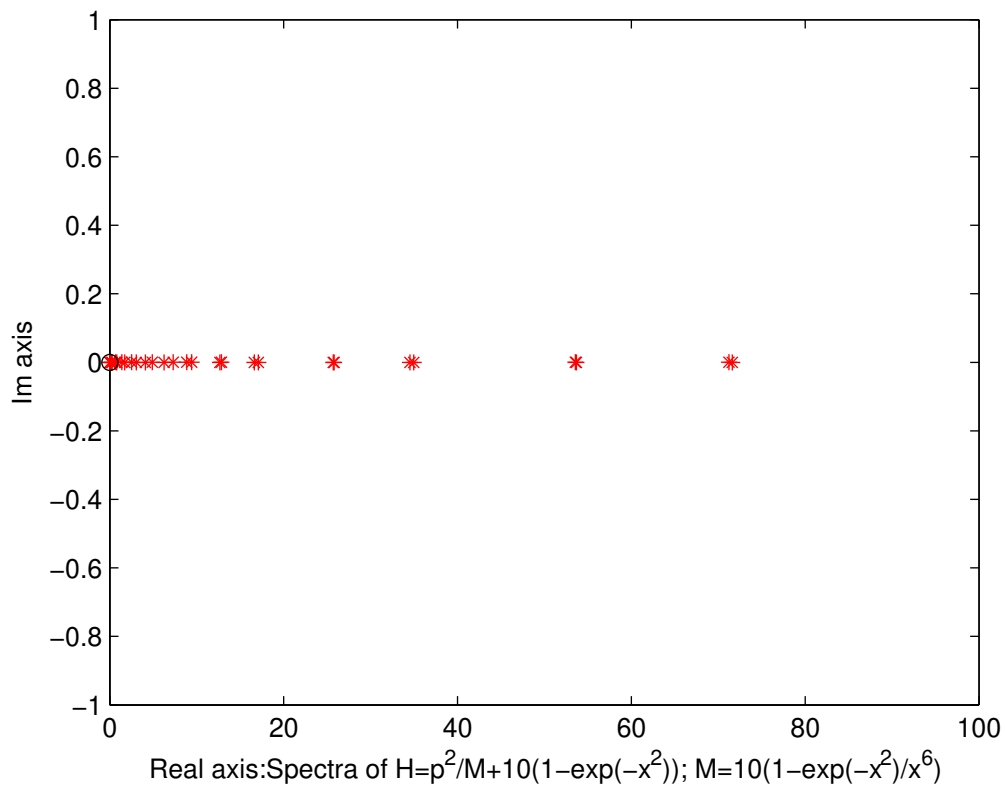Figure 6: Energy levels of $H = p^2/M + Mx^6$;   with $M = 10\frac{(1-exp(-x^2))}{x^6}$

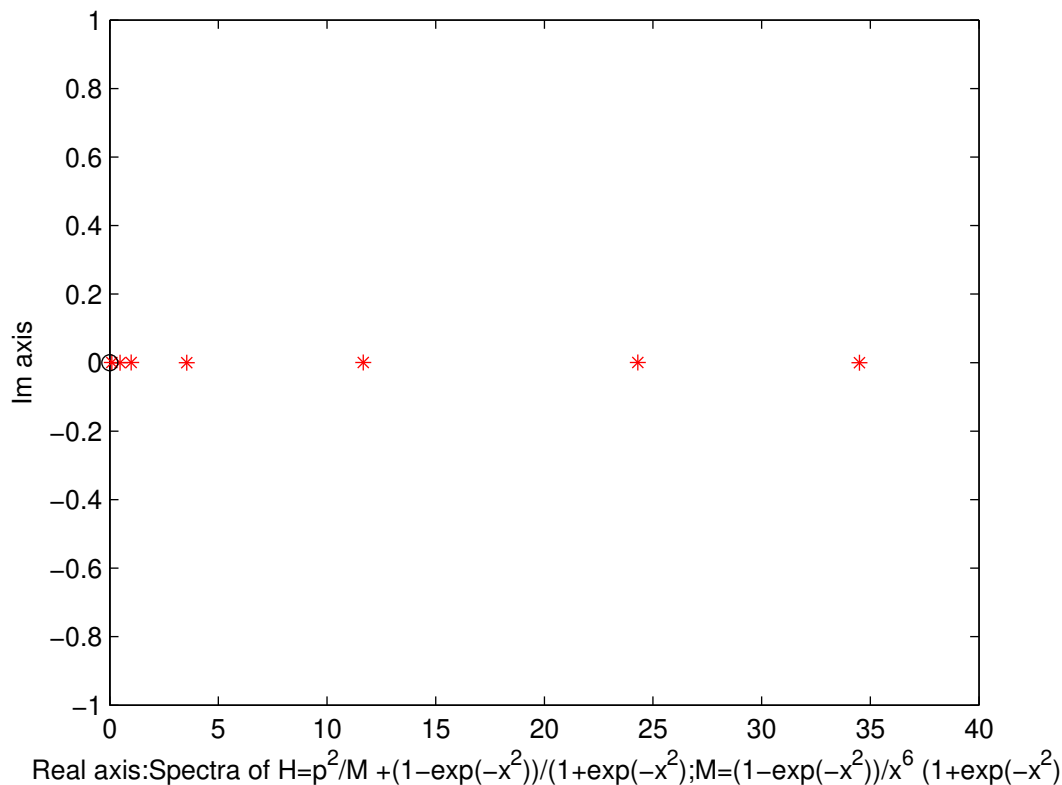

Figure 10: Energy levels of $H = p^2/M + Mx^6$; $M = \frac{(1-exp(-x^2))}{x^6(1+\exp -x^2)}$
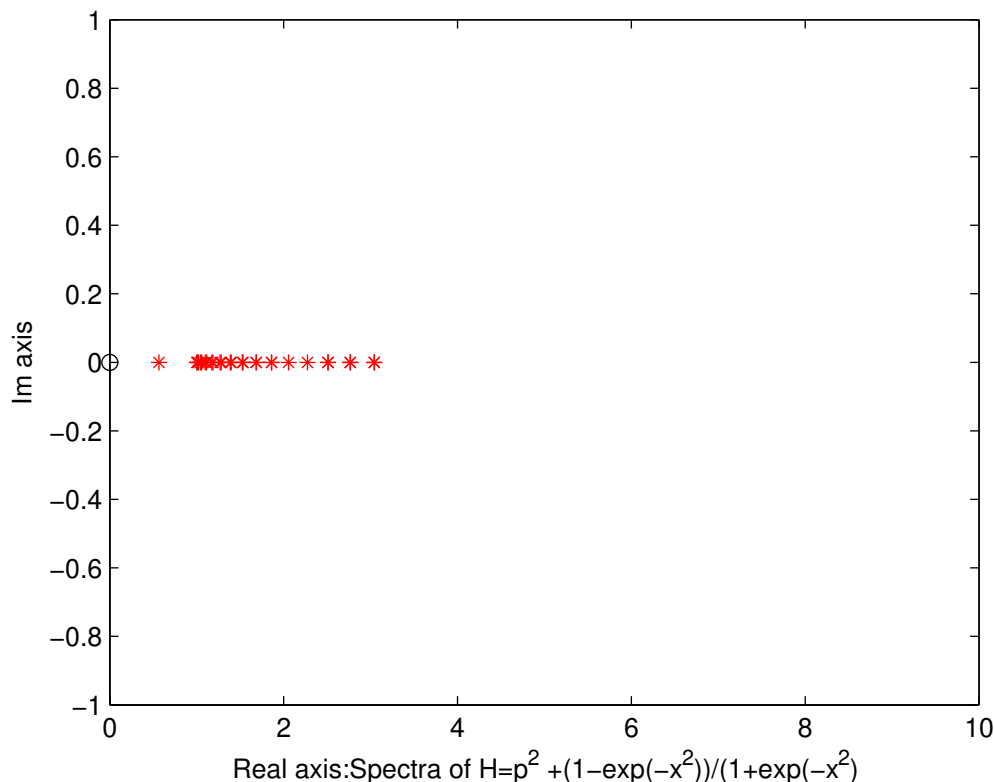
Figure 7: Energy levels of $H = p^2 + \frac{(1-\exp -x^2)}{(1+\exp -x^2)}$

3. Method of calculation

In order to calculate the energy levels of above Hamiltonians, we use matrix diagonalisation method [10, 12] to mirror convergent energy levels, on solving the eigenvalue relation

$$H|\Psi >= E|\Psi > \tag{18}$$

with

$$|\Psi >= \sum_m A_m|m > \tag{19}$$

where $|m >$ satisfies the relation

$$[H_0 = p^2 + x^2]|m >= (2m+1)|m > \tag{20}$$

4. Conclusion

In this paper we have shown that DDL quantum systems can reflect infinite distinct discrete energy levels under suitable form of position dependent mass. The model mass

Table 1 : Fractional Harmonic Oscillator and energy levels comparison

n	g	Present	Previous [6]
0	0.1	1.380 531 8	1.380 53
1		4.079 883 0	4.079 8
2		6.667 919 1	6.667
3		9.166 567 4	
0	1	1.232 350 7	1.232 35
1		3.507 388 3	3.507 38
2		5.589 778 9	5.589 77
		7.648 201 2	

was actually proposed by Cruz et al. [15] and used by others [1, 3, 7, 9]. It is worth mentioning that only this typical form of T can be derived [8]. No other form of von Roos model Hamiltonian [13, 14] has that advantage. In other words, all other forms can only be based on approximation [5]. This universality feature has been exploited in the above model Hamiltonians. All figures have plotted from the respective numerical calculation of energy levels. It should be borne in mind that all the figures on PDM actually we have plotted with T i.e $\frac{p^2}{M} = T$. In order to convince the reader about our method [1, 3], we consider two simple models as

$$H_{Mitra}^{(1)} = p^2 + x^2 + \frac{x^2}{(1 + 0.1x^2)} \tag{21}$$

and

$$H_{Mitra}^{(2)} = p^2 + x^2 + \frac{x^2}{(1 + x^2)} \tag{22}$$

and compare the present result with that of earlier computation by Mitra [6] in Table 1.

Summarizing the above, we conclude that FDL(Finite Discrete Levels) $\Longrightarrow$ IDLPDM(Infinite Discrete Levels in Position Dependent Mass) in quantum operators.

5. Acknowledgements

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