EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 15, No. 3, 2022, 897-915
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Computer Virus Fractional Order Model with Effects of Internal and External Storage Media

Muhammad Farman[1], Ali Akgul[2], H. Shanak[3], Jihad Asad[3,*], Aqeel Ahmad[4]

[1] *Department of Mathematics, Khawaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Pakistan*
[2] *Art and Science Faculty, Department of Mathematics, Siirt University, 56100 Siirt Turkey*
[3] *Department of Physics, Faculty of Applied Sciences, Palestine Technical University- Kadoorie, Tulkarm 305, West Bank, Palestine*
[4] *Department of Mathematics, Ghazi University, D G Khan-32000, Pakistan*

Abstract. In this work, we focus on the implementation of epidemic techniques on computer virus and study the dynamic transmission of several viruses to minimize the destruction of computers. We aim to make and analyze computer viruses through the Atangana-Baleanu sense and the Atangana-Taufik scheme, which is used for the fractional derivative model for the computer virus epidemic. It contained infected external computer effects and removable storage media on the computer viruses. For the validation of the model, we also discussed its positivity and boundedness. Fixed point theory and the iterative methods helped a lot to find out the existence and uniqueness of the model. In the case of numerical simulation, we used Atanagana-Taufik technique to illustrate the effects of varying the fractional order. The graphical results support our theoretical results from which, we analyze the infected external computer effects and removable storage media on the computer viruses.

2020 Mathematics Subject Classifications: 65P40, 68M07

Key Words and Phrases: Computer virus, Atangana-Baleanu sense, Atangan-Taufik, Fixed point theory, positivity and boundedness, Existence and uniqueness.

1. Introduction

The malicious and destructive results that can be obtained from program codes are known as computer viruses. They are automated programs that, against the user's wishes, replicate themselves to spread to new targets and infect computers. A lot of time and effort has gone into researching how to avoid harmful actions. To effectively control the spread of computer viruses, it is critical to understand how malicious codes spread over the Internet. To reduce the threat of virus several techniques can be proposed with the help of epidemic

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v15i3.4378

Email addresses: `j.asad@ptuk.edu.ps` (Jihad Asad)

models. Computer viruses have different types ranging from host dependent viruses and network worms, which cause a significant threat to our daily life and work [35]. The interconnected networks are the crucial channel of the fast spread of computer viruses. The network security community provides long and continuous attention to the virus diffusion. In Cohen [13] and Murray [26] noticed the analogy between computer viruses, their biological counterparts, and different techniques that investigate the dynamics of computer sciences. The first dynamical model for computer viruses was proposed by Kephart and White [20]. After that, different models for viruses on the computer were proposed [14, 18, 24, 25, 31, 33, 38, 39, 42]. There was no infectivity in the previous models. The original SLBS (susceptible-latent-breaking-susceptible) model for the computer virus was organized by Yang et al [40].

In recent years the fractional calculus has fascinated the attention of researchers and the various features of that study under investigation. Genetic mutations is dominant tool for defining the dynamic function of various body systems. The power of these component operators is their non-local features that are not in the integer separator operator. The real-world problems include characterization of memory and hereditary properties, while fractional order problems include integration and transects differentiation that can be understandable through fractional calculus [6, 10]. Riemann Liouville proposed idea of fractional derivative. Scientist used the latest fractional order derivative in the exponential kernel [7, 17]. The approaches of epidemic model shows different problems of non-singular kernel, which includes trigonometric and exponential functions [15, 16, 34]. The COVID-19 disease conceptual model which effectively catches the proposed outbreak of this virus [4, 37, 41]. The nonlinear fractional differential equation containing Atanagana-Baleanu fractional derivative is solved by Toufik and Atangana [36]. The most repetitive feature of these models is their global (non-local) features that include fractional application are also discuss in [1–3, 5, 29]. Khan and co-authors investigated several fractional models to study the dynamics of the zika virus, pine with disease, covid-19, and gonorrhea with optimal control [9, 21–23]. A new fractional derivative with a nonsingular kernel involving exponential, Mittag-leffler, power functions, and some advanced approaches for epidemic models have been elaborated in [8, 11, 27, 27, 28, 30]. The Lie group method is used in [32] to obtain the Lie symmetry algebra admitted by the time fractional Black-Scholes equation. In [12], authors build a proper extension of the classical prolongation formula of conformable derivative point transformations. This method was demonstrated and used to build a symmetry group admitting conformable ordinary and partial differential equations. A new dynamical model was developed in [18]. The authors thoroughly examined the model, and discovered that the unique (viral) equilibrium is globally asymptotically stable. The writers [19] presented an approximation algorithm that transforms the given system of equations into a nonlinear matrix equation by representing the unknown solutions and their derivatives in matrix forms along with the collocation points. A combination of the idea of quasi-linearization and the Bessel/Legendre-collocation method was applied to the original nonlinear system in addition to the direct Bessel or Legendre-collocation method.

In this work, we apply the Atangana-Baleanu fractional derivative with sumudu transform and Atangan-Tufik scheme with Mittag-Leffler kernel to a non-integer order for the computer virus model. We also discussed the positivity and boundedness of the fractional order system. The existence and uniqueness of the solutions of the proposed factional scheme are reputable using fixed-point theory and an iterative method. Lastly, simulation are made to see actual behaviour of this physical phenomena.

2. Preliminaries

Definition 1. *For any function $\Psi(t)$ over a set, the* ***Sumudu transform***

$$Z = \left\{ \Psi(t) : \exists\, \phi, such\ that\ \phi_1, \phi_2 > 0, |\Psi(t)| < \phi e^{\frac{|\phi|}{\phi_i}}, if\ t \in (-1)^j \times [0, \infty) \right\}$$

is defined by

$$A(r) = ST[\phi] = \int_0^\infty exp(-\phi)\Psi(rt)d\phi \qquad r \in (-\phi_1, \phi_2)$$

Definition 2. *The* ***Atangana Baleanu in Caputo sense*** *in [18, 25] for function $\Psi(t)$ is given as*

$$^{ABC}{}_{\sigma}D^{\sigma}_{\tau}(\Psi(t)) = \frac{AB(\sigma)}{m-\sigma}\int_a^t \frac{d^m}{dw^m}h(w)E_\sigma\left\{-\frac{\sigma(t-w)^\sigma}{m-\sigma}\right\}dw \qquad m-1<\sigma<m \tag{1}$$

we have

$$\left[^{ABC}{}_{\sigma}D^{\sigma}_{\tau}\Psi(t)\right](s) = \frac{AB(\sigma)}{1-\alpha}\frac{s^\sigma L\left[\Psi(t)\right](s) - s^{\sigma-1}\Psi(0)}{s^\sigma + \frac{\sigma}{1-\sigma}} \tag{2}$$

The sumudu transform for (1), we have

$$ST\left[^{ABC}{}_{\sigma}D^{\sigma}_{\tau}\Psi(t)\right](s) = \frac{B(\sigma)}{1-\sigma}\left\{\sigma\Gamma(\alpha+1)E_\sigma\left(-\frac{w^\sigma}{1-\sigma}\right)\right\} \times \left[ST(\Psi(t)) - \Psi(0)\right] \tag{3}$$

Definition 3. *A function $\Psi(t)$ for ABC fractional order σ is given by*

$$^{ABC}{}_{\sigma}I^{\sigma}_{\tau}(\Psi(t)) = \frac{(1-\sigma)\Psi(t)}{B-\sigma} + \frac{\alpha}{B(\sigma)\Gamma(\sigma)}\int_\sigma^t \Psi(s)(t-s)^{\sigma-1}ds \tag{4}$$

3. Fractional Order Computer Virus Model

The classical computer virus model given in [41], having four compartments $S(t)$,$L(t)$,$B(t)$ and $R(t)$ which represent the susceptible, latent computers, breaking-out computers and recovered computer after infection with internal and external sources with time t respectively. Besides the subsequent basis assumptions are created as: the exploit rate of each compartment is positive constant μ; the coming into rates of the four compartments are b_1,b_2,b_3,b_4 are positive constants respectively; each computer in the S-compartment is diseased with L (or B) computers with a chance of $\beta_1 L$ (or $\beta_2 B$), where β_1, β_2 are favorable times; each computer in S-room is diseased with changeable storage media that may have

θ; each computer in the L-compartment breaks down with α; each computer in the R-compartment loses potential protection η; because of fixing and modernizing the antivirus software well-timed, each computer on the Internet is likely to receive γ_1. As a result of re-installing the operating system, each computer in room L (or B) is at risk of chance γ_2 or γ_3, the actual complete order model obtained from [41]. We have following system of differential equations

$$\begin{cases} \frac{dS}{dt} = & b_1 + \gamma_2 L + \gamma_3 B + \eta R - \mu S - \gamma_1 S - \beta_1 LS - \beta_2 BS - \theta S, \\ \frac{dL}{dt} = & b_2 + \beta_1 LS + \beta_2 BS + \theta S - \gamma_1 L - \gamma_2 L - \mu L - \alpha L, \\ \frac{dB}{dt} = & b_3 + \alpha L - \mu B - \gamma_1 B - \gamma_3 B, \\ \frac{dR}{dt} = & b_4 + \gamma_1 S + \gamma_1 L + \gamma_1 B - \eta R - \mu R. \end{cases} \tag{5}$$

We can express the above system in ABC fractional order form as:

$$\begin{cases} {}^{ABC}{}_0D_t^\alpha S(t) = & b_1 + \gamma_2 L + \gamma_3 B + \eta R - \mu S - \gamma_1 S - \beta_1 LS - \beta_2 BS - \theta S, \\ {}^{ABC}{}_0D_t^\alpha L(t) = & b_2 + \beta_1 LS + \beta_2 BS + \theta S - \gamma_1 L - \gamma_2 L - \mu L - \alpha L, \\ {}^{ABC}{}_0D_t^\alpha B(t) = & b_3 + \alpha L - \mu B - \gamma_1 B - \gamma_3 B, \\ {}^{ABC}{}_0D_t^\alpha R(t) = & b_4 + \gamma_1 S + \gamma_1 L + \gamma_1 B - \eta R - \mu R. \end{cases} \tag{6}$$

with given initial conditions

$$S(0), L(0), B(0), R(0) \geq 0 \tag{7}$$

3.1. Analysis of the Model:

Theorem 1. *The solution of the proposed system (6) is bounded and unique in R_+^4 according to initial conditions.*

Proof. The existence and uniqueness of the solution of system of the equations (6) on the time interval $(0, \infty)$ can be obtained in the region R_+^4 is positively invariant. From model of system of the equations (6), we find

$$\begin{cases} {}^{ABC}{}_0D_t^\alpha S(t)|_{S=0} = & b_1 + \gamma_2 L + \gamma_3 B + \eta R \geq 0, \\ {}^{ABC}{}_0D_t^\alpha L(t)|_{L=0} = & b_2 + \beta_2 BS + \theta S \geq 0, \\ {}^{ABC}{}_0D_t^\alpha B(t)|_{B=0} = & b_3 + \alpha L \geq 0, \\ {}^{ABC}{}_0D_t^\alpha R(t)|_{R=0} = & b_4 + \gamma_1 S + \gamma_1 L + \gamma_1 B \geq 0. \end{cases} \tag{8}$$

If $\{S(0), L(0), B(0), R(0)\} \in R_+^4$, then according to above equations, the solution $\{S(t), L(t), B(t), R(t)\}$ cannot escape from the hyperplanes $S = 0, L = 0, B = 0$ and $R = 0$. Solution's lie in the domain R_+^4 is a positively invariant set.

Theorem 2. *The region*

$$\begin{aligned} A = \{ & {}^{ABC}{}_0D_t^\alpha S(t), {}^{ABC}{}_0D_t^\alpha L(t), {}^{ABC}{}_0D_t^\alpha B(t), {}^{ABC}{}_0D_t^\alpha R(t) \in R_+^4|_0 \\ & \textit{and } {}^{ABC}{}_0D_t^\alpha S(t) + {}^{ABC}{}_0D_t^\alpha L(t) + {}^{ABC}{}_0D_t^\alpha B(t) + {}^{ABC}{}_0D_t^\alpha R(t) \leq \frac{\Lambda}{\mu} \} \end{aligned}$$

is a positive invariant set for the system (6).

Proof. For the verification of the results used the system (6), we have

$$^{ABC}{}_0D_t^\alpha N(t) = (b_1 + b_2 + b_3 + b_4) - \mu N(t)$$

let $\Lambda = (b_1 + b_2 + b_3 + b_4)$, then

$$^{ABC}{}_0D_t^\alpha N(t) = \Lambda - \mu N(t)$$

Using the Laplace transformation, we get

$$s^\alpha N(t) - s^{\alpha-1}N(0) = \frac{\Lambda}{s} - \mu N(s)$$

which further gives

$$N(s) = \frac{s^{-1}\Lambda}{s^\alpha + \mu} - \frac{s^{\alpha-1}N(0)}{s^\alpha + \mu}$$

From equations (6), we infer that if $(S_0, L_0, B_0, R_0) \in R_+^4$, then

$$\begin{aligned} N(t) &= \Lambda t^\alpha E_{\alpha,\alpha+1}(-\mu t^\alpha) + E_{\alpha,1}(-\mu t^\alpha) \\ &= \frac{(\Omega - \delta)\left(\mu t^\alpha E_{\alpha,\alpha+1}(-\mu t^\alpha)\right)}{\mu} + E_{\alpha+1}(-\mu t^\alpha) \\ &\leq \frac{\Lambda}{\mu\Gamma(1)} \\ &\leq \frac{\Lambda}{\mu} \end{aligned}$$

and

$$\leq \frac{(b_1 + b_2 + b_3 + b_4)}{\mu}$$

Hence the solution is bounded in the given domain for sub-compartments of the system. This proved the results.

3.2. Equilibrium Point:

By substituting the left hand side of the system equal to zero, we get equilibrium point of the system. Let $^{ABC}{}_0D_t^\alpha N(t) ={}^{ABC}{}_0D_t^\alpha S(t) +{}^{ABC}{}_0D_t^\alpha L(t) +{}^{ABC}{}_0D_t^\alpha B(t) +{}^{ABC}{}_0D_t^\alpha R(t)$ and $b = b_1 + b_2 + b_3 + b_4$. By solving the system (6), we have $\lim_{t\to\infty} N = N^*$ and $\lim_{t\to\infty} R = R^*$, where

$$N^* = \frac{b}{\mu} \quad R^* = \frac{\mu b_4 + b\gamma_1}{\mu(\gamma_1 + \eta + \mu)}$$

We get

$$\begin{cases} \dot{L} = b_2 + (\beta_1 L + \beta_2 B + \theta)(N^* - R^* - L - B) - (\gamma_1 + \gamma_2 + \mu + \alpha)L \\ \dot{B} = b_3 + \alpha L - (\mu + \gamma_1 + \gamma_3)B \end{cases}$$

with initial conditions $(L(0), B(0)) \in \zeta$, where

$$\zeta = \{(L, B) | L \geq 0, B \geq 0, 0 \leq L + B \leq N^* - R^*\}$$

Hence it is invariant.

4. Computer Virus Model with ABC Derivative

In this section, consider the system (6) with ABC derivative and definition of sumudu transform, we get

$$B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{-w^\alpha}{1-\alpha}\right)ST[S(t)-S(0)] = ST\left[b_1+\gamma_2 L+\gamma_3 B+\eta R-\mu S-\gamma_1 S-\beta_1 LS-\beta_2 BS-\theta S\right] \tag{9}$$

$$B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{-w^\alpha}{1-\alpha}\right)ST[L(t)-L(0)] = ST\left[b_2+\beta_1 LS+\beta_2 BS+\theta S-\gamma_1 L-\gamma_2 L-\mu L-\alpha L\right] \tag{10}$$

$$B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{-w^\alpha}{1-\alpha}\right)ST[B(t)-B(0)] = ST\left[b_3+\alpha L-\mu B-\gamma_1 B-\gamma_3 B\right] \tag{11}$$

$$B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{-w^\alpha}{1-\alpha}\right)ST[R(t)-R(0)] = ST\left[b_4+\gamma_1 S+\gamma_1 L+\gamma_1 B-\eta R-\mu R\right] \tag{12}$$

By reorganizing the system of equations(9)-(12), we have

$$ST[S(t)] = S(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)}\times ST[b_1+\gamma_2 L+\gamma_3 B+\eta R-\mu S-\gamma_1 S-\beta_1 LS-\beta_2 BS-\theta S] \tag{13}$$

$$ST[L(t)] = L(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)}\times ST[b_2+\beta_1 LS+\beta_2 BS+\theta S-\gamma_1 L-\gamma_2 L-\mu L-\alpha L] \tag{14}$$

$$ST[B(t)] = B(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)}\times ST[b_3+\alpha L-\mu B-\gamma_1 B-\gamma_3 B] \tag{15}$$

$$ST[R(t)] = R(0)+\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)}\times ST[b_4+\gamma_1 S+\gamma_1 L+\gamma_1 B-\eta R-\mu R] \tag{16}$$

we get

$$\begin{aligned} S(t) =& S(0)+ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)}\times ST[b_1+\gamma_2 L+\gamma_3 B+\eta R-\mu S \\ &-\gamma_1 S-\beta_1 LS-\beta_2 BS-\theta S]\} \end{aligned} \tag{17}$$

$$L(t) = L(0)+ST^{-1}\left\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)}\times ST[b_2+\beta_1 LS+\beta_2 BS+\theta S-\gamma_1 L-\gamma_2 L-\mu L-\alpha L]\right\} \tag{18}$$

$$B(t) = B(0)+ST^{-1}\left\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)}\times ST[b_3+\alpha L-\mu B-\gamma_1 B-\gamma_3 B]\right\} \tag{19}$$

$$R(t) = R(0)+ST^{-1}\left\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_4+\gamma_1 S+\gamma_1 L+\gamma_1 B-\eta R-\mu R]\right\} \tag{20}$$

Therefore, the following is obtained

$$\begin{aligned} S_{(n+1)}(t) =& S(0) + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_1+\gamma_2 L_n+\gamma_3 B_n+\eta R_n - \\ &(\mu+\gamma_1+\theta)S_n-\beta_1 L_n S_n-\beta_2 B_n S_n]\} \end{aligned} \tag{21}$$

$$\begin{aligned} L_{(n+1)}(t) =& L(0) + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_2+\beta_1 L_n S_n+\beta_2 B_n S_n+\theta S_n \\ &-(\gamma_1+\gamma_2+\mu+\alpha)L_n]\} \end{aligned} \tag{22}$$

$$B_{(n+1)}(t) = B(0)+ST^{-1}\left\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_3+\alpha L_n-(\mu+\gamma_1+\gamma_3)B_n]\right\} \tag{23}$$

$$R_{(n+1)}(t) = R(0)+ST^{-1}\left\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_4+\gamma_1 S_n+\gamma_1 L_n+\gamma_1 B_n-(\eta+\mu)R_n]\right\} \tag{24}$$

And obtained solution of the system of equations(21)-(24) is presented as $S(t) = \lim_{n\to\infty} S_n(t)$; $L(t) = \lim_{n\to\infty} L_n(t)$; $B(t) = \lim_{n\to\infty} B_n(t)$ and $R(t) = \lim_{n\to\infty} R_n(t)$

Theorem 3. *Let $(X, |.|)$ be a Banach space and H a self-map of X satisfying*

$$\|Hx - Hr\| \leq \theta\|X - Hx\| + \theta\|x - r\|$$

$\forall x, r \in X$, where $0 \leq \theta < 1$. Assume that H is a picard H-stable. Let us consider the system of equations (21)-(24) and we obtained

$$\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{-w^\alpha}{1-\alpha}\right)} \tag{25}$$

It is also known as fractional Lagrange multiplier.

Theorem 4. *Express K be a self-map is specified by*

$$\begin{aligned} K[S_{(n+1)}(t)] =& S_{(n+1)}(t) = S(0) + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_1+\gamma_2 L_n \\ &+\gamma_3 B_n+\eta R_n-(\mu+\gamma_1+\theta)S_n-\beta_1 L_n S_n-\beta_2 B_n S_n]\} \end{aligned} \tag{26}$$

$$K[L_{(n+1)}(t)] = L_{(n+1)}(t) = L(0) + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_2 + \beta_1 L_n S_n + \beta_2 B_n S_n + \theta S_n - (\gamma_1+\gamma_2+\mu+\alpha)L_n]\} \tag{27}$$

$$K[B_{(n+1)}(t)] = B_{(n+1)}(t) = B(0) + ST^{-1}\left\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_3 + \alpha L_n - (\mu+\gamma_1+\gamma_3)B_n]\right\} \tag{28}$$

$$K[R_{(n+1)}(t)] = R_{(n+1)}(t) = R(0) + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \times ST[b_4 + \gamma_1 S_n + \gamma_1 L_n + \gamma_1 B_n - (\eta+\mu)R_n]\} \tag{29}$$

Proof. Considering the norm properties and triangular inequalities, we get

$$\begin{aligned}\|K[S_n(t)] - K[S_m(t)]\| \leq & \|S_n(t) - S_m(t)\| + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \\ & \times ST[b_1 + \gamma_2(L_n(t) - L_m(t)) + \gamma_3(B_n(t) - B_m(t)) + \eta(R_n(t) - R_m(t)) \\ & - (\theta+\mu+\gamma_1)(S_n(t) - S_m(t)) - \beta_1(L_n(t) - L_m(t))(S_n(t) - S_m(t)) \\ & - \beta_2(B_n(t) - B_m(t))(S_n(t) - S_m(t))]\}\end{aligned}$$

$$\begin{aligned}\|K[L_n(t)] - K[L_m(t)]\| \leq & \|L_n(t) - L_m(t)\| + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \\ & \times ST[b_2 + \beta_1(L_n(t) - L_m(t))(S_n(t) - S_m(t)) \\ & + \beta_2(B_n(t) - B_m(t))(S_n(t) - S_m(t)) + \theta(S_n(t) - S_m(t)) \\ & - (\gamma_1+\gamma_2+\mu+\alpha)(L_n(t) - L_m(t))]\}\end{aligned}$$

$$\begin{aligned}\|K[B_n(t)] - K[B_m(t)]\| \leq & \|B_n(t) - B_m(t)\| + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \\ & \times ST[b_3 + \alpha(L_n(t) - L_m(t)) - (\mu+\gamma_1+\gamma_3)(B_n(t) - B_m(t))]\}\end{aligned}$$

$$\begin{aligned}\|K[R_n(t)] - K[R_m(t)]\| \leq & \|R_n(t) - R_m(t)\| + ST^{-1}\{\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha\left(\frac{w^\alpha}{1-\alpha}\right)} \\ & \times ST[b_4 + \gamma_1(S_n(t) - S_m(t)) + \gamma_1(L_n(t) - L_m(t)) \\ & + \gamma_1(B_n(t) - B_m(t)) - (\eta+\mu)(R_n(t) - R_m(t))]\}\end{aligned}$$

K fulfills the condition associated with theorem (3) when

$$\theta = (0, 0, 0, 0)$$

$$\begin{aligned}\theta =&\|S_n(t)-S_m(t)\| \times \|-(S_n(t)+S_m(t))\| + b_1 + \gamma_2\|L_n(t)-L_m(t)\| + \gamma_3\|B_n(t)-B_m(t)\|\\&+\eta\|R_n(t)-R_m(t)\| - (\theta+\mu+\gamma_1)\|S_n(t)-S_m(t)\| - \beta_1\|L_n(t)-L_m(t)\|\|S_n(t)-S_m(t)\|\\&-\beta_2\|B_n(t)-B_m(t)\|\|S_n(t)-S_m(t)\|\|L_n(t)-L_m(t)\|\|-(L_n(t)+L_m(t))\| + b_2\\&+\beta_1\|L_n(t)-L_m(t)\|\|S_n(t)-S_m(t)\| + \beta_2\|B_n(t)-B_m(t)\|\|S_n(t)-S_m(t)\|\\&+\theta\|S_n(t)-S_m(t)\| - (\gamma_1+\gamma_2+\mu+\alpha)\|L_n(t)-L_m(t)\|\|B_n(t)-B_m(t)\|\|-(B_n(t)+B_m(t))\|\\&+b_3+\alpha\|L_n(t)-L_m(t)\| - (\mu+\gamma_1+\gamma_3)\|B_n(t)-B_m(t)\|\|R_n(t)-R_m(t)\|\|-(R_n(t)+R_m(t))\|\\&+b_4+\gamma_1\|S_n(t)-S_m(t)\| + \gamma_1\|L_n(t)-L_m(t)\| + \gamma_1\|B_n(t)-B_m(t)\| - (\eta+\mu)\|R_n(t)-R_m(t)\|\end{aligned}$$

Hence proved its stable according to defied condition in theorem (3)

Theorem 5. *Uniqueness of the system (6) solution obtained with iteration method. Concern the following Hilbert space* $H = L^2((p,q)\times(0,r))$

$$h:(p,q)\times[0,T]\to R, \quad \int\int ghdghd < \infty$$

In this regard, the following operations are considered

$$\theta = (0,0,0,0)$$

$$\begin{aligned}\theta =&b_1+\gamma_2L(t)+\gamma_3B(t)+\eta R(t)-\mu S(t)-\gamma_1S(t)-\beta_1L(t)S(t)-\beta_2B(t)S(t)-\theta S(t)\\&+b_2+\beta_1LS+\beta_2BS+\theta S-\gamma_1L-\gamma_2L-\mu L-\alpha L+b_3+\alpha L-\mu B-\gamma_1B-\gamma_3B\\&+b_4+\gamma_1S+\gamma_1L+\gamma_1B-\eta R-\mu R\end{aligned} \tag{30}$$

We establish that the inner product of

$$T(S_{11}(t)-S_{12}(t), L_{11}(t)-L_{12}(t), B_{11}(t)-B_{12}(t), R_{11}(t)-R_{12}(t), (v_1,v_2,v_3,v_4))$$

where $\{S_{11}(t)-S_{12}(t), L_{11}(t)-L_{12}(t), B_{11}(t)-B_{12}(t), R_{11}(t)-R_{12}(t)\}$ *are the special solutions of the system.*

$$\begin{aligned}&[b_1+\gamma_2(L_{21}(t)-L_{22})+\gamma_3(B_{31}(t)-B_{32}(t))+\eta(R_{41}(t)-R_{42}(t))-(\theta+\mu+\gamma_1)(S_{11}(t)-S_{12}(t))\\&-\beta_1(L_{21}(t)-L_{22}(t))(S_{11}(t)-S_{12}(t))-\beta_2(B_{31}(t)-B_{32}(t))(S_{11}(t)-S_{12}(t)),v_1]\\&\le[b_1+\gamma_2\|L_{21}(t)-L_{22}(t)\|\|v_1\|+\gamma_3\|B_{31}(t)-B_{32}(t)\|\|v_1\|+\eta\|R_{41}(t)-R_{42}(t)\|\|v_1\|\\&-(\theta+\mu+\gamma_1)\|S_{11}(t)-S_{22}(t)\|\|v_1\|-\beta_1\|L_{21}(t)-L_{22}(t)\|\|S_{11}(t)-S_{12}(t)\|\|v_1\|\\&-\beta_2\|B_{31}(t)-B_{32}(t)\|\|S_{11}(t)-S_{12}(t)\|\|v_1\|]\end{aligned}$$

$$\begin{aligned}&[b_2+\beta_1(L_{21}(t)-L_{22}(t))(S_{11}(t)-S_{12}(t))+\beta_2(B_{31}(t)-B_{32}(t))(S_{11}(t)-S_{12}(t))\\&+\theta(S_{11}(t)-S_{12}(t))-(\gamma_1+\gamma_2+\mu+\alpha)(L_{21}(t)-L_{22}(t)),v_2]\\&\le[b_2+\beta_1\|(L_{21}(t)-L_{22}(t))\|\|(S_{11}(t)-S_{12}(t))\|\|v_2\|+\beta_2\|(B_{31}(t)-B_{32}(t))\|\|(S_{11}(t)-S_{12}(t))\|\|v_2\|\\&+\theta\|(S_{11}(t)-S_{12}(t))\|\|v_2\|-(\gamma_1+\gamma_2+\mu+\alpha)\|(L_{21}(t)-L_{22}(t))\|\|v_2\|]\end{aligned}$$

$$[b_3+\alpha(L_{21}(t)-L_{22}(t))-(\mu+\gamma_1+\gamma_3)(B_{31}(t)-B_{32}(t)),v_3]$$
$$\leq [b_3+\alpha\|(L_{21}(t)-L_{22}(t))\|\|v_3\|-(\mu+\gamma_1+\gamma_3)\|(B_{31}(t)-B_{32}(t))\|\|v_3\|]$$

$$[b_4+\gamma_1(S_{11}(t)-S_{12}(t))+\gamma_1(L_{21}(t)-L_{22}(t))+\gamma_1(B_{31}(t)-B_{32}(t))-(\eta+\mu)(R_{41}(t)-R_{42}(t)),v_4]$$
$$\leq [b_4+\gamma_1\|(S_{11}(t)-S_{12}(t))\|\|v_4\|+\gamma_1\|(L_{21}(t)-L_{22}(t))\|\|v_4\|+\gamma_1\|(B_{31}(t)-B_{32}(t))\|\|v_4\|$$
$$-(\eta+\mu)\|(R_{41}(t)-R_{42}(t))\|\|v_4\|]$$

Hence it is converge with topological concepts in parameters $(\chi_{e_1},\chi_{e_2},\chi_{e_3},\chi_{e_4})$

$$\|S(t)-S_{11}(t)\|,\|S(t)-S_{12}(t)\|\leq\frac{\chi_{e_1}}{w}$$
$$\|L(t)-L_{21}(t)\|,\|L(t)-L_{22}(t)\|\leq\frac{\chi_{e_2}}{\delta}$$
$$\|B(t)-B_{31}(t)\|,\|B(t)-B_{32}(t)\|\leq\frac{\chi_{e_3}}{\zeta}$$
$$\|R(t)-R_{41}(t)\|,\|R(t)-R_{42}(t)\|\leq\frac{\chi_{e_4}}{h}$$

where

$$\begin{aligned}w=&4(b_1+\gamma_2\|L_{21}(t)-L_{22}(t)\|+\gamma_3\|B_{31}(t)-B_{32}(t)\|+\eta\|R_{41}(t)-R_{42}(t)\|\\&-(\theta+\mu+\gamma_1)\|S_{11}(t)-S_{22}(t)\|-\beta_1\|L_{21}(t)-L_{22}(t)\|\|S_{11}(t)-S_{12}(t)\|\\&-\beta_2\|B_{31}(t)-B_{32}(t)\|\|S_{11}(t)-S_{12}(t)\|)\|v_1\|\end{aligned}$$

$$\begin{aligned}\delta=&4(b_2+\beta_1\|(L_{21}(t)-L_{22}(t))\|\|(S_{11}(t)-S_{12}(t))\|+\beta_2\|(B_{31}(t)-B_{32}(t))\|\|(S_{11}(t)-S_{12}(t))\|\\&+\theta\|(S_{11}(t)-S_{12}(t))\|-(\gamma_1+\gamma_2+\mu+\alpha)\|(L_{21}(t)-L_{22}(t))\|)\|v_2\|\end{aligned}$$

$$\zeta=4(b_3+\alpha\|(L_{21}(t)-L_{22}(t))\|-(\mu+\gamma_1+\gamma_3)\|(B_{31}(t)-B_{32}(t))\|)\|v_3\|$$

$$\begin{aligned}h=&4(b_4+\gamma_1\|(S_{11}(t)-S_{12}(t))\|+\gamma_1\|(L_{21}(t)-L_{22}(t))\|+\gamma_1\|(B_{31}(t)-B_{32}(t))\|\\&-(\eta+\mu)\|(R_{41}(t)-R_{42}(t))\|)\|v_4\|\end{aligned}$$

where

$$\begin{aligned}\leq&(b_1+\gamma_2\|L_{21}(t)-L_{22}(t)\|+\gamma_3\|B_{31}(t)-B_{32}(t)\|+\eta\|R_{41}(t)-R_{42}(t)\|\\&-(\theta+\mu+\gamma_1)\|S_{11}(t)-S_{22}(t)\|-\beta_1\|L_{21}(t)-L_{22}(t)\|\|S_{11}(t)-S_{12}(t)\|\\&-\beta_2\|B_{31}(t)-B_{32}(t)\|\|S_{11}(t)-S_{12}(t)\|)\neq 0\end{aligned}$$

$$\begin{aligned}\leq&(b_2+\beta_1\|(L_{21}(t)-L_{22}(t))\|\|(S_{11}(t)-S_{12}(t))\|+\beta_2\|(B_{31}(t)-B_{32}(t))\|\|(S_{11}(t)-S_{12}(t))\|\\&+\theta\|(S_{11}(t)-S_{12}(t))\|-(\gamma_1+\gamma_2+\mu+\alpha)\|(L_{21}(t)-L_{22}(t))\|)\neq 0\end{aligned}$$

$$\leq(b_3+\alpha\|(L_{21}(t)-L_{22}(t))\|-(\mu+\gamma_1+\gamma_3)\|(B_{31}(t)-B_{32}(t))\|)\neq 0$$

$$\begin{aligned}\leq&(b_4+\gamma_1\|(S_{11}(t)-S_{12}(t))\|+\gamma_1\|(L_{21}(t)-L_{22}(t))\|+\gamma_1\|(B_{31}(t)-B_{32}(t))\|\\&-(\eta+\mu)\|(R_{41}(t)-R_{42}(t))\|)\neq 0\end{aligned}$$

Where $\{\|v_1\|,\|v_2\|,\|v_3\|,\|v_4\|\}\neq 0$ $\|(S_{11}(t)-S_{12}(t))\|=0$, $\|(L_{21}(t)-L_{22}(t))\|=0$, $\|(B_{31}(t)-B_{32}(t))\|=0$, $\|(R_{41}(t)-R_{42}(t))\|=0$ *so* $S_{11}(t)=S_{12}(t)$, $L_{21}(t)=L_{22}(t)$, $B_{31}(t)=B_{32}(t)$, $R_{41}(t)=R_{42}(t)$ *This complete proof is uniqueness.*

5. Advanced Numerical Scheme

Here in this section considering the numerical scheme is defined in [36], we have

$$\begin{cases} {}^{ABC}{}_0Dy(t) = h(t, w(t)) \\ w(0) = w_0 \end{cases} \tag{31}$$

We have

$$w(t) - w(0) = \frac{1-\sigma}{ABC(\sigma)} h(t, w(t)) + \frac{\alpha}{\Gamma(\sigma) \times ABC(\sigma)} \int_0^t h(\tau, w(\tau))(t-\tau)^{\sigma-1} d\tau \tag{32}$$

At a given point $t_{(m+1)}, m = 0, 1, 2, 3, 4, 5, ,$ we can write above equation

$$w(t_{m+1}) - w(0) = \frac{1-\sigma}{ABC(\sigma)} h(t_m, w(t_m)) + \frac{\alpha}{\Gamma(\sigma) \times ABC(\sigma)} \int_0^{t_{m+1}} h(\tau, w(\tau))(t_{m+1}-\tau)^{\sigma-1} d\tau \tag{33}$$

$$= \frac{1-\sigma}{ABC(\sigma)} h(t_m, w(t_m)) + \frac{\alpha}{\Gamma(\sigma) \times ABC(\sigma)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} h(\tau, w(\tau))(t_{m+1}-\tau)^{\sigma-1} d\tau \tag{34}$$

By using interval $[t_k, t_(k+1)]$, the function $h(\tau, y(\tau))$, with the help of two-steps Lagrange polynomial interpolation, we have

$$\begin{aligned} P_k(\tau) &= \frac{\tau - t_{k-1}}{t_k - t_{k-1}} h(t_k, w(t_k)) - \frac{\tau - t_k}{t_k - t_{k-1}} h(t_{k-1}, w(t_{k-1})) \\ &= \frac{h(t_k, w(t_k))}{h}(\tau, t_{k-1}) - \frac{h(t_{k-1}, w(t_{k-1}))}{h}(\tau, t_k) \\ &\cong \frac{h(t_k, w(t_k))}{h}(\tau, t_{k-1}) - \frac{h(t_{k-1}, w(t_{k-1}))}{h}(\tau, t_k) \end{aligned} \tag{35}$$

By using (34), we get

$$\begin{aligned} w_{m+1} =& w_0 + \frac{1-\sigma}{ABC(\sigma)} h(t_m, w(t_m)) + \frac{\alpha}{\Gamma(\sigma) \times ABC(\sigma)} \sum_{k=o}^{m} \\ & \left(\frac{h(t_k, y_k)}{h} \int_{t_k}^{t_{k+1}} (\tau - t_{k-1})(t_{n+1}-\tau)^{\sigma-1} d\tau - \frac{h(t_{k-1}, y_{k-1})}{h} \int_{t_k}^{t_{k+1}} (\tau - t_k)(t_{m+1}-\tau)^{\sigma-1} d\tau\right) \end{aligned} \tag{36}$$

For simplification, we will consider

$$B_{\sigma,k,1} = \int_{t_k}^{t_{k+1}} (\tau - t_{k-1})(t_{m+1}-\tau)^{\sigma-1} d\tau \tag{37}$$

similarly

$$B_{\sigma,k,2} = \int_{t_k}^{t_{k+1}} (\tau - t_k)(t_{m+1}-\tau)^{\sigma-1} d\tau \tag{38}$$

$$B_{\sigma,k,1} = h^{\alpha+1}\frac{(m+1-k)^{\sigma}(m-k+2+\sigma)-(m-k)^{\sigma}(m-k+2+2)}{\alpha(\alpha+1)}$$

$$B_{\sigma,k,2} = h^{\alpha+1}\frac{(m+1-k)^{\sigma+1}-(m-k)^{\sigma}(m-k+1+\sigma)}{\alpha(\alpha+1)}$$

By putting (37) and (38), we get

$$\begin{aligned} w_{m+1} =& w_0 + \frac{1-\sigma}{ABC(\sigma)} h(t_m, y(t_m)) + \frac{\sigma}{ABC(\sigma)} \sum_{k=0}^{n} (\frac{h^{\sigma} h(t_k, y_k)}{\Gamma(\alpha+2)}((q_3)^{\sigma} q_2 - (q_4)^{\sigma} q_1) \\ & - \frac{h^{\sigma} h(t_{k-1}, y_{k-1})}{\Gamma(\alpha+2)}((q_3)^{\sigma+1} - (q_4)^{\sigma} q_5)) \end{aligned} \tag{39}$$

We obtain the following for the system of equations (6).

$$\begin{aligned} S_{m+1} =& S_0 + \frac{1-\sigma}{ABC(\sigma)} f(t_m, S(t_m)) + \frac{\sigma}{ABC(\sigma)} \sum_{k=0}^{n} (\frac{h^{\sigma} f(t_k, S_k)}{\Gamma(\alpha+2)}((q_3)^{\sigma} q_2 - (q_4)^{\sigma} q_1) \\ & - \frac{h^{\sigma} f(t_{k-1}, S_{k-1})}{\Gamma(\alpha+2)}((q_3)^{\sigma+1} - (q_4)^{\sigma} q_5)) \end{aligned} \tag{40}$$

$$\begin{aligned} L_{m+1} =& L_0 + \frac{1-\sigma}{ABC(\sigma)} f(t_m, L(t_m)) + \frac{\sigma}{ABC(\sigma)} \sum_{k=0}^{n} (\frac{h^{\sigma} f(t_k, L_k)}{\Gamma(\alpha+2)}((q_3)^{\sigma} q_2 - (q_4)^{\sigma} q_1) \\ & - \frac{h^{\sigma} f(t_{k-1}, L_{k-1})}{\Gamma(\alpha+2)}((q_3)^{\sigma+1} - (q_4)^{\sigma} q_5)) \end{aligned} \tag{41}$$

$$\begin{aligned} B_{m+1} =& B_0 + \frac{1-\sigma}{ABC(\sigma)} f(t_m, B(t_m)) + \frac{\sigma}{ABC(\sigma)} \sum_{k=0}^{n} (\frac{h^{\sigma} f(t_k, B_k)}{\Gamma(\alpha+2)}((q_3)^{\sigma} q_2 - (q_4)^{\sigma} q_1) \\ & - \frac{h^{\sigma} f(t_{k-1}, B_{k-1})}{\Gamma(\alpha+2)}((q_3)^{\sigma+1} - (q_4)^{\sigma} q_5)) \end{aligned} \tag{42}$$

$$\begin{aligned} R_{m+1} =& R_0 + \frac{1-\sigma}{ABC(\sigma)} f(t_m, R(t_m)) + \frac{\sigma}{ABC(\sigma)} \sum_{k=0}^{n} (\frac{h^{\sigma} f(t_k, R_k)}{\Gamma(\alpha+2)}((q_3)^{\sigma} q_2 - (q_4)^{\sigma} q_1) \\ & - \frac{h^{\sigma} f(t_{k-1}, R_{k-1})}{\Gamma(\alpha+2)}((q_3)^{\sigma+1} - (q_4)^{\sigma} q_5)) \end{aligned} \tag{43}$$

Where $q_1 = m - k + 2 + 2\sigma$, $q_2 = m - k + 2 + \sigma$, $q_3 = m + 1 - k$, $q_4 = m - k$ and $q_5 = m - k + 1 + \sigma$

6. Results and Discussion

The mathematical analysis of the epidemic computer virus model with the effect of external and internal storage media is proposed with the new fractional operator with given parameters details in [41]. Numerical simulation carried out with ABC fractional derivative for computer virus model. The graphs of the approximate solutions in different fractional order are provided in Figures 1-8, which show the memory effect. It can be observed that internal and external storage media are the rich source of the spread of the virus. Also, we found that external storage media has significant effect for the viral the infection. From figures (1) and (5), we can see that susceptible computers decreases due to the rapid increase in latent, breaking-out, and recovered computers. In fig (2), the simulation show the dynamics of latent computers increase with time while decreasing with time. In fig (3), we can see that the break out of computers also growing fast and causing rapid and severe damage to computers. In fig (4), the recovery of computers is smoothly increasing due to the effect of fractional operator. From figures 6-8, the simulation shows the quick increase in latent and break out computers with time t and causes the disturbance rapidly and slow the recovery rate of computers.

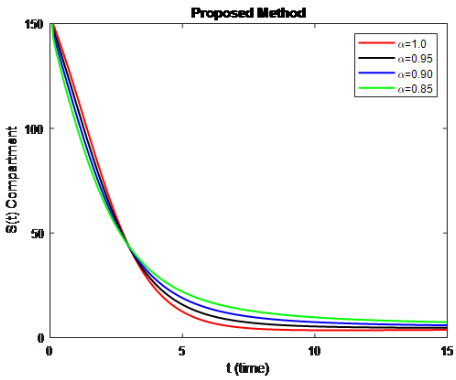

Figure 1: Simulation of S(t) Compartment with proposed fractional order scheme

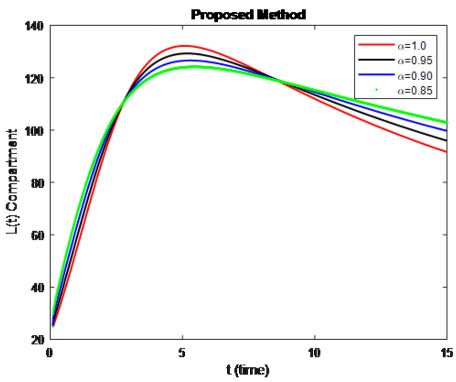

Figure 2: Simulation of L(t) Compartment with proposed fractional order scheme

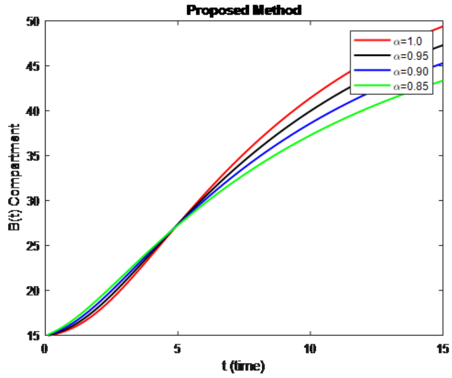

Figure 3: Simulation of B(t) Compartment with proposed fractional order scheme

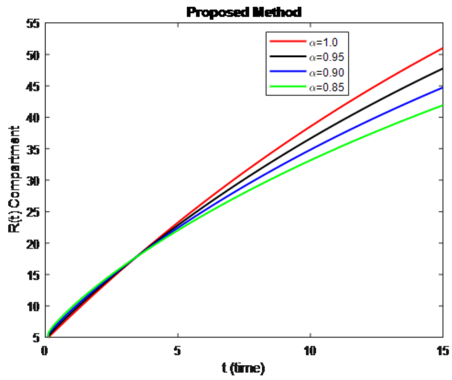

Figure 4: Simulation of R(t) Compartment with proposed fractional order scheme

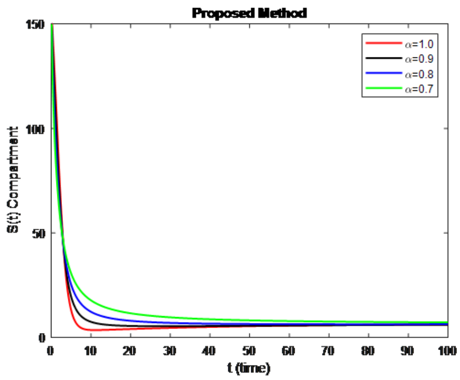

Figure 5: Simulation of S(t) Compartment with proposed fractional order scheme

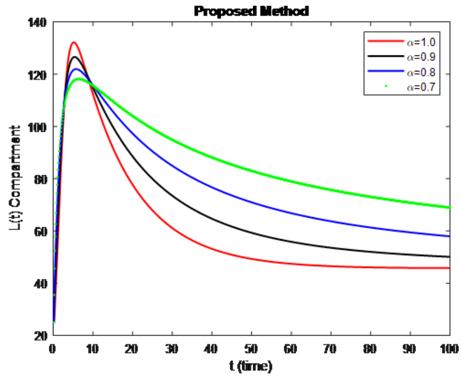

Figure 6: Simulation of L(t) Compartment with proposed fractional order scheme

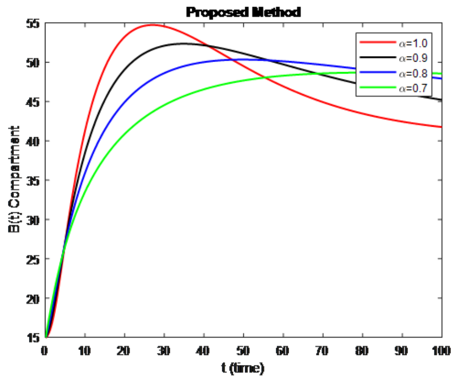

Figure 7: Simulation of B(t) Compartment with proposed fractional order scheme

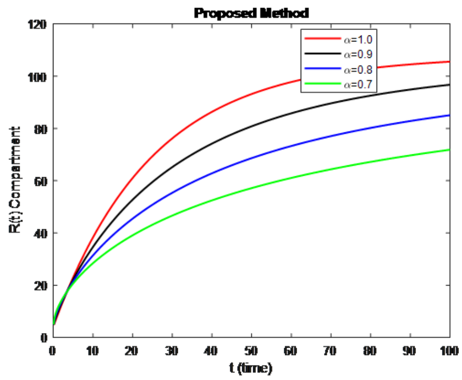

Figure 8: Simulation of R(t) Compartment with proposed fractional order scheme

7. Conclusion

In this work, we presented the new result for the fractional order computer virus model with ABC fractional derivative and the Atangana-Toufik scheme. Qualitative analysis with positivity and boundedness of the proposed system was also discussed. Additionally, the uniqueness and stability of the proposed scheme of iterative results are proved by using the fixed point theorem. Finally, numerical simulations at different fractional orders are obtained, which support the theoretical results of the computer virus model. It is observed that internal and external storage media are the cause of the spread of the virus. Also, we found that all external storage media has a greater effect on viral infection with increasing the time t. These results are very helpful to control the virus infection in computers to overcome the threats on results. In future work, we intend to expand the modeling of a computer virus in the stochastic fractional-order derivatives as well as partial differential equation.

Acknowledgements

The authors Hussein Shanak and Jihad Asad would like to thank Palestine Technical University- Kadoorie for supporting this project.

References

[1] Ali Akgül. A novel method for a fractional derivative with non-local and non-singular kernel. *Chaos, Solitons & Fractals*, 114:478–482, 2018.

[2] Esra Karatas Akgül. Solutions of the linear and nonlinear differential equations within the generalized fractional derivatives. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 29(2):023108, 2019.

[3] A Atangana, A Akgül, and KM Owolabi. Analysis of fractal fractional differential equations, alexandria eng. *J*, 59:1117–1134, 2020.

[4] A Atangana, E Bonyah, and AA Elsadany. A fractional order optimal 4d chaotic financial model with mittag-leffler law. *Chinese Journal of Physics*, 65:38–53, 2020.

[5] Abdon Atangana and Ali Akgül. Can transfer function and bode diagram be obtained from sumudu transform. *Alexandria Engineering Journal*, 59(4):1971–1984, 2020.

[6] Abdon Atangana and Badr Saad T Alkahtani. Analysis of the keller–segel model with a fractional derivative without singular kernel. *Entropy*, 17(6):4439–4453, 2015.

[7] Abdon Atangana and Dumitru Baleanu. New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model. *arXiv preprint arXiv:1602.03408*, 2016.

[8] LF Ávalos-Ruiz, JF Gomez-Aguilar, A Atangana, and Kolade M Owolabi. On the dynamics of fractional maps with power-law, exponential decay and mittag–leffler memory. *Chaos, Solitons & Fractals*, 127:364–388, 2019.

[9] E Bonyah, MA Khan, KO Okosun, and JF Gómez-Aguilar. Modelling the effects of heavy alcohol consumption on the transmission dynamics of gonorrhea with optimal control. *Mathematical biosciences*, 309:1–11, 2019.

[10] Michele Caputo and Mauro Fabrizio. A new definition of fractional derivative without singular kernel. *Progress in Fractional Differentiation & Applications*, 1(2):73–85, 2015.

[11] Y Chatibi, EH El Kinani, and A Ouhadan. Variational calculus involving nonlocal fractional derivative with mittag–leffler kernel. *Chaos, Solitons & Fractals*, 118:117–121, 2019.

[12] Youness Chatibi, Abdelaziz Ouhadan, et al. Lie symmetry analysis of conformable differential equations. *AIMS Mathematics*, 4(4):1133–1144, 2019.

[13] Fred Cohen. Computer viruses: theory and experiments. *Computers and security*, 6(1):22–35, 1987.

[14] Alberto d'Onofrio. A note on the global behaviour of the network-based sis epidemic model. *Nonlinear Analysis: Real World Applications*, 9(4):1567–1572, 2008.

[15] Muhammad Farman, Ali Akgül, Dumitru Baleanu, Sumaiyah Imtiaz, and Aqeel Ahmad. Analysis of fractional order chaotic financial model with minimum interest rate impact. *Fractal and Fractional*, 4(3):43, 2020.

[16] Muhammad Farman, Muhammad Umer Saleem, Aqeel Ahmad, Sumaiyah Imtiaz, Muhammad Farhan Tabassum, Sana Akram, and MO Ahmad. A control of glucose level in insulin therapies for the development of artificial pancreas by atangana baleanu derivative. *Alexandria Engineering Journal*, 59(4):2639–2648, 2020.

[17] Muhammad Farman, Muhammad Umer Saleem, MF Tabassum, Aqeel Ahmad, and MO Ahmad. A linear control of composite model for glucose insulin glucagon pump. *Ain Shams Engineering Journal*, 10(4):867–872, 2019.

[18] Chenquan Gan, Xiaofan Yang, Wanping Liu, and Qingyi Zhu. A propagation model of computer virus with nonlinear vaccination probability. *Communications in Nonlinear Science and Numerical Simulation*, 19(1):92–100, 2014.

[19] Mohammad Izadi, Maryam Seifaddini, and Mehdi Afshar. Approximate solutions of a sir epidemiological model of computer viruses. *Advanced Studies: Euro-Tbilisi Mathematical Journal*, 14(4):203–219, 2021.

[20] JO Kephart. Direct-graph epidemiological models of computer virus. In *Proceedings of IEEE Symposium on Security and Privacy, May 20-22, 1991*. IEEE, 1991.

[21] Meraj A Khan, Armin Farahvash, David N Douda, Johann-Christoph Licht, Hartmut Grasemann, Neil Sweezey, and Nades Palaniyar. Jnk activation turns on lps-and gram-negative bacteria-induced nadph oxidase-dependent suicidal netosis. *Scientific Reports*, 7(1):1–16, 2017.

[22] Muhammad Altaf Khan, Saif Ullah, and Muhammad Farhan. The dynamics of zika virus with caputo fractional derivative. *AIMS Math*, 4(1):134–146, 2019.

[23] Muhammad Altaf Khan, Saif Ullah, and Sunil Kumar. A robust study on 2019-ncov outbreaks through non-singular derivative. *The European Physical Journal Plus*, 136(2):1–20, 2021.

[24] Bimal Kumar Mishra and Navnit Jha. Fixed period of temporary immunity after run of anti-malicious software on computer nodes. *Applied Mathematics and Computation*, 190(2):1207–1212, 2007.

[25] Bimal Kumar Mishra and Navnit Jha. Seiqrs model for the transmission of malicious objects in computer network. *Applied Mathematical Modelling*, 34(3):710–715, 2010.

[26] William Hugh Murray. The application of epidemiology to computer viruses. *Computers and Security*, 7(2):139–145, 1988.

[27] Kolade M Owolabi and Abdon Atangana. Computational study of multi-species fractional reaction-diffusion system with abc operator. *Chaos, Solitons & Fractals*, 128:280–289, 2019.

[28] Kolade M Owolabi and Abdon Atangana. Mathematical analysis and computational experiments for an epidemic system with nonlocal and nonsingular derivative. *Chaos, Solitons & Fractals*, 126:41–49, 2019.

[29] Kolade M Owolabi, Abdon Atangana, and Ali Akgul. Modelling and analysis of fractal-fractional partial differential equations: application to reaction-diffusion model. *Alexandria Engineering Journal*, 59(4):2477–2490, 2020.

[30] Kolade M Owolabi, JF Gómez-Aguilar, and Berat Karaagac. Modelling, analysis and simulations of some chaotic systems using derivative with mittag–leffler kernel. *Chaos, Solitons & Fractals*, 125:54–63, 2019.

[31] José Roberto C Piqueira and Vanessa O Araujo. A modified epidemiological model for computer viruses. *Applied Mathematics and Computation*, 213(2):355–360, 2009.

[32] SU Rehman, Aly R Seadawy, M Younis, and STR Rizvi. On study of modulation instability and optical soliton solutions: the chiral nonlinear schrödinger dynamical equation. *Optical and Quantum Electronics*, 53(8):1–17, 2021.

[33] Jianguo Ren, Xiaofan Yang, Qingyi Zhu, Lu-Xing Yang, and Chunming Zhang. A novel computer virus model and its dynamics. *Nonlinear Analysis: Real World Applications*, 13(1):376–384, 2012.

[34] Muhammad Umer Saleem, Muhammad Farman, Aqeel Ahmad, Ehsan Ul Haque, and MO Ahmad. A caputo fabrizio fractional order model for control of glucose in insulin therapies for diabetes. *Ain Shams Engineering Journal*, 11(4):1309–1316, 2020.

[35] Peter Szor. *The Art of Computer Virus Research and Defense.* 2005.

[36] M Toufik and A Atangana. Nonlocal continuum theories of beam for the analysis of carbon nanotubes. *Eur. Phys. J. Plus*, 132(10):1–16, 2017.

[37] Khalil Ullah, Muhammad Aslam, and Tabassum Naz Sindhu. Bayesian analysis of the weibull paired comparison model using informative prior. *Alexandria Engineering Journal*, 59(4):2371–2378, 2020.

[38] Fangwei Wang, Yunkai Zhang, Changguang Wang, Jianfeng Ma, and SangJae Moon. Stability analysis of a seiqv epidemic model for rapid spreading worms. *Computers and Security*, 29(4):410–418, 2010.

[39] John C Wierman and David J Marchette. Modeling computer virus prevalence with a susceptible-infected-susceptible model with reintroduction. *Computational statistics and data analysis*, 45(1):3–23, 2004.

[40] Lu-Xing Yang, Xiaofan Yang, Luosheng Wen, and Jiming Liu. A novel computer virus propagation model and its dynamics. *International Journal of Computer Mathematics*, 89(17):2307–2314, 2012.

[41] Xulong Zhang. Modeling the spread of computer viruses under the effects of infected external computers and removable storage media. *International Journal of Security and Its Applications*, 10(3):419–428, 2016.

[42] Qingyi Zhu, Xiaofan Yang, Lu-Xing Yang, and Xulong Zhang. A mixing propagation model of computer viruses and countermeasures. *Nonlinear Dynamics*, 73(3):1433–1441, 2013.