EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 15, No. 1, 2022, 281-289
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Almost bi interior ideal in semigroups and their fuzzifications

T. Gaketem

Fuzzy Algebras and Decision-Making Problems Research Unit, Department of Mathematics /School of Science, University of Phayao, Phayao 56000, Thailand

Abstract. In this paper, we define the concepts of almost bi-interior ideal and fuzzy almost bi-interior ideal in semigroups. Moreover, we prove that relation between almost bi-interior ideal and fuzzy almost bi-interior ideal.

2020 Mathematics Subject Classifications: 20M12, 06F05
Key Words and Phrases: bi-interior ideal, fuzzy bi-interior ideal, weakly bi-interior ideal, weakly fuzzy bi-interior ideal

1. Introduction

Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague, which introduced by Zadeh in 1965 [9]. Fuzzy set theory became a phenomenon since its logic can deal with information that is imprecise, vague, partially true, or without sharp boundaries. The reader may for a compilation of articles on fuzzy sets, fuzzy logic and their applications. In 1979 Kuroki used fuzzy set in semigroup and chracterizations properties of fuzzy semigroup. The almost ideal theory in semigroups studied by Grosek and Satko in 1980 [3]. In 1981, Bogdanvic, [4] established definitions of almost bi-ideals in semigroups and studies properties of almost bi-ideals in semigroups. Later, Chinram give definition the definitions of types of alomst ideals in semigroups such that almost quasi-ideal [7]. In 2020, N. Kaopusek et al. [6] discussed almost interior ideals and weakly almost interior ideals of semigroups by using the concepts of almost ideals and interior ideals of semigroups and investigated their properties. Recently, R. Chinram and W. Nakkhasen [2] sutdied properteis of almost bi-quasi-interior ideals and their fuzzy bi-interior ideals in semigroups. Moreover, the concept of almost interior ideal has been discussed in other research sucht that [1], [8]. Krishna and Rao gave definition of bi-interior ideal in semigroups in 2018 [5].

In this paper, we give definition of almost bi-interior ideal and fuzzy almost bi-interior ideal in semigroups. Moreover, we prove that relation between almost bi-interior ideal and fuzzy almost bi-interior ideal.

DOI: https://doi.org/10.29020/nybg.ejpam.v15i1.4279

Email address: thiti.ga@up.ac.th (T. Gaketem)

 © 2022 EJPAM All rights reserved.

2. Preliminaries

In this section we give some concepts and results, which will be helpful in later sections.

Definition 1. *[5] A non-empty subset M of semigroup $\mathcal{S}$ is called*

(1) *a subsemigroup of $\mathcal{S}$ if $M^2 \subseteq M$,*

(2) *a left (right) ideal of $\mathcal{S}$ if $\mathcal{S}M \subseteq M \quad (M\mathcal{S} \subseteq M)$. By an* ideal *$M$ of a semigroup $\mathcal{S}$ we mean a left ideal and a right ideal of $\mathcal{S}$,*

(3) *a bi-ideal of $\mathcal{S}$ if M is a subsemigroup of $\mathcal{S}$ and $M\mathcal{S}M \subseteq M$,*

(4) *an interior ideal of $\mathcal{S}$ if M is a subsemigroup of $\mathcal{S}$ and $\mathcal{S}M\mathcal{S} \subseteq M$,*

(5) *a* quasi-ideal *of $\mathcal{S}$ if $M\mathcal{S} \cap \mathcal{S}M \subseteq M$,*

(6) *a* left (right) almost ideal *of $\mathcal{S}$ if $aM \cap M \neq \emptyset$ $(Ma \cap M \neq \emptyset)$ for all $a \in \mathcal{S}$. By an* almost ideal *M of a semigroup $\mathcal{S}$ we mean a left almost ideal and a right almost ideal of $\mathcal{S}$,*

(7) *a* almost bi-ideal *of $\mathcal{S}$ if $MaM \cap M \neq \emptyset$ for all $a \in \mathcal{S}$,*

(8) *a* almost interior ideal *of $\mathcal{S}$ if $aMb \cap M \neq \emptyset$ for all $a, b \in \mathcal{S}$.*

(9) *a* almost quasi ideal *of $\mathcal{S}$ if $(aM \cap Ma) \cap M \neq \emptyset$ for all $a, b \in \mathcal{S}$.*

A subsemigroup M of a semigroup $\mathcal{S}$ is said to be *left (right) bi-quasi ideal* of $\mathcal{S}$ if $\mathcal{S}M \cap M\mathcal{S}M \subseteq M(M\mathcal{S} \cap M\mathcal{S}M \subseteq M)$. A subsemigroup M of a semigroup $\mathcal{S}$ is said to be *bi-quasi ideal* of $\mathcal{S}$ if it is both a left bi-quasi and right bi-quasi ideal of $\mathcal{S}$. A subsemigroup M of a semigroup $\mathcal{S}$ is said to be *bi-interior ideal* of $\mathcal{S}$ if M is a subsemigroup of $\mathcal{S}$ and $\mathcal{S}M\mathcal{S} \cap M\mathcal{S}M \subseteq M$. [5]. We note here that the properties is hold:

(1) Every left ideal is a bi-interior ideal of $\mathcal{S}$.

(2) Every right ideal is a bi-interior ideal of $\mathcal{S}$.

(3) Every ideal is a bi-interior ideal of $\mathcal{S}$.

(4) Every quasi ideal is a bi-interior ideal of $\mathcal{S}$.

(5) The arbitrary intersection of bi-interior of $\mathcal{S}$ is also bi-interior ideal of $\mathcal{S}$.

(6) If M a bi-interior ideal of $\mathcal{S}$, then $M\mathcal{S}$ and $\mathcal{S}M$ are bi-interior ideals of $\mathcal{S}$.

For any $h_i \in [0,1]$, $i \in \mathcal{F}$, define

$$\underset{i\in\mathcal{F}}{\vee} h_i := \sup_{i\in\mathcal{F}}\{h_i\} \quad \text{and} \quad \underset{i\in\mathcal{F}}{\wedge} h_i := \inf_{i\in\mathcal{F}}\{h_i\}.$$

We see that for any $h, r \in [0,1]$, we have

$$h \vee r = \max\{h, r\} \quad \text{and} \quad h \wedge r = \min\{h, r\}.$$

A *fuzzy set (fuzzy subset)* of a non-empty set E is a function $\varphi : E \rightarrow [0, 1]$.

For any two fuzzy sets φ and ξ of a non-empty set E, define the symbol as follows:

(1) $\varphi \geq \xi \Leftrightarrow \varphi(h) \geq \xi(h)$ for all $h \in E$,

(2) $\varphi = \xi \Leftrightarrow \varphi \geq \xi$ and $\xi \geq \varphi$,

(3) $(\varphi \wedge \xi)(h) = \min\{\varphi(h), \xi(h)\} = \varphi(h) \wedge \xi(h)$ and $(\varphi \vee \xi)(h) = \max\{\varphi(h), \xi(h)\} = \varphi(h) \vee \xi(h)$ for all $h \in E$,

(4) $\varphi \subseteq \xi$ if $\varphi(h) \leq \xi(h)$,

(5) $(\varphi \cup \xi)(h) = \max\{\varphi(h), \xi(h)\}$ and $(\varphi \cap \xi)(h) = \min\{\varphi(h), \xi(h)\}$ for all $h \in E$.

(6) the *support* of φ instead of $\text{supp}(\varphi) = \{h \in E \mid \varphi(h) \neq 0\}$.
For the symbol $\varphi \leq \xi$, we mean $\xi \geq \varphi$.

For any two fuzzy sets φ and ξ of a semigroup $\mathcal{S}$. The product of fuzzy subsets φ and ξ of $\mathcal{S}$ is defined as follow, for all $h \in \mathcal{S}$

$$(\varphi \circ \xi)(h) = \begin{cases} \bigvee\limits_{h=yz} \{\varphi(y) \wedge \xi(z)\} & \text{if} \quad h = yz, \\ 0 & \text{otherwise.} \end{cases}$$

The characteristic function of a subset M of a nonempty set $\mathcal{S}$ is a fuzzy set of $\mathcal{S}$

$$\lambda_M(h) = \begin{cases} 1 & \text{if} \quad h \in M \\ 0 & \text{if} \quad h \notin M. \end{cases}$$

for all $h \in S$.

Definition 2. *[5] A fuzzy set φ of a semigroup $\mathcal{S}$ is said to be*

(1) *a fuzzy subsemigroup of $\mathcal{S}$ if $\varphi(hr) \geq \varphi(h) \wedge \varphi(r)$, for all $h, r \in \mathcal{S}$,*

(2) *a fuzzy left (right) ideal of $\mathcal{S}$ if $\varphi(hr) \geq \varphi(r)$ $(\varphi(hr) \geq \varphi(h))$, for all $h, r \in S$. A fuzzy ideal of $\mathcal{S}$ if it is both a fuzzy left ideal and a fuzzy right ideal of $\mathcal{S}$,*

(3) *a fuzzy bi-ideal of $\mathcal{S}$ if φ is a fuzzy subsemigroup of $\mathcal{S}$ and $\varphi(hrk) \geq \varphi(h) \wedge \varphi(k)$ for all $h, r, k \in \mathcal{S}$,*

(4) *a fuzzy interior ideal of $\mathcal{S}$ if φ is a fuzzy subsemigroup of $\mathcal{S}$ and $\varphi(hrk) \geq \varphi(r)$ for all $h, r, k \in \mathcal{S}$,*

(5) *a fuzzy quasi-ideal of $\mathcal{S}$ if $\varphi(h) \geq (\mathcal{S} \circ \vartheta)(h) \wedge (\varphi \circ \mathcal{S})(h)$ for all $h \in S$ where $\mathcal{S}$ is a fuzzy subset of $\mathcal{S}$ mapping every element of $\mathcal{S}$ to 1,*

(6) *a fuzzy bi-interior ideal of* $\mathcal{S}$ *if* $(\lambda_M \circ \varphi \circ \lambda_M) \wedge (\varphi \circ \lambda_M \circ \varphi) \subseteq \varphi$.

The definition of fuzzy point of a set. For $h \in \mathcal{S}$ and $t \in (0, 1]$, a fuzzy point p_δ of a set $\mathcal{S}$ is a fuzzy subset of $\mathcal{S}$ defined by

$$p_\delta(h) = \begin{cases} \delta & \text{if } h = r \\ 0 & \text{if } r \neq h. \end{cases}$$

Definition 3. *[2] A fuzzy set* φ *of a semigroup* $\mathcal{S}$ *is said to be*

(1) *a fuzzy left (right) almost ideal of* $\mathcal{S}$ *if* $(p_\delta \circ \varphi) \wedge \varphi \neq 0$ $((\varphi \circ p_\delta) \wedge \varphi \neq 0)$ *for all fuzzy point* p_δ*. A fuzzy almost ideal of* $\mathcal{S}$ *if it is both a fuzzy left almost ideal and a fuzzy right almost ideal of* $\mathcal{S}$*,*

(2) *a fuzzy almost bi-ideal of* $\mathcal{S}$ *if* $(p_\delta \circ \varphi \circ p_\delta) \wedge \varphi \neq 0$ *for all fuzzy point* p_δ*.*

(3) *a fuzzy almost interior ideal of* $\mathcal{S}$ *if* $(\varphi \circ p_\delta \circ \varphi) \wedge \varphi \neq 0$ *for all fuzzy point* p_δ*.*

(4) *a fuzzy almost quasi-ideal of* $\mathcal{S}$ *if* $[(p_\delta \circ \varphi) \wedge (\varphi \circ p_\delta)] \wedge \varphi \neq 0$ *for all fuzzy point* p_δ*.*

3. Almost Bi-interior Ideals in semigroups.

In this section, we define the notions of almost bi-interior ideals, weakly almost bi-interior ideals in ordered semigroups and some properties of them are investigated.

Definition 4. *A nonempty set* M *of a semigroup* $\mathcal{S}$ *is called a*

(1) *almost bi-interior ideal of* $\mathcal{S}$ *if* $(hMr \cap MnM) \cap M \neq \emptyset$*, for all* $h, r, n \in \mathcal{S}$

(2) *weakly almost bi-interior ideal of* $\mathcal{S}$ *if* $(hMh \cap MhM) \cap M \neq \emptyset$*, for all* $h \in \mathcal{S}$*.*

Theorem 1. *Let* $\mathcal{S}$ *be a semigroup. Then the following statement hold.*

(1) *Every bi-interior ideal of* $\mathcal{S}$ *is an almost bi-interior ideal of* $\mathcal{S}$*.*

(2) *Every weak bi-interior ideal of* $\mathcal{S}$ *is an weak almost bi-interior ideal of* $\mathcal{S}$*.*

Proof. Suppose that M is a bi-interior ideal of $\mathcal{S}$ and let $h, r, n \in \mathcal{S}$. Then $(hMr \cap MnM) \neq \emptyset$. Thus $(hMr \cap MnM) \subseteq (\mathcal{S}M\mathcal{S} \cap M\mathcal{S}M) \subseteq M$. It implies that $(hMr \cap MnM) \cap M \subseteq (\mathcal{S}M\mathcal{S} \cap M\mathcal{S}M) \neq \emptyset$. Hence M is an almost bi-interior ideal of $\mathcal{S}$.

The proof of the other similar to the proof (1).

Theorem 2. *Let* M *and* L *be nonempty subsets a semigroup of* $\mathcal{S}$ *with* $M \subseteq L$*. Then the following statement hold.*

(1) *If* M *is an almost bi-interior ideal of* $\mathcal{S}$*, then* L *is an almost bi-interior ideal of* $\mathcal{S}$*.*

(2) *If* M *is a weak almost bi-interior ideal of* $\mathcal{S}$*, then* L *is a weak almost bi-interior ideal of* $\mathcal{S}$*.*

Proof. Suppose that M is an almost bi-interior ideal of $\mathcal{S}$ and $h, n, r \in \mathcal{S}$. Then $(hMr \cap MnM) \neq \emptyset$. Thus $(hMr \cap MnM) \subseteq (kLr \cap LnL) \neq \emptyset$. By assumption, $(hMr \cap MnM) \cap M \neq \emptyset$. It implies that $\emptyset \neq (hMr \cap MnM) \cap M \subseteq (hLr \cap LnL) \cap L$. Thus $(hLr \cap LnL) \cap L \neq \emptyset$. Hence L is an almost bi-interior ideal of $\mathcal{S}$.

The proof of the other similar to the proof (1).

Corollary 1. *Let $\mathcal{S}$ be a semigroup. Then the following statement hold.*

(1) *The finite union almost bi-interior ideal of $\mathcal{S}$ is an almost bi-interior ideal of $\mathcal{S}$.*

(2) *The finite union weak almost bi-interior ideal of $\mathcal{S}$ is an weak almost bi-interior ideal of $\mathcal{S}$.*

4. Fuzzy Almost Bi-Interior Ideals and weak Fuzzy Almost Bi-Interior Ideals.

In this section, we define the notions of fuzzy almost bi-interior ideals and weakly fuzzy bi-interior ideal in semigroups and some properties of them are investigated.

Definition 5. *A nonzero fuzzy set φ of a semigroup $\mathcal{S}$ is called a*

(1) *fuzzy almost bi-interior ideal of $\mathcal{S}$ if $((p_\delta \circ \varphi \circ q_{\delta_1}) \wedge (\varphi \circ g_{\delta_2} \circ \varphi)) \wedge \varphi \neq 0$, for fuzzy point p_δ, q_{δ_1} and g_{δ_2} of $\mathcal{S}$.*

(2) *weakly almost interior ideal of $\mathcal{S}$ if $((p_\delta \circ \varphi \circ p_{\delta_1}) \wedge (\varphi \circ p_{\delta_2} \circ \varphi)) \wedge \varphi \neq 0$, for fuzzy point p_δ, p_{δ_1} and p_{δ_2} of $\mathcal{S}$.*

The following theorem easily to prove.

Theorem 3. *Let φ be a nonzero fuzzy subset of a semigroup $\mathcal{S}$. Then the following statement hold.*

(1) *Every fuzzy bi-interior ideal of $\mathcal{S}$ is a fuzzy almost bi-interior ideal of $\mathcal{S}$.*

(2) *Every weak fuzzy bi-interior ideal of $\mathcal{S}$ is a weak fuzzy almost bi-interior ideal of $\mathcal{S}$.*

Theorem 4. *Let φ and ν be a nonzero fuzzy sets of a semigroup $\mathcal{S}$ with $\varphi \preceq \xi$. Then the following statement hold.*

(1) *If φ is a fuzzy almost bi-interior ideal of $\mathcal{S}$, then ξ is a fuzzy almost bi-interior ideal of $\mathcal{S}$.*

(2) *If φ is a weakly fuzzy almost bi-interior ideal of $\mathcal{S}$, then ξ is a weakly almost bi-ideal of $\mathcal{S}$.*

Proof. Suppose that φ is a fuzzy almost bi-interior ideal of $\mathcal{S}$ and $p_\delta, q_{\delta_1}, g_{\delta_2} \in (0, 1]$. Then $(((p_\delta \circ \varphi \circ q_{\delta_1}) \wedge (\varphi \circ g_{\delta_2} \circ \varphi)) \wedge \varphi \neq 0$. By assumption, $(((p_\delta \circ \varphi \circ q_{\delta_1}) \wedge (\varphi \circ g_{\delta_2} \circ \varphi)) \wedge \varphi \preceq (((p_\delta \circ \xi \circ q_{\delta_1}) \wedge (\xi \circ g_{\delta_2} \circ \xi)) \wedge \xi \neq 0$. Thus $(((p_\delta \circ \xi \circ q_{\delta_1}) \wedge (\xi \circ g_{\delta_2} \circ \xi)) \wedge \xi \neq 0$. Hence ξ is a fuzzy almost bi-interior ideal of $\mathcal{S}$.

The proof of the other similar to the proof (1).

Corollary 2. *Let $\mathcal{S}$ be an ordered semigroup. Then the following statement hold.*

(1) *The finite union fuzzy almost bi-interior ideal of $\mathcal{S}$ is a fuzzy almost bi-interior ideal of $\mathcal{S}$.*

(2) *The finite union weakly fuzzy almost bi-interior ideal of $\mathcal{S}$ is a weakly fuzzy almost bi-interior ideal of $\mathcal{S}$.*

Theorem 5. *Let M be a nonempty subset of an ordered semigroup $\mathcal{S}$. Then the following statement hold.*

(1) *M is an almost bi-interior ideal of $\mathcal{S}$ if and only if λ_M is a fuzzy almost bi-interior ideal of $\mathcal{S}$.*

(2) *M is a weakly almost interior ideal of $\mathcal{S}$ if and only if λ_M is a weakly fuzzy almost bi-interior ideal of $\mathcal{S}$.*

Proof. Suppose that M is an almost bi-interior ideal of $\mathcal{S}$, $h, r, n \in \mathcal{S}$ and $\delta, \delta_1, \delta_2 \in (0, 1]$. Then $(hMr \cap MnM) \cap M \neq \emptyset$. Thus there exists $c \in \mathcal{S}$ such that $c \in (hMr \cap MnM)$ and $c \in M$ So $c = hm_1r$ and $c = m_2nm_3$ for some $m_1, m_2, m_3 \in M$. It follows that

$$\begin{aligned}(p_\delta \circ \lambda_M \circ q_{\delta_1}) \wedge (\lambda_M \circ g_{\delta_2} \circ \lambda_M)(c) &= \Big(\bigvee_{c=hm_1r} \{p_\delta(h) \wedge \lambda_M(m_1) \wedge q_{\delta_1}(r)\}\Big) \\ &\wedge \Big(\bigvee_{c=m_2nm_3} \{\lambda_M(m_2) \wedge g_{\delta_2}(n) \wedge \lambda_M(m_3)\}\Big) \\ &\neq 0,\end{aligned}$$

and $\lambda_M(c) = 1$. Thus $(p_\delta \circ \lambda_M \circ q_{\delta_1}) \wedge (\lambda_M \circ g_{\delta_2} \circ \lambda_M) \wedge \lambda_M \neq 0$.
Hence λ_M is a fuzzy almost bi-interior ideal of $\mathcal{S}$.

For the converse, assume that λ_M is a fuzzy almost bi-interior ideal of $\mathcal{S}$, let $h, r, n \in \mathcal{S}$ and $\delta, \delta_1, \delta_2 \in (0, 1]$. Then $(p_\delta \circ \lambda_M \circ q_{\delta_1}) \wedge (\lambda_M \circ g_{\delta_2} \circ \lambda_M) \wedge \lambda_M \neq 0$. Thus there exists $c \in S$ such that $(p_\delta \circ \lambda_M \circ q_{\delta_1}) \wedge (\lambda_M \circ g_{\delta_2} \circ \lambda_M)(c) \neq 0$ and $\lambda_M(c) \neq 0$. So $c \in (hMr \cap MnM)$ and $c \in M$ implies that $(hMr \cap MnM) \cap M \neq \emptyset$. Therefore M is an almost bi-interior ideal of $\mathcal{S}$.

The proof of the other similar to the proof (1).

Theorem 6. *Let φ be a nonzero fuzzy set of a semigroup $\mathcal{S}$. Then the following statement hold.*

(1) *φ is a fuzzy almost bi-interior ideal of $\mathcal{S}$ if and only if supp(φ) is an almost bi-interior ideal of $\mathcal{S}$.*

(2) *φ is a weakly fuzzy almost bi-interior ideal of $\mathcal{S}$ if and only if supp(φ) is a weakly almost bi-interior ideal of $\mathcal{S}$.*

Proof. Suppose that φ is a fuzzy almost bi-interior ideal of $\mathcal{S}$, $p, q, g \in \mathcal{S}$ and $\delta, \delta_1, \delta_2 \in (0, 1]$. Then $(p_\delta \circ \varphi \circ q_{\delta_1}) \wedge (\varphi \circ g_{\delta_2} \circ \varphi) \wedge \varphi \neq 0$. Thus there exist $c \in S$ such that $(p_\delta \circ \varphi \circ q_{\delta_1}) \wedge (\varphi \circ g_{\delta_2} \circ \varphi)(c) \neq 0$ and $\varphi(c) \neq 0$. So there exist h, r, n such that $c = hm_1r$ and $c = m_2nm_3$ It follows that

$$(\bigvee_{c=hm_1r}\{p_\delta(h)\wedge\varphi(m_1)\wedge q_{\delta_1}(r)\})\wedge(\bigvee_{c=m_2nm_3}\{\varphi(m_2)\wedge g_{\delta_2}(n)\wedge\varphi(m_3)\})$$
$$=(p_\delta\circ\varphi\circ q_{\delta_1})\wedge(\varphi\circ g_{\delta_2}\wedge\varphi)\neq 0.$$

Thus $\varphi(m_1)\neq 0$, $\varphi(m_2)\neq 0$, $\varphi(m_3)\neq 0$ so $m_1,m_2,m_3\in\text{supp}(\varphi)$ implies that $(p_\delta\circ\lambda_{supp(\varphi)}\circ q_{\delta_1})\wedge(\lambda_{supp(\varphi)}\circ g_{\delta_2}\wedge\lambda_{supp(\varphi)})(c)\neq 0$. Thus $\lambda_{supp(\varphi)}$ is a fuzzy almost bi-interior ideal of $\mathcal{S}$. By Theorem 5, supp(φ) is an almost bi-interior ideal of $\mathcal{S}$.

For the converse, assume that supp(φ) is an almost bi-interior ideal of $\mathcal{S}$. By Theorem 5, $\lambda_{supp(\varphi)}$ is a fuzzy almost bi-interior ideal of $\mathcal{S}$. Thus for any fuzzy point $p_\delta, q_{\delta_1}, g_{\delta_2}$ of $\mathcal{S}$ such that

$$(p_\delta\circ\lambda_{supp(\varphi)}\circ q_{\delta_1})\wedge(\lambda_{supp(\varphi)}\circ g_{\delta_2}\circ\lambda_{supp(\varphi)})\neq 0.$$

So there exists $c\in\mathcal{S}$ such that $(p_\delta\circ\lambda_{supp(\varphi)}\circ q_{\delta_1})\wedge(\lambda_{supp(\varphi)}\circ g_{\delta_2})(c)\neq 0$ and $\lambda_{supp(\varphi)}(c)\neq 0$. Thus there exist $h,r,n\in\mathcal{S}$ such that $c=hm_1r$ and $c=m_2nm_3$ It follows that

$$(\bigvee_{c=hm_1r}\{p_\delta(h)\wedge\lambda_{supp(\varphi)}(m_1)\wedge q_{\delta_1}(r)\})\wedge(\bigvee_{c=m_2nm_3}\{\lambda_{supp(\varphi)}(m_2)\wedge g_{\delta_2}(n)\wedge\lambda_{supp(\varphi)}(m_3)\})$$
$$\neq 0.$$

So $\lambda_{supp(\varphi)}(m_1)\neq 0$, $\lambda_{supp(\varphi)}(m_2)\neq 0$, $\lambda_{supp(\varphi)}(m_3)\neq 0$ implies that
$\varphi(m_1)\neq 0$, $\varphi(m_2)\neq 0$, $\varphi(m_3)\neq 0$. Hence

$$(p_\delta\circ\lambda_{supp(\varphi)}\circ q_{\delta_1})\wedge(\lambda_{supp(\varphi)}\circ g_{\delta_2})(c)$$
$$=(\bigvee_{c=hm_1r}\{p_\delta(h)\wedge\varphi(m_1)\wedge q_{\delta_1}(r)\})\wedge(\bigvee_{c=m_2nm_3}\{\varphi(m_2)\wedge g_{\delta_2}(n)\wedge\varphi(m_3)\})\neq 0.$$

It follows that $(p_\delta\circ\varphi\circ q_{\delta_1})\wedge(\varphi\circ g_{\delta_2}\circ\varphi)\wedge\varphi\neq 0$ for any fuzzy point $p_\delta, q_{\delta_1}, g_{\delta_2}$ of $\mathcal{S}$ Therefore φ is a fuzzy almost bi-interior ideal of $\mathcal{S}$.

The proof of the other similar to the proof (1).

Next, we define minimal fuzzy almost bi-interior ideals in semigroups and study the between minimal almost bi-interior ideals and minimal fuzzy almost bi-interior ideals of semigroups.

Definition 6. *A fuzzy almost bi-interior ideal (weak bi-interior ideal) φ of a semigroup $\mathcal{S}$ is called* minimal *if for any fuzzy almost bi-interior ideal (weak bi-interior ideal) ξ of $\mathcal{S}$ if whenever $\xi\subseteq\varphi$, then sup(ξ) =sup(φ).*

Theorem 7. *Let M be a nonempty subset of a semigroup $\mathcal{S}$. Then*

(1) *M is a minimal almost bi-interior ideal of $\mathcal{S}$ if and only if λ_M is a minimal fuzzy almost bi-interior ideal of $\mathcal{S}$.*

(2) *M is a minimal weak almost bi-interior ideal of $\mathcal{S}$ if and only if λ_M is a minimal fuzzy weak almost bi-interior ideal of $\mathcal{S}$.*

Proof. Assume that M is a minimal almost bi-interior ideal of $\mathcal{S}$. By Theorem 5, λ_M is a fuzzy almost bi-interior ideal of $\mathcal{S}$. Let ξ be a fuzzy almost bi-interior ideal of $\mathcal{S}$ such that $\xi \subseteq \lambda_M$ Then $\text{supp}(\xi) \subseteq \text{supp}(\lambda_M) = M$. By Theorem 6, $\text{supp}(\xi)$ is an almost bi-interior ideal of $\mathcal{S}$. Since K is minimal we have $\text{supp}(\xi) = K = \text{supp}(\lambda_K)$. Therefore, λ_K is minimal of $\mathcal{S}$.

Conversely, suppose that λ_K is a minimal BF almost interior ideal of $\mathcal{S}$.
By Theorem 5, K is an almost bi-interior ideal of $\mathcal{S}$. Let M be an almost bi-interior ideal of $\mathcal{S}$ such that $M \subseteq K$. Then λ_K is a fuzzy almost interior ideal of $\mathcal{S}$ such that $\lambda_M \subseteq \lambda_K$. Hence, $M = \text{supp}(\lambda_M) = \text{supp}(\lambda_K) = K$. Therefore, K is minimal of $\mathcal{S}$.

The proof of the other similar to the proof (1).

5. Conclusion

The union of two almost bi-interior ideal, and weakly bi-interior ideal is also an almost bi-interior ideal and weakly bi-interior ideal respectively in semigroups and results in class fuzzifications is the same. We prove relationship between almost bi-interior ideal, weakly bi-interior ideal and class fuzzifications. In the future work, we can study bi-interior ideals and their fuzzifications in algebraic structures.

References

[1] W. Yonthanthum A. Simuen and R. Chinram. Almost interior gamma ideals and fuzzy almost interior gamma ideals in gamma semigroups. *Mathematics and Staistics*, 9:302–308, 2021.

[2] R. Chinram and W. Nakkhasen and. Almost bi-quasi-interior ideals and fuzzy almost bi-quasiinterior ideals of semigroups. *Journal of Mathematics and Computer Science*, 26:128–136, 2022.

[3] O. Grosek and L. Satko. A new notion in the theory of semigroup. *Semigroup Forum*, 20:233–240, 1980.

[4] O. Grosek and L. Satko. On minimal A-ideals of semigroups. *Semigroup Forum*, 23:283–295, 1981.

[5] M. Krishna and M. Rao. Bi-interior ideals of semigroups. *Discussiones Mathematicae General algebra and applications*, 38:69–78, 2018.

[6] T. Kaewnoi N. Kaopusek and R. Chinram. On almost interior ideals and weakly almost interior ideals in semigroups. *Journal of Discrecte Mathematical Science and Cryptography*, 2020.

[7] K. Wattanatripop S. Suebsung and R. Chinram. Characterizing almost quasi-Γ-ideals and fuzzy almost quasi-Γ-ideals of Γ-semigroups. *Communications in Mathematics and Applications*, 11:233–240, 2020.

[8] A. Simuen R. Chinram W. Krailoet and P. Petchkaew. A note on fuzzy almost interior ideals in semigroups. *Mathematics and Staistics*, 9:302–308, 2021.

[9] L.A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.