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Weakly (Λ, sp)-continuous multifunctions

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Abstract. This paper is deals with the concept of weakly (Λ, sp)-continuous multifunctions. In particular, some characterizations of weakly (Λ, sp)-continuous multifunctions are investigated.

2020 Mathematics Subject Classifications: 54C08, 54C60

Key Words and Phrases: (Λ, sp)-open set, weakly (Λ, sp)-continuous multifunction

1. Introduction

The branch of mathematics called topology is concerned with all questions directly or indirectly related to continuity. The topological structures of set theories dealing with uncertainities were first introduced by Chang [5]. Lashin et al. [9] investigated topological spaces by generalizing rough set theory. The concept of soft topological spaces defined by Shabir and Naz [17] on an initial universe with a fixed set of parameters. Şenel and Çağman [7] extended the concept of bitopological spaces to soft bitopological spaces. Şenel [6] presented the notion of soft bitopological Hausdorff spaces and introduced some new notions in soft bitopological spaces such as SBT points, SBT continuous functions and SBT homeomorphisms. Continuity is a basic concept for the study in topological spaces. Semi-open sets [11], preopen sets [12] and β-open sets [8] play an important role in the researching of generalizations of continuity in topological spaces. By using these sets many authors introduced and studied various types of weak forms of continuity for functions and multifunctions. Levine [10] introduced the concept of weakly continuous functions. Popa [14] and Smithson [18] independently introduced the notion of weakly continuous multifunctions. In [15], the present authors introduced a class of multifunctions called weakly α-continuous multifunctions. Some characterizations of weakly α-continuous multifunctions are investigated in [4] and [15]. Popa and Noiri [16] investigated several characterizations of weakly β-continuous multifunctions. In 1983, Abd El-Monsef et al. [8] introduced a weak form of open sets called β-open sets. This notion was also called semi-preopen sets in the sense of Andrijević [1]. In 2004, Noiri and Hatir [13] introduced the notion of

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Λ_{sp}-sets in terms of the concept of β-open sets and investigated the notion of Λ_{sp}-closed sets by using Λ_{sp}-sets. In [3], the author introduced the concepts of (Λ, sp)-open sets and (Λ, sp)-closed sets which are defined by utilizing the notions of Λ_{sp}-sets and β-closed sets. The purpose of the present paper is to introduce the notion of weakly (Λ, sp)-continuous multifunctions. Furthermore, several characterizations of weakly (Λ, sp)-continuous multifunctions are discussed.

2. Preliminaries

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *β-open* [8] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$. The complement of a β-open set is called *β-closed*. The family of all β-open sets of a topological space (X, τ) is denoted by $\beta(X, \tau)$. A subset $\Lambda_{sp}(A)$ [13] is defined as follows: $\Lambda_{sp}(A) = \cap\{U \mid A \subseteq U, U \in \beta(X, \tau)\}$. A subset B of a topological space (X, τ) is called a *Λ_{sp}-set* [13] if $B = \Lambda_{sp}(B)$. A subset A of a topological space (X, τ) is called *(Λ, sp)-closed* [3] if $A = T \cap C$, where T is a Λ_{sp}-set and C is a β-closed set. The complement of a (Λ, sp)-closed set is called *(Λ, sp)-open*. The family of all (Λ, sp)-open sets in a topological space (X, τ) is denoted by $\Lambda_{sp}O(X, \tau)$. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, sp)-cluster point* [3] of A if $A \cap U \neq \emptyset$ for every (Λ, sp)-open set U of X containing x. The set of all (Λ, sp)-cluster points of A is called the *(Λ, sp)-closure* [3] of A and is denoted by $A^{(\Lambda,sp)}$. The union of all (Λ, sp)-open sets contained in A is called the *(Λ, sp)-interior* [3] of A and is denoted by $A_{(\Lambda,sp)}$.

Lemma 1. [3] *Let A and B be subsets of a topological space (X, τ). For the (Λ, sp)-closure, the following properties hold:*

(1) $A \subseteq A^{(\Lambda,sp)}$ and $[A^{(\Lambda,sp)}]^{(\Lambda,sp)} = A^{(\Lambda,sp)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,sp)} \subseteq B^{(\Lambda,sp)}$.

(3) $A^{(\Lambda,sp)} = \cap\{F | A \subseteq F$ and F is (Λ, sp)-closed$\}$.

(4) $A^{(\Lambda,sp)}$ is (Λ, sp)-closed.

(5) A is (Λ, sp)-closed if and only if $A = A^{(\Lambda,sp)}$.

Lemma 2. [3] *Let A and B be subsets of a topological space (X, τ). For the (Λ, sp)-interior, the following properties hold:*

(1) $A_{(\Lambda,sp)} \subseteq A$ and $[A_{(\Lambda,sp)}]_{(\Lambda,sp)} = A_{(\Lambda,sp)}$.

(2) If $A \subseteq B$, then $A_{(\Lambda,sp)} \subseteq B_{(\Lambda,sp)}$.

(3) $A_{(\Lambda,sp)}$ is (Λ, sp)-open.

(4) *A is (Λ, sp)-open if and only if $A_{(\Lambda,sp)} = A$.*

(5) $[X - A]^{(\Lambda,sp)} = X - A_{(\Lambda,sp)}$.

(6) $[X - A]_{(\Lambda,sp)} = X - A^{(\Lambda,sp)}$.

A subset A of a topological space (X, τ) is said to be $s(\Lambda, sp)$-*open* (resp. $p(\Lambda, sp)$-*open*, $r(\Lambda, sp)$-*open*, $\alpha(\Lambda, sp)$-*open*, $\beta(\Lambda, sp)$-*open*) if $A \subseteq [A_{(\Lambda,sp)}]^{(\Lambda,sp)}$ (resp. $A \subseteq [A^{(\Lambda,sp)}]_{(\Lambda,sp)}$, $A = [A^{(\Lambda,sp)}]_{(\Lambda,sp)}$, $A \subseteq [[A_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)}$, $A \subseteq [[A^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}$) [3]. The complement of a $s(\Lambda, sp)$-open (resp. $p(\Lambda, sp)$-open, $r(\Lambda, sp)$-open, $\alpha(\Lambda, sp)$-open, $\beta(\Lambda, sp)$-open) set is said to be $s(\Lambda, sp)$-*closed* (resp. $p(\Lambda, sp)$-*closed*, $r(\Lambda, sp)$-*closed*, $\alpha(\Lambda, sp)$-*closed*, $\beta(\Lambda, sp)$-*closed*). The family of all $s(\Lambda, sp)$-open (resp. $p(\Lambda, sp)$-open, $r(\Lambda, sp)$-open, $\alpha(\Lambda, sp)$-open, $\beta(\Lambda, sp)$-open) sets in a topological space (X, τ) is denoted by $s\Lambda_{sp}O(X, \tau)$ (resp. $p\Lambda_{sp}O(X, \tau)$, $r\Lambda_{sp}O(X, \tau)$, $\alpha\Lambda_{sp}O(X, \tau)$, $\beta\Lambda_{sp}O(X, \tau)$).

By a multifunction $F : (X, \tau) \to (Y, \sigma)$, following [2], we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is,

$$F^+(B) = \{x \in X \mid F(x) \subseteq B\}$$

and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$ and for each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$. Let $\mathcal{P}(Y)$ be the collection of all nonempty subsets of Y. For any (Λ, sp)-open set V of a topological space (Y, σ), we denote $V^+ = \{B \in \mathcal{P}(Y) \mid B \subseteq V\}$ and $V^- = \{B \in \mathcal{P}(Y) \mid B \cap V \neq \emptyset\}$.

3. Characterizations of weakly (Λ, sp)-continuous multifunctions

In this section, we introduce the notion of weakly (Λ, sp)-continuous multifunctions. Furthermore, several characterizations of weakly (Λ, sp)-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be weakly (Λ, sp)-continuous if, for each $x \in X$ and each (Λ, sp)-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, there exists a (Λ, sp)-open set U of X containing x such that $F(U) \subseteq V_1^{(\Lambda,sp)}$ and $F(z) \cap V_2^{(\Lambda,sp)} \neq \emptyset$ for every $z \in U$.*

Theorem 1. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *F is weakly (Λ, sp)-continuous;*

(2) $F^+(V_1) \cap F^-(V_2) \subseteq [F^+(V_1^{(\Lambda,sp)}) \cap F^-(V_2^{(\Lambda,sp)})]_{(\Lambda,sp)}$ *for every (Λ, sp)-open sets V_1, V_2 of Y;*

(3) $[F^-([K_1]_{(\Lambda,sp)}) \cup F^+([K_2]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(K_1) \cup F^+(K_2)$ *for every (Λ, sp)-closed sets K_1, K_2 of Y;*

(4) $[F^-([B_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([B_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(B_1^{(\Lambda,sp)}) \cup F^+(B_2^{(\Lambda,sp)})$ *for every subsets* B_1, B_2 *of* Y*;*

(5) $F^+([B_1]_{(\Lambda,sp)}) \cap F^-([B_2]_{(\Lambda,sp)}) \subseteq [F^+(B_1^{(\Lambda,sp)}) \cap F^-(B_2^{(\Lambda,sp)})]_{(\Lambda,sp)}$ *for every subsets* B_1, B_2 *of* Y*;*

(6) $[F^-(V_1) \cup F^+(V_2)]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)})$ *for every* (Λ, sp)*-open sets* V_1, V_2 *of* Y*.*

Proof. (1) ⇒ (2): Let V_1, V_2 be any (Λ, sp)-open sets of Y such that

$$x \in F^+(V_1) \cap F^-(V_2).$$

Then, $F(x) \in V_1^+ \cap V_2^-$ and hence there exists a (Λ, sp)-open set U of X containing x such that $F(U) \subseteq V_1^{(\Lambda,sp)}$ and $F(z) \cap V_2^{(\Lambda,sp)} \neq \emptyset$ for each $z \in U$. Thus,

$$x \in U \subseteq F^+(V_1^{(\Lambda,sp)}) \cap F^-(V_2^{(\Lambda,sp)})$$

and hence $x \in [F^+(V_1^{(\Lambda,sp)}) \cap F^-(V_2^{(\Lambda,sp)})]_{(\Lambda,sp)}$. This shows that

$$F^+(V_1) \cap F^-(V_2) \subseteq [F^+(V_1^{(\Lambda,sp)}) \cap F^-(V_2^{(\Lambda,sp)})]_{(\Lambda,sp)}.$$

(2) ⇒ (3): Let K_1, K_2 be any (Λ, sp)-closed sets of Y. Then, $Y - K_1$ and $Y - K_2$ are (Λ, sp)-open sets in Y, by (2),

$$\begin{aligned} X - (F^-(K_1) \cup F^+(K_2)) &= (X - F^-(K_1)) \cap (X - F^+(K_2)) \\ &= F^+(Y - K_1) \cap F^-(Y - K_2) \\ &\subseteq [F^+([Y - K_1]^{(\Lambda,sp)}) \cap F^-([Y - K_2]^{(\Lambda,sp)})]_{(\Lambda,sp)} \\ &= [(X - F^-([K_1]_{(\Lambda,sp)})) \cap (X - F^+([K_2]_{(\Lambda,sp)}))]_{(\Lambda,sp)} \\ &= [X - [F^-([K_1]_{(\Lambda,sp)}) \cup F^+([K_2]_{(\Lambda,sp)})]]_{(\Lambda,sp)} \\ &= X - [F^-([K_1]_{(\Lambda,sp)}) \cup F^+([K_2]_{(\Lambda,sp)})]^{(\Lambda,sp)} \end{aligned}$$

and hence $[F^-([K_1]_{(\Lambda,sp)}) \cup F^+([K_2]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(K_1) \cup F^+(K_2)$.

(3) ⇒ (4): Let B_1, B_2 be any subsets of Y. Then, $B_1^{(\Lambda,sp)}$ and $B_2^{(\Lambda,sp)}$ are (Λ, sp)-closed in Y and by (3),

$$[F^-([B_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([B_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(B_1^{(\Lambda,sp)}) \cup F^+(B_2^{(\Lambda,sp)}).$$

(4) ⇒ (5): Let B_1, B_2 be any subsets of Y. By (4), we have

$$\begin{aligned} &F^-([B_1]_{(\Lambda,sp)}) \cap F^+([B_2]_{(\Lambda,sp)}) \\ &= X - [F^+([Y - B_1]^{(\Lambda,sp)}) \cup F^-([Y - B_2]^{(\Lambda,sp)})] \\ &\subseteq X - [F^+([[Y - B_1]^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^-([[Y - B_2]^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \end{aligned}$$

$$= X - [F^+(Y - [[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^-(Y - [[B_2]_{(\Lambda,sp)}]^{(\Lambda,sp)})]^{(\Lambda,sp)}$$
$$= X - [(X - F^-([[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)})) \cup (X - F^+([[B_2]_{(\Lambda,sp)}]^{(\Lambda,sp)}))]^{(\Lambda,sp)}$$
$$= X - [X - [F^-([[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cap F^+([[B_2]_{(\Lambda,sp)}]^{(\Lambda,sp)})]]^{(\Lambda,sp)}$$
$$= [F^-([[B_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}) \cap F^+([[B_2]_{(\Lambda,sp)}]^{(\Lambda,sp)})]_{(\Lambda,sp)}.$$

Thus, $F^+([B_1]_{(\Lambda,sp)}) \cap F^-([B_2]_{(\Lambda,sp)}) \subseteq [F^+(B_1^{(\Lambda,sp)}) \cap F^-(B_2^{(\Lambda,sp)})]_{(\Lambda,sp)}$.

(5) ⇒ (2): The proof is obvious.

(2) ⇒ (1): Let V_1, V_2 be any (Λ, sp)-open sets of Y such that $x \in F^+(V_1) \cap F^-(V_2)$. By (2), $x \in F^+(V_1) \cap F^-(V_2) \subseteq [F^+(V_1^{(\Lambda,sp)}) \cap F^-(V_2^{(\Lambda,sp)})]_{(\Lambda,sp)}$. Then, there exists a (Λ, sp)-open set U of X such that $x \in U \subseteq F^+(V_1^{(\Lambda,sp)}) \cap F^-(V_2^{(\Lambda,sp)})$. Thus, $F(U) \subseteq V_1^{(\Lambda,sp)}$ and $F(z) \cap V_2^{(\Lambda,sp)} \neq \emptyset$ for every $z \in U$. This shows that F is weakly (Λ, sp)-continuous.

(4) ⇒ (6): Let V_1, V_2 be any (Λ, sp)-open sets of Y. By (4), we have

$$[F^-(V_1) \cup F^+(V_2)]^{(\Lambda,sp)} \subseteq [F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)}$$
$$\subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)}).$$

(6) ⇒ (2): Let V_1, V_2 be any (Λ, sp)-open sets of Y. Thus, by (6),

$$F^+(V_1) \cap F^-(V_2) \subseteq F^+([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^-([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})$$
$$= X - [F^-([Y - V_1^{(\Lambda,sp)}]^{(\Lambda,sp)}) \cup F^+([Y - V_2^{(\Lambda,sp)}]^{(\Lambda,sp)})]$$
$$\subseteq X - [F^-(Y - V_1^{(\Lambda,sp)}) \cup F^+(Y - V_2^{(\Lambda,sp)})]^{(\Lambda,sp)}$$
$$= [F^+(V_1^{(\Lambda,sp)}) \cap F^-(V_2^{(\Lambda,sp)})]_{(\Lambda,sp)}.$$

Definition 2. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly (Λ, sp)-continuous if, for each $x \in X$ and each (Λ, sp)-open set V of Y containing $f(x)$, there exists a (Λ, sp)-open set U of X containing x such that $f(U) \subseteq V^{(\Lambda,sp)}$.*

Corollary 1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly (Λ, sp)-continuous;

(2) $f^{-1}(V) \subseteq [f^-(V^{(\Lambda,sp)})]_{(\Lambda,sp)}$ for every (Λ, sp)-open set V of Y;

(3) $[f^{-1}([K_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(K)$ for every (Λ, sp)-closed set K of Y;

(4) $[f^{-1}([B^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(B^{(\Lambda,sp)})$ for every subset B of Y;

(5) $f^{-1}(B_{(\Lambda,sp)}) \subseteq [f^{-1}(B^{(\Lambda,sp)})]_{(\Lambda,sp)}$ for every subset B of Y;

(6) $[f^{-1}(V)]^{(\Lambda,sp)} \subseteq f^-(V^{(\Lambda,sp)})$ for every (Λ, sp)-open set V of Y.

Definition 3. [3] *Let A be a subset of a topological space (X, τ). The $\theta(\Lambda, sp)$-closure of A, $A^{\theta(\Lambda,sp)}$, is defined as follows:*

$$A^{\theta(\Lambda,sp)} = \{x \in X \mid A \cap\ U^{(\Lambda,sp)} \neq \emptyset \text{ for each } U \in \Lambda_{sp}O(X,\tau) \text{ containing } x\}.$$

Lemma 3. [3] *For a subset A of a topological space (X, τ), the following properties hold:*

(1) If A is (Λ, sp)-open in X, then $A^{(\Lambda,sp)} = A^{\theta(\Lambda,sp)}$.

(2) $A^{\theta(\Lambda,sp)}$ is (Λ, sp)-closed.

Theorem 2. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is weakly (Λ, sp)-continuous;

(2) $[F^-([B_1^{\theta(\Lambda,sp)}]_{(\Lambda,sp)}) \cap F^+([B_2^{\theta(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(B_1^{\theta(\Lambda,sp)}) \cup F^+(B_2^{\theta(\Lambda,sp)})$ for every subsets B_1, B_2 of Y;

(3) $[F^-([B_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([B_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(B_1^{\theta(\Lambda,sp)}) \cup F^+(B_2^{\theta(\Lambda,sp)})$ for every subsets B_1, B_2 of Y;

(4) $[F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)})$ for every (Λ, sp)-open sets V_1, V_2 of Y;

(5) $[F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)})$ for every $p(\Lambda, sp)$-open sets V_1, V_2 of Y;

(6) $[F^-([K_1]_{(\Lambda,sp)}) \cup F^+([K_2]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(K_1) \cup F^+(K_2)$ for every $r(\Lambda, sp)$-closed sets K_1, K_2 of Y.

Proof. (1) ⇒ (2): Let B_1, B_2 be any subsets of Y. Then, $B_1^{\theta(\Lambda,sp)}$ and $B_2^{\theta(\Lambda,sp)}$ are (Λ, sp)-closed in Y, by Theorem 1,

$$[F^-([B_1^{\theta(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([B_2^{\theta(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(B_1^{\theta(\Lambda,sp)}) \cup F^+(B_2^{\theta(\Lambda,sp)}).$$

(2) ⇒ (3): This is obvious since $B^{(\Lambda,sp)} \subseteq B^{\theta(\Lambda,sp)}$ for every subset B of Y.

(3) ⇒ (4): This is obvious since $V^{(\Lambda,sp)} = V^{\theta(\Lambda,sp)}$ for every (Λ, sp)-open set V of Y.

(4) ⇒ (5): Let V_1, V_2 be any $p(\Lambda, sp)$-open sets of Y. Since $V_i \subseteq [V_i^{(\Lambda,sp)}]_{(\Lambda,sp)}$, we have $V_i^{(\Lambda,sp)} = [[V_i^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}$ for $i = 1, 2$. Now, put $U_i = [V_i^{(\Lambda,sp)}]_{(\Lambda,sp)}$, then U_i is (Λ, sp)-open in Y and $U_i^{(\Lambda,sp)} = V_i^{(\Lambda,sp)}$, by (4),

$$[F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)}).$$

(5) ⇒ (6): Let K_1, K_2 be any $r(\Lambda, sp)$-closed sets of Y. Then, $[K_1]_{(\Lambda,sp)}$ and $[K_2]_{(\Lambda,sp)}$ are $p(\Lambda, sp)$-open in Y and by (5),

$$[F^-([K_1]_{(\Lambda,sp)}) \cup F^+([K_2]_{(\Lambda,sp)})]^{(\Lambda,sp)}$$

$$= [F^-([[[K_1]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([[[K_2]_{(\Lambda,sp)}]^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)}$$
$$\subseteq F^-(K_1) \cup F^+(K_2).$$

(6) $\Rightarrow$ (1): Let V_1, V_2 be any (Λ, sp)-open sets of Y. Then, $V_1^{(\Lambda,sp)}$ and $V_2^{(\Lambda,sp)}$ are $r(\Lambda, sp)$-closed in Y and by (6), we have

$$[F^-(V_1) \cup F^+(V_2)]^{(\Lambda,sp)} \subseteq [F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)}$$
$$\subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)}).$$

It follows from Theorem 1 that F is weakly (Λ, sp)-continuous.

Corollary 2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly (Λ, sp)-continuous;

(2) $[f^{-1}([B^{\theta(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(B^{\theta(\Lambda,sp)})$ for every subset B of Y;

(3) $[f^{-1}([B^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(B^{\theta(\Lambda,sp)})$ for every subset B of Y;

(4) $[f^{-1}([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ for every (Λ, sp)-open set V of Y;

(5) $[f^{-1}([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ for every $p(\Lambda, sp)$-open set V of Y;

(6) $[f^{-1}(K_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(K)$ for every $r(\Lambda, sp)$-closed set K of Y.

Theorem 3. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is weakly (Λ, sp)-continuous;

(2) $[F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)})$ for every $\beta(\Lambda, sp)$-open sets V_1, V_2 of Y;

(3) $[F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)})$ for every $s(\Lambda, sp)$-open sets V_1, V_2 of Y.

Proof. (1) $\Rightarrow$ (2): Let V_1, V_2 be any $\beta(\Lambda, sp)$-open sets of Y. Then, we have

$$V_i \subseteq [[V_i^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}$$

and $V_i^{(\Lambda,sp)} = [[V_i^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}$ for $i = 1, 2$. Since $V_1^{(\Lambda,sp)}$ and $V_2^{(\Lambda,sp)}$ are $r(\Lambda, sp)$-closed in Y, by Theorem 2,

$$[F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)}).$$

(2) $\Rightarrow$ (3): This is obvious since every $s(\Lambda, sp)$-open set is $\beta(\Lambda, sp)$-open.

(3) $\Rightarrow$ (1): Let V_1, V_2 be any $\beta(\Lambda, sp)$-open sets of Y. Then, $V_1^{(\Lambda,sp)}$ and $V_2^{(\Lambda,sp)}$ are $r(\Lambda, sp)$-closed sets of Y and hence $V_1^{(\Lambda,sp)}$ and $V_2^{(\Lambda,sp)}$ are $s(\Lambda, sp)$-open in Y, by (3), we have $[F^-([V_1^{(\Lambda,sp)}]_{(\Lambda,sp)}) \cup F^+([V_2^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq F^-(V_1^{(\Lambda,sp)}) \cup F^+(V_2^{(\Lambda,sp)})$ and by Theorem 2, F is weakly (Λ, sp)-continuous.

Corollary 3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly (Λ, sp)-continuous;

(2) $[f^{-1}([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ for every $\beta(\Lambda, sp)$-open set V of Y;

(3) $[f^{-1}([V^{(\Lambda,sp)}]_{(\Lambda,sp)})]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ for every $s(\Lambda, sp)$-open set V of Y.

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