EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 15, No. 1, 2022, 1-14
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

On m-I-Continuous Multifunctions

Takashi Noiri[1,*], Valeriu Popa[2]

[1] *2949-1 Shiokita-cho, Hinagu, Yatsushiro-shi, Kumamoto-ken, 869-5142 Japan*
[2] *Department of Mathematics, University Vasile Alecsandri of Bacau, 600 115-Bacau, Romania*

Abstract. Let $mIO(X)$ be the family of $\star$-open (resp. α-I-open, pre-I-open, semi-I-open, β-I-open, etc.) sets in an ideal topological space (X, τ, I). By using $mIO(X)$, we introduce and investigate the notions of an m-I-continuous multifunction $F : (X, \tau, I) \to (Y, \sigma)$ and $mi^\star$-continuous multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$. The notion of $mi^\star$-continuity is a generalization of m-I-continuity and $i^\star$-continuity [9].

2020 Mathematics Subject Classifications: 54C08, 54C60

Key Words and Phrases: Minimal structure, ideal topological space, m-I-open, m-I-continuous, $mi^\star$-continuous, multifunction

1. Introduction

Semi-open sets, pre-open sets, α-open sets, b-open sets and β-open sets play an important role in the research of generalizations of continuity for functions and multifunctions. In 1961, Marcus [23] introduced the notion of quasicontinuity in topological spaces. Neubrunnova [26] showed that quasicontinuity is equivalent to semi-continuity due to Levine [21]. Bânzaru [6] and Bânzaru and Crivăţ [7] extended it to the notion of quasicontinuity for multifunctions. Properties of quasicontinuous multifunctions are further investigated in [13], [33], and [39].

The present authors introduced and studied α-continuous multifunctions [36], precontinuous multifunctions [39], β-continuous multifunctions [37]. Przemski [46] also introduced the notions of α-continuity, precontinuity and presemi-continuity for multifunctions. It is poved in [36] (resp. [39], [37]) that the notion of α-continuity (resp. precontinuity, β-continuity) for multifunctions in the sense of Popa and Noiri is equivalent to that of α-continuity (resp. precontinuity, presemi-continuity) in the sense of Przemski.

The notions of minimal structure, m-continuity, M-continuity are introduced in [40] and [41]. By using these notions, the present authors unified theory of continuity in [42],

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v15i1.4207

Email addresses: `t.noiri@nifty.com` (T. Noiri), `vpopa@ub.ro` (V. Popa)

[44], and [28] and other papers. The upper/lower m-continuous (resp. M-continuous) multifunctions are introduced and investigated in [42], [44] (resp. [28], [29]) and other papers.

The notion of ideal topological spaces was introduced in [20], [47]. As generelarizations of open sets, the notions of semi-I-open sets, pre-I-open sets, α-I-open sets, b-I-open sets and β-I-open sets are inroduced and studied. The notion of upper/lower-Icontinuous multifunctions is introduced in [2]. Quite recently other results are obtained in [8], [9], [4], [31] and other papers.

In this paper, by $mIO(X)$ we denote the family of $\star$-open (resp. semi-I-open, pre-I-open, α-I-open, b-I-open, β-I-open, etc.) sets in an ideal topological space (X, τ, I). Then we introduce and investigate the notion of an m-I-continuous multifunction $F : (X, \tau, I) \to (Y, \sigma)$ which generalizes the results obtained in [36], [37] and [39]. Furthermore, we introduce the notion of an $mi^{\star}$-continuous multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ which generalizes the notions of $i^{\star}$-continuous multifunctions [9] and m-I-continuous multifunctions.

2. Preliminaries

Let (X, τ) be a topological spacce and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively.

Definition 1. A subset A of a topological space (X, τ) is said to be
(1) *α-open* [27] if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$,
(2) *semi-open* [21] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,
(3) *preopen* [24] if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,
(4) *b-open* [3] if $A \subset \mathrm{Cl}(\mathrm{Int}(A)) \cup \mathrm{Int}(\mathrm{Cl}(A))$,
(5) *β-open* [1] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$.

The family of all semi-open (resp. preopen, α-open, b-open, β-open) sets in (X, τ) is denoted by $\mathrm{SO}(X)$ (resp. $\mathrm{PO}(X)$, $\alpha(X)$, $\mathrm{BO}(X)$, $\beta(X)$).

Throughout the present paper, spaces (X, τ) and (Y, σ) always mean topological spaces and $F : (X, \tau) \to (Y, \sigma)$ presents a multivalued function. For a multifunction, we shall denote the upper and lower inverses of a subset B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is,

$$F^+(B) = \{x \in X : F(x) \subset B\} \text{ and } F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}.$$

Let $\mathcal{P}(Y)$ be the collection of all nonempty subsets of Y. For any open set V of Y, we denote $V^+ = \{B \in \mathcal{P}(Y) : B \subset V\}$ and $V^- = \{B \in \mathcal{P}(Y) : B \cap V \neq \emptyset\}$ [46].

Definition 2. A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be *quasi-continuous* [6], [7], [33] (resp. *precontinuous* [39], *α-continuous* [36], *β-continuous* [37]) at a point $x \in X$ if for each open sets G_1, G_2 of Y such that $F(x) \in G_1^+ \cap G_2^-$, there exists a semi-open (resp. preopen, α-open, β-open) set U of X containing x such that $F(u) \in G_1^+ \cap G_2^-$ for every $u \in U$. A multifunction is said to be *quasi-continuous* (resp. *precontinuous*, *α-continuous*, *β-continuous*) if it has this property at each point of $x \in X$.

3. m-continuous multifunctions

Definition 3. A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a *minimal structure* (briefly *m-structure*) on X if $\emptyset \in m_X$ and $X \in m_X$. Each member of m_X is said to be *m_X-open* (briefly *m-open*) and the complement of an m_X-open set is said to be *m_X-closed*. (briefly *m-closed*). A set X with an m_X-structure m_X is called an m-space and is denoted by (X, m_X)

Remark 1. Let (X, τ) be a topological space. Then the families τ, $\alpha(X)$, SO(X), PO(X), BO(X), $\beta(X)$ are all m-structures on X.

Definition 4. Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the *m_X-closure* of A and the *m_X-interior* of A are defined in [22] as follows:
(1) $\mathrm{mCl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$,
(2) $\mathrm{mInt}(A) = \cup\{U : U \subset A, U \in m_X\}$.

Remark 2. Let (X, τ) be a topological space and A a subset of X. If $m_X = \tau$ (resp. SO(X), PO(X), BO(X), $\alpha(X)$, $\beta(X)$), then we have
(1) $\mathrm{mCl}(A) = \mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\mathrm{bCl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_\beta\mathrm{Cl}(A)$),
(2) $\mathrm{mInt}(A) = \mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\mathrm{bInt}(A)$, $\alpha\mathrm{Int}(A)$, ${}_\beta\mathrm{Int}(A)$).

Lemma 1. ([22]). *Let (X, m_X) be an m-space. For subsets A and B of X, the following properties hold:*
(1) $\mathrm{mCl}(X - A) = X - \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X - A) = X - \mathrm{mCl}(A)$,
(2) If $(X - A) \in m_X$, *then* $\mathrm{mCl}(A) = A$ *and if* $A \in m_X$, *then* $\mathrm{mInt}(A) = A$,
(3) $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,
(4) If $A \subset B$, *then* $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,
(5) $A \subset \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(A) \subset A$,
(6) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Definition 5. A minimal structure m_X on a nonempty set X is said to have *property* $\mathcal{B}$ [22] if the union of any family of subsets belonging to m_X belongs to m_X.

Remark 3. Let (X, τ) be a topological space. Then the families τ, SO(X), PO(X), $\alpha(X)$, BO(X) and $\beta(X)$ are all minimal structures having property $\mathcal{B}$.

Lemma 2. *Let X be a nonempty set and m_X an m-structure with property $\mathcal{B}$. Then, the following properties are hold:*
(1) $\mathrm{mInt}(A) = A$ *if and only if* $A \in m_X$,
(2) $\mathrm{mCl}(A) = A$ *if and only if A is m-closed,*
(3) mInt(A) $\in m_X$ *and mCl(A) is m-closed.*

Definition 6. A multifunction $F : (X, m_X) \to (Y, \sigma)$ is said to be *m-continuous* at $x \in X$ [42] if for each open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, there exists $U \in m_X$ containing x such that $F(u) \in V_1^+ \cap V_2^-$ for every $u \in U$. F is said to be *m-continuous* if it has the property at each point of X.

Remark 4. Let $F : (X, m_X) \to (Y, \sigma)$ be a multifunction. If $m_X = \mathrm{SO}(X)$ (resp. $\mathrm{PO}(X)$, $\alpha(X)$, $\mathrm{BO}(X)$, $\beta(X)$), then F is quasi-continuous (resp. precontinuous, α-continuous, b-continuous, β-continuous).

Theorem 1. ([44]). *For a multifunction $F : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is m-continuous at $x \in X$;

(2) $F(x) \in V_1^+ \cap V_2^-$ implies $x \in mInt[F^+(V_1) \cap F^-(V_2)]$ for every open sets V_1, V_2 of Y;

(3) $x \in \mathrm{mCl}(F^-(B_1) \cup F^+(B_2))$ implies $x \in F^-(Cl(B_1)) \cup F^+(Cl(B_2))$ for every subsets B_1, B_2 of Y;

(4) $x \in F^-(Int(B_1)) \cap F^+(Int(B_2))$ implies $x \in Int(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

Theorem 2. ([42]). *For a multifunction $F : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is m-continuous;

(2) $F^+(G_1) \cap F^-(G_2) = \mathrm{mInt}(F^+(G_1) \cap F^-(G_2))$ for every open sets G_1, G_2 of Y;

(3) $F^-(K_1) \cup F^+(K_2) = \mathrm{mCl}(F^-(K_1) \cup F^+(K_2))$ for every closed sets K_1, K_2 of Y;

(4) $\mathrm{mCl}(F^-(B_1) \cup F^+(B_2)) \subset F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))$ for every subsets B_1, B_2 of Y;

(5) $F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2)) \subset \mathrm{mInt}(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

For a multifunction $F : (X, m_X) \to (Y, \sigma)$, we define $D_m(F)$ as follows:
$D_m(F) = \{x \in X : F \text{ is not } m\text{-continuous at } x\}$.

Theorem 3. ([44]). *For a multifunction $F : (X, m_X) \to (Y, \sigma)$, the following equalities hold:*

$$\begin{aligned} D_m(F) &= \textstyle\bigcup_{G_1,G_2 \in \sigma}\{F^+(G_1) \cap F^-(G_2) - \mathrm{mInt}(F^+(G_1) \cap F^-(G_2))\} \\ &= \textstyle\bigcup_{B_1,B_2 \in \mathcal{P}(Y)}\{F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2)) - \mathrm{mInt}(F^-(B_1) \cap F^+(B_2))\} \\ &= \textstyle\bigcup_{B_1,B_2 \in \mathcal{P}(Y)}\{\mathrm{mCl}(F^-(B_1) \cup F^+(B_2)) - [F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))]\} \\ &= \textstyle\bigcup_{H_1,H_2 \in \mathcal{F}}\{\mathrm{mCl}(F^-(H_1) \cup F^+(H_2)) - [F^-(H_1) \cup F^+(H_2)]\}, \end{aligned}$$

where $\mathcal{F}$ is the family of closed sets of (Y, σ).

Definition 7. ([42]). Let (X, m_X) be an m-space. For a subset A of X, the *m_X-frontier* $\mathrm{mFr}(A)$ of A is defined as follows:

$$\mathrm{mFr}(A) = \mathrm{mCl}(A) \cap \mathrm{mCl}(X - A).$$

Theorem 4. ([42]). *The set of all points $x \in X$ at which a multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is not m-continuous is identical with the union of the m_X-frontiers of the intersections of upper/lower inverse images of open sets containing/meeting $F(x)$.*

Definition 8. A subset B of a topological space (Y, σ) is said to be

(1) *α-regular* [19] if for each $b \in B$ and any open set U containing b, there exists an

open set G of Y such that $b \in G \subset \mathrm{Cl}(G) \subset U$,

(2) *α-paracompact* [48] if every σ-open cover of B has a σ-open refinement which covers B and is locally finite for each point of Y.

For a multifunction $F : (X, m_X) \to (Y, \sigma)$, by $\mathrm{Cl}(F) : X \to Y$ [6] we denote a multifunction defined as follows: $\mathrm{Cl}(F)(x) = \mathrm{Cl}(F(x))$ for each $x \in X$. Similarly, $\mathrm{sCl}(F)$ (resp. $\mathrm{pCl}(F)$, $\alpha\mathrm{Cl}(F)$, $\mathrm{bCl}(F)$, $\beta\mathrm{Cl}(F)$) is defined in [32] (resp. [34], [35], [8], [38]).

Theorem 5. ([42]). *Let $F : (X, m_X) \to (Y, \sigma)$ be a multifunction such that $F(x)$ is α-regular and α-paracompact for each $x \in X$. Then the following properties are equivalent:*

(1) F is m-continuous;

(2) G is m-continuous, where $G = \mathrm{Cl}(F)$, $\mathrm{sCl}(F)$, $\mathrm{pCl}(F)$, $\alpha\mathrm{Cl}(F)$, $\mathrm{bCl}(F)$, and $\beta\mathrm{Cl}(F)$.

Definition 9. ([42]). A multifunction $F : (X, m_X) \to (Y, \sigma)$ is said to be

(1) *upper m-continuous* at $x \in X$ if for each open set V containing $F(x)$, there exists $U \in m_X$ containing x such that $F(U) \subset V$,

(2) *lower m-continuous* at $x \in X$ if for each open set V meeting $F(x)$, there exists $U \in m_X$ containing x such that $F(u) \cap V \neq \emptyset$ for every $u \in U$,

(3) *upper/lower m-continuous* if it has this property at each point $x \in X$.

Theorem 6. ([42]). *Let X be a nonempty set with two m-structures m^1_X and m^2_X satisfying property $\mathcal{B}$ such that $V_1 \in m^1_X$ and $V_2 \in m^2_X$ implies $V_1 \cap V_2 \in m^1_X$. If a multifunction $F : (X, m^1_X) \to (Y, \sigma)$ is upper m-continuous and $F : (X, m^2_X) \to (Y, \sigma)$ is lower m-continuous, then $F : (X, m^1_X) \to (Y, \sigma)$ is m-continuous.*

Theorem 7. ([42]). *Let X be a nonempty set with two m-structures m^1_X and m^2_X satisfying property $\mathcal{B}$ such that $V_1 \in m^1_X$ and $V_2 \in m^2_X$ implies $V_1 \cap V_2 \in m^1_X$. If a multifunction $F : (X, m^1_X) \to (Y, \sigma)$ is lower m-continuous and $F : (X, m^2_X) \to (Y, \sigma)$ is upper m-continuous, then $F : (X, m^1_X) \to (Y, \sigma)$ is m-continuous.*

4. Ideal topological spaces

Let (X, τ) be a topological space. The notion of ideals has been introduced in [20] and [47] and further investigated in [18]

Definition 10. A nonempty collection I of subsets of a set X is called an *ideal on X* if it satisfies the following two conditions:

(1) $A \in I$ and $B \subset A$ implies $B \in I$,

(2) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

A topological space (X, τ) with an ideal I on X is called an *ideal topological space* and is denoted by (X, τ, I). Let (X, τ, I) be an ideal topological space. For any subset A of X, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau : x \in U\}$, is called the *local function* of A with respect to τ and I [18]. Hereafter $A^\star(I, \tau)$ is simply denoted by $A^\star$. It is well known that $\mathrm{Cl}^\star(A) = A \cup A^\star$ defines a Kuratowski closure operator on X and the topology generated by $\mathrm{Cl}^\star$ is denoted by $\tau^\star$.

Lemma 3. *Let (X, τ, I) be an ideal topological space and A, B be subsets of X. Then the following properties hold:*

(1) $A \subset B$ *implies* $\mathrm{Cl}^\star(A) \subset \mathrm{Cl}^\star(B)$,

(2) $\mathrm{Cl}^\star(X) = X$ *and* $\mathrm{Cl}^\star(\emptyset) = \emptyset$,

(3) $\mathrm{Cl}^\star(A) \cup \mathrm{Cl}^\star(B) \subset \mathrm{Cl}^\star(A \cup B)$.

Definition 11. Let (X, τ, I) be an ideal topological space. A subset A of X is said to be

(1) *α-I-open* [16] if $A \subset \mathrm{Int}(\mathrm{Cl}^\star(\mathrm{Int}(A)))$,

(2) *semi-I-open* [16] if $A \subset \mathrm{Cl}^\star(\mathrm{Int}(A))$,

(3) *pre-I-open* [10] if $A \subset \mathrm{Int}(\mathrm{Cl}^\star(A))$,

(4) *b-I-open* [5] if $A \subset \mathrm{Int}(\mathrm{Cl}^\star(A)) \cup \mathrm{Cl}^\star(\mathrm{Int}(A))$,

(5) *β-I-open* [17] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}^\star(A)))$,

(6) *weakly semi-I-open* [14] if $A \subset \mathrm{Cl}^\star(\mathrm{Int}(\mathrm{Cl}(A)))$,

(7) *weakly b-I-open* [25] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}^\star(A))) \cup \mathrm{Cl}^\star(\mathrm{Int}(\mathrm{Cl}(A)))$,

(8) *strongly β-I-open* [15] if $A \subset \mathrm{Cl}^\star(\mathrm{Int}(\mathrm{Cl}^\star(A)))$,

(9) *semi*-I-open* [12] if $A \subset \mathrm{Cl}(\mathrm{Int}^\star(A))$,

(10) *pre*-I-open* [11] if $A \subset \mathrm{Int}^\star(\mathrm{Cl}(A))$,

(11) *$\beta_I^\star$-open* [11] if $A \subset \mathrm{Cl}(\mathrm{Int}^\star(\mathrm{Cl}(A)))$.

The family of all α-I-open (resp. semi-I-open, pre-I-open, b-I-open, β-I-open, weakly semi-I-open, weakly b-I-open, strongly β-I-open, *semi**-I-open, pre*-I-open, $\beta_I^\star$-open) sets in an ideal topological space (X, τ, I) is denoted by $\alpha\mathrm{IO}(X)$ (resp. $\mathrm{SIO}(X)$, $\mathrm{PIO}(X)$, $\mathrm{BIO}(X)$, $\beta\mathrm{IO}(X)$, $\mathrm{WSIO}(X)$, $\mathrm{WBIO}(X)$, $\mathrm{S}\beta\mathrm{IO}(X)$, $\mathrm{S^\star IO(X)}$, $\mathrm{P^\star IO(X)}$, $\beta_I\mathrm{O(X)}$).

Definition 12. By $\mathrm{mIO}(X)$, we denote each one of the families $\tau^\star$, $\alpha\mathrm{IO}(X)$, $\mathrm{SIO}(X)$, $\mathrm{PIO}(X)$, $\mathrm{BIO}(X)$, $\beta\mathrm{IO}(X)$, $\mathrm{WSIO}(X)$, $\mathrm{WBIO}(X)$, $\mathrm{S}\beta\mathrm{IO}(X)$, $\mathrm{S^\star IO}(X)$, $\mathrm{P^\star IO}(X)$, $\beta^\star\mathrm{IO}(X)$.

Lemma 4. *Let (X, τ, I) be an ideal topological space. Then* $\mathrm{mIO}(X)$ *is a minimal structure and has property $\mathcal{B}$.*

Definition 13. Let (X, τ, I) be an ideal topological space. For a subset A of X, $\mathrm{mCl_I}(A)$ and $\mathrm{mInt_I}(A)$ are defined as follows:

(1) $\mathrm{mCl_I}(A) = \cap\{F : A \subset F, X \setminus F \in \mathrm{mIO}(X)\}$,

(2) $\mathrm{mInt_I}(A) = \cup\{U : U \subset A, U \in \mathrm{mIO}(X)\}$.

Let (X, τ, I) be an ideal topological space and $\mathrm{mIO}(X)$ the m_X-structure on X. If $\mathrm{mIO}(X) = \tau^\star$ (resp. $\alpha\mathrm{IO}(X)$, $\mathrm{SIO}(X)$, $\mathrm{PIO}(X)$, $\mathrm{BIO}(X)$, $\beta\mathrm{IO}(X)$, $\mathrm{WSIO}(X)$, $\mathrm{WBIO}(X)$, $\mathrm{S}\beta\mathrm{IO}(X)$), $\mathrm{S^\star IO}(X)$, $\mathrm{P^\star IO}(X)$, $\beta^\star\mathrm{IO}(X)$, then we have the following:

(1) $\mathrm{mCl_I}(A) = \mathrm{Cl}^\star(A)$ (resp. $\alpha\mathrm{Cl_I}(A)$, $\mathrm{sCl_I}(A)$, $\mathrm{pCl_I}(A)$, $\mathrm{bCl_I}(A)$, $\beta\mathrm{Cl_I}(A)$, $\mathrm{wsCl_I}(A)$, $\mathrm{wbCl_I}(A)$, $\mathrm{s}\beta\mathrm{Cl_I}(A)$, $\mathrm{s^\star Cl_I}(A)$, $\mathrm{p^\star Cl_I}(A)$, $\beta^\star\mathrm{Cl_I}(A)$),

(2) $\mathrm{mInt_I}(A) = \mathrm{Int}^\star(A)$ (resp. $\alpha\mathrm{Int_I}(A)$, $\mathrm{sInt_I}(A)$, $\mathrm{pInt_I}(A)$, $\mathrm{bInt_I}(A)$, $\beta\mathrm{Int_I}(A)$, $\mathrm{wsInt_I}(A)$, $\mathrm{wbInt_I}(A)$, $\mathrm{s}\beta\mathrm{Int_I}(A)$, $\mathrm{s^\star Int_I}(A)$, $\mathrm{p^\star Int_I}(A)$, $\beta^\star\mathrm{Int_I}(A)$).

5. m-I-continuous multifunctions

Definition 14. A multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is said to be *m-I-continuous* at $x \in X$ if for each open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, there exists $U \in \mathrm{mIO}(X)$

containing x such that $F(u) \in V_1^+ \cap V_2^-$ for every $u \in U$. F is said to be *m-I-continuous* if it has the property at each point of X.

By Theorem 1 and Definition 13, we obtain the following theorem.

Theorem 8. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is m-I-continuous at $x \in X$;

(2) $F(x) \in V_1^+ \cap V_2^-$ implies $x \in \mathrm{mInt_I}[F^+(V_1) \cap F^-(V_2)]$ for every open sets V_1, V_2 of Y;

(3) $x \in \mathrm{mCl_I}(F^-(B_1) \cup F^+(B_2))$ implies $x \in F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))$ for every subsets B_1, B_2 of Y;

(4) $x \in F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2))$ implies $x \in \mathrm{mInt_I}(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

By Theorem 2 and Definition 13, we obtain the following theorem:

Theorem 9. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is m-I-continuous;

(2) $F^+(G_1) \cap F^-(G_2) \in \mathrm{mIO}(X)$ for every open sets G_1, G_2 of Y;

(3) $F^-(K_1) \cup F^+(K_2)$ is m-I-closed for every closed sets K_1, K_2 of Y;

(4) $\mathrm{mCl_I}(F^-(B_1) \cup F^+(B_2)) \subset F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))$ for every subsets B_1, B_2 of Y;

(5) $F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2)) \subset \mathrm{mInt_I}(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

Let $\mathrm{mIO}(X) = \tau^\star$, then by Theorem 9, we obtain the following corollary:

Corollary 1. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is $\tau^\star$-continuous;

(2) $F^+(G_1) \cap F^-(G_2) \in \tau^\star$ for every open sets G_1, G_2 of Y;

(3) $F^-(K_1) \cup F^+(K_2)$ is $\tau^\star$-closed for every closed sets K_1, K_2 of Y;

(4) $\mathrm{Cl}^\star(F^-(B_1) \cup F^+(B_2)) \subset F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))$ for every subsets B_1, B_2 of Y;

(5) $F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2)) \subset \mathrm{Int}^\star(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

Let $\mathrm{mIO}(X) = \mathrm{SIO}(X)$, then by Theorem 9, we obtain the following corollary:

Corollary 2. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is semi-I-continuous;

(2) $F^+(G_1) \cap F^-(G_2) \in \mathrm{SIO}(X)$ for every open sets G_1, G_2 of Y;

(3) $F^-(K_1) \cup F^+(K_2)$ is semi-I-closed for every closed sets K_1, K_2 of Y;

(4) $\mathrm{sCl_I}(F^-(B_1) \cup F^+(B_2)) \subset F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))$ for every subsets B_1, B_2 of

Y;

(5) $F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2)) \subset \mathrm{sInt_I}(F^-(B_1) \cap F^+(B_2))$ *for every subsets* B_1, B_2 *of* Y*.*

For a multifunction $F : (X, \tau, I) \to (Y, \sigma)$, we define $D_{mI}(F)$ as follows:

$$D_{mI}(F) = \{x \in X : F \text{ is not } m\text{-}I\text{-continuous at } x\}.$$

Theorem 10. *For a multifunction* $F : (X, \tau, I) \to (Y, \sigma)$*, the following equalities hold:*

$$\begin{aligned} D_m(F) &= \textstyle\bigcup_{G_1,G_2\in\sigma}\{F^+(G_1) \cap F^-(G_2) - \mathrm{mInt_I}(F^+(G_1) \cap F^-(G_2))]\} \\ &= \textstyle\bigcup_{B_1,B_2\in\mathcal{P}(Y)}\{F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2)) - \mathrm{mInt_I}(F^-(B_1) \cap F^+(B_2))\} \\ &= \textstyle\bigcup_{B_1,B_2\in\mathcal{P}(Y)}\{\mathrm{mCl_I}(F^-(B_1) \cup F^+(B_2)) - [F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))]\} \\ &= \textstyle\bigcup_{H_1,H_2\in\mathcal{F}}\{\mathrm{mCl_I}(F^-(H_1) \cup F^+(H_2)) - [F^-(H_1) \cup F^+(H_2)]\}, \end{aligned}$$

where $\mathcal{F}$ *is the family of closed sets of* (Y, σ)*.*

Let $\mathrm{mIO}(X) = \mathrm{SIO}(X)$, then by Theorem 10 we obtain the following corollary.

Corollary 3. *For a multifunction* $F : (X, \tau, I) \to (Y, \sigma)$*, the following equalities hold:*

$$\begin{aligned} D_m(F) &= \textstyle\bigcup_{G_1,G_2\in\sigma}\{F^+(G_1) \cap F^-(G_2) - \mathrm{sInt_I}(F^+(G_1) \cap F^-(G_2))]\} \\ &= \textstyle\bigcup_{B_1,B_2\in\mathcal{P}(Y)}\{F^-(\mathrm{Int}(B_1)) \cap F^+(\mathrm{Int}(B_2)) - \mathrm{sInt_I}(F^-(B_1) \cap F^+(B_2))\} \\ &= \textstyle\bigcup_{B_1,B_2\in\mathcal{P}(Y)}\{\mathrm{sCl_I}(F^-(B_1) \cup F^+(B_2)) - [F^-(\mathrm{Cl}(B_1)) \cup F^+(\mathrm{Cl}(B_2))]\} \\ &= \textstyle\bigcup_{H_1,H_2\in\mathcal{F}}\{\mathrm{sCl_I}(F^-(H_1) \cup F^+(H_2)) - [F^-(H_1) \cup F^+(H_2)]\}, \end{aligned}$$

where $\mathcal{F}$ *is the family of closed sets of* (Y, σ)*.*

Definition 15. Let (X, τ, I) be an ideal topological space. For a subset A of X, the *m_I-frontier* $m_I\mathrm{Fr}(A)$ of A is defined as follows:

$$m_I\mathrm{Fr}(A) = mCl_I(A) \cap mCl_I(X - A).$$

Theorem 11. *The set of all points* $x \in X$ *at which a multifunction* $F : (X, \tau, I) \to (Y, \sigma)$ *is not* m*-*I*-continuous is identical with the union of the* m_I*-frontiers of the intersection of upper/lower inverse images of open sets containing/meeting* $F(x)$*.*

Proof. The proof follows from Definition 13 and Theorem 4.

If $\mathrm{mIO}(X) = \tau^\star$, then we obtain the following corollary:

Corollary 4. *The set of all points* $x \in X$ *at which a multifunction* $F : (X, \tau, I) \to (Y, \sigma)$ *is not* $\tau^\star$*-continuous is identical with the union of the* $\tau^\star$*-frontiers of the intersection of upper/lower inverse images of open sets containing/meeting* $F(x)$*.*

If $\mathrm{mIO}(X) = \mathrm{SIO}(X)$, then we obtain the following corollary:

Corollary 5. *The set of all points* $x \in X$ *at which a multifunction* $F : (X, \tau, I) \to (Y, \sigma)$ *is not semi-*I*-continuous is identical with the union of the* S_I*-frontiers of the intersection of upper/lower inverse images of open sets containing/meeting* $F(x)$*.*

Theorem 12. *Let $F : (X, \tau, I) \to (Y, \sigma)$ be a multifunction such that $F(x)$ is α-regular and α-paracompact for each $x \in X$. Then the following properties are equivalent:*

(1) F is m-I-continuous;

(2) G is m-I-continuous, where $G = Cl(F(x))$, $sCl(F)$, $pCl(F)$, $\alpha Cl(F)$, $bCl(F)$, $\beta Cl(F)$.

Proof. The proof follows from Theorem 5.

Definition 16. A multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is said to be

(1) *upper m-I-continuous* at $x \in X$ if for each open set V containing $F(x)$, there exists $U \in \text{mIO}(X)$ containing x such that $F(U) \subset V$,

(2) *lower m-I-continuous* at $xinX$ if for each open set V meeting $F(x)$, there exists $U \in \text{mIO}(X)$ containing x such that $F(u) \cap V \neq \emptyset$ for every $u \in U$,

(3) *upper/lower m-I-continuous* if it has this property at each point $x \in X$.

Theorem 13. *Let X be a nonempty set with two m-structures m_X^1 and m_X^2 satisfying property $\mathcal{B}$ such that $V_1 \in m_X^1$ and $V_2 \in m_X^2$ implies $V_1 \cap V_2 \in m_X^1$. If a multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is upper m_X^1-I-continuous and $F : (X, \tau, I) \to (Y, \sigma)$ is lower m_X^2-I-continuous, then $F : (X, \tau, I) \to (Y, \sigma)$ is m_X^1-I-continuous.*

Theorem 14. *Let X be a nonempty set with two m-structures m_X^1 and m_X^2 satisfying property $\mathcal{B}$ such that $V_1 \in m_X^1$ and $V_2 \in m_X^2$ implies $V_1 \cap V_2 \in m_X^1$. If a multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is lower m_X^1-continuous and $F : (X, \tau, I) \to (Y, \sigma)$ is upper m_X^2-continuous, then $F : (X, \tau, I) \to (Y, \sigma)$ is m_X^1-I-continuous.*

6. $mi^\star$-continuous multifunctions

A multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is said to be *$i^\star$-continuous* [9] if for each $x \in X$ and each $\sigma^\star$-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, there exists a $\tau^\star$-open set U containing x such that $F(U) \subset V_1^+$ and $F(u) \cap V_2 \neq \emptyset$ for every $u \in U$.

Definition 17. A multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is said to be *$mi^\star$-continuous* if for each $x \in X$ and each $\sigma^\star$-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, there exists an mIO($X$)-open set U containing x such that $F(U) \subset V_1^+$ and $F(u) \cap V_2 \neq \emptyset$ for every $u \in U$.

Remark 5. For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, we have the following properties:

(1) If $\text{mIO}(X) = \tau^\star$, then every $mi^\star$-continuous multifunction is $i^\star$-continuous. Therefore, the notion of $mi^\star$-continuity is a generalization of $i^\star$-continuity.

(2) If $J = \{\emptyset\}$, then $\sigma^\star = \sigma$. Therefore, the notion of $mi^\star$-continuity is a generalization of m-I-continuity.

Theorem 15. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following properties are equivalent:*

(1) F is $mi^\star$-continuous;

(2) For each point $x \in X$ and each $\sigma^\star$-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, $x \in \mathrm{mInt_I}(F^+(V_1) \cap F^-(V_2))$;

(3) $F^+(V_1) \cap F^-(V_2) \in \mathrm{mIO}(X)$ for every $\sigma^\star$-open sets V_1, V_2 of Y;

(4) $F^-(K_1) \cup F^+(K_2)$ is m-I-closed for every $\sigma^\star$-closed sets K_1, K_2 of Y;

(5) $\mathrm{mCl_I}(F^-(B_1) \cup F^+(B_2)) \subset F^-(\mathrm{Cl}^\star(B_1)) \cup F^+(\mathrm{Cl}^\star(B_2))$ for every subsets B_1, B_2 of Y;

(6) $F^-(\mathrm{Int}^\star(B_1)) \cap F^+(\mathrm{Int}^\star(B_2)) \subset \mathrm{mInt_I}(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

Proof. (1) => (2): Let $x \in X$ and V_1, V_2 be any $\sigma^\star$-open sets of Y such that $F(x) \in V_1^+ \cap V_2^-$. Then there exists $U \in \mathrm{mIO}(X)$ containing x such that $F(U) \in V_1^+ \cap V_2^-$. Therefore, $U \subset F^+(V_1) \cap F^-(V_2)$ and hence $x \in \mathrm{mInt_I}(F^+(V_1) \cap F^-(V_2))$.

(2) => (3): Let V_1, V_2 be any $\sigma^\star$-open sets of Y and $x \in F^+(V_1) \cap F^-(V_2)$. Then $F(x) \subset V_1$ and $F(x) \cap V_2 \neq \emptyset$. By (2), we have $x \in \mathrm{mInt_I}(F^+(V_1) \cap F^-(V_2))$ and $F^+(V_1) \cap F^-(V_2) \subset \mathrm{mInt_I}(F^+(V_1) \cap F^-(V_2))$. This shows that $F^+(V_1) \cap F^-(V_2) \in \mathrm{mIO}(X)$.

(3) => (4): This easily follows from the fact that $F^-(Y - B) = X - F^+(B)$ and $F^+(Y - B) = X - F^-(B)$ for every subset B of Y.

(4) => (5): B_1, B_2 be any subsets of Y. Then $\mathrm{Cl}^\star(B_1)$ and $\mathrm{Cl}^\star(B_2)$ are $\sigma^\star$-closed. By (4), $\mathrm{mCl_I}(F^-(B_1) \cup F^+(B_2)) \subset \mathrm{mCl_I}(F^-(\mathrm{Cl}^\star(B_1)) \cup F^+(\mathrm{Cl}^\star(B_2))) = (F^-(\mathrm{Cl}^\star(B_1)) \cup F^+(\mathrm{Cl}^\star(B_2)))$.

(5) => (6): B_1, B_2 be any subsets of Y. By (5), we have

$$X - \mathrm{mInt}_I(F^-(B_1) \cap F^+(B_2)) = \mathrm{mCl}_I(X - (F^-(B_1) \cap F^+(B_2))) = \mathrm{mCl}_I((X - F^-(B_1)) \cup (X - F^+(B_2))) = \mathrm{mCl}_I(F^+(Y - B_1) \cup F^-(Y - B_2)) \subset F^+(\mathrm{Cl}^\star(Y - B_1)) \cup F^-(\mathrm{Cl}^\star(Y - B_2)) = (X - F^-(\mathrm{Int}^\star(B_1))) \cup (X - F^+(\mathrm{Int}^\star(B_2))) = X - (F^-(\mathrm{Int}^\star(B_1)) \cap F^+(\mathrm{Int}^\star(B_2))).$$

Therefore, we obtain $F^-(\mathrm{Int}^\star(B_1)) \cap F^+(\mathrm{Int}^\star(B_2)) \subset \mathrm{mInt_I}(F^-(B_1) \cap F^+(B_2))$.

(6) => (1): Let $x \in X$ and V_1, V_2 be any $\sigma^\star$-open sets of Y such that $F(x) \in V_1^+ \cap V_2^-$. By (6), $F^-(V_1) \cap F^+(V_2) \subset \mathrm{mInt_I}(F^-(V_1) \cap F^+(V_2))$. This shows that $F^-(V_1) \cap F^+(V_2) \in \mathrm{mIO}(X)$. And put $U = F^-(V_1) \cap F^+(V_2)$. Then U is an $\mathrm{mIO}(X)$-open set containing x such that $F(U) \subset V_1^+$ and $F(u) \cap V_2^- \neq \emptyset$ for every $u \in U$. Therefore, F is $mi^\star$-continuous.

If $\mathrm{mIO}(X) = \mathrm{SIO}(X)$, by Theorem 15 we obtain the following corollary:

Corollary 6. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following properties are equivalent:*

(1) F is $si^\star$-continuous;

(2) For each point $x \in X$ and each $\sigma^\star$-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, $x \in \mathrm{sInt_I}(F^+(V_1) \cap F^-(V_2))$;

(3) $F^+(V_1) \cap F^-(V_2) \in \mathrm{SIO}(X)$ for every $\sigma^\star$-open sets V_1, V_2 of Y;

(4) $F^-(K_1) \cup F^+(K_2)$ is semi-I-closed for every $\sigma^\star$-closed sets K_1, K_2 of Y;

(5) $\mathrm{sCl_I}(F^-(B_1) \cup F^+(B_2)) \subset F^-(\mathrm{Cl}^\star(B_1)) \cup F^+(\mathrm{Cl}^\star(B_2))$ for every subsets B_1, B_2 of Y;

(6) $F^-(\mathrm{Int}^\star(B_1)) \cap F^+(\mathrm{Int}^\star(B_2)) \subset \mathrm{sInt_I}(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

If $\mathrm{mIO}(X) = \mathrm{PIO}(X)$, by Theorem 15 we obtain the following corollary:

Corollary 7. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following properties are equivalent:*

(1) F is pi*-continuous;

(2) For each point $x \in X$ and each $\sigma^\star$-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, $x \in \mathrm{pInt_I}(F^+(V_1) \cap F^-(V_2))$;

(3) $F^+(V_1) \cap F^-(V_2) \in \mathrm{PIO}(X)$ for every $\sigma^\star$-open sets V_1, V_2 of Y;

(4) $F^-(K_1) \cup F^+(K_2)$ is pre-I-closed for every $\sigma^\star$-closed sets K_1, K_2 of Y;

(5) $\mathrm{pCl_I}(F^-(B_1) \cup F^+(B_2)) \subset F^-(\mathrm{Cl}^\star(B_1)) \cup F^+(\mathrm{Cl}^\star(B_2))$ for every subsets B_1, B_2 of Y;

(6) $F^-(\mathrm{Int}^\star(B_1)) \cap F^+(\mathrm{Int}^\star(B_2)) \subset \mathrm{pInt_I}(F^-(B_1) \cap F^+(B_2))$ for every subsets B_1, B_2 of Y.

Theorem 16. *The set of all points $x \in X$ at which a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is not mi*-continuous is identical with the union of the m_I-frontiers of the intersection of upper/lower inverse images of $\star$-open sets containing/meeting $F(x)$.*

Proof. The proof follows similarly from Theorem 11.

If $\mathrm{mIO}(X) = \tau^\star$, then we obtain the following corollary:

Corollary 8. *The set of all points $x \in X$ at which a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is not $\tau^\star$-continuous is identical with the union of the $\tau^\star$-frontiers of the intersection of upper/lower inverse images of $\star$-open sets containing/meeting $F(x)$.*

If $\mathrm{mIO}(X) = \mathrm{SIO}(X)$, then we obtain the following corollary:

Corollary 9. *The set of all points $x \in X$ at which a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is not si*-continuous is identical with the union of the S_I-frontiers of the intersection of upper/lower inverse images of $\star$-open sets containing/meeting $F(x)$.*

References

[1] M. E. Abd El-Monsef, S. N. El-Deep and R. A. Mahmoud, *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ. **12** (1983), 77–90.

[2] M. Akdag, *On upper and lower I-continuous multifunctions*, Far East J. Math. Sci. **25** (2007), 48–57.

[3] D. Andrijević, *On b-open sets,* Mat. Vesnik **48** (1996), 59–64.

[4] C. Arivazhagi and N. Rajesh, *Nearly I-continuous multifunctions*, Bol. Soc. Paran. Mat. **37** (2019), 33–38.

[5] G. Aslim and A. Cakusu Culer, *b-I-open sets and decompositions of continuity via idealizations* Proc. Inst. Math. Acad. Nat. Acad. Sci. Azerbaidjan **22** (2003), 27–32.

[6] T. Bânzaru, *Multifunctions and M-product spaces (Romanian),* Bul. St. Tehn. Inst. Politehn. "Traian. Vuia", Timişoara, Ser. Mat. Fiz. Mec. Teor. Appl. **17**(**31**) (1972), 17–23.

[7] T. Bânzaru et N. Crivăţ, *Structures uniformes sur l'espace des parties d'un espace uniform et quasicontinuité des applications multivoques,* Bul. St. Tehn. Inst. Politehn. "Traian. Vuia", Timişoara, Ser. Mat. Fiz. Mec. Teor. Appl. **20**(**34**) (1975), 135–136.

[8] C. Boonpok, *On continuous multifunctions in ideal topological spaces*, Lobacewscki J. Math. **1** (2009), 24–35.

[9] C. Boonpok and P. Pue-on *Continuity for multifunctions in ideal topological spaces*, WSEAS Trans. Math. **19** (2020), 624–631.

[10] J. Dontchev, *On pre-$\mathcal{I}$-open sets and a decomposition of $\mathcal{I}$-continuity*, Banyan Math. J. **2** (1996).

[11] E. Ekici, *On AC_I-sets, BC_I-sets, $\beta_I^\star$-open sets and decomopositions of continuity in ideal topological spaces* Creat. Math. Inform. **20**(1) (2011), 47–54.

[12] E. Ekici and T. Noiri, *$\star$-hyperconnected ideal topological spaces*, Anal. St. Univ. Alexandru I. Cusi, Iasi (N.S.) Mat. **58**(1) (2012), 121–129.

[13] J. Ewert and T. Neubrunn, *On quasicontinuous multivalued maps,* Demonstratio Math. **21** (1988), 697–711.

[14] E. Hatir and S. Jafari, *On weakly semi-I-open sets and other decomposition of continuity via ideals*, Sarajevo J. Math. **14** (2006), 107–114.

[15] E. Hatir, A. Keskin and T. Noiri, *On a new decomposition of continuity via idealization*, JP J. Geometry Toplogy **3**(1) (2003), 53–64.

[16] E. Hatir and T. Noiri, *On decompositions of continuity via idealization*, Acta Math. Hungar. **96**(4) (2002), 341–349.

[17] E. Hatir and T. Noiri, *On β-I-open sets and decompositions of almost-I-continuity*, Bull. Malays. Math. Sci. Soc. (2) **29**(1) (2006), 119–124.

[18] D. Janković and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly **97** (1990), 295–310.

[19] I. Kovačević, *Subsets and paracompactness,* Univ. u Novom Sadu, Zb. Rad. Prirod. Mat. Fac. Ser. Mat. **14** (1984), 79–87.

[20] K. Kuratowski, Topology, Vol. I, Acadmic Press, New York, 1966.

[21] N. Levine, *Semi-open sets and semi-continuity in topological spaces,* Amer. Math. Monthly **70** (1963), 36–41.

[22] H. Maki, C. K. Rao and A. Nagoor Gani, *On generalizing semi-open and preopen sets*, Pure Appl. Math. Sci. **49** (1999), 17–29.

[23] S. Marcus, *Sur les fonctions quasicontinues au sens de S. Kempisty*, Colloq. Math. **8** (1961), 47–53.

[24] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, *On precontinuous and weak precontinuous mappings,* Proc. Math. Phys. Soc. Egypt **53** (1982), 47–53.

[25] J. M. Mustafa, S. Al Ghour and K.Al Zoubi, *Weakly b-I-open sets and weakly b-I-continuous functiions*, Ital. J. Pure Appl. Math. **N.30** (2013), 23–32.

[26] T. Neubrunnova, *On certain generalizations of the notion of continuity*, Mat. Casopis **23** (1973), 374–380.

[27] O. Njåstad, *On some classes of nearly open sets,* Pacific J. Math. **15** (1965), 961–970.

[28] T. Noiri and V. Popa, *On upper and lower M-continuous multifunctions*, Filomat **14** (2000), 73–86.

[29] T. Noiri and V. Popa, *A unified theory on the points of discontinuity for multifunctions*, Bull. Math. Soc. Sci. Math. Roumanie **45**(**93**)(1-2) (2002), 97–107.

[30] T. Noiri and V. Popa, *A unified theory of contra-I-continuous functions in ideal topological spaces,* Anal. Univ. Sci. Budapest. Math. **63** (2020), 3–17.

[31] T. Noiri and V. Popa, *Faintly m-I-continuous multifunctions,* Eur. Bull. Math. **3**(2) (2020), 103–113.

[32] V. Popa, *Multifonctions semi-continues,* Rev. Roumaine Math. Pures Appl. **27** (1982), 807–815.

[33] V. Popa, *Some characterizations of quasicontinuous and weakly continuous multifunctions (Romanian)*, Stud. Cerc. Mat. **37** (1985), 77–82.

[34] V. Popa, *Some properties of H-almost continuous multifunctions,* Problemy Mat. **10** (1988), 9–26.

[35] V. Popa and T. Noiri, *On upper and lower α-continuous multifunctions*, Math. Slovaca **43** (1993), 477–491.

[36] V. Popa and T. Noiri, *Characterizations of α-continuous multifunctions*, Univ. u Novom Sadu, Zb. Rad. Prirod Mat. Fac. Ser. Mat. **23** (1993), 29–38.

[37] V. Popa and T. Noiri, *Some properties of β-continuous multifunctions,* Anal. St. Univ. "Al. I. Cuza" Iaşi 42, Supl. s.I.a, Mat. (1996), 207–215.

[38] V. Popa and T. Noiri, *On upper and lower β-continuous multifunctions,* Real Anal. Exchange **22** (1996/97), 362–376.

[39] V. Popa and T. Noiri, *A note on precontinuity and quasicontinuity for multifunctions*, Demonstratio Math. **30** (1997), 271–278.

[40] V. Popa and T. Noiri, *On M-continuous functions,* Anal. Univ. "Dunărea de Jos" Galaţi, Ser. Mat. Fiz. Mec. Teor., Fasc. II **18** (**23**) (2000), 31–41.

[41] V. Popa and T. Noiri, *On the definitions of some generalized forms of continuity under minimal conditions*, Mem. Fac. Sci. Kochi Univ. Math. Ser. **22** (2001), 9–19.

[42] V. Popa and T. Noiri, *On m-continuous multifunctions,* Bul. St. Univ. Politeh. Timisoara, Ser. Mat. Fiz. **46** (**60**)(2) (2001), 1–12.

[43] V. Popa and T. Noiri, *A unified theory of weak continuity for functions,* Rend. Circ. Mat. Palermo (2) **51** (2002), 439–464.

[44] V. Popa and T. Noiri, *A unified theory of the points of continuity and discontinuity for multifunctions*, Annal. Univ. de Vest Timisoara Ser. Math. Inform. **61**(1) (2003), 9–19.

[45] V. Popa and T. Noiri, *Upper and lower m-I-continuous multifunctions*, Sci. Stud. Res. Math. Inform. **29**(2) (2019), 51–64.

[46] M. Przemski, *Some generalizations of continuity and quasicontinuity of multivalued maps,* Demonstratio Math. **26** (1993), 381–400.

[47] R. Vaidyanathaswani, *The localization theory in set-topology*, Proc. Indian Acad. Sci. **20** (1945), 51–61.

[48] J. D. Wine, *Locally paracompact spaces,* Glasnik Mat. Ser. III **10**(**30**) (1975), 351–357.