EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 15, No. 3, 2022, 1180-1188
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Slight (Λ, sp)-continuity and Λ_{sp}-extremally disconnectedness

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Abstract. This paper is concerned with the concepts of upper and lower slightly (Λ, sp)-continuous multifunctions. Moreover, some characterizations of upper and lower slightly (Λ, sp)-continuous multifunctions are established.

2020 Mathematics Subject Classifications: 54C08, 54C60, 54G05

Key Words and Phrases: Upper slight (Λ, sp)-continuity, lower slight (Λ, sp)-continuity, Λ_{sp}-extremally disconnectedness

1. Introduction

Stronger and weaker forms of open sets play an important role in the researches of generalizations of continuity for functions and multifunctions in topological spaces. The concept of slightly continuous functions was first introduced by Jain [6]. In 1995, Nour [11] defined slightly semi-continuous functions as a weak form of slight continuity and investigated several characterizations of slightly semi-continuous functions. Noiri and Chae [8] further investigated slight semi-continuity. Pal and Bhattacharya [12] defined a function to be faintly precontinuous if the preimage of each clopen set of the codomain is preopen and obtained some properties of such functions. Slight continuity implies both slight semi-continuity and faint precontinuity. In 2001, Noiri [7] introduced the concept of slight β-continuity which is implies by both slight semi-continuity and faint precontinuity. A unified theory of slight continuity is presented in [14], the present authors introduced and investigated the concept of slightly m-continuous functions. Noiri and Popa [10] introduced the notion of slightly m-continuous multifunctions and studied the relationships among m-continuity, almost m-continuity, weak m-continuity and slight m-continuity for multifunctions.

The concept of β-open sets due to Abd El-Monsef et al. [5] or semi-preopen sets in the sense of Andrijević [1] plays a significant role in general topology. In 2004, Noiri and Hatir

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DOI: https://doi.org/10.29020/nybg.ejpam.v15i3.4369

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[9] introduced the notion of Λ_{sp}-sets in terms of the concept of β-open sets and investigated the notion of Λ_{sp}-closed sets by using Λ_{sp}-sets. In [3], the author introduced the concepts of (Λ, sp)-open sets and (Λ, sp)-closed sets which are defined by utilizing the notions of Λ_{sp}-sets and β-closed sets. In particular, some characterizations of Λ_{sp}-extremally disconnected spaces are investigated in [3]. The purpose of the present paper is to introduce the notions of upper and lower slightly (Λ, sp)-continuous multifunctions. Furthermore, some characterizations of upper and lower slightly (Λ, sp)-continuous multifunctions are discussed.

2. Preliminaries

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *β-open* [5] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$. The complement of a β-open set is called *β-closed.* The family of all β-open sets of a topological space (X, τ) is denoted by $\beta(X, \tau)$. A subset $\Lambda_{sp}(A)$ [9] is defined as follows: $\Lambda_{sp}(A) = \cap\{U \mid A \subseteq U, U \in \beta(X, \tau)\}$. A subset A of a topological space (X, τ) is called a *Λ_{sp}-set* [9] if $A = \Lambda_{sp}(A)$. A subset A of a topological space (X, τ) is called *(Λ, sp)-closed* [3] if $A = T \cap C$, where T is a Λ_{sp}-set and C is a β-closed set. The complement of a (Λ, sp)-closed set is called *(Λ, sp)-open.* The family of all (Λ, sp)-open sets in a topological space (X, τ) is denoted by $\Lambda_{sp}O(X, \tau)$. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, sp)-cluster point* [3] of A if $A \cap U \neq \emptyset$ for every (Λ, sp)-open set U of X containing x. The set of all (Λ, sp)-cluster points of A is called the *(Λ, sp)-closure* [3] of A and is denoted by $A^{(\Lambda,sp)}$. The union of all (Λ, sp)-open sets contained in A is called the *(Λ, sp)-interior* [3] of A and is denoted by $A_{(\Lambda,sp)}$.

Lemma 1. [3] *Let A and B be subsets of a topological space (X, τ). For the (Λ, sp)-closure, the following properties hold:*

(1) $A \subseteq A^{(\Lambda,sp)}$ and $[A^{(\Lambda,sp)}]^{(\Lambda,sp)} = A^{(\Lambda,sp)}$.

(2) If $A \subseteq B$, then $A^{(\Lambda,sp)} \subseteq B^{(\Lambda,sp)}$.

(3) $A^{(\Lambda,sp)} = \cap\{F | A \subseteq F$ and F is (Λ, sp)-closed$\}$.

(4) $A^{(\Lambda,sp)}$ is (Λ, sp)-closed.

(5) A is (Λ, sp)-closed if and only if $A = A^{(\Lambda,sp)}$.

Lemma 2. [3] *Let A and B be subsets of a topological space (X, τ). For the (Λ, sp)-interior, the following properties hold:*

(1) $A_{(\Lambda,sp)} \subseteq A$ and $[A_{(\Lambda,sp)}]_{(\Lambda,sp)} = A_{(\Lambda,sp)}$.

(2) If $A \subseteq B$, then $A_{(\Lambda,sp)} \subseteq B_{(\Lambda,sp)}$.

(3) $A_{(\Lambda,sp)}$ *is* (Λ, sp)*-open.*

(4) A *is* (Λ, sp)*-open if and only if* $A_{(\Lambda,sp)} = A$.

(5) $[X - A]^{(\Lambda,sp)} = X - A_{(\Lambda,sp)}$.

(6) $[X - A]_{(\Lambda,sp)} = X - A^{(\Lambda,sp)}$.

A subset A of a topological space (X, τ) is said to be $s(\Lambda, sp)$*-open* (resp. $p(\Lambda, sp)$*-open*, $r(\Lambda, sp)$*-open*, $\beta(\Lambda, sp)$*-open*) if $A \subseteq [A_{(\Lambda,sp)}]^{(\Lambda,sp)}$ (resp. $A \subseteq [A^{(\Lambda,sp)}]_{(\Lambda,sp)}$, $A = [A^{(\Lambda,sp)}]_{(\Lambda,sp)}$, $A \subseteq [[A^{(\Lambda,sp)}]_{(\Lambda,sp)}]^{(\Lambda,sp)}$) [3]. The complement of a $s(\Lambda, sp)$-open (resp. $p(\Lambda, sp)$-open, $r(\Lambda, sp)$-open, $\beta(\Lambda, sp)$-open) set is called $s(\Lambda, sp)$*-closed* (resp. $p(\Lambda, sp)$*-closed*, $r(\Lambda, sp)$*-closed*, $\beta(\Lambda, sp)$*-closed*).

The family of all $s(\Lambda, sp)$-open (resp. $p(\Lambda, sp)$-open, $r(\Lambda, sp)$-open, $\beta(\Lambda, sp)$-open) sets in a topological space (X, τ) is denoted by $s\Lambda_{sp}O(X, \tau)$ (resp. $p\Lambda_{sp}O(X, \tau)$, $r\Lambda_{sp}O(X, \tau)$, $\beta\Lambda_{sp}O(X, \tau)$). A subset A of a topological space (X, τ) is called (Λ, sp)*-clopen* [4] if A is both (Λ, sp)-open and (Λ, sp)-closed.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [2] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$. Then, F is said to be a *surjection* if $F(X) = Y$, or equivalently, if for each $y \in Y$, there exists an $x \in X$ such that $y \in F(x)$. Moreover, $F : X \to Y$ is called *upper semi-continuous* (resp. *lower semi-continuous*) if $F^+(V)$ (resp. $F^-(V)$) is open in X for every open set V of Y [13].

3. Upper and lower slightly (Λ, sp)-continuous multifunctions

In this section, we introduce of the concepts of upper and lower slightly (Λ, sp)-continuous multifunctions. Moreover, some characterizations of upper and lower slightly (Λ, sp)-continuous multifunctions are discussed.

Definition 1. *A multifunction* $F : (X, \tau) \to (Y, \sigma)$ *is said to be:*

(i) upper slightly (Λ, sp)*-continuous if, for each* $x \in X$ *and each* (Λ, sp)*-clopen set* V *of* Y *such that* $F(x) \subseteq V$*, there exists a* (Λ, sp)*-open set* U *of* X *containing* x *such that* $F(U) \subseteq V$*;*

(ii) lower slightly (Λ, sp)*-continuous if, for each* $x \in X$ *and each* (Λ, sp)*-clopen set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$*, there exists a* (Λ, sp)*-open set* U *of* X *containing* x *such that* $F(z) \cap V \neq \emptyset$ *for each* $z \in U$*.*

Theorem 1. *For a multifunction* $F : (X, \tau) \to (Y, \sigma)$*, the following properties are equivalent:*

(1) F is upper slightly (Λ, sp)-continuous;

(2) $F^+(V)$ is (Λ, sp)-open in X for every (Λ, sp)-clopen set V of Y;

(3) $F^-(V)$ is (Λ, sp)-closed in X for every (Λ, sp)-clopen set V of Y.

Proof. (1) $\Rightarrow$ (2): Let V be any (Λ, sp)-clopen set of Y and let $x \in F^+(V)$. Then, $F(x) \subseteq V$. Since F is upper slightly (Λ, sp)-continuous, there exists $U \in \Lambda_{sp}O(X, \tau)$ containing x such that $F(U) \subseteq V$. Thus, $x \in U \subseteq F^+(V)$ and hence $x \in [F^+(V)]_{(\Lambda,sp)}$. Therefore, $F^+(V) \subseteq [F^+(V)]_{(\Lambda,sp)}$. This shows that $F^+(V)$ is (Λ, sp)-open in X.

(2) $\Rightarrow$ (3) and (3) $\Rightarrow$ (2): The proofs are obvious.

(2) $\Rightarrow$ (1): Let $x \in X$ and let V be any (Λ, sp)-clopen set of Y containing $F(x)$. Then, $x \in F^+(V)$, by (2), $F^+(V)$ is (Λ, sp)-open in X. Put $U = F^+(V)$, then U is a (Λ, sp)-open set of X containing x such that $F(U) \subseteq V$. Thus, F is upper slightly (Λ, sp)-continuous.

Theorem 2. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) F is lower slightly (Λ, sp)-continuous;

(2) $F^-(V)$ is (Λ, sp)-open in X for every (Λ, sp)-clopen set V of Y;

(3) $F^+(V)$ is (Λ, sp)-closed in X for every (Λ, sp)-clopen set V of Y.

Proof. The proof is similar to that of Theorem 1.

Definition 2. *A function $f : (X, \tau) \to (Y, \sigma)$ is called slightly (Λ, sp)-continuous if, for each $x \in X$ and each (Λ, sp)-clopen set V of Y containing $f(x)$, there exists a (Λ, sp)-open set U of X containing x such that $f(U) \subseteq V$.*

Corollary 1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is slightly (Λ, sp)-continuous;

(2) $f^{-1}(V)$ is (Λ, sp)-open in X for every (Λ, sp)-clopen set V of Y;

(3) $f^{-1}(V)$ is (Λ, sp)-closed in X for every (Λ, sp)-clopen set V of Y.

Definition 3. *Let A be a subset of a topological space (X, τ). The (Λ, sp)-frontier of A, denoted by (Λ, sp)-$Fr(A)$, (Λ, sp)-$Fr(A) = A^{(\Lambda,sp)} \cap [X - A]^{(\Lambda,sp)} = A^{(\Lambda,sp)} - A_{(\Lambda,sp)}$.*

Theorem 3. *The set of all points $x \in X$ at which a multifunction $F : (X, \tau) \to (Y, \sigma)$ is not upper slightly (Λ, sp)-continuous is identical with the union of (Λ, sp)-frontiers of the upper inverse images of (Λ, sp)-clopen sets containing $F(x)$.*

Proof. Suppose that F is not upper slightly (Λ, sp)-continuous at $x \in X$. Then, there exists a (Λ, sp)-clopen set V of Y containing $F(x)$ such that $U \cap (X - F^+(V)) \neq \emptyset$ for every $U \in \Lambda_{sp}O(X, \tau)$ containing x. Thus, $x \in [X - F^+(V)]^{(\Lambda,sp)}$. On the other hand, we have $x \in F^+(V) \subseteq [F^+(V)]^{(\Lambda,sp)}$ and hence $x \in (\Lambda, sp)\text{-}Fr(F^+(V))$.

Conversely, suppose that F is upper slightly (Λ, sp)-continuous at $x \in X$. Let V be any (Λ, sp)-clopen set of Y containing $F(x)$. Then, there exists $U \in \Lambda_{sp}O(X, \tau)$ containing x such that $U \subseteq F^+(V)$; hence $x \in [F^+(V)]_{(\Lambda,sp)}$. Thus, $x \notin (\Lambda, sp)\text{-}Fr(F^+(V))$ for every (Λ, sp)-clopen set V of Y containing $F(x)$.

Theorem 4. *The set of all points $x \in X$ at which a multifunction $F : (X, \tau) \to (Y, \sigma)$ is not lower slightly (Λ, sp)-continuous is identical with the union of (Λ, sp)-frontiers of the lower inverse images of (Λ, sp)-clopen sets meeting $F(x)$.*

Proof. The proof is similar to that of Theorem 3.

Definition 4. [3] *A topological space (X, τ) is called Λ_{sp}-extremally disconnected if $V^{(\Lambda,sp)}$ is (Λ, sp)-open in X for every (Λ, sp)-open set V of X.*

Theorem 5. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is a Λ_{sp}-extremally disconnected space, the following properties are equivalent:*

(1) F is upper slightly (Λ, sp)-continuous;

(2) $[F^-(V)]^{(\Lambda,sp)} \subseteq F^-(V^{(\Lambda,sp)})$ for every (Λ, sp)-open set V of Y;

(3) $F^+(K_{(\Lambda,sp)}) \subseteq [F^+(K)]_{(\Lambda,sp)}$ for every (Λ, sp)-closed set K of Y.

Proof. (1) ⇒ (2): Let V be any (Λ, sp)-open set of Y. Since (Y, σ) is Λ_{sp}-extremally disconnected, $V^{(\Lambda,sp)}$ is (Λ, sp)-open in Y. Thus, $V^{(\Lambda,sp)}$ is (Λ, sp)-clopen in Y. By Theorem 1, $F^-(V^{(\Lambda,sp)})$ is (Λ, sp)-closed and hence $[F^-(V)]^{(\Lambda,sp)} \subseteq [F^-(V^{(\Lambda,sp)})]^{(\Lambda,sp)} = F^-(V^{(\Lambda,sp)})$.

(2) ⇒ (3): Let K be any (Λ, sp)-closed set of Y. Then, $Y - K$ is (Λ, sp)-open in Y, by (2), we have

$$\begin{aligned} X - [F^+(K)]_{(\Lambda,sp)} &= [X - F^+(K)]^{(\Lambda,sp)} \\ &= [F^-(Y - K)]^{(\Lambda,sp)} \\ &\subseteq F^-([Y - K]^{(\Lambda,sp)}) \\ &= F^-(Y - K_{(\Lambda,sp)}) \\ &= X - F^+(K_{(\Lambda,sp)}) \end{aligned}$$

and hence $F^+(K_{(\Lambda,sp)}) \subseteq [F^+(K)]_{(\Lambda,sp)}$.

(3) ⇒ (1): Let $x \in X$ and let V be any (Λ, sp)-clopen set of Y containing $F(x)$. Thus, by (3), $x \in F^+(V) = F^+(V_{(\Lambda,sp)}) \subseteq [F^+(V)]^{(\Lambda,sp)}$. Then, there exists $U \in \Lambda_{sp}O(X, \tau)$ such that $x \in U \subseteq F^+(V)$; hence $F(U) \subseteq V$. This shows that F is upper slightly (Λ, sp)-continuous.

Theorem 6. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is a Λ_{sp}-extremally disconnected space, the following properties are equivalent:*

(1) F is lower slightly (Λ, sp)-continuous;

(2) $[F^+(V)]^{(\Lambda,sp)} \subseteq F^+(V^{(\Lambda,sp)})$ for every (Λ, sp)-open set V of Y;

(3) $F^-(K_{(\Lambda,sp)}) \subseteq [F^-(K)]_{(\Lambda,sp)}$ for every (Λ, sp)-closed set K of Y.

Proof. The proof is similar to that of Theorem 5.

Lemma 3. [3] *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is Λ_{sp}-extremally disconnected.

(2) The (Λ, sp)-closure of every $s(\Lambda, sp)$-open set of X is (Λ, sp)-open.

(3) The (Λ, sp)-closure of every $p(\Lambda, sp)$-open set of X is (Λ, sp)-open.

(4) The (Λ, sp)-closure of every $r(\Lambda, sp)$-open set of X is (Λ, sp)-open.

Theorem 7. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is a Λ_{sp}-extremally disconnected space, the following properties are equivalent:*

(1) F is upper slightly (Λ, sp)-continuous;

(2) $[F^-(V)]^{(\Lambda,sp)} \subseteq F^-(V^{(\Lambda,sp)})$ for every $s(\Lambda, sp)$-open set V of Y;

(3) $F^+(K_{(\Lambda,sp)}) \subseteq [F^+(K)]_{(\Lambda,sp)}$ for every $s(\Lambda, sp)$-closed set K of Y.

Proof. The proof is similar to that of Theorem 5 and it follows from Theorem 1 and Lemma 3.

Theorem 8. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is a Λ_{sp}-extremally disconnected space, the following properties are equivalent:*

(1) F is lower slightly (Λ, sp)-continuous;

(2) $[F^+(V)]^{(\Lambda,sp)} \subseteq F^+(V^{(\Lambda,sp)})$ for every $s(\Lambda, sp)$-open set V of Y;

(3) $F^-(K_{(\Lambda,sp)}) \subseteq [F^-(K)]_{(\Lambda,sp)}$ for every $s(\Lambda, sp)$-closed set K of Y.

Proof. The proof is similar to that of Theorem 6 and it follows from Theorem 2 and Lemma 3.

Corollary 2. *For a function $f : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is a Λ_{sp}-extremally disconnected space, the following properties are equivalent:*

(1) f is slightly (Λ, sp)-continuous;

(2) $[f^{-1}(V)]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ *for every* $s(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $f^{-1}(K_{(\Lambda,sp)}) \subseteq [f^{-1}(K)]_{(\Lambda,sp)}$ *for every* $s(\Lambda, sp)$*-closed set* K *of* Y*.*

Theorem 9. *For a multifunction* $F : (X, \tau) \to (Y, \sigma)$*, where* (Y, σ) *is a* Λ_{sp}*-extremally disconnected space, the following properties are equivalent:*

(1) F *is upper slightly* (Λ, sp)*-continuous;*

(2) $[F^{-}(V)]^{(\Lambda,sp)} \subseteq F^{-}(V^{(\Lambda,sp)})$ *for every* $p(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $F^{+}(K_{(\Lambda,sp)}) \subseteq [F^{+}(K)]_{(\Lambda,sp)}$ *for every* $p(\Lambda, sp)$*-closed set* K *of* Y*.*

Proof. The proof is similar to that of Theorem 5 and it follows from Theorem 1 and Lemma 3.

Theorem 10. *For a multifunction* $F : (X, \tau) \to (Y, \sigma)$*, where* (Y, σ) *is a* Λ_{sp}*-extremally disconnected space, the following properties are equivalent:*

(1) F *is lower slightly* (Λ, sp)*-continuous;*

(2) $[F^{+}(V)]^{(\Lambda,sp)} \subseteq F^{+}(V^{(\Lambda,sp)})$ *for every* $p(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $F^{-}(K_{(\Lambda,sp)}) \subseteq [F^{-}(K)]_{(\Lambda,sp)}$ *for every* $p(\Lambda, sp)$*-closed set* K *of* Y*.*

Proof. The proof is similar to that of Theorem 6 and it follows from Theorem 2 and Lemma 3.

Corollary 3. *For a function* $f : (X, \tau) \to (Y, \sigma)$*, where* (Y, σ) *is a* Λ_{sp}*-extremally disconnected space, the following properties are equivalent:*

(1) f *is slightly* (Λ, sp)*-continuous;*

(2) $[f^{-1}(V)]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ *for every* $p(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $f^{-1}(K_{(\Lambda,sp)}) \subseteq [f^{-1}(K)]_{(\Lambda,sp)}$ *for every* $p(\Lambda, sp)$*-closed set* K *of* Y*.*

Theorem 11. *For a multifunction* $F : (X, \tau) \to (Y, \sigma)$*, where* (Y, σ) *is a* Λ_{sp}*-extremally disconnected space, the following properties are equivalent:*

(1) F *is upper slightly* (Λ, sp)*-continuous;*

(2) $[F^{-}(V)]^{(\Lambda,sp)} \subseteq F^{-}(V^{(\Lambda,sp)})$ *for every* $\beta(\Lambda, sp)$*-open set* V *of* Y*;*

(3) $F^{+}(K_{(\Lambda,sp)}) \subseteq [F^{+}(K)]_{(\Lambda,sp)}$ *for every* $\beta(\Lambda, sp)$*-closed set* K *of* Y*.*

Proof. The proof is similar to that of Theorem 5 and it follows from Theorem 1 and Lemma 3.

Theorem 12. *For a multifunction $F : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is a Λ_{sp}-extremally disconnected space, the following properties are equivalent:*

(1) F is lower slightly (Λ, sp)-continuous;

(2) $[F^+(V)]^{(\Lambda,sp)} \subseteq F^+(V^{(\Lambda,sp)})$ for every $\beta(\Lambda, sp)$-open set V of Y;

(3) $F^-(K_{(\Lambda,sp)}) \subseteq [F^-(K)]_{(\Lambda,sp)}$ for every $\beta(\Lambda, sp)$-closed set K of Y.

Proof. The proof is similar to that of Theorem 6 and it follows from Theorem 2 and Lemma 3.

Corollary 4. *For a function $f : (X, \tau) \to (Y, \sigma)$, where (Y, σ) is a Λ_{sp}-extremally disconnected space, the following properties are equivalent:*

(1) f is slightly (Λ, sp)-continuous;

(2) $[f^{-1}(V)]^{(\Lambda,sp)} \subseteq f^{-1}(V^{(\Lambda,sp)})$ for every $\beta(\Lambda, sp)$-open set V of Y;

(3) $f^{-1}(K_{(\Lambda,sp)}) \subseteq [f^{-1}(K)]_{(\Lambda,sp)}$ for every $\beta(\Lambda, sp)$-closed set K of Y.

4. Conclusion

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. This paper deals with the concepts of upper and lower slight (Λ, sp)-continuity. In particular, some characterizations of upper and lower slightly (Λ, sp)-continuous multifunctions are obtained. The ideas and results of this paper may motivate further research.

Acknowledgements

This research project was financially supported by Mahasarakham University.

References

[1] D. Andrijević. On b-open sets. *Matematički Vesnik*, 48:59–64, 1996.

[2] C. Berge. *Espaces topologiques fonctions multivoques.* Dunod, Paris, 1959.

[3] C. Boonpok. (Λ, sp)-closed sets and related topics in topological spaces. *WSEAS Transactions on Mathematics*, 19:321–322, 2020.

[4] C. Boonpok and J. Khampakdee. (Λ, sp)-open sets in topological spaces. *European Journal of Pure and Applied Mathematics*, 15(2):572–588, 2022.

[5] M. E. Abd El-Monsef, S. N. El-Deeb, and R. A. Mahmoud. β-open sets and β-continuous mappings. *Bulletin of the Faculty of Science. Assiut University.*, 12:77–90, 1983.

[6] R. C. Jain. *The role of regularly open sets in general topology.* Ph.D. Thesis, Meerut University, Meerut, 1980.

[7] T. Noiri. On slightly β-continuous functions. *International Journal of Mathematics and Mathematical Sciences*, 28:469–478, 2001.

[8] T. Noiri and G. I. Chae. A note on slightly semi-continuous functions. *Bulletin of the Calcutta Mathematical Society*, 92:87–92, 2000.

[9] T. Noiri and E. Hatir. Λ_{sp}-sets and some weak separation axioms. *Acta Mathematica Hungarica*, 103(3):225–232, 2004.

[10] T. Noiri and V. Popa. Slightly m-continuous multifunctions. *Bulletin of the Institute of Mathematics Academia Sinica (New Series)*, 1(4):485–501, 2006.

[11] T. M. Nour. Slightly semi-continuous functions. *Bulletin of the Calcutta Mathematical Society*, 87:187–190, 1995.

[12] M. C. Pal and P. Bhattacharya. Faint precontinuous functions. *Soochow Journal of Mathematics*, 21:273–289, 1995.

[13] V. I. Ponomarev. Properties of topological spaces preserved under multivalued continuous mapping on compacta. *American Mathematical Society Translations*, 38(2):119–140, 1964.

[14] V. Popa and T. Noiri. Slightly m-continuous functions. *Radovi Matematički*, 10:1–14, 2001.