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Some Properties of g-Groups

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Abstract. A nonempty set G is a g-group [with respect to a binary operation $*$] if it satisfies the following properties: $(g1)$ $a*(b*c) = (a*b)*c$ for all $a, b, c \in G$; $(g2)$ for each $a \in G$, there exists an element $e \in G$ such that $a*e = a = e*a$ (e is called an *identity element* of a); and, $(g3)$ for each $a \in G$, there exists an element $b \in G$ such that $a*b = e = b*a$ for some identity element e of a.
In this study, we gave some important properties of g-subgroups, homomorphism of g-groups, and the zero element. We also presented a couple of ways to construct g-groups and g-subgroups.

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1. Introduction

A *binary operation* $*$ on a set G is a function from $G \times G$ to G. The image of (a, b) under $*$ will be denoted the by $a*b$. A nonempty set G is a g-group with respect to a binary operation $*$ if it satisfies the following properties: $(g1)$ $a*(b*c) = (a*b)*c$ for all $a, b, c \in G$ (in this case, we say that $*$ is associative); $(g2)$ for each $a \in G$, there exists an element $e \in G$ (called an *identity element*) such that $a*e = a = e*a$; and, $(g3)$ for each $a \in G$, there exists an element $b \in G$ (called an *inverse* of g) such that $a*b = e = b*a$ for some identity element e of a. In this case, we write $(G, *)$ to denote the algebraic structure. If $a*b = b*a$ for all $a, b \in G$, then we say that G is an Abelian g-group. An element with a unique identity element is called a *unit*, otherwise we say that a is *non-unit*.

The singleton sets $\{0\}$ and $\{1\}$ with respect to multiplication $\times$ are g-groups (the two are called trivial g-groups). Tables 1 and 2 may be helpful in seeing this.

Also, the set $\{0, 1\}$ is also a g-group under multiplication as shown in Table 3.

The introduction of the algebraic structure g-group was motivated by the intention of presenting a structure having a unique operation which generalizes the properties of the operation multiplication in a field.

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$\times$	0
0	0

Table 1: The g-group $\{0\}$

$\times$	1
1	1

Table 2: The g-group $\{1\}$

Let G be a non-empty set. An e-group is an algebra $(G;*;A)$ where $*$ is a binary operation in G and A is a non-empty subset of G which satisfies the following axioms: $(E1)$ $x*(y*z) = (x*y)*z$ for all $x, y, z \in G$; $(E2)$ For every $x \in G$ there exists an element $a \in A$ such that $x * a = a * x = x$ (the existence of an identity element corresponding to every element of G); And, $(E3)$ For every $x \in G$ there exists an element $y \in G$ such that $x * y, y * x \in A$ [6].

Let U be a non-empty set, and $*$ be a binary operation in U. The couple $\langle U, *\rangle$ is an *Ubat-space* if the following properties hold: $(U1)$ $x * (y * z) = (x * y) * z$ for all $x, y, z \in U$; $(U2)$ There exists $y \in U$ such that $x * y = y * x = y$ for all $x \in U$; And, $(U3)$ There exists $z \in U$ such that $x * z = z * x = x$ for all $x \in U$.

For example, the singleton set $\{0\}$ with respect to multiplication $\times$ in the Table 1, the singleton set $\{1\}$ with respect to multiplication $\times$ in the Table 2 and the set $\{0, 1\}$ under multiplication in Table 3 are also *Ubat*-spaces.

A nonempty set G is a *generalized group* with respect to a binary operation $*$ if it satisfies the following properties. $(M1)$ $f * (g * h) = (f * g) * h$ for all $f, g, h \in G$; $(M2)$ for each $g \in G$, there exists a unique element $e(g)$ such that $g * e(g) = g = e(g) * g$; And, $(M3)$ for each $g \in G$, there exists an element $h \in G$ such that $g * h = h * g = e(g)$.

Hereafter, please refer to [3] for the other concepts.

This paper is a sequel of a previously published study [1] where a new algebraic structure called g-group was introduced. Additional properties of such structure is presented and shown in this paper.

In the early twentieth century, algebra had evolved into a study of axiomatic systems. It was then referred to as abstract algebra [5]. Since then, mathematicians have introduced and explored various algebraic structures. Some were found related to another and others were found to be entirely different.

One particular concept that captured the attention of many researchers is that of groups. Several group-related structures such as quasigroups, generalized groups and similar structures became the favorite topic of algebra enthusiasts [6], [2], [7]. Findings from these studies were found to be applicable in other branches of mathematics such as Number Theory, Geometry, Analysis [4], Computer Science [1], etc.

Unlike in groups, distinct elements of a g-group may have different identity elements. Also, the identity element as well as the inverse may not be unique. A g-group is generally not a group, but groups are necessarily g-groups. Distinctions of g-groups from other group-like structures like generalized group and e-group are established in [1].

Ubat et al. [1] presented Figure 1, which briefly summarizes the relationship of the

$\times$	0	1
0	0	0
1	0	1

Table 3: The g-group $\{0, 1\}$

different algebraic structure. Solid arcs represent the fact that the family in the tail is a subset of the one in the head. While dashed arcs represent the idea 'can be made'. For example, a dashed line is drawn from the family of Ubat-spaces to the family of e-groups since, although *Ubat*-spaces $(G, *)$ and e-groups are non comparable structures, but a subset A from G can be chosen, so that $(G; *; A)$ is an e-group.

$3 = 2.2pt = 2.1pt(38,0)(68,0)(86,4)(62,26)(-30,4)(0,26)(27,4)(3,26)(49,30)(11,30)(83,4)(5,26)$

e$-groupat030\,Ubat - spaceat - 300groupat300g-groupat6030generalizedgroupat900$

Figure 1: Relationship in terms of set theoretic inclusion of the classes of groups, g-groups, e-groups, generalized groups and *Ubat*-spaces

The structure g-group may have important applications in microprocessor design. Specifically, it can be used to minimize digital circuits which uses AND gates only. For example, consider the digital circuit with three inputs, A, B, and C, given by $(A \vee B) \vee (A \vee C)$. By inspection, the expression $(A \vee B) \vee (A \vee C)$ suggest that a digital circuit needs three AND gates to give the desired output. However, using some properties of the g-group $(\mathbb{Z}_2, \cdot)$, the circuit can be minimized as follows. Identifying $\cdot$ with $\vee$, we have $(A \vee B) \vee (A \vee C) = (A \cdot B) \cdot (A \cdot C) = [(A \cdot B) \cdot A] \cdot C = [A \cdot (B \cdot A)] \cdot C = [A \cdot (A \cdot B)] \cdot C = [(A \cdot A) \cdot B] \cdot C = (A \cdot B) \cdot C = (A \vee B) \vee C$. Note that the expression $(A \vee B) \vee C$ uses two AND gates only, but still performs the same function as $(A \vee B) \vee (A \vee C)$. This simplifies the design of the circuit.

In this study, we gave some important properties of g-groups, and provided a couple of ways of constructing them.

2. Preliminary Results

The following statements are found in [1]. We shall be using them for the succeeding properties.

Remark 1. *An inverse of a unit is also a unit. In addition, the two (the unit and its inverse) have the same identity element.*

Remark 2. *A unit has a unique inverse.*

Remark 3. *The identity of a unit is also a unit.*

Remark 4. *In an Abelian g-group, the identity of the product of two units is equal to the product of their corresponding identities.*

Let G be an Abelian g-group, and H be the set of all units of G, that is $H = \{h \in G : h \text{ is a unit}\}$. In the succeeding discussions, the set H refer to $H = \{h \in G : h \text{ is a unit}\}$. We say that H has a unique identity (or a *trunk*) if all the elements of H have the same identity element.

Remark 5. *If $x \in G \backslash H$, then x has a unique identity element for which it has an inverse.*

3. g-Subgroups

In this section, we introduce the concept g-subgroups, and provide some means of constructing them. A very distinctive property of some types of g-subgroups is that their complements are also g-subgroups, which is not always the case in other structures. Most of the discussions in this section is focused on showing that the set of all units of a particular g-group possesses the said property.

The next statement, Definition 1, defines what a g-subgroup is.

Definition 1. *Let G be a g-group. A non-empty subset J of G is called a g-subgroup of G if J is a g-group with respect to the operation of G.*

Example 1 exhibits some of g-subgroups.

Theorem 1. *Let G be an Abelian g-group. If $H \neq \varnothing$, then H is a g-subgroup of G.*

Proof. $(g1)$ is satisfied by the fact that H is a subset of G. $(g2)$ follows from Remark 3, and $(g3)$ follows from Remark 1 and Remark 2. What remains to be shown is the fact that the operation is a binary operation in H, that is, the product of two units is itself a unit. Let $a, b \in H$. If e' is an identity of ab, then by Remark 4, $e' = e_a e_b = e_{ab}$. This shows that ab has a unique identity, that is, ab is a unit. □

The next statement, Lemma 1, says that in an Abelian group every element of H has the same identity element, that is H is a trunk.

Lemma 1. *Let G be an Abelian g-group. If $a, b \in H$, then $e_a = e_b$.*

Proof. Let $a, b \in H$. Then by Theorem 1, ab is a unit. Observe that $e_a ab = ab$, whence, $e_a = e_{ab}$. Similarly, observe that $abe_b = ab$, whence, $e_b = e_{ab}$. Since a, b, and ab are units, we have $e_a = e_{ab} = e_b$. □

The next statement follows from Theorem 1 and Lemma 1.

Recall, a *group* is a non-empty set G together with a binary operation $*$, satisfying the following axioms: $(G1)$ For all $a, b, c \in G$, we have $(a * b) * c = a * (b * c)$; $(G2)$ There is an element e in G such that for all $x \in G$, we have $e * x = x = x * e$; and, $(G3)$ for each $a \in G$, there exists an element b in G such that $a * b = e = b * a$.

Corollary 1. *Let G be an Abelian g-group. If $H = \varnothing$, then H is a group.*

Proof. $(G1)$ is satisfied by the fact that H is a subset of G. $(G2)$ follows from Lemma 1, and $(G3)$ follows from Remark 1 and Remark 2. In the same sense as in the proof of Theorem 1 the operation can be shown to be a binary operation in H. $\square$

The next statement, Lemma 2, says that $G\backslash H$ is closed with respect to the binary operation in G.

Lemma 2. *Let G be an Abelian g-group. If $a \in G\backslash H$, then $ab \in G\backslash H$ for all $b \in G$.*

Proof. Let $a \in G\backslash H$ and $ab \in H$. If $a \in G\backslash H$, then there exist e and e' such that $ea = a = e'a$ where $e \neq e'$. Hence, $eab = ab = e'ab$. Since ab is a unit, we must have $e = e'$. This is a contradiction. $\square$

The next statement, Corollary 2, follows directly from Lemma 2.

Corollary 2. *Let G be an Abelian g-group. If $a, b \in G\backslash H$, then $ab \in G\backslash H$.*

The next statement, Lemma 3, says that every element in $G\backslash H$ has an identity element in $G\backslash H$, that is, $G\backslash H$ satisfies $(g2)$.

Lemma 3. *Let G be an Abelian g-group. If $a \in G\backslash H$, then a has an identity in $G\backslash H$.*

Proof. If $a \in G\backslash H$, then a has two or more identity elements, say e and e' are two of its distinct identities. By Remark 5 a has a unique identity, say e_a, such that $aa^{-1} = e_a$. Note that $e_a e = a^{-1}ae = a^{-1}a = e_a$, that is e is an identity of e_a. Similarly, note that $e_a e' = a^{-1}ae' = a^{-1}a = e_a$, that is e' is an identity of e_a. Since $e \neq e'$, e_a is not a unit, that is $e_a \in G\backslash H$. $\square$

The next statement, Lemma 4, says that inverse of a non-unit is a non-unit, that is, $G\backslash H$ satisfies $(g3)$.

Lemma 4. *Let G be an Abelian g-group. If $a \in G\backslash H$, then a has an inverse $G\backslash H$.*

Proof. If $a \in G\backslash H$, then by Remark 5, a has a unique identity such that a has an inverse, say the inverse is b. Suppose that $b \in H$. Then by Remark 2, b has a unique inverse, which by Remark 1 must be in H. This is a contradiction since a (which is in $G\backslash H$) is also an inverse of b. $\square$

Finally, the next statement, Theorem 2, provides a way of constructing a g-subgroup. It says that if H is the set of all units, then its complement is a g-subgroup also.

Theorem 2. *Let G be an Abelian g-group. If $H \neq G$, then $G\backslash H$ is a g-subgroup of G.*

Proof. By Corollary 2, the operation in G is a binary operation in $G\backslash H$. $(g1)$ follows from the fact that $G\backslash H$ is a subset of G the operation is associative in G. $(g2)$ follows from Lemma 3, while $(g3)$ follows from Lemma 4. $\square$

Theorem 1 and Theorem 2 implies that g-groups may be partitioned into g-subgroups. In particular, the set of all units of a g-group and its complement are both g-subgroups. This is not the case for groups.

$\times_6$	1	5
1	1	5
5	5	1

Table 4: The g-subgroup H under $\times_6$

$\times_6$	0	2	3	4
0	0	0	0	0
2	0	4	0	2
3	0	0	3	0
4	0	2	0	4

Table 5: The g-subgroup $G\backslash H = \{0, 2, 3, 4\}$ under $\times_6$

Example 1. *Consider the g-group $S_6 = \{0, 1, 2, 3, 4, 5\}$ under multiplication modulo 6. Note that the subsets $H = \{1, 5\}$ and $G\backslash H = \{0, 2, 3, 4\}$ are g-subgroups of $(S_6, \times_6)$. Table 4 and Table 5 may be helpful in seeing this.*

If e is an identity of an element in G, say x, such that there an element y with $xy = e$, then we say that e is a *leaf*. The next theorem, Theorem 3, provides another way of constructing a g-subgroup. It says that the set of all leaf in a g-group is a g-subgroup.

Theorem 3. *Let G be an Abelian g-group. If $E = \{e \in G : e \text{ is a leaf}\}$, then E is a g-subgroup of G.*

Proof. $(g1)$ follows from the fact that E is a subset of G. Next, since each element, say e, of E is a leaf, it is an identity of some element, say x, with a property that there exist $y \in G$ with $xy = e$. Since $ex = x$, we have $exy = xy$. And so, $ee = e$. Hence, e has an identity (which is itself) and an inverse (which is itself also). This shows that E satisfies $(g2)$ and $(g3)$. Accordingly, E is a g-subgroup of G. □

This section is culminated with two obvious remarks, Remark 6 and Remark 7. Remark 6 is evident from Example 1. This is not always the case for other structures.

Remark 6. *The intersection of g-subgroups may not be a g-subgroup.*

Remark 7. *The union of g-subgroups is not always a g-subgroup.*

To see this, we note that $H_1 = \{2, 4\}$ and $H_2 = \{1, 3\}$ are g-subgroups of $S_6 = (\{0, 1, 2, 3, 4, 5\}, \times_6)$, but their union $H_1 \cup H_2$ is not a g-subgroup.

4. Homomorphism

In this section, we present some [homomorphism] conditions that we will impose on a function so that it will be able to identify the units of one g-group with the units of another. The main objective here is to provide another way of constructing a g-subgroup.

The next statement, Definition 2, describes what a homomorphism is.

Definition 2. *Let G and J be g-groups with binary operations $*$ and $*'$, respectively. A function $f : G \to J$ is a homomorphism if $f(a * b) = f(a) *' f(b)$.*

Example 2. *Consider again the g-group $S_6 = \{0, 1, 2, 3, 4, 5\}$ under multiplication modulo 6. In Example 1, the subsets $H = \{1, 5\}$ and $G \backslash H = \{0, 2, 3, 4\}$ are both g-subgroups of S_6. Now, define $f : H \to G \backslash H$ by $1 \mapsto 4$ and $5 \mapsto 2$. Then it is easy to see that f is a homomorphism.*

Let G be an Abelian g-group. Since by Corollary 1, $H = \{h \in G : h \text{ is a unit}\}$ is a group, all the established properties of a homomorphism $f : H \to H$ for groups should hold.

The next theorem, Theorem 4, say that under a homomorphism the image of an identity is an identity in the co-domain, and the image of an inverse is an inverse in the co-domain.

Theorem 4. *Let G and J be Abelian g-groups and $a \in G$ with identity e and inverse b. If $f : G \to J$ is a homomorphism, then*

a.) $f(e)$ is an identity of $f(a)$, and

b.) $f(b)$ is an inverse of $f(a)$.

Proof. (a.) Since f is a homomorphism, we have $f(a)f(e) = f(ae) = f(a)$. Hence, $f(e)$ is also an identity of $f(a)$. (b.) In the same token, since f is a homomorphism, we have $f(e) = f(ab) = f(a)f(b)$. Now, since in (a.) $f(e)$ is an identity, $f(b)$ is also an inverse of $f(a)$. □

For the next theorem, Theorem 5, we let G and J be Abelian g-groups. In addition, we let $H = \{h \in G : h \text{ is a unit}\}$ and $H' = \{h \in J : h \text{ is a unit}\}$.

Theorem 5. *If both $f : G \to J$ and $f^{-1} : J \to G$ are homomorphisms, then $h \in H$ if and only if $f(h) \in H'$, that is $f(H) = H'$.*

Proof. It suffices to show that if $h \in H$, then $f(h) \in H'$. Assume that $h \in H$ and $f(h) \notin H'$. By Theorem 4(a), $f(e_h)$ is an identity of $f(h)$. If $f(h) \notin H'$, then there is another identity of $f(h)$, say e, in J with $e \neq f(e_h)$. Since f^{-1} is also a homomorphism, we have $h = f^{-1}(f(h)) = f^{-1}(f(h)e) = f^{-1}(f(h))f^{-1}(e) = hf^{-1}(e)$, that is $f^{-1}(e)$ is another identity of h. This is a contradiction. □

Remark 8 maybe worth noting. We say that a homomorphism is a *monomorphism* if it is injective.

Remark 8. *Let G and J be Abelian g-groups, and $f : G \to J$ be a monomorphism. If g is not unit of G, then $f(g)$ is not a unit of J.*

To see this, let e and e' be distinct identities of a. By Theorem 4(a), $f(e)$ and $f(e')$ are identities of $f(a)$. Since f is injective, $f(e) \neq f(e')$. Thus, $f(a)$ is not a unit.

The next statement, Corollary 3, follows from Remark 8. We let $H = \{h \in G : h \text{ is a unit}\}$.

Corollary 3. *Let G be an Abelian g-group, and $f : G \to G$ be a monomorphism. If $a \in G \backslash H$, then so is $f(a)$.*

The next statement, Theorem 6, also provides a way of constructing a g-subgroups via a homomorphism.

Theorem 6. *Let G and J be Abelian g-groups. If $f : G \to J$ is a homomorphism, then $f(G)$ Abelian g-subgroup of J.*

Proof. Since the operation is closed, associative, and commutative in G, the conditions for homomorphism should imply the closure, the associativity, and the commutativity of the operation in $f(G)$. Hence, the closure property, stated in ($g1$), and the commutativity requirements are satisfied. Finally, ($g2$) and ($g3$) follow from Theorem 4. □

5. The Zero Element

In this section, we give some important properties of zero elements and zero-divisors. At the end of this section, we presented two corollaries that provide another way of constructing a g-subgroup.

The next statement, Definition 3, describes what a zero element is.

Definition 3. *Let G be a g-group. An element $0 \in G$ is called a zero if $a0 = 0a = 0$ for all $a \in G$.*

Example 3. *Consider the g-group $(S_6, \times_6)$ in Example 1. Note that the element 0 is the zero.*

Remark 9. *A g-group may or may not have a zero.*

To see this, consider again the g-group $(S_6, \times_6)$ in Example 1. Note that its g-subgroup $H = \{1, 5\}$ does not have a zero element while $G \backslash H$ has.

It is easy to see that a zero in a non-trivial g-group is not a unit, since all the other elements is its identity.

The next statement, Theorem 7, says that the zero element is unique if it exists.

Theorem 7. *A g-group can have at most one zero element.*

Proof. Let G be a g-group with zeros 0 and $0'$. Then $a0 = 0 = 0a$ and $a0' = 0' = 0'a$ for all $a \in G$. Thus, $0 = 00' = 0'0 = 0$. □

The next statement, Theorem 8, says that the image of the zero element under a homomorphism is the zero element.

Theorem 8. *Let G and G' be both g-groups with zeros 0 and $0'$, respectively. If $f : G \to G'$ is a homomorphism, then $f(0) = 0'$.*

Proof. Observe that for all $a \in G$, $f(a)f(0) = f(a0) = f(0) = f(0a) = f(0)f(a)$. □

The next statement, Definition 4, describes what a zero divisor is.

Definition 4. *Let G be a g-group with a zero element 0, and $a \in G$ with $a \neq 0$. We say that a is a zero divisor if there exists $b \in G$ with $b \neq 0$ such that $ab = 0 = ba$.*

Example 4. *In the g-group $S_6 = \{0, 1, 2, 3, 4, 5\}$, 2, 3 and 4 are zero divisors since $2(3) = 3(4) = 0$.*

The next statement, Theorem 9, says that the image of a zero divisor under a homomorphism is a zero divisor.

Theorem 9. *Let G and G' be both g-groups with zeros 0 and $0'$, respectively. If $f : G \to G'$ is a homomorphism, then the images of the zero divisors in G are zero divisors in G'.*

Proof. Let a be a zero divisor in G. Then there exists $b \neq 0$ such that $ab = 0 = ba$. Note that by Theorem 8, $f(0)$ is the zero of the domain. Since f is a homomorphism, $f(a)f(b) = f(ab) = f(0) = f(ba) = f(b)f(a)$. □

The next statement, Theorem 10, says that an identity of a zero divisor is also a zero divisor.

Theorem 10. *Let G be a g-group with a zero element 0, and $a \in G$ with identity e for which a has an inverse b. If a is a zero divisor, then so is e.*

Proof. Note that $e \neq 0$, otherwise $a = ea = 0a = 0$, which is a contradiction since a is a zero divisor. Also, there exist $c \in G$ with $c \neq 0$ and $ac = 0 = ca$. Hence, $ec = bac = b0 = 0$. Thus, e is also a zero divisor. □

This section is culminated with two corollaries, Corollary 4 and Corollary 5 providing another way of constructing a g-subgroup.

Corollary 4. *Let G be a g-group with a zero element 0, and $x \in G$ with identity e and inverse y. If x is a unit and a zero divisor, then so is y.*

Proof. Note that $y \neq 0$, otherwise $e = xy = x0 = 0$, which is a contradiction to Theorem 10. Since x is a zero divisor, there exist $w \in G$ with $w \neq 0$ and $xw = 0 = wx$. Also, since x is a unit, e is also an identity of its inverse, y. Hence, $yw = yew = yyxw = yy0 = 0$. Thus, y is also a zero divisor. □

Corollary 5. *Let G be a g-group with a zero element. The subset $D = \{d \in G : d \text{ is a unit zero divisor}\} \cup \{0\}$ is a g-subgroup of G.*

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