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Origami wrapping with an equilateral triangular prism

Attawut Wongpradit[1], Prathomjit Khachorncharoenkul[2,*]

[1] *Division of Mathematics and Statistics, Faculty of Science and Technology, Thammasat University, Bangkok, 10330, Thailand*
[2] *School of Science, Walailak University, Nakhon Si Thammarat, 80160, Thailand*

Abstract. We study a geometric model to find the relationship of the surface area of rectangular packaging that encapsulates an equilateral triangular prism in a Thai traditional tall wrap pattern and find the minimum area of a rectangular wrapping paper.

2020 Mathematics Subject Classifications: 14G22, 52B10

Key Words and Phrases: Origami, geometric model, package

1. Introduction

Traditional food packaging has been developed into a practical method to protect a meal during both storage and transportation. Due to changes in technology and large quantities of human consumption, containers from natural materials are considered non practical resources in modern manufacturing [1, 5]. Instead of fading from history, local wrapping knowledge is one of the most widely studied topics due to the beauty of its complex design [4, 6, 7], the convenience of the user [3], the efficiency of folding techniques [5], and the eco-friendly reason [2, 5]. Especially from a usage viewpoint, food packed in the traditional pattern is tightly sealed compared to the quantity of the material used. To analyze the relation between the amount of material and the size of the contained goods, we begin with the Soongsung style, a Thai tall form banana leaf wrapping style that is still currently utilized in Thai rural areas. This paper focuses, through geometric investigations, on the folding pattern of this tall wrapping style, which always begins from a banana leaf that is cut into a rectangular shape to cover Khanom Saisai, a coconut milk custard that is steamed in the form of a triangular prism, as shown in Figure 1.

In this paper, an equilateral triangular prism is used to represent a triangular prism carried in the package to serve the convenient purpose of deriving a proof. The aim is to determine the optimal size of the wrapping rectangle that varies by the given size of the equilateral triangular prism contained, as shown in Theorem 1 and Theorem 2.

*Corresponding author.
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Email addresses: attawutt@tu.ac.th (A. Wongpradit), prathomjit@gmail.com (P. Khachorncharoenkul)


Figure 1: Khanom Saisai. Left figure: https://shorturl.at/elvyV Right figure: https://shorturl.at/gwAP7

2. Thai traditional tall wrapping pattern

Let $\square A_1A_2A_3A_4$ be a rectangular wrapping paper with $|\overline{A_1A_4}| > |\overline{A_1A_2}|$. Let $B_1B_2B_3B_4B_5B_6$ be an equilateral triangular prism such that each cross-section $B_1B_2B_6$ and $B_3B_4B_5$ is an equilateral triangle; see Figure 2.

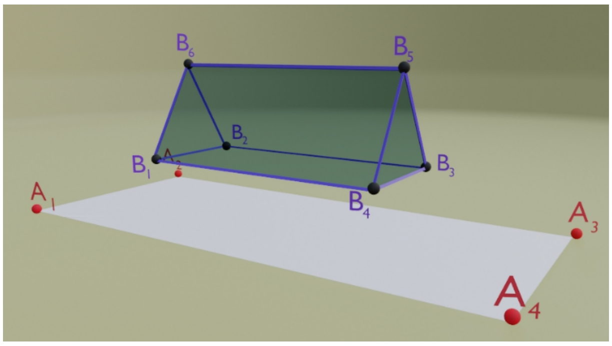


Figure 2: Size of a prism and a wrapping paper.

Let B_1', B_2', B_3' and B_4' be interior points of $\square A_1A_2A_3A_4$ such that $\overline{B_i'B_j'}$ is parallel to $\overline{A_iA_j}$ for every i, j, where $\overline{A_iA_j}$ is a perimeter of $\square A_1A_2A_3A_4$. In addition, the intersection point of $\overline{B_1'B_3'}$ and $\overline{B_2'B_4'}$ and the intersection point of $\overline{A_1A_3}$ and $\overline{A_2A_4}$ coincide (see Figure 3).

We will define an origami pattern of $\square A_1A_2A_3A_4$ that encapsulates the prism $B_1B_2B_3B_4B_5B_6$ where the points B_i and B_i' coincide for any $i = 1, 2, 3, 4$.

Step 1: Construct B_5', B_6', C_i and C_i'. The paper is folded upon the equilateral triangle face of a prism along $\overline{B_3'B_4'}$ and $\overline{B_1'B_2'}$. Let B_5' and B_6' be the points on the wrapping paper where B_5' and B_5 coincide and where B_6' and B_6 coincide, see Figure 4.

Let k be the length of lateral edge B_1B_4,

l be the length of prism edge B_1B_6,

K be the length of paper edge A_1A_4,

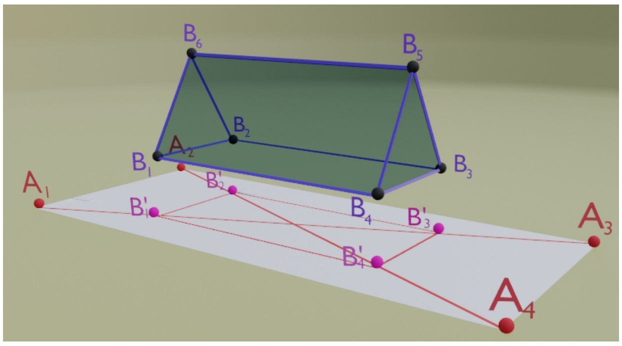


Figure 3: The positions that create the prism on the wrapping paper.

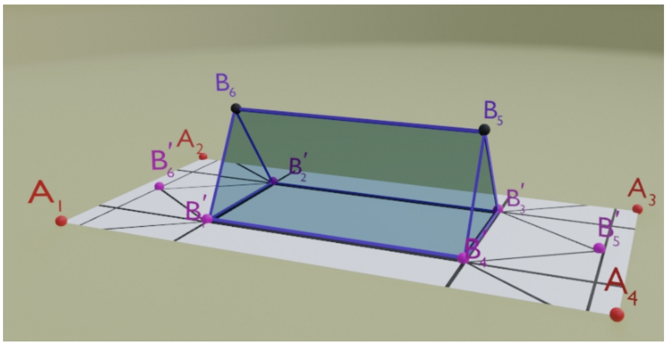


Figure 4: Step 1.

L be the length of paper edge A_1A_2,
s be the distance between the paper edge $\overline{A_1A_2}$ and the point B_6',
and t be the distance between the paper edge $\overline{A_1A_4}$ and the lateral edge B_1B_4. For clarity, see Figure 5.

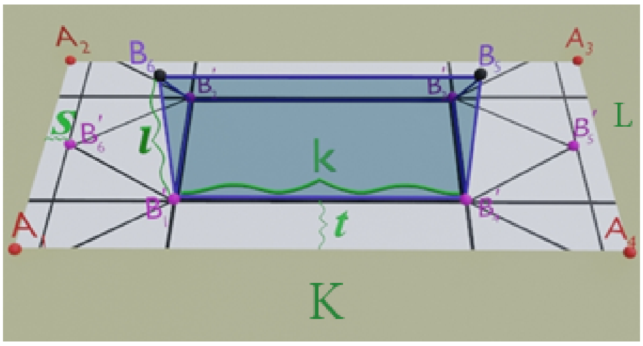


Figure 5: variable positions.

From the symmetrical wrapping of an equilateral triangular prism, we obtain $k = |B_1B_4| = |B_2B_3|$ and $l = |B_1B_2| = |B_3B_4| = |B_1B_6| = |B_2B_6| = |B_3B_5| = |B_4B_5|$. From the origami pattern, we obtain that s is equal to the distance between the paper edge $\overline{A_3A_4}$ and the point B_5' and t is equal to the distance between the paper edge $\overline{A_2A_3}$ and the lateral edge B_2B_3.

Let C_1 be a point on the line $\overline{B_1B_6}$, which is t units away from B_1. Let C_2 be on line $\overline{B_2B_6}$, t units away from B_2. Points C_3 and C_4 follow a similar pattern. In addition, let C_i' be the point on the edge of paper $\square A_1A_2A_3A_4$, where we fold C_i' onto C_i along lateral edges for each $i = 1, 2, 3, 4$; see Figure 6.

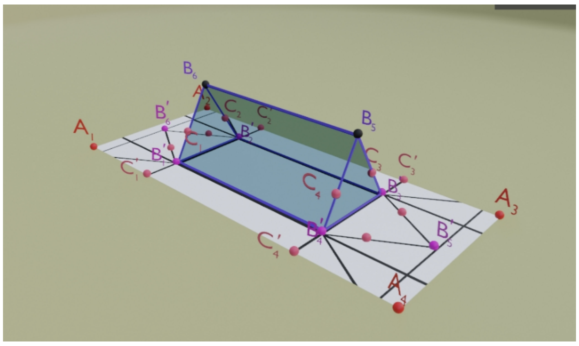


Figure 6: Position of points C_i and C_i'.

Step 2: Construct C_i''. Fold the paper upon the prism along the lines $\overline{B_1'B_4'}$ and $\overline{B_2'B_3'}$ such that each point C_i' on paper and each point C_i on the lateral face of the prism coincide, as shown in Figure 7.

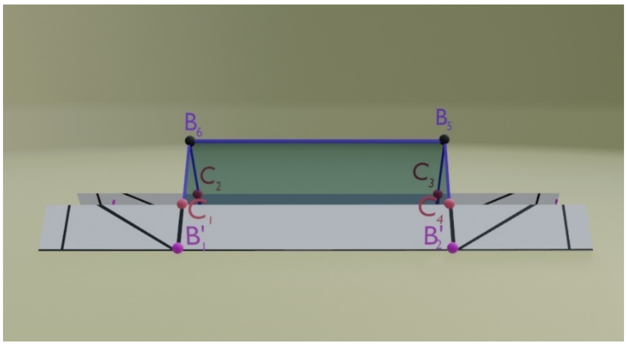


Figure 7: Step 2.

Now, we consider both equilateral triangle faces. The paper is folded upon the prism along the lines $\overline{B_1'B_2'}$ and $\overline{B_3'B_4'}$. Let C_1'' be the point on line $\overline{B_1'B_6'}$ that coincide with C_1'' and C_1'. A similar assumption is made for points C_2'', C_3'', C_4''; see Figure 8.

Step 3: Construct E_i, E_i' and D_i. From Figure 8, let the point E_1 be located on the lateral face of the prism, on the line $\overline{C_1C_4}$ at $\sqrt{3}t$ units away from C_1. A similar assumption is made for points E_2, E_3, E_4. Next, let E_1' be the point on the edge of the paper $\overline{A_1C_1'}$ such that the distance between E_1' and C_1' is $\sqrt{3}t$ units. A similar assumption is made for points E_2', E_3', E_4'; see Figure 9.

Next, from Figure 8, E_i' is folded to E_i so that $B_i'C_i'$ is creased, and we see that E_i and E_i' coincide for any $i = 1, 2, 3, 4$, which indicates a completed folding. Moreover, the intersection point of the paper edge and the lateral edge B_5B_6 is called point D_i, where

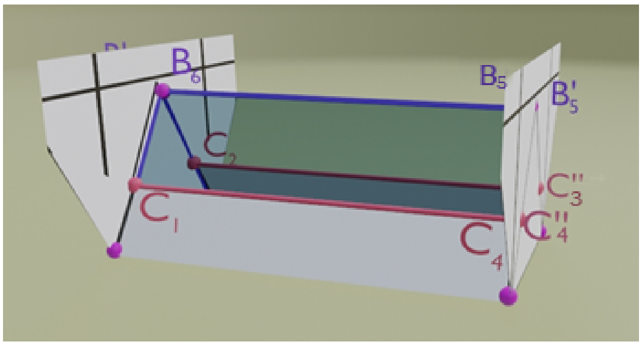


Figure 8: Position of point C_i''.

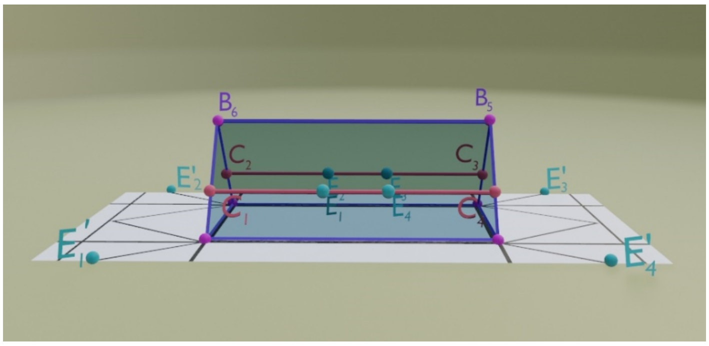


Figure 9: Position of points E_i and E_i'.

D_i corresponds to the point A_i for each $i = 1, 2, 3, 4$, as shown in Figure 10.

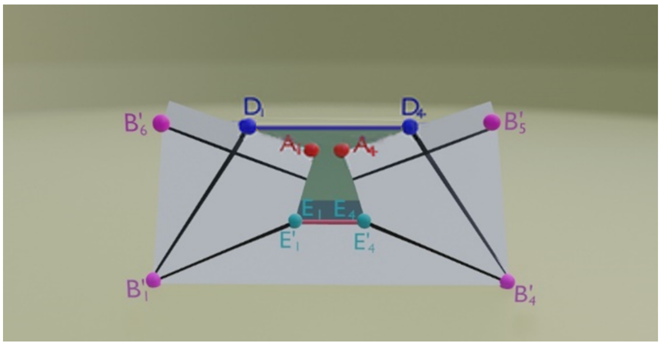
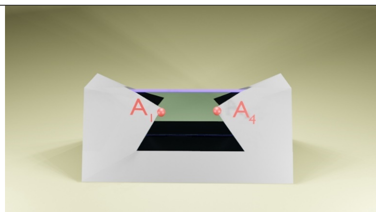
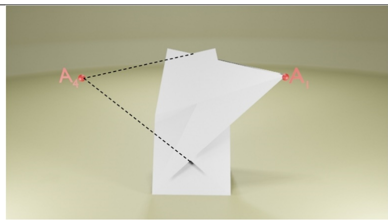
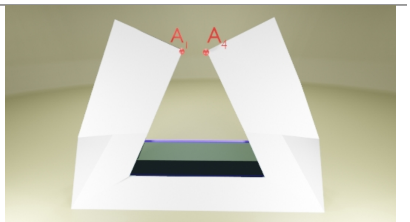
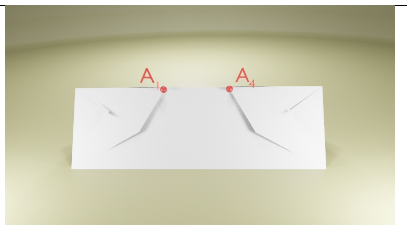
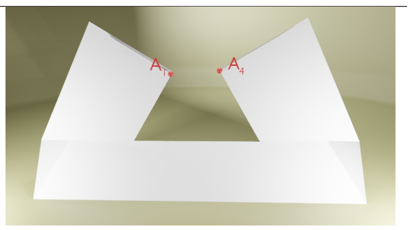


Figure 10: Completed folding.

By observing Figure 10, which shows a completed folding, we can see that the format of the results obtained by wrapping according to the above procedure will differ depending on the size of the inner prism and the size of the wrapper. Examples of results obtained from wrapping different sizes are shown in Table 1.

For the next section, we will consider a wrapping pattern that does not contain a gap between the inner prism and the wrapping paper and find the minimum area of a rectangular wrapping paper, that is, find the value of $K \times L$ in terms of k and l.

Table 1: Results obtained from wrapping different sizes.

Case	K	L	k	l	Result
1	13.9	6.5	$\sqrt{3}(3.5)$	3.5	
2	13.95	7.5	7	3.5	
3	21.12	10.5	10	3.5	
4	21.06	8.5	7	3.5	
5	15.6	10.5	3.5	3.5	
6	22	6.5	10	3.5	
7	16.06	10.5	10	3.5	
8	21.24	10.5	10	3.5	

3. Encased wrapping

An encased wrapping is a gapless wrapping that makes the contours inside invisible, that is, all results of folding the paper for wrapping that can completely encapsulate the prism.

The above definition of wrapping is used to describe a wrapping that can completely encapsulate the prism. However, it still cannot represent a practical wrapping style due to its pattern. In some theoretical cover patterns, when used as a prototype of the actual wrapping, it was found that there is not enough cover paper to move the position of the package at all because the inner object and the wrapping paper are not held together by any force as shown in Figure 11. Hence, we need to consider a wrapping pattern that has enough overlap for the purpose of package wrapping, as shown in Figure 12.

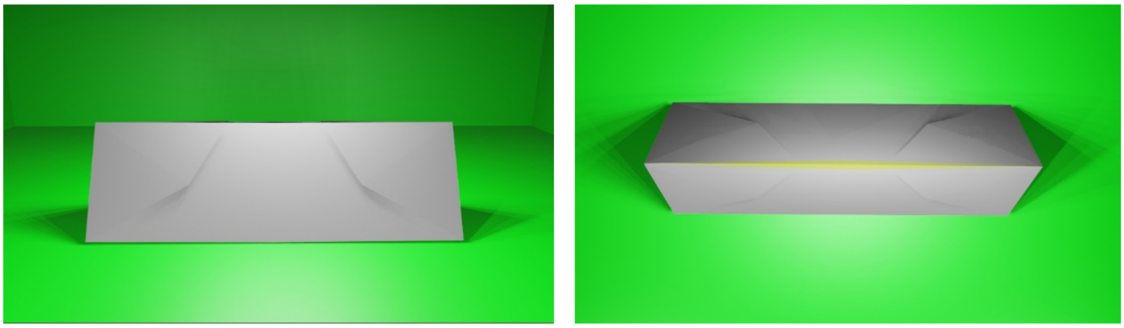


Figure 11: Wrapping that is not practical as packaging.

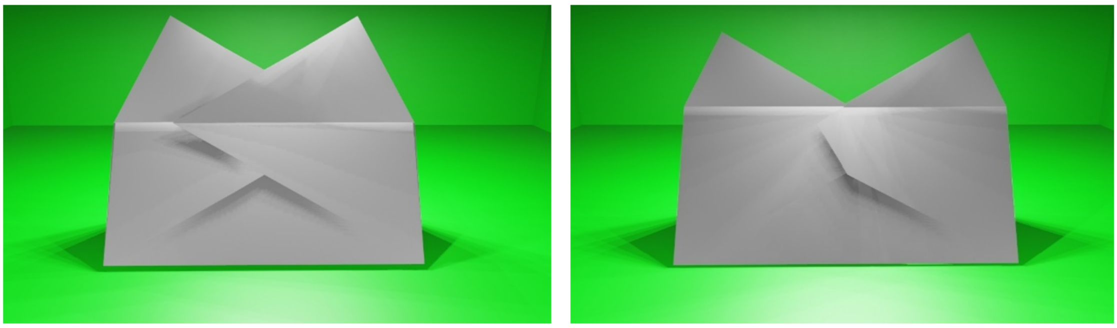


Figure 12: Wrapping that can be used as packaging.

Therefore, we develop terms of coverage that are practical as packaging. We will begin by considering the boundary of the paper relative to the size of an equilateral cross-section prism. We begin from the smallest size of paper in which the wrap will occur.

From the tall wrapping described in Section 2, the tall, encased wrapping by rectangular paper in tall wrapping is achieved by increasing the paper size from the projection of an equilateral cross-section prism shape, as shown in Figure 13. The distance beyond the projection of the prism is called *the distance between the edges of the paper* $\overline{A_1A_2}$ *and* *the point* B_6', denoted by s. *The distance between the edges of paper* $\overline{A_1A_4}$ *and the lateral edge of prism* B_1B_4 is denoted by t, as shown in Figure 14.

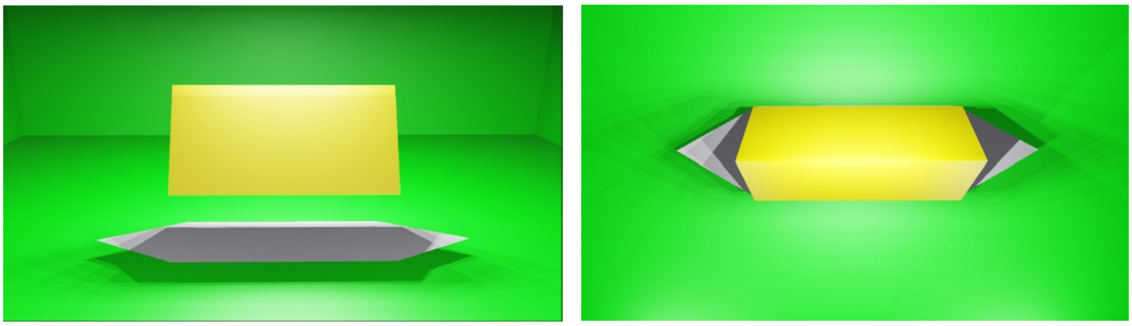


Figure 13: Projection of the prism.

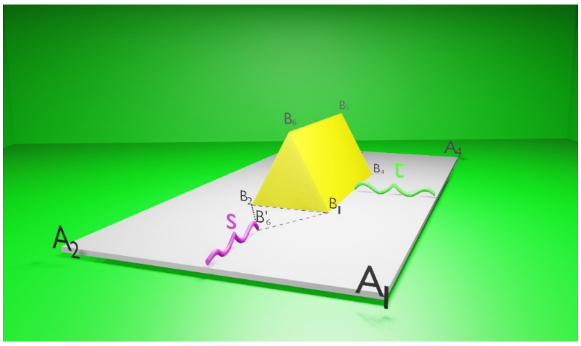


Figure 14: The distance between the edge of the paper and the prism.

By the definitions of s and t, we obtain the relationship of the variables given above as follows:

$$s = \frac{K - k - \sqrt{3}l}{2} \quad \text{and} \quad t = \frac{L - l}{2}.$$

Consider the smallest rectangular paper that contains the unfolded prism. The rectangle has the values $K = k + \sqrt{3}l$ and $L = 3l$, as shown in Figure 15.

However, this is not considered a size of paper that can be wrapped in a tall wrapping to cover a prism because when folding the paper in a tall wrapping form, there is still a gap to see the prism. In terms of s and t, the size of the above paper gives $s = 0$ and $t = 0$. Hence, the first condition of the required paper size has appeared; that is, s and t must both be greater than zero.

The remainder of this section is devoted to finding the values of s and t that will cause wrapping that can be folded as a tall, encased wrapping to lead to the main results.

If a rectangular paper has $t > l$, then the result of wrapping is an overlap of the paper in the area above the lateral edge $\overline{B_5B_6}$, as shown in Figure 16. Thus, the boundary of possible t that prevents excess paper after wrapping is $t \in (0, l]$. However, an equilateral triangular prism that has a side length of l units and a height of k units where k and l

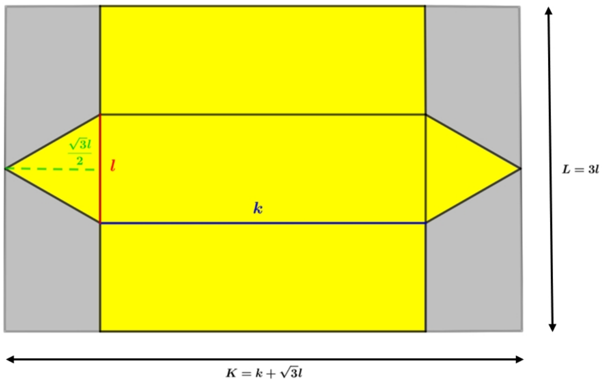


Figure 15: The unfolded prism.

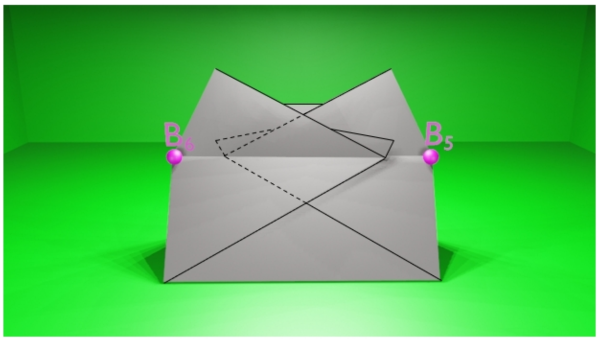


Figure 16: A wrapping to cover a prism when $t > l$.

are any positive real numbers cannot be used as a tall wrap at all k and l values. From Figure 16, we begin to consider ways in which coverage can be achieved. If a rectangular paper has $t = l$ to wrap a prism in a tall wrapping, then such wrapping is not considered as a tall wrap that can be used as packaging because when wrapped according to the steps given above, points A_1 and A_4 do not overlap. It is incapable of functioning as packaging because even though there is no gap to be seen inside when wrapped successfully, moving such packages may form gaps. Paper convergence is really a fit convergence. There is no surface to hold all the packages together.

From the reasons mentioned above, we will find the relationship between the values of k, l, s and t. We begin by considering the reasons for practical encapsulation as packaging, as shown in Figure 17.

Case $t = l$. We find that the only reason this wrapping is not an encased wrapping for packaging is the gap between A_1 and A_4 after folding is completed. There is a chance that a gap will form on the lateral edge of the prism, although the lengths of the papers meet exactly, as shown in Figure 17.

Consequently, we calculate the K-value of the paper so that there is no gap between points A_1 and A_4 after folding.

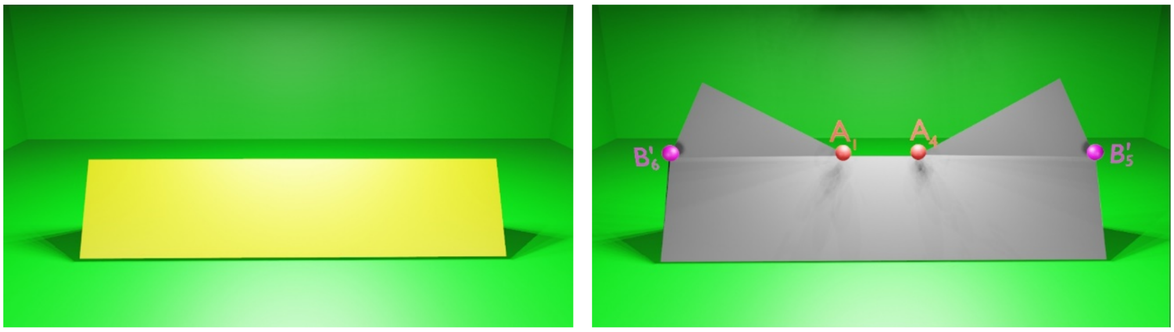


Figure 17: A tall, encased wrapping that is not as packaging.

Lemma 1. *Let $B_1B_2B_3B_4B_5B_6$ be an equilateral triangular prism such that the cross-section has height k units and side length l units and $\square A_1A_2A_3A_4$ be a rectangular wrapping paper with width L units and length K units such that $L \geq 3l$. If $K \geq \frac{3k}{2} + \sqrt{3}l$ and $k \leq 2\sqrt{3}l$, then this wrapping is a tall, encased wrapping that can be used as packaging.*

Proof. Since $L \geq 3l$ and the definition of $t = \frac{L-l}{2}$, we obtain $t \geq \frac{3l-l}{2} = l$. To find the smallest paper size that allows for tall, encased wrapping that can be used as packaging, we determine the case where $t = l$. From Figure 18, an unfolded paper obtained by wrapping the prism, we know that the three angles of the cross-sections $B_1'B_2'B_6'$ on the equilateral triangular prism are each 60 degrees. It forces $\angle B_1'B_2'B_6' = \angle B_2'B_6'B_1' = \angle B_6'B_1'B_2' = 60$. Since $|\overline{B_1'B_2'}| = |\overline{B_1'B_6'}| = |\overline{B_2'B_6'}| = l$, we have $\sin 60 = \sin(\angle B_6'B_1'B_2') = \frac{|\overline{B_6'X}|}{l}$ and then $|\overline{B_6'X}| = \frac{\sqrt{3}l}{2}$. Consider that creases, $\triangle A_1B_1'B_6'$ and $\triangle A_1B_1'C_1'$ can be folded together so that each point can be made coincident with another point. It follows that the two triangles are equal in all respects and $\angle B_1'B_6'A_1 = 90$

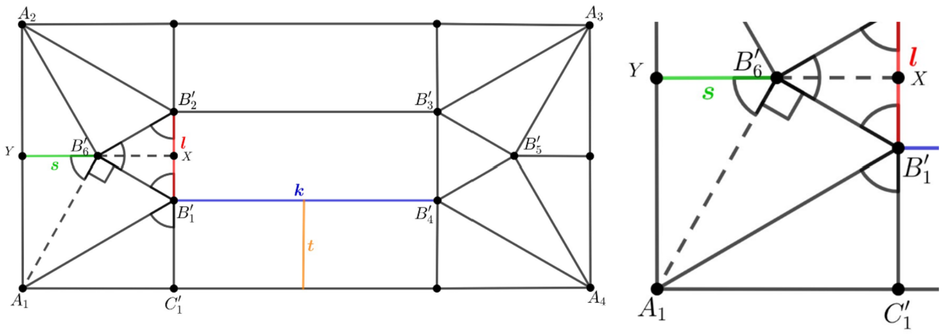


Figure 18: An unfolded paper obtained by wrapping the prism in case $t = l$.

Since $\angle XB_6'B_1' = 30$, we have

$$\angle YB_6'A_1 = 180 - \angle XB_6'B_1' - \angle B_1'B_6'A_1 = 180 - 30 - 90 = 60.$$

In addition, we know $|\overline{YB_6'}| = s$, which implies for $\triangle YB_6'A_1$ that $\cos(\angle YB_6'A_1) = \frac{|\overline{YB_6'}|}{|\overline{B_6'A_1}|}$. Thus, $|\overline{B_6'A_1}| = 2s$. From Figure 17, to achieve wrapping in a tall, encased wrap that can be used as packaging, it is necessary to move the points A_1 and A_4 to converge. Since $|\overline{B_6'A_1}| = |\overline{B_5'A_4}| = 2s$, we obtain $4s \geq k$. Then, we have a requirement that $s \geq \frac{k}{4}$. From the paper length $K = 2s + \frac{2\sqrt{3}l}{2} + k$ and the above requirement, we obtain $K \geq \frac{3k}{2} + \sqrt{3}l$. We can conclude that if $K \geq \frac{3k}{2} + \sqrt{3}l$, then the wrapping that is achieved is a tall, encased wrap that can be used as packaging.

Next, from Figure 18, points A_1, A_2, A_3 and A_4 are brought to line $\overline{B_5B_6}$. Then, the four corners of the paper are exactly on the lateral edge of the prism; moreover, $\angle YB_6'A_1 = 60$ and $|\overline{YA_1}| = |\overline{XC_1'}| = \frac{l}{2} + t$. Thus, $\sqrt{3} = \tan(60) = \tan(\angle YB_6'A_1) = \frac{\frac{l}{2}+t}{s}$. Hence, $s = \frac{\sqrt{3}l}{2}$. We must determine that the boundary of the prism height is no more than the confluence of the paper angles on the lateral edge of the prism or, in other words, $k \leq 4s = 2\sqrt{3}l$. $\square$

Since the possible boundaries of t are $(0, l]$, another case that we must consider is $t < l$.

Case $t < l$. When we wrap packages with $t < l$, we find that uncovered wrapping can occur because the area of the paper covering the lateral faces of the prism is too small as shown in Figure 19. We can fix this by increasing the size of the paper so that the points D_1 and D_4 or the points E_1' and E_4' that appear after the fold converge. The size of the paper is according to the following lemma.

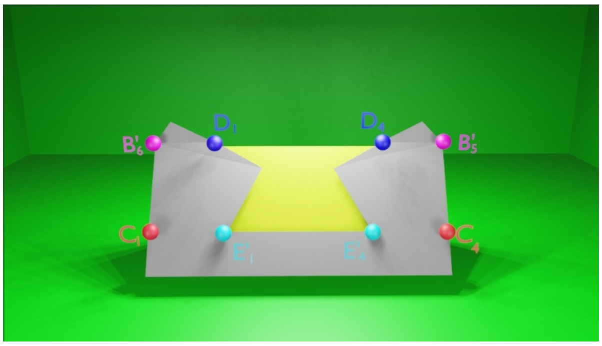


Figure 19: point position.

Lemma 2. *Let $B_1B_2B_3B_4B_5B_6$ be an equilateral triangular prism such that the cross-section has height k units and side length l units and $\square A_1A_2A_3A_4$ be a rectangular wrapping paper with width L units and length K units such that $L < 3l$. If $K \geq \frac{3k}{2} + \sqrt{3}l$ and $k < 2\sqrt{3}l$, then this wrapping is a tall, encased wrapping that can be used as packaging.*

Proof. Since $L < 3l$ and the definition of $t = \frac{L-l}{2}$, we obtain $t < \frac{3l-l}{2} = l$. To find the smallest paper size that allows for a tall, encased wrapping that can be used as packaging, we consider the unfolded paper obtained by wrapping the prism, as shown in Figure 20. In the same manner as for the proof of lemma 1, we obtain $\angle B_1'B_2'B_6' = \angle B_2'B_6'B_1' = \angle B_6'B_1'B_2' = 60$ and $|\overline{B_6'X}| = \frac{\sqrt{3}l}{2}$. Moreover, $\angle C_1B_1'C_1' = 120$. Consider that crease, $\triangle C_1E_1'B_1'$ and $\triangle C_1'E_1'B_1'$ can be folded together so that each point can be made coincident with another point and $|\overline{C_1'B_1'}| = |\overline{C_1B_1'}| = t$. It follows that the two triangles are equal in all respects and $\angle C_1B'_1E'_1 = 60$.By $\triangle C_1E'_1B'_1$, we get $\tan(\angle C_1B'_1E'_1) = \frac{|\overline{C_1E'_1}|}{|\overline{C_1B'_1}|}$. Thus, $|\overline{C_1E'_1}| = \sqrt{3}t$. Similarly, $|\overline{C_4E'_4}| = \sqrt{3}t$.

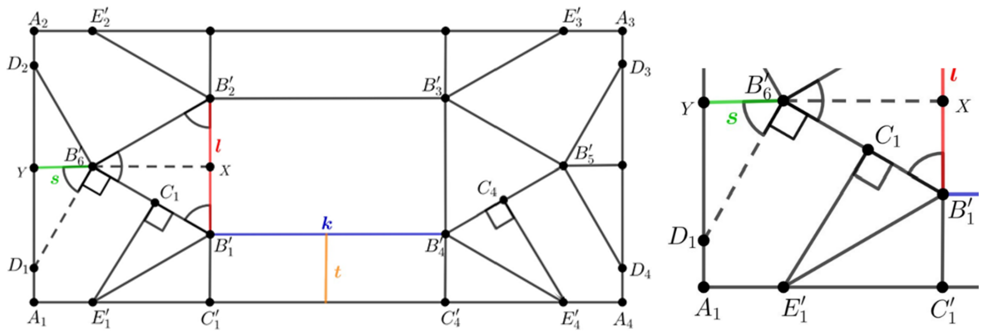


Figure 20: An unfolded paper obtained by wrapping the prism in case $t < l$.

From Figure 19, to achieve wrapping in a tall, encased wrap that can be used as packaging, it is necessary to fold the point D_1 to D_4 or fold the point E'_1 to E'_4. This forces the following conditions to be met $\min\{|\overline{C_1E'_1}|, |\overline{B'_6D_1}|\} \geq \frac{k}{2}$. Thus, $|\overline{C_1E'_1}| + |\overline{C_4E'_4}| \geq |\overline{B'_1B'_4}|$. It follows that $\sqrt{3}t + \sqrt{3}t \geq k$. Then, we obtain a requirement that $2\sqrt{3}t \geq k$. Since $t < l$, it implies that $k < 2\sqrt{3}l$.

Moreover, since$|\overline{B'_6D_1}| + |\overline{B'_5D_4}| \geq |\overline{B'_1B'_4}|$, we get a requirement that $s \geq \frac{k}{4}$. Since the paper length $K = 2s + \frac{2\sqrt{3}l}{2} + k$ and the above requirements, we have $K \geq \frac{3k}{2} + \sqrt{3}l$. We can conclude that if $K \geq \frac{3k}{2} + \sqrt{3}l$, then the wrapping achieves a tall, encased wrap that can be used as packaging. □

Now, we have lemmas that show how to calculate the size of paper to wrap a tall, encased wrapping as packaging. However, for any prism of height k, it is not always certain that we can take the tall, encased wrapping formin order to wrap an equilateral triangular prism. An appropriate t-value must be selected.

Let t^* be the smallest t-value that causes wrapping as a package and s^* be the smallest s-value that causes wrapping as a package. This means that if $t < t^*$, then the wrapping formed by the t-value rectangular paper is not a tall, encased wrapping as a package because the s value that we define later will not affect the wrapping to cover the prism as shown in Figure 21. We find that no matter how much we increase the s the result is not tall, encased wrapping if the given value of t does not reach the value of t^*. The following lemma finds the values of t and s to analyze the smallest boundary of the paper.

Theorem 1. *Let $B_1B_2B_3B_4B_5B_6$ be an equilateral triangular prism such that the cross-section has height k units and side length l units and $\square A_1A_2A_3A_4$ be a rectangular wrapping paper with width L units and length K units such that $L < 3l$.*
Let t be the distance between the paper edge $\overline{A_1A_4}$ and the lateral edge B_1B_4, that is, $t = \frac{L-l}{2}$,
t^* be the smallest t-value that causes wrapping as a package,
s be the distance between the paper edge $\overline{A_1A_2}$ and the point B'_6, that is, $s = \frac{K-k-\sqrt{3}l}{2}$,

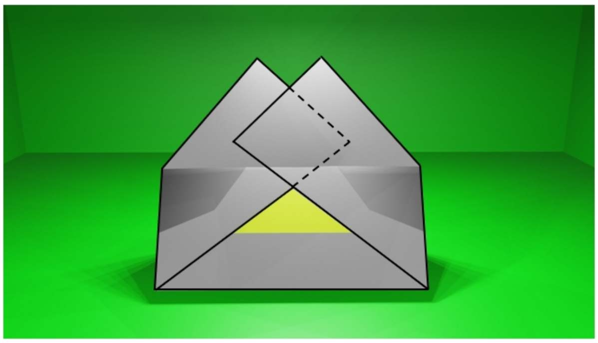


Figure 21: Folding paper in case $t < t^*$.

and s^* be the smallest s-value that causes wrapping as a package. Then $t^* = \frac{k}{2\sqrt{3}}$ and $s^* = \frac{k}{4}$.

Proof. Assume the wrapping is a tall, encased wrapping used as packaging. By Lemma 2, we have $K \geq \frac{3k}{2} + \sqrt{3}l$. We consider the folding scope of the unfolded prism, as shown in Figure 15; then, the size of the paper that fits the prism is $\frac{2\sqrt{3}l}{2} + k$. It follows that $2s^* = 2(\frac{k}{4})$, that is, $s^* = \frac{k}{4}$. According to the proof of Lemma 2, $2\sqrt{3}t \geq k$, as a result, $t^* = \frac{k}{2\sqrt{3}}$. □

Now, the boundary of s and t are $s^* \leq s$ and $t^* \leq t \leq l$, respectively to achieve wrapping in a tall, encased wrap that can be used as packaging. Thus, the smallest of s and t are s^* and t^*, respectively. Hence, the paper that can be completely wrapped within a minimal region is $s = s^*$ and $t = t^*$ as the following theorem.

Theorem 2. *Let k and l be positive real numbers such that $k \leq 2\sqrt{3}l$. The smallest paper that can be completely wrapped contains $K = \frac{3k}{2} + \sqrt{3}l$ and $L = l + \frac{k}{\sqrt{3}}$, and the paper with the least area $K \times L = (\frac{3k}{2} + \sqrt{3}l)(l + \frac{k}{\sqrt{3}})$.*

We can see more clearly illustrated by the following examples.

Example 1. *Let $k = 3.5(\sqrt{3}), l = 3.5, s = 1.52$ and $t = 1.75$. From Figure 22, when folded paper is the specified size, it is a tall, encased wrapping that can be used as packaging and corresponds to Theorem 1 that $t = t^* = \frac{k}{2\sqrt{3}}$ and $s = s^* = \frac{k}{4}$. In addition, the area is 106.12 units^2 corresponding to Theorem 2.*

Example 2. *Let $k = 3.5(\sqrt{3}), l = 3.5, s = 1.3$ and $t = 1.5$. From Figure 23, when folded paper is the specified size, it is a tall, uncovered wrapping that can be used as packaging because it does not correspond to Lemma 2 and Theorem 1.*

Remark 1. *From Figure 24, Khanom Saisai, which is a Thai dessert, resembles an equilateral triangular prism. We see that if k is less than l, then it makes the s-value larger than the k-value. Besides, when we fold both wings of the banana leaves together to*

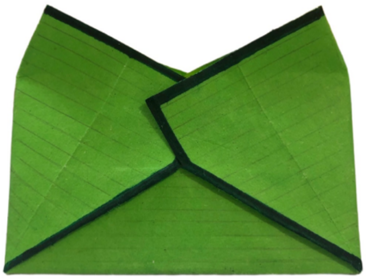


Figure 22: Folding paper in case $t = t^*$ and $s = s^*$.

Figure 23: Folding paper in case where $t < t^*$ and $s < s^*$.

wrap Khanom Saisai, the tall shape required to tightly wrap Khanom Saisai is not obtained. Therefore, it is necessary to cut the banana leaves in the collision. Thus, the banana leaves that are wrapped are not rectangular. It can be seen that wrapping Khanom Saisai is a form of tall, encased wrapping, corresponding to the definition of a tall, encased wrapping as packaging.

Figure 24: Steamed flour with coconut filling (Thai dessert) - Khanom Saisai.

4. Discussion

This research may be applied to the application of Khanom Saisai to find the smallest banana leaves as packaging to help reduce business costs. Our further work may be considered in origami wrapping with an isosceles triangle prism.

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