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An Approach of Estimating the Value at Risk of Heavy-tailed Distribution using Copulas

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Abstract. The value at risk (VaR) plays a fundamental role in modeling risk in financial studies. We propose a approach in estimating the VaR for heavy-tailed distribution by taking into account the effects of certain covariates on the variable of interest. This method, involves estimating the extreme conditional quantiles by using the assciated copula. Morever, we use Bernstein copulas to estimate the intermediate conditional quantile in a non-parametric approach of the direct method. Then, the extreme conditional quantile is also estimated and we study the asymptotic properties of this new estimator.

2020 Mathematics Subject Classifications: 91G70, 60G65, 62G32, 62H05

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1. Introduction

The Value at Risk (VaR) is one of the best known risk measures in many fields such as finance and insurance. This measure quantifies the maximum loss that a portfolio manager can incur during a certain time horizon at a given confidence level. For a given level of confidence, the corresponding VaR for a random variable Y with distribution function F_0 is given by

$$VaR_Y(\alpha) = Q_Y(\alpha) = \inf\{y \in \mathbb{R} : F_0(y) \geq \alpha\}. \tag{1}$$

However, in some domains such as finance or actuarial science, the usual methods of estimating the VaR can be affected by factors such as interest rates or inflation. In the same vein, in many applications, the tail quantiles of the variable of interest Y depend on some covariate X from whom the VaR depends on. But, in practice its estimation does not take into account the effect of covariates on the variable of interest. So, it turns out to be necessary in the estimation of certain risk measures such as VaR, to take into account the influence of covariates.

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A VaR of level α being a quantile of order α, the estimation of a value at risk in the presence of covariates consists in the estimation of a conditional quantile. Let (Y, X) be a pair of random variables such that Y is the univariate response or interest variable, with marginal distribution function F_0 and X the covariate is a random vector with marginal distribution function F; the conditional distribution function will be noted $F_Y(\bullet \mid X)$. The VaR under the effects of covariates of level α is defined by

$$CoVaR_Y(\alpha) = Q_Y(\alpha \mid X = x) = \inf\{y : F_Y(y \mid x) \geq \alpha\}, \tag{2}$$

where x is a prespecified covariate vector.

Several methods of estimating extreme conditional quantiles based on extreme values theory (EVT) exist. More particularly, J. Beirlant et al.[1] used univariate extreme theory and quantile regression to estimate conditional quantiles. Based on the nearest neighbours method, Gardes and al. [2] have proposed a method for estimating extreme conditional quantiles. However, this method suffers from the lack of data in the local neighbourhood. Based also on EVT and quantile regression, Wang and al. [3] proposed an estimation method assuming the linear dependence model. Furthermore, in 2013, Wang and al.[4] proposed another estimation method by relaxing the linearity assumption while Noh et al. [5] focused on a semi-parametric method for estimating conditional quantiles based on quantile regression and copulas. In 2017, Nasri & Bouezmarni [6] proposed two parametric and semi-parametric estimation methods based on copulas.

In this study, we consider a random vector $X = (X_1; X_2; ...; X_m)^T$ of dimension m with joint distribution function F and marginal distribution function $F_1; ...; F_m$. According to Sklar's theorem [7] we can find a copula C such that

$$F(x_1; ...; x_m) = C(F_1(x_1); ...; F_m(x_m)). \tag{3}$$

If the marginals distributions functions $F_1, ..., F_m$ are continuous, then the copula C is unique. Conversely, if C is a m-dimensional copula an $F_1, ..., F_m$ univariate distributions functions, then the function F defined by (3) is a distribution function with margins $F_1, ..., F_m$ and

$$C(u_1, ..., u_m) = F(F_1^{-1}(u_1), ..., F_m^{-1}(u_m)). \tag{4}$$

More generaly, the copula C and the marginal distribution functions are not known and are estimated. Several methods of estimating copulas have been proposed. Parametric methods estimate the copula and the marginal distribution functions in a parametric way such as in Oakes [13], Romano and Joe [17]. Semi-parametric methods assume instead a parametric model for the copula and a non-parametric model for the marginal functions, see [10], [11], [14] and [23]. Finally there are the non-parametric estimation methods. It assumes a non-parametric model for the copulas and the marginal distributions. We can cite the work of Deuhevels [9], Gijbels and Mielniczuk [12].

In this work we use a non-parametric copula estimation method based on Bernstein polynomials to estimate the intermediate conditional quantile. Sancetta and Satchell [18] proposed in 2004 a method for estimating the copula function based on Bernstein polynomials called Bernstein copula. In the following, we consider the univariate case, i.e. $m = 1$, and we will use a non-parametric estimation of copulas for the estimation of our intermediate quantile.

Our method of estimating extreme conditional quantiles is based on univariate EVT and the relationship between copulas and marginal distribution functions. The method consists in to use the copulas to estimate the intermediate conditional quantile and then using the EVT to estimate the extreme conditional quantile. EVT provides a elegant mathematicals tool for analyzing rares events. Our method of estimating the extreme conditional quantile differs from existing methods in that by using copulas namely Bernstein copulas, the dependence relationship between the variable of interest and the covariate is general. In particular, Bernstein copulas provide a more adequate estimate of the underlying dependence structure.

The paper is structured as follows. In the second section, we will discuss the various preliminaries that concern extreme values theory and copulas. In the section 3 we present our estimation method and study the asymptotic behaviour of our estimator. The last section consists of the conclusion followed by a discussion.

2. Materials and methods

In the following sections, we estimate the intermediate conditional quantile based on a copulas and we introduce the notions of copulas with a focus on Bernstein ones.

2.1. An overview of extreme conditional quantiles.

We suppose in the remainder of this work that $F_Y(\bullet \mid x)$ belongs to the domain of attraction of a distribution of extreme values G_γ $(\gamma \in R)$. Let $Z_1; ...; Z_n$ be a sample of random variables with distribution F and let $M_n = \max_{1 \leq i \leq n} Z_i$. The law of M_n suitably normalised converges to

$$G_\gamma(x) = \exp\left\{-(1+\gamma z)^{\frac{-1}{\gamma}}\right\}; \tag{5}$$

when $n \to \infty$ and for $1 + \gamma z > 0$

According to the Fisher-Tippet-Gnedenko theorem (see [8]), G_γ belongs to one of the three following types of distributions:

$$G_\gamma(x) = \begin{cases} \exp\left[-(x)^{\frac{-1}{\gamma}}\right] \; for \; x \geq 0 \text{ and } \; \gamma > 0 & (Fréchet\ model) \\ \exp\left[-\exp(-x)\right] \; for \; x \in \mathbb{R} \; and \; \gamma = 0 & (Gumbel\ model) \\ \exp\left[-(-x)^{\frac{-1}{\gamma}}\right] \; for \; \; x < 0 & (Weibull\ Negative\ model) \end{cases}.$$

We are particularly interested in the case where $\gamma > 0$ i.e. the conditional distribution function $F_Y(\bullet \mid x)$ belongs to the Fréchet attraction domain. In this case the extreme conditional quantile is defined by:

$$q_Y\left(\alpha_n \mid X=x\right)=\left(\frac{1-\beta_n}{1-\alpha_n}\right)^{\gamma(x)} q_Y\left(\beta_n \mid x\right); \tag{6}$$

with β_n such that $\beta_n \to 1$ and $n(1-\beta_n) \to 0$ when $n \to \infty$ and the index of extreme values γ depends on the covariate X.

2.2. On the concept of Bernstein copulas

The use of Bernstein Copulas in our estimation have many reasons.:

- Firstly, it can approximate any behavior in the tail, and it recently has attracted attention in insurance modeling and is thus a natural candidate for further analysis. The Bernstein copulas are therefore relevant for the estimation of the CoVaR.
- Secondly, the Bernstein copula is attractive from a modeling perspective.
- Thirdly, the Bernstein copula is also suitable in higher dimensions, which is a major advantage compared to other parametric and non-parametric estimators.
- Fourthly, its mathematical properties are interesting as the Bernstein estimator converges to the underlying dependence structure, provides a higher rate of consistency than other common nonparametric estimators [22].

In what follows, we first deal with some important result on copulas, the notion of Bernstein copulas and the relationship between the conditional distribution function and copulas. Consider (Y, X) a pair of random variables with marginal distribution functions F_0 and F and joint distribution function H. According to relation (3) if F_0 and F are continuous, then C is unique and we have $C(u,v) = H(F_0^{-1}(u), F^{-1}(v))$. Given a sample of random variables $(Y_1, X_1), ..., (Y_n, X_n)$, the empirical estimator of the copula C is defined by, for all $u, v \in [0,1]$ by

$$C_n(u,v) = H_n(F_{0n}^{-1}(u), F_n^{-1}(v))$$

with

$$H_n(x,y) = n^{-1}\sum_{i=1}^{n} I(X_i \le x, Y_i \le y);$$

$$F_n(x) = H_n(x,\infty) = n^{-1}\sum_{i=1}^{n} I(X_i \le x);$$

and

$$F_{0n}(y) = H_n(\infty, y) = n^{-1}\sum_{i=1}^{n} I(Y_i \le y).$$

While modeling insurance tools, Sancetta & Satchelle [18] proposed the Bernstein copula function defined below as an estimator of the copula C

$$B_p(u,v) = \sum_{k=0}^{p}\sum_{l=0}^{p} C\left(\frac{k}{p},\frac{l}{p}\right) B(p,k,u)B(p,l,v) \tag{7}$$

where $B(p,k,u) = C_p^k u^k(1-u)^{p-k}$ is a binomial probability called Beirnstein's polynomial. Note that $\lim_{p\to\infty} B_p(u,v) = C(u,v)$ is uniform over $[0,1]^2$ since C is continuous over $[0,1]^2$. However, this estimator depends on the copula C which is always unknown. They thus proposed in the above estimator to replace the copula C by its empirical estimator C_n [18]. This estimator is given by

$$C_{p,n}(u,v) = \sum_{k=0}^{p}\sum_{l=0}^{p} C_n\left(\frac{k}{p},\frac{l}{p}\right) B(p,k,u)B(p,l,v). \tag{8}$$

In the same vein, in the following paper [19] it has been showed that the almost certain convergence and the asymptotic normality of this estimator, see [22] and [23].

2.2.1. Copulas and conditional distribution function

The conditional quantile estimation based on copulas is based on quantile regression and the asymptotic distribution of this estimator has been proven to be Gaussian (see Noh et al [5]). Authors such as Kraus and Czado [20], Nasri and Bouezmarni [21] have used copulas based on the plug-in method. More recently, Remillard, Nasri and Bouezmarni [6] have used copulas and the direct estimation method of conditional quantiles to estimate the conditional quantile. The asymptotic normality of their estimator has been studied. We consider Y the one-dimensional random variable corresponding to the response variable of marginal distribution F_0 and $X = (X_1, X_2; ...; X_m)^T$ a random vector of dimension m and of marginal distribution function $F(x) = (F_1(x_1); ...; F_m(x_m))$. We note the conditional distribution of $Y \mid X$ by

$$F_Y(y \mid x) = \mathbb{P}(Y \le y \mid X = x), \quad y \in \mathbb{R}, \quad x = (x_1, ..., x_m) \in \mathbb{R}^m. \tag{9}$$

We can establish a relation between the conditional distribution function, the copula function and the marginal distributions. This relation was established in its first version, i.e. in the case $m = 1$ in Bouyé & Salomon 14. In the case of dimension $m \ge 2$ [6], the relation is given for all real x and y by:

$$F_Y(y \mid x) = \tilde{C}(F_0(y), F(x)); \tag{10}$$

where $\tilde{C}$ is the copula such that [16]

$$\tilde{C}(F_0(y), F(x)) = \frac{\partial F_1(x_1)...\partial F_m(x_m) C(F_0(y), F_1(x_1), ..., F_m(x_m))}{\partial x_1 ... \partial x_m C(1, F_1(x_1), ..., F_m(x_m))}. \tag{11}$$

Now by definition of the copula $C(1, F(x)) = F(x)$. So, equation(11) becomes

$$\tilde{C}(F_0(y), F(x)) = \frac{\partial F_1(x_1)...\partial F_m(x_m)C(F_0(y), F_1(x_1), ..., F_m(x_m))}{f(x_1, ..., x_m)} \tag{12}$$

In the rest of our work, we consider that $m = 1$, i.e. that X is a one-dimensional random variable. We therefore consider the pair of random variables (Y, X) of marginal distribution resp F_0 and F and of conditional distribution $F_Y(\bullet \mid X)$. The relation between the copulas and the conditional distribution is now established as follows:

$$F_Y(y \mid x) = \tilde{C}(F_0(y), F(x)) = \frac{\partial C(F_0(y), F(x))}{\partial F(x)}. \tag{13}$$

The conditional quantile function can be defined as the inverse of the conditional distribution function. For a conditional quantile of order α we have

$$Q_Y(\alpha \mid x) = F_0^{-1}(\Gamma(\alpha, F(x))) \tag{14}$$

where $\Gamma(\alpha, v)$ is the quantile of order α of the distribution function $\tilde{C}(u, v)$ that is to say $\Gamma(\alpha, v) = \inf\{u \in [0, 1] \mid \Gamma(u, v) = \alpha\}$ and F_0^{-1} the generalized inverse of F_0. In general, neither the copula $\tilde{C}$, nor the marginal distribution functions F_0 and F are known. They must therefore be estimated. Using this method of estimating the conditional quantile we will estimate the intermediate quantile. Concretely, we will use the Bernstein copula estimator to estimate the copula $\tilde{C}$ and the empirical estimate for the marginal distribution functions.

3. Mains Results

3.1. Intermediate conditional quantile estimating by copulas

Let α_n be the order of the quantile such that $\alpha_n \to 1$ when $n \to \infty$. The conditional quantile is said to be intermediate if $n(1 - \alpha_n) \to \infty$ and extreme if $n(1 - \alpha_n) \to K$ where K is a constant. In this work, we estimate an extreme VaR under the effects of covariates and therefore an extreme conditional quantile. Extreme value theory provides an ideal framework for these types of estimates.

One considers a sequence (β_n) defined such that $\beta_n \to 1$ and $n(1-\beta_n) \to \infty$ when $n \to \infty$, the conditional quantile of order β_n is an intermediate quantile noted $Q_Y(\beta_n \mid X = x)$. We will thus estimate this intermediate conditional quantile by using the relation established between the copula functions, the marginal distribution functions and the conditional quantile functions.

According to the equation (14),

$$Q_Y(\beta_n \mid X = x) = F_0^{-1}(\Gamma(\beta_n, F(x))$$

with Γ is the quantile of order β_n of the distribution function $\tilde{C}$, and F_0^{-1} is the generalized inverse of F_0. We will first estimate the copula $\tilde{C}$ which corresponds to the conditional distribution and then we will estimate its quantile and finish by estimating the generalized inverse of the partial distribution F_0.

We first estimate the copula of the conditional distribution. By definition, we have:

$$F_Y\left(y \mid x\right) = \tilde{C}(F_0(y), F(x)) = \frac{\partial C(F_0(y), F(x))}{\partial F(x)}.$$

Like the copula C and the marginal distribution functions are generally not known, we will estimate them by using respectively Bernstein's copulas for the copula C and empirical distribution functions for the marginal distribution functions. We call F_{0n} and F_n the empirical distribution functions corresponding to the marginal distributions F_0 and F

The estimator of C using Bernstein's copulas is given by

$$C_{p,n}(F_{0n}(y), F_n(x)) = \sum_{k=0}^{p}\sum_{l=0}^{p} C_n\left(\frac{k}{p}, \frac{l}{p}\right) B(p, k, F_{0n}(y))B(p, l, F_n(x)) \tag{15}$$

where C_n is an empirical estimator of the copula C and $B(p, k, F_{0n}(y))$ (resp) $B(p, l, F_n(x)$ the Bernstein polynomials of order p in $F_{0n}(y)$ (resp) $F_{0n}(x)(x)$. We have $B(p, l, F_n(x)) = C_p^l(F_n(x))^l(1 - F_n(x))^{p-l}$.

By partially derivating $C_{p,n}$ with respect to $F_n(x)$, we obtain the copula $\tilde{C}_{p,n}$ given by the following result.

Proposition 1. *The estimator of the copule of conditional distribution is given by*

$$\tilde{C}_{p,n}(F_{0n}(y), F_n(x)) = A \times C_{p,n}(F_{0n}(y), F_n(x)). \tag{16}$$

with A defined by $A = -\dfrac{p^2F_n(x) + 5pF_n(x) - 2p}{2F_n(x)(1 - F_n(x))}$.

If $p = p(n) \to \infty$ and $\frac{n}{p \log\log n} \to k \geq 0$ then

$$\parallel \tilde{C}_{p,n} - AC \parallel = \mid A \mid \times O(n^{-\frac{1}{2}}(\log\log n)^{\frac{1}{2}})\ a.s\ n \to \infty \tag{17}$$

avec $\parallel . \parallel$ the supremum norm

The following proof is for the proposition (1)

Proof. $\parallel \tilde{C}_{p,n} - AC \parallel = \mid A \mid\parallel C_{p,n} - C \parallel$. According to the theorem 1 of [19] $\parallel C_{p,n} - C \parallel = O(n^{-\frac{1}{2}}(\log\log n)^{\frac{1}{2}}$. We then deduce the result

3.2. Estimation of the partial inverse of the copula $\tilde{C}$

This consists in finding the partial inverse of the copula $\tilde{C}(F_0(y), F(x))$. It is defined by

$$\Gamma(\beta(n), F(x)) = \inf\{F_0(y) \mid \tilde{C}(F_0(y), F(x)) = \beta_n\}.$$

The intermediate conditional quantile is given by

$$Q_Y(\beta_n \mid X = x) = F_0^{-1}(\Gamma(\beta(n), F(x))); \tag{18}$$

where F_0^{-1} is the generalized inverse of F_0.

Proposition 2. *Let β_n be the parameter of the intermediate conditional quantile. We fix $\beta_n = 1 - \frac{k}{n}$ such that $k = k(n) \to \infty$ when $n \to \infty$. The non-parametric estimator of the intermediate conditional quantile of order β_n is defined by*

$$\hat{Q}_Y(\beta_n \mid X = x) = F_{0n}^{-1}\left\{C_{p,n}^{-1}\left(-\beta_n A^{-1}, F_n(x)\right)\right\}. \tag{19}$$

Proof. Recall that for a Bernstein polynomial $B(m, k, z)$ it comes that:

$$\frac{d}{dz}B(m,k,z) = m\left[B(m-1,k-1,z) - B(m-1,kz)\right] \tag{20}$$

and that

$$C_{p,n}(F_0(y), F(x)) = F_0(y)F(x)\sum_{k=0}^{p}\sum_{l=0}^{p} C_n\left(\frac{k}{p},\frac{l}{p}\right)B(p-1,k-1,F_0(y))B(p-1,l-1,F(x)). \tag{21}$$

By definition we have

$$\tilde{C}_{p,n}(F_0(y), F(x)) = \sum_{k=0}^{p}\sum_{l=0}^{p} C_n\left(\frac{k}{p},\frac{l}{p}\right)B(p,k,F_0(y))\frac{d}{dF(x)}B(p,l,F(x)). \tag{22}$$

Moreover from (20) we have equation (22) wich becomes:

$$\tilde{C}_{p,n}(F_0(y), F(x)) = \sum_{k=0}^{p}\sum_{l=0}^{p} C\left(\frac{k}{p},\frac{l}{p}\right)B(p,k,F_0(y))p\left[B(p-1,l-1,F(x)) - B(p-1,l,F(x))\right] \tag{23}$$

Expanding and simplifying (23), we find

$$\tilde{C}_{p,n}(F_0(y), F(x)) = B - D; \tag{24}$$

where

$$B = p\left(1 - \frac{F(x)}{1-F(x)}\right)\sum_{k=0}^{p}\sum_{l=0}^{p} C_n\left(\frac{k}{p},\frac{l}{p}\right)B(p,k,F_0(y))B(p-1,l-1,F(x)) - B(p-1,l,F(x));$$

and

$$D = \frac{F(x)}{1-F(x)}\sum_{k=0}^{p}\sum_{l=0}^{p} lC_n\left(\frac{k}{p},\frac{l}{p}\right)B(p,k,F_0(y))B(p-1,l-1,F(x)).$$

So, since we have

$$\sum_{k=0}^{p}\sum_{l=0}^{p} C_n\left(\frac{k}{p},\frac{l}{p}\right)B(p,k,F_0(y))B(p-1,l-1,F(x)) = \frac{1}{F(x)}C_{p,n}(F_0(y),F(x));$$

Then, it comes that

$$B = p\left(1 - \frac{F(x)}{1-F(x)}\right)\frac{1}{F(x)}C_{p,n}(F_0(y),F(x))$$

and

$$D = \frac{p(p+1)}{2(1-F(x))} C_{p,n}(F_0(y), F(x)).$$

So, we obtain

$$\tilde{C}_{p,n}(F_0(y), F(x)) = B + D = AC_{p,n}(F_0(y), F(x)). \tag{25}$$

Hence the value of A obtained in the previous definition. Using the formula

$$\Gamma(\beta(n), F(x)) = \inf\{F_0(y) \mid \tilde{C}(F_0(y), F(x)) = \beta_n\}$$

we obtain

$$F_0(y) = C_{p,n}^{-1}(\beta_n A^{-1}, F(x)). \tag{26}$$

Moreoer by inversion of F_0 we obtain:

$$y = F_0^{-1}(C_{p,n}^{-1}(\beta_n A^{-1}, F(x))) \tag{27}$$

Hence the value of the conditional quantile intermediate

Let us make the following assumption.

Hypothesis 1. *We assume that the index of the extreme values γ of the conditional quantile is a constant and consequently does not depend on the covariate X*

Proposition 3. *Consider a sample of n random variables for the response variable $(Y_1, X_1), (Y_2, X_2), ..., (Y_n, X_n$ Let α_n be defined such that $\alpha_n \to 1$ and $n(1-\alpha_n) \to K$ where K is a constant. The non-parametric estimator of the extreme conditional quantile of order α_n is defined by*

$$\hat{Q}_Y(\alpha_n \mid X = x) = \left(\frac{1-\alpha_n}{1-\beta_n}\right)^{\gamma_{F_0}} F_0^{-1}\left\{C_{p,n}^{-1}\left(-\beta_n A^{-1}, F_n(x)\right)\right\} \tag{28}$$

where γ_{F_0} is Hill estimator define by $\gamma_{F_0} = \frac{1}{k}\sum_{k=0}^{n} \log(Y_{n-i,n}) - \log(Y_{n-k,n})$.

3.3. Asymptotic results

In this section, we establish the limiting behaviour of our different estimators.

Proposition 4. *Assuming the conditions of Theorem 2 in [19] are satisfied*

- *If $\frac{\sqrt{n}}{p} \to 0$ then for all $(u, v) \in [0; 1]^2$*

$$\sqrt{n}A(\tilde{C}_{p,n}(u, v) - C(u, v)) \longrightarrow_D \mathbb{N}(0, A^2\sigma^2(u, v)).$$

- *If $\frac{\sqrt{n}}{p} \to d$ avec $0 < d < \infty$ alors pour tout $(u, v) \in [0; 1]^2$*

$$\sqrt{n}A(\tilde{C}_{p,n}(u, v) - C(u, v)) \longrightarrow_D \mathbb{N}(db(u, v), A^2\sigma^2(u, v));$$

with

$$\sigma^2(u,v) = C(u,v)(1-C(u,v)) + u(1-u)C_u^2(u,v) + v(1-v)C_v^2(u,v) \quad (29)$$

$$-2(1-u)C(u,v)C_u(u,v) \quad -2(1-v)C(u,v)C_v(u,v)$$

$$+2C_u(u,v)C_v(u,v)\left[C(u,v)-uv\right]$$

and

$$b(u,v) = \frac{1}{2}\left[u(1-u)C_{uu}(u,v) + v(1-v)C_{vv}(u,v)\right]$$

where

$$C_{uu} = \frac{\partial^2}{\partial u^2}C(u,v) \text{ and } C_u = \frac{\partial}{\partial u}C(u,v)$$

Proof. According to the theorem 2 of [19], we have

- If $\frac{\sqrt{n}}{p} \to 0$ then for all $(u,v) \in [0;1]^2$

$$\sqrt{n}(C_{p,n}(u,v) - C(u,v)) \longrightarrow_D \mathbb{N}(0, \sigma^2(u,v))$$

  we deduce the precedent result

- If $\frac{\sqrt{n}}{p} \to d$ avec $0 < d < \infty$ alors pour tout $(u,v) \in [0;1]^2$

$$\sqrt{n}(C_{p,n}(u,v) - C(u,v)) \longrightarrow_D \mathbb{N}(db(u,v), \sigma^2(u,v)),$$

  we also deduce the second result

4. Conclusion

This paper allowed us to propose a new approach while estimating the extreme value at risk for heavy-tailed distribution using the associated copula which by taking into account the effects of certain covariates on the variable of interest. So, the extreme conditional quantiles have been estimated for this subfamily of distributions. The model of Bernstein copulas made it possible to estimate the intermediate conditional quantile. Then, the extreme conditional quantile have been estimated and we provided the asymptotic properties of this estimator.

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