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Derivations in differentially δ-prime rings

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Abstract. Let R be an associative ring with identity. In this paper we extend the J.H. Maynes results, which he treatised in [27]. In particular, we prove that if R is a δ-prime ring with $char R \neq 2$ and I is a nonzero δ-ideal of R, where $0 \neq \delta \in \mathfrak{D}$, $c \in R$ and $[c, \delta(c)]$ in the center of R, then R is commutative.

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1. Introduction

It shall be assumed throughout here, that R is an associative ring with respect to the addition $(+)$ and the multiplication $(\cdot)$ with an identity, $\mathfrak{D}$ is the set of all derivations in R. Consider Lie multiplication "$[-,-]$" on R, which is defined by $[c, d] = cd - dc$, where $[c, d]$ is called a Lie commutator of elements c, d of R. The set $[C, D]$ of the additive group R^+ of a ring R is the Lie commutator subgroup generated by all $[c, d]$ such that $c \in C$ and $d \in D$. Obsreve that $Z(R)$ is the center of R, $C(R)$ is the commutator ideal generated by the set $\{[c, d] : c, d \in R\}$, $annT = \{r \in R : rT = 0 = Tr\}$ the annihilator of $T \subseteq R$.

An additive subgroup T of R is called a Lie ideal of R if $[c, d] \in T$, for all $c \in T$ and $d \in R$. An aditive map $\delta : R \to R$ is called *a derivation* on R if

$$\delta(cd) = \delta(c)d + c\delta(d)$$

for all $c, d \in R$.

Furthermore, the map $\partial_a : R \to R$ defined by, $\partial_a(c) = ac - ca$, where $c \in R$ is a partial derivation generated by $a \in R$ i.e.,

$$\partial_a(I) = [a, I] = \{[a, i] : i \in I\}.$$

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In addition, the map δ is called centralizing on a subset I of R if

$$[c, \delta(c)] = c\delta(c) - \delta(c)c \in Z(R)\ \forall c \in I$$

.

Assume that Δ is a non-empty subset of $\mathfrak{D}$. An ideal I of R, where $\delta(I) \subseteq I$ for $\delta \in \Delta$ is called a δ-ideal. A ring R is called δ-prime if, for any two δ-ideals I, J of R, the condition $IJ = 0$ implies that $I = 0$ or $J = 0$.

The special case where a centralizing automorphism is a commuting automorphism is defined by $cd(c) = d(c)c, c \in R$. Likewise, d is called a semi commuting automorphism if $cd(c) = d(c)c$ or $cd(c) = -d(c)c$ holds for any $c \in R$. All other definitions and facts are standard and they can be found in [17], [18], [19] and [23].

Recall that, the first theorem of Posner informs us that if R is a prime ring with $charR \neq 2$, then a composition of two nonzero derivations is not a derivation.

Many authors generalized Posner's theorem in various ways as Bergen [6], Chebotar [11], Chuang [12] , [13], Hirano [21], Lanski [24] and Martindale [25]. Furthermore, Creedon[15] generalized Posner's first theorem to semiprime algebras, i.e. he showed that the composition of two nonzero derivations in any algebra S is a derivation. On the other hand, from the second Posner theorem which states: if R is a prime ring with centralizing derivation $d \neq 0$ on R, then R is commutative.

In fact, this theorem extremely helped some researches to study the commuting derivations, because every centralizing derivation is a commuting derivation. Recall that, the assumption of primeness in the second Posner theorem is necessarily, since if we take for example the ring $R = S \times T$ such that S is a commutative ring with derivation d_1 and T is a non-commutative ring, then we can prove that the derivation on R given by $d(s, t) = (d_1(s), 0)$ is a non zero commuting derivation, but R is not a commutative ring.

In the last fifty years, a lot of results have been obtained about commuting and centralizing derivations d (d satisfies the condition $[d(c), c] \in Z(R)$ for all $c \in R$). However, many authors extended it by taking a centralizing map on a ring only. In 1973, Awtar [4] studied the centralizing derivation on Lie ideals and Jordan ideals. In particular, he proved that if R is a prime ring of $charR \neq 2$ and $T \neq \{0\}$ is a Lie ideal or Jordan ideal and subring in R, with $d \neq 0$ being a derivation on R, if $[c, d(c)] \in Z(R)$, for all $c \in T$ then R is commutative.

In addition, if we assume that either T is a Lie (Jordan) ideal or a subring , then R is not necessarily commutative. That can be shown as follows: let R be a prime ring with $charR \neq 2$ and $d \neq 0$ is a derivation of R. If T is a Lie or Jordan ideal and a subring of R and if $[c, d(c)] \in Z(R)$, for all $c \in T$, then the ring R is commutative.

Mayne [26] got the same result, i.e. if R is a prime ring and $d \neq 0$ is a centralizing automorphism, then R is an integral domain. Furthermore, Mayne [27] generalized the previous results for a derivation d or an automorphism,

Moreover, Mayne [28] showed that if there exists a centralizing derivation $d \neq \{0\}$ or a centralizing automorphis on an ideal $T \neq 0$ of a prime ring R, then R is commutative.

Also, Awtar [4] extended the derivation case on a prime ring with any characteristic. Whereas, McCrimmon [29] proved that the automorphism in Mayne's theorem did not generalize for a semiprime ring.

Likewise, Vukman [32] has extended Posner's second theorem by proving that if $d \neq 0$ is a derivation on prime ring with $charR \neq 2$ and $[[d(c), c], c] = 0$, for all $c \in R$, then either $d = 0$ or R is commutative. In fact, this theorem has merely showed that d is commuting. In addition, in 1992, Vukman extended the second Posner's result for an automorphism or a centralizing derivation on a Lie ideal $T \neq \{0\}$. Whereas, in 1993 Bresar [8] showed that an additive map is not centralizing on determined subsets of prime and non-commutative ring.

Futhermore, Some generalizations of these results for a prime ring are contained in [20–22]. As for a semiprime ring we refer the reader to [10], [7], [9], [32] and [33].

In our current research we shall generalize the theorem of Mayne [27] and theorem of Hirano and Tominaga [21], so this generalization of the two theorems give us a new wider class of δ-prime rings and we prove the following

Theorem 1. *Let R be a δ-prime ring of charastristic $\neq 2$ and I be a nonzero δ-ideal of R, where $0 \neq \delta \in \mathfrak{D}$. If*

$$[c, \delta(c)] \in Z(R) \ \forall c \in I.$$

Then R is commutative.

2. Preliminaries

Many authors have been studying the centralizing automorphisms and derivation on ring R.

C. R. Miers [30] has considered the map defined on C^* Algebra.

Moreover, in [4] R.A. Awtar showed if existence a nonzero centerlizing derivation on a prime ring, then R is commutative, so he gives a shorter proof of Posner's theorem [31].

Awtar in [4] proved that if R is a prime ring with $charR \neq 2$ havig a derivation d on a Jordan ideal $J \neq \{0\}$, where the derivation is centralizing on J, implies $J \subseteq Z(R)$.

In [14] L.O. Chung and J.Luh showed the equivalence between semi-commuting automorphism and commuting automorphism on a prime ring. If the prime ring R has a nontrivial semicommuting automorphism and R with $charR \neq 2$ or $Z(R) \neq \{0\}$, this implies the commutativity of the ring R.

In [16] N. Divinsky proved that if the simple Artinian ring has a nontrivial centralizing automorphism, then R is a field.

On the other hand in [21] it has been proved that if R has a nontrivial automorphism, then R is a field.

In [1, 2] it has been proved the commutativity of a prime and semiprime rings.

Now willing to prove our theorem, we will need to state some lemmas:

Lemma 1. *Let $I \neq \{0\}$ be δ-ideal of a δ-prime ring R. If $\delta(I) = 0$, then $\delta(R) = 0$.*

Proof. Since $RI \subseteq I$ and $IR \subseteq I$. Then

$$\delta(RI) = \delta(R)I = 0 = \delta(IR) = I\delta(R).$$

Thus we deduce that $\delta(R) \subseteq annI$, but I is a δ-ideal and so

$$\delta(R) = 0.$$

Lemma 2. *Let $\delta \neq 0$ be a derivation on a ring R and $I \neq \{0\}$ be δ-ideal of R. If R is δ-prime such that*

$$[\delta(a), a] = 0 \ \forall a \in I. \qquad (2-1)$$

then R is commutative.

Proof. Linearizing the equation (2-1) on I, then we have for all $a, b, c \in I$

$$0 = [\delta(a+b), a+b] = [\delta(a), a] + [\delta(a), b] +$$

$$[\delta(b), a] + [\delta(b), b] = [\delta(a), b] + [\delta(b), a].$$

Thus we conclude that

$$[\delta(a), b] = [a, \delta(b)]. \qquad (2-2)$$

Now replacing the right side in (2-2) $\delta(b)$ by $a\delta(b)$ we have

$$[a, a\delta(b)] = a[a, \delta(b)] = a[\delta(a), b] =$$

$$a\delta(a)b - ab\delta(a) = \delta(a)ab - ab\delta(a) =$$

$$[\delta(a), ab] = [a, \delta(ab)] = [a, \delta(a)b] + [a, a\delta(b)].$$

Hence

$$\delta(a)[a, b] = [a, \delta(a)b] = 0. \qquad (2-3)$$

Now replacing b by cb in (2-3) we obtain that

$$0 = \delta(a)[a, cb] = \delta(a)[a, c]b + \delta(a)c[a, b] = \delta(a)c[a, b].$$

Consequently,

$$\delta(a)I[a, b] = 0.$$

Thus by using the δ-primenes we get either $a = 0$ or $[a, b] = 0$. Since $I \neq \{0\}$, then we deduce that $[a, b] = 0$ and therefore, I is commutative. Then we have

$$I^2C(R) = 0,$$

and so $C(R) = 0$. Hence R is commutative.

3. δ-derivation on δ-ideal

First of all in the next lemma we give a generalization of Lemma 1 from [31]

Lemma 3. *Let $\delta \neq 0$ be a derivation of a ring R . If R is δ-prime ring such that*

$$a[\delta^n(a), R] = 0,$$

$$(respectively\ [\delta^n(b), R]a = 0)\ \forall a, b \in R,$$

and for all integers $n \geq 0$, then either $a = 0$ or $b \in Z(R)$.

Proof. Suppose that $x, y \in R$ and n, k are a nonnegative integers. From [31] we have

$$a\partial_{\delta^n(b)}(R) = 0,$$

then

$$0 = a\partial_{\delta^n(b)}(xy) = a\partial_{\delta^n(b)}(x)y + ax\partial_{\delta^n(b)}(y) =$$

$$ax\partial_{\delta^n(b)}(y).$$

This means that

$$aR[\delta^n(b), y] = 0.$$

Consequently,

$$aR\delta^k([\delta^n(b), y]) = 0,$$

what forces that $a = 0$ or $[\delta^n(b), y] = 0$ (and then $b \in Z(R)$).

Lemma 4. *Let $I \neq \{0\}$ be a right δ-ideal of a δ-prime ring R. If I is commutative, then R is commutative.*

Proof. Suppose that $a \in I$. Then $\partial_a(I) = 0$ and so $\partial_a(R) \subseteq annI$.
Since $annI$ is a δ-ideal, then we see that

$$annI = 0,$$

and so $a \in Z(R)$. Hence $I \subseteq Z(R)$. Then $IC(R) = 0$ and hence

$$C(R) = 0.$$

Lemma 5. *Let $\delta \neq 0$ be a nonzero derivation of a δ-prime ring R. If*

$$[b, \delta^n(a)b] \in Z(R),$$

and $0 \neq b \in R$ for all integers $n \geq 0$, then $a \in Z(R)$.

Proof. Since for all $x \in R$ we have

$$0 = [\delta^n(a)b, x] = \delta^n(a)[b, x] + [\delta^n(a), x]b = [\delta^n(a), x]b.$$

Then by Lemma 3 we see that $a \in Z(R)$.

Lemma 6. *Let R be a $\delta-$prime ring of characterstic $\neq 2$ and I be a $\delta-$ideal of R. If*

$$[x, \delta(x)] \in Z(R) \ \forall x \in I, \qquad (3-1)$$

then $[x, \delta(x)] = 0$.

Proof. Suppose that $x, y \in I$. Now replace x by $x + y$ in (3-1) we get

$$[x + y, \delta(x + y)] = [x, \delta(x)] + [x, \delta(y)] + [y, \delta(x)] + [y, \delta(y)],$$

and so

$$[x, \delta(y)] + [y, \delta(x)] \in Z(R). \qquad (3-2).$$

Now substituting y by x^2 in (3-2), we obtain

$$4x[x, \delta(x)] = [x, \delta(x^2)] + [x^2, \delta(x)] \in Z(R).$$

Consequently,

$$x[x, \delta(x)] \in Z(R) \qquad (3-3).$$

Then

$$0 = [x[x, \delta(x)], \delta(x)] = [x, \delta(x)]^2.$$

Obviously that

$$\delta([x, \delta(x)]) \in Z(R).$$

and

$$\delta([x, \delta(x)]) = [\delta(x), \delta(x)] + [x, \delta^2(x)] =$$

$$[x, \delta^2(x)]$$

and

$$\delta([x, \delta^2(x)]) \in Z(R).$$

$$\delta([x, \delta^2(x)]) = [\delta(x), \delta^2(x)] + [x, \delta^3(x)].$$

Hence $[\delta(x), \delta^2(x)] \in Z(G)$ we have

$$[x, \delta^3(x)] \in Z(R).$$

In addition, by induction on n we obtain that

$$[x, \delta^n(x)] \in Z(R), \qquad (3-4).$$

Now substituting y by $x\delta^n(x)$ in (3-2) we get (since $[x, \delta(x\delta^n(x)] + [x\delta^n(x), \delta(x)] \in Z(G)$)

$$[x, \delta(x\delta^n(x)] + [x\delta^n(x), \delta(x)] =$$

$$[x, \delta(x)\delta^n(x)] + [x, x\delta^{n+1}(x)] - [\delta(x), x\delta^n(x)] =$$

$$[x, \delta(x)]\delta^n(x) + \delta(x)[x, x\delta^n(x)] +$$

$$+x[x, x\delta^{n+1}(x)] - [\delta(x), x]\delta^n(x) -$$

$$-x[\delta(x), \delta^n(x)] =: T.$$

Then,

$$0 = [T, \delta^n(x)] = [\delta(x), \delta^n(x)].[x, \delta^n(x)] +$$

$$[x, \delta^n(x)].[x, \delta^{n+1}(x)] - [x, \delta^n(x)].[\delta(x), \delta^n(x)] =$$

$$[x, \delta^n(x)][x, \delta^{n+1}(x)]. \qquad (3-5)$$

Substituting y by $x^2\delta^n(x)$ in (3-2), we will obtain

$$[x, \delta(x^2\delta^n(x))] + [x^2\delta^n(x), \delta(x)] \in Z(R))$$

$$[x, \delta(x^2\delta^n(x))] + [x^2\delta^n(x), \delta(x)] =$$

$$[x, \delta(x)x\delta^n(x)] + [x, x\delta(x)\delta^n(x)]+$$

$$+[x, x^2\delta^{n+1}(x)] - [\delta(x), x^2\delta^n(x)] =$$

$$[x, \delta(x)x]\delta^n(x) + [x, \delta^n(x)]\delta(x)x+$$

$$[x, x\delta(x)]\delta^n(x) + [x, \delta^n(x)]x\delta(x)+$$

$$[x, x^2]\delta^{n+1}(x) + [x, \delta^{n+1}(x)]x^2-$$

$$-[\delta(x), x^2]\delta^n(x) - [\delta(x), \delta^n(x)]x^2 =$$

$$[x, \delta(x)]x\delta^n(x) + [x, x]\delta(x)\delta^n(x)+$$

$$[x, \delta^n(x)]\delta(x)x + [x, x]\delta(x)\delta^n(x)+$$

$$[x, \delta(x)]x\delta^n(x) + [x, \delta^n(x)]x\delta(x)+$$

$$[x, \delta^{n+1}(x)]x^2 - 2[\delta(x), x]x\delta^n(x)-$$

$$-[\delta(x), \delta^n(x)]x^2 =$$

$$4[x, \delta(x)]x\delta^n(x) + [x, \delta^n(x)]x\delta(x)+$$

$$+[x, \delta^n(x)]x\delta(x)x + [x, \delta^{n+1}(x)]x^2-$$

$$+[\delta(x), \delta^n(x)]x^2 =: Q. \qquad (3-6)$$

multiplying Q by $[x, \delta^n(x)]$ in (3-6)
A in view of (3-2) we obtain

$$[x, \delta^n(x)]^2x\delta(x) + [x, \delta^n(x)]^2\delta(x)x-$$

$$-[\delta(x), \delta^n(x)][x, \delta^n(x)]x^2 \in Z(R).$$

Then,

$$0 = [\delta^n(x), F] = [\delta^n(x), x\delta(x)][x, \delta^n(x)]^2+$$

$$+[\delta^n(x), \delta(x)x][x\delta^n(x)]^2-$$

$$-[\delta(x), \delta^n(x)][x, \delta^n(x)][\delta^n(x), x^2] =$$

$$= [\delta^n(x), x][x, \delta^n(x)]^2\delta(x)+$$

$$= [\delta^n(x), \delta(x)][x, \delta^n(x)]^2\delta(x)+$$

$$+[\delta^n(x), \delta(x)][x, \delta^n(x)]^2x+$$

$$+[\delta^n(x), \delta(x)][x, \delta^n(x)]^2x+$$

$$+[\delta^n(x), x][x, \delta^n(x)]^2\delta(x)-$$

$$-2[\delta(x), \delta^n(x)][x, \delta^n(x)][\delta^n(x), x]x =$$

$$2[\delta^n(x), \delta(x)][x, \delta^n(x)]^2-$$

$$-2[x, \delta^n(x)]^3\delta(x) + 2[\delta(x), \delta^n(x)][x, \delta^n(x)]^2x =$$

$$= 2[x, \delta^n(x)]^3\delta(x)] =: X$$

Then

$$[x, \delta^n(x)]^3\delta(x)] = 0$$

Thus

$$[x, \delta^{n+1}(x)]^3\delta(x)] = 0$$

Hence

$$[x, \delta^{n+1}(x)]^3[\delta(x), \delta^n(x)] =$$

$$[x, \delta^{n+1}(x)]^3\delta(x)\delta^n(x)-$$

$$-\delta^n(x)[x, \delta^{n+1}(x)]^3\delta(x) = 0. \qquad (3-7)$$

Multiplying (3-7) by $[x, \delta^{n+1}(x)]^2$ we obtain

$$[x, \delta^{n+1}(x)]^4 = 0.$$

Thus

$$([x, \delta^{n+1}(x)]R^4 = 0.$$

this means that

$$A = \sum_{n=1}^{\infty}\sum_{x\in I}[x, \delta^n(x)]R.$$

$$(\delta([x, \delta^{n+1}(x)]R) = [\delta(x), \delta^{n+1}(x)]R+$$

$$[x, \delta^{n+2}(x)]R + [x, \delta^n(x)]\delta(R) \subseteq A)$$

is a sum of nilpotent ideals and I is a nil ideal, since A is δ-ideal, we deduce that

$$A = 0.$$

This gives that

$$[x, \delta(x)] = 0.$$

Lemma 7. *Let R be δ-prime ring of char$R \neq 2$ and*

$$[\delta(x), x] \in Z(R)\ \forall x \in R.$$

Then R is commutative.

Proof. It is well known that $[R, R]$ is a Lie ideal of R. Moreover,

$$\delta([R, R]) \subseteq [R, R].$$

Now on the one hand if $[R, R]$ is commutative, then by Lemma (1-7) in [5] $C(R)$ is a nil ideal, Thus

$$C(R) = 0,$$

and R is commutative.

On the other hand by Lemma 13 [3] $[R, R]$ contains a nonzro δ-ideal I of R. Thus by(3-2) we have

$$[\delta(x), y] \in Z(R)\ \forall x, y \in R.$$

This means that $\delta(I) \subseteq Z(R)$.Then for all $a \in I$

$$[\delta(a), a] = 0,$$

and by Lemma 2 R is commutative.

proof of Theorem (1)

Since $[x, \delta(x)] \in Z(R)$ for all $x \in I$, then by Lemma 7 we get

$$[x, \delta(x)] = 0.$$

Now using Lemma 2 and since R is A δ-prime ring and $[x, \delta(x)] = 0$, then R is commutative.

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