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Fractional Order Techniques for Stiff Differential Equations Arising from Chemistry Kinetics

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Abstract. In this paper, we consider the stiff systems of ordinary differential equations arising from chemistry kinetics. We develop the fractional order model for chemistry kinetics problems by using the Caputo Fabrizio and Atangana-Baleanu derivatives in Caputo sense. We apply the Sumudu transform to obtain the solutions of the models. Uniqueness and stability analysis of the problem are also established by using the fixed point theory results. Numerical results are obtained by using the proposed schemes which supports theoretical results. These concepts are very important for using the real-life problems like Brine tank cascade, Recycled Brine tank cascade, pond pollution, home heating and biomass transfer problem. These results are crucial for solving the nonlinear model in chemistry kinetics.

2020 Mathematics Subject Classifications: 65P40, 68M07

Key Words and Phrases: Chemistry kinetics, fractional technique, stability, uniqueness, Sumudu transform.

1. Introduction

Some problems with the fractional derivatives which include the trigonometric and exponential functions [3–8, 12, 14] show some related approaches for models of epidemic. Different fractional operator is used in literature to solve the real life problems [2, 9, 10, 13, 15] . The chemical reaction has been introduced by Robertson as [1]:

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$$A \xrightarrow{k_1} B. \tag{1}$$

$$B + B \xrightarrow{k_2} C + B. \tag{2}$$

$$B + C \xrightarrow{k_3} A + C. \tag{3}$$

The problem has three equations, where k_1,k_2 and k_3 describe the rate constants and A,B and C are the chemical species contained. By using the mass action law, the get

$$\begin{aligned} y_1' &= -M_1 y_1 + M_3 y_2 y_3, \\ y_2' &= M_1 y_1 - M_2 y_2^2 - M_3 y_2 y_3, \\ y_3' &= M_2 y_2^2. \end{aligned} \tag{4}$$

In this system $y_1(t), y_2(t)$ and $y_3(t)$ are the concentrations of the chemical species A,B and C. The initial time $t = 0$ can be given by $(y_{01}, y_{02}, y_{03})^T$. Where $M_1 = 0.04, M_2 = 3 \times 10^7$ and $M_3 = 10^4$, and the initial concentrations were $y_{01} = 1/100000, y_{02} = 0$ and $y_{03} = 0$. Our new Caputo–Fabrizio fractional model for Robertson problem can therefore be written as follows:

$$\begin{aligned} {}_0^{CF}D_t^{\rho} y_1 &= -M_1 y_1 + M_3 y_2 y_3, \\ {}_0^{CF}D_t^{\rho} y_2 &= M_1 y_1 - M_2 y_2^2 - M_3 y_2 y_3, \\ {}_0^{CF}D_t^{\rho} y_3 &= M_2 y_2^2. \end{aligned} \tag{5}$$

2. Basic Definitions

Some basic definitions are given in this section [3–5, 12].

Definition 1. *Sumudu transform for any function $\phi(t)$ over a set is given as,* $A = \{\phi(t) : \textit{there exist}\ \Lambda, \tau_1, \tau_2 > 0,\ |\phi(t)| < \Lambda \exp(|t|/\tau_i),\ \textit{if}\ t \in (-1)^i \times [0, \infty)\}$ *is described by*

$$F(u) = ST[\phi(t)] = \int_0^\infty \exp(-t)\phi(ut)dt, u \in (-\tau_1, \tau_2). \tag{6}$$

Definition 2. *Atangana-Baleanu derivative in Caputo sense is described as [13] :*

$${}_a^{ABC}D_\tau^\alpha \phi(t) = \frac{AB(\alpha)}{n-\alpha} \int_a^t \frac{d^n}{dw^n}\phi(w) E_\alpha \frac{-\alpha(t-w)^\alpha}{(n-\alpha)} dw,\ n-1 < \alpha < n. \tag{7}$$

The Laplace transform of equation (7) is acquired as:

$$L[{}_a^{ABC}D_\tau^\alpha \phi(t)](s) = \frac{AB(\alpha)}{1-\alpha} \frac{(s^\alpha L[\phi(t)](s) - s^{\alpha-1}\phi(0))}{s^\alpha + \frac{\alpha}{1-\alpha}}. \tag{8}$$

By using Sumudu transform (ST) for equation (7), we obtain

$$ST[{}_0^{ABC}D_\tau^\alpha \phi(t)](s) = \frac{B(\alpha)}{1-\alpha} \alpha\Gamma(\alpha+1) E_\alpha(\frac{-1}{1-\alpha} w^\alpha)[ST(\phi(t)) - \phi(0)]. \tag{9}$$

3. Caputo Fabrizio Derivative

By using Sumudu Transform on system (5), we have

$$\begin{aligned} M(\rho)\frac{ST(y_1(t))-y_1(0)}{1-\rho+\rho u} &= ST[-M_1y_1+M_3y_2y_3],\\ M(\rho)\frac{ST(y_2(t))-y_2(0)}{1-\rho+\rho u} &= ST[M_1y_1-M_2y_2^2-M_3y_2y_3],\\ M(\rho)\frac{ST(y_3(t))-y_3(0)}{1-\rho+\rho u} &= ST[M_2y_2^2]. \end{aligned} \tag{10}$$

Rearranging the above equations yields:

$$\begin{aligned} ST(y_1(t)) &= y_1(0)+\frac{1-\rho+\rho u}{M(\rho)}ST[-M_1y_1+M_3y_2y_3],\\ ST(y_2(t)) &= y_2(0)+\frac{1-\rho+\rho u}{M(\rho)}ST[M_1y_1-M_2y_2^2-M_3y_2y_3],\\ ST(y_3(t)) &= y_3(0)+\frac{1-\rho+\rho u}{M(\rho)}ST[M_2y_2^2]. \end{aligned} \tag{11}$$

Taking inverse transform for system (11) gives:

$$\begin{aligned} y_1(t) &= y_1(0)+ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(-M_1y_1+M_3y_2y_3)],\\ y_2(t) &= y_2(0)+ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(M_1y_1-M_2y_2^2-M_3y_2y_3)],\\ y_3(t) &= y_3(0)+ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(M_2y_2^2)]. \end{aligned} \tag{12}$$

We get this recursive form as:

$$\begin{aligned} y_{1(n+1)}(t) &= y_{1(n)}(0)+ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(-M_1y_{1(n)}+M_3y_{2(n)}y_{3(n)})],\\ y_{2(n+1)}(t) &= y_{2(n)}(0)+ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(M_1y_{1(n)}-M_2y_{2(n)}^2-M_3y_{2(n)}y_{3(n)})],\\ y_{3(n+1)}(t) &= y_{3(n)}(0)+ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(M_2y_{2(n)}^2)]. \end{aligned} \tag{13}$$

And the solution of system (13) is obtained as:

$$y_1(t)=\lim_{n\to\infty}y_{1(n)}(t),\ \ y_2(t)=\lim_{n\to\infty}y_{2(n)}(t),\ \ y_3(t)=\lim_{n\to\infty}y_{3(n)}(t). \tag{14}$$

3.1. Stability Analysis of the Problem

Theorem 1. *Let $(X_1,.)$ be a Banach space and P be a self-map of X_1 satisfying*

$$\|P_x-P_y\|\leq C\|x-P_x\|+c\|x-y\|$$

for all $x, y \in X_1$*, where* $0 \le C, 0 \le c < 1$*. Consider that* P *is a P-Stable. We have*

$$\begin{aligned} y_{1(n+1)}(t) &= y_{1(n)}(0) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(-M_1 y_{1(n)} + M_3 y_{2(n)} y_{3(n)})], \\ y_{2(n+1)}(t) &= y_{2(n)}(0) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(M_1 y_{1(n)} - M_2 y_{2(n)}^2 - M_3 y_{2(n)} y_{3(n)})], \\ y_{3(n+1)}(t) &= y_{3(n)}(0) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(M_2 y_{2(n)}^2)]. \end{aligned} \quad (15)$$

Where $\frac{1-\rho+\rho u}{M(\rho)}$ *is the fractional Lagrange multiplier.*

Theorem 2. *Let us describe a self-map* P *as*

$$\begin{aligned} &P(y_{1(n)}(t)) = y_{1(n+1)}(t) = y_{1(n)}(0) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(-M_1 y_{1(n)} + M_3 y_{2(n)} y_{3(n)})], \\ &P(y_{2(n)}(t)) = y_{2(n+1)}(t) = y_{2(n)}(0) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} \\ &ST(M_1 y_{1(n)} - M_2 y_{2(n)}^2 - M_3 y_{2(n)} y_{3(n)})], \\ &P(y_{3(n)}(t)) = y_{3(n+1)}(t) = y_{3(n)}(0) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(M_2 y_{2(n)}^2)]. \end{aligned} \quad (16)$$

is P-Stable in $L^1(a, b)$ *if*

$$\begin{aligned} &C = ([1 - M_1 f(\gamma) + M_3(K+L)h(\gamma)], [1 + M_1 f(\gamma) - M_2 g(\gamma) - M_3(K+L)h(\gamma)], \\ &[1 + M_2 g(\gamma)]), \\ &c = (0, 0, 0). \end{aligned} \quad (17)$$

Proof. We prove that P has a fixed point. For this, we evaluate the following for all $(m, n) \in \mathbb{N} \times \mathbb{N}$.

$$\begin{aligned} &P(y_{1(n)}(t)) - P(y_{1(m)}(t)) = y_{1(n)}(t) - y_{1(m)}(t) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(-M_1 y_{1(n)} + M_3 y_{2(n)} y_{3(n)})] \\ &- ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(-M_1 y_{1(m)} + M_3 y_{2(m)} y_{3(m)})], \\ &P(y_{2(n)}(t)) - P(y_{2(m)}(t)) = y_{2(n)}(t) - y_{2(m)}(t) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(M_1 y_{1(n)} - M_2 y_{2(n)}^2 \\ &+ M_3 y_{2(n)} y_{3(n)})] - ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(M_1 y_{1(m)} - M_2 y_{2(m)}^2 - M_3 y_{2(m)} y_{3(m)})], \\ &P(y_{3(n)}(t)) - P(y_{3(m)}(t)) = y_{3(n)}(t) - y_{3(m)}(t) + ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(M_2 y_{2(n)}^2)] \\ &- ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)} ST(M_2 y_{2(m)}^2)] \end{aligned}$$

. (18)

Considering equation (18) , we get

$$\begin{aligned} \|P(y_{1(n)}(t)) - P(y_{1(m)}(t))\| = \|y_{1(n)}(t) - y_{1(m)}(t)| \\ +ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(-M_1 y_{1(n)} + M_3 y_{2(n)} y_{3(n))}] \\ -ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(-M_1 y_{1(m)} + M_3 y_{2(m)} y_{3(m)})]\|. \end{aligned} \tag{19}$$

Using triangular inequality for equation (19), we get

$$\begin{aligned} \|P(y_{1(n)}(t)) - P(y_{1(m)}(t))\| = \|y_{1(n)}(t) - y_{1(m)}(t)\| \\ + \|ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(-M_1 y_{1(n)} + M_3 y_{2(n)} y_{3(n)})] \\ - ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(-M_1 y_{1(m)} + M_3 y_{2(m)} y_{3(m)})]\| \end{aligned} \tag{20}$$

Upon further simplification gives:

$$\begin{aligned} &\|P(y_{1(n)}(t)) - P(y_{1(m)}(t))\| = \|y_{1(n)}(t) - y_{1(m)}(t)\| \\ &+ \|ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(-M_1 y_{1(n)} + M_3 y_{2(n)} y_{3(n)} + M_1 y_{1(m)} \\ &- M_3 y_{2(m)} y_{3(m)})]\| \end{aligned} \tag{21}$$

$$\begin{aligned} &\|P(y_{1(n)}(t)) - P(y_{1(m)}(t))\| = \|y_{1(n)}(t) - y_{1(m)}(t)\| \\ &+ ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(\| - M_1(y_{1(n)} - y_{1(m)})\| + \|M_3 y_{2(n)}(y_{3(n)} - y_{3(m)})\| \\ &+ \|M_3 y_{3(m)}(y_{2(n)} - y_{2(m)})\|)]. \end{aligned} \tag{22}$$

$\|y_{1(n)}(t) - y_{1(m)}(t)\| \approx \|y_{2(n)}(t) - y_{2(m)}(t)\|$
$\|y_{1(n)}(t) - y_{1(m)}(t)\| \approx \|y_{3(n)}(t) - y_{3(m)}(t)\|$
Replacing this in equation (22) gives:

$$\begin{aligned} \|P(y_{1(n)}(t)) - P(y_{1(m)}(t))\| = \|y_{1(n)}(t) - y_{1(m)}(t)\| \\ +ST^{-1}[\frac{1-\rho+\rho u}{M(\rho)}ST(\| - M_1(y_{1(n)} - y_{1(m)})\| + \|M_3 y_{2(n)}(y_{1(n)} - y_{1(m)})\| \\ +\|M_3 y_{3(m)}(y_{1(n)} - y_{1(m)})\|)] \\ \|P(y_{1(n)}(t)) - P(y_{1(m)}(t))\| = \|y_{1(n)} - y_{1(m)}\|[1 + ST^{-1}ST(\frac{1-\rho+\rho u}{M(\rho)})\| - M_1\| + \\ ST^{-1}(ST\frac{1-\rho+\rho u}{M(\rho)})\|M_3(y_{2(n)} + y_{3(m)})\|] \end{aligned} \tag{23}$$

Since $y_{2(n)}, y_{3(m)}$ are bounded as they are convergent sequence, therefore, we can obtain two different constants K,L for all "t" such that

$$\|y_{2(n)}\| < K, \ \|y_{3(m)}\| < L, \ (m, n) \in \mathbb{N} \times \mathbb{N}. \tag{24}$$

Now considering equation (22) with equation (23), we get

$$\|P(y_{1(n)}(t)) - P(y_{1(m)}(t))\| = (1 - M_1 f(\gamma) + M_3(K + L)h(\gamma))\|y_{1(n)} - y_{1(m)}\| \tag{25}$$

Where f,g and h are functions from $ST^{-1}ST(\frac{1-\rho+\rho u}{M(\rho)})$. Similarly we get

$$\|P(y_{2(n)}(t)) - P(y_{2(m)}(t))\| = (1 + M_1 f(\gamma) - M_2 g(\gamma) - M_3(K + L)h(\gamma))\|y_{2(n)} - y_{2(m)}\| \tag{26}$$

$$\|P(y_{3(n)}(t)) - P(y_{3(m)}(t))\| = (1 + M_2 g(\gamma))\|y_{3(}(n)) - y_{3(m)}\| \tag{27}$$

Where

$1 - M_1 f(\gamma) + M_3(K + L)h(\gamma) < 1$,
$1 + M_1 f(\gamma) - M_2 g(\gamma) - M_3(K + L)h(\gamma) < 1$,
$1 + M_2 g(\gamma) < 1$,

Then, we get $c = (0, 0, 0)$,
$C = (1 - M_1 f(\gamma) + M_3(K + L)h(\gamma)$,
$1 + M_1 f(\gamma) - M_2 g(\gamma) - M_3(K + L)h(\gamma)$,
$1 + M_2 g(\gamma))$.

4. Atangana-Baleanu Derivative in Caputo Sense

We consider

$$\begin{aligned}
{}_0^{ABC}D_t^\alpha y_1 &= -M_1 y_1 + M_3 y_2 y_3, \\
{}_0^{ABC}D_t^\alpha y_2 &= M_1 y_1 - M_2 y_2^2 - M_3 y_2 y_3, \\
{}_0^{ABC}D_t^\alpha y_3 &= M_2 y_2^2.
\end{aligned} \tag{28}$$

By using Sumudu transform, we have

$$\begin{aligned}
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{(1-\alpha)} E_\alpha(-\frac{1}{(1-\alpha)} w^\alpha) ST(y_1(t) - y_1(0)) = ST[-M_1 y_1 + M_3 y_2 y_3], \\
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{(1-\alpha)} E_\alpha(-\frac{1}{(1-\alpha)} w^\alpha) ST(y_2(t) - y_2(0)) = ST[M_1 y_1 - M_2 y_2^2 - M_3 y_2 y_3], \\
&\frac{B(\alpha)\alpha\Gamma(\alpha+1)}{(1-\alpha)} E_\alpha(-\frac{1}{(1-\alpha)} w^\alpha) ST(y_3(t) - y_3(0)) = ST[M_2 y_2^2].
\end{aligned} \tag{29}$$

Rearranging the above equations gives:

$$\begin{aligned} ST(y_1(t)) &= y_1(0) + \frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times ST[-M_1y_1 + M_3y_2y_3], \\ ST(y_2(t)) &= y_2(0) + \frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times ST[M_1y_1 - M_2y_2^2 - M_3y_2y_3], \\ ST(y_3(t)) &= y_3(0) + \frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times ST[M_2y_2^2]. \end{aligned} \tag{30}$$

$$\begin{aligned} y_1(t) &= y_1(0) + ST^{-1}[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times ST(-M_1y_1 + M_3y_2y_3)], \\ y_2(t) &= y_2(0) + ST^{-1}[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times ST(M_1y_1 - M_2y_2^2 - M_3y_2y_3)], \\ y_3(t) &= y_3(0) + ST^{-1}[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times ST(M_2y_2^2)]. \end{aligned} \tag{31}$$

Then, we get

$$\begin{aligned} & y_{1(n+1)}(t) = y_{1(n)}(0) + ST^{-1}[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times \\ & ST(-M_1y_{1(n)} + M_3y_{2(n)}y_{3(n)})], \\ & y_{2(n+1)}(t) = y_{2(n)}(0) + ST^{-1}[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times \\ & ST(M_1y_{1(n)} - M_2y_{2(n)}^2 - M_3y_{2(n)}y_{3(n)})], \\ & y_{3(n+1)}(t) = y_3(n))(0) + ST^(-1)[\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \times ST(M_2y_{2(n)}^2)]. \end{aligned} \tag{32}$$

And the solution of system (32) is obtained as:

$$y_1(t) = \lim_{n\to\infty} y_{1(n)}(t), \;\; y_2(t) = \lim_{n\to\infty} y_{2(n)}(t), \;\; y_3(t) = \lim_{n\to\infty} y_{3(n)}(t). \tag{33}$$

4.1. Stability and Uniqueness of the Proposed Scheme

Assume that $(X, |.|)$ is a Banach space and H a self-map of X. Let $r_{n+1} = f(Hr_n)$ be specific recursive procedure. The following condition must be fulfilled for $r_{n+1} = Hr_n$

- The fixed point set of H possesses at least one element.
- r_n converges to a point $p \in F(H)$.
- $\lim_{n\to\infty} x_n(t) = p$.

.

Theorem 3. *Suppose that* $(X,|.|)$ *is a Banach space and* H *a self-map of* X *satisfying*

$$\|H_x - H_r\| \leq \Theta\|x - H_x\| + \theta\|x - r\|,$$

for all $x, r \in X$*, where* $0 \leq \Theta$*,* $0 \leq \theta < 1$*. Then* H *is Picard H-Stable.*

Theorem 4. *Describe* H *as a self-map:*

$$\begin{aligned}
H[y_{1(n+1)}(t)] = y_{1(n+1)}(t) = y_{1(n)}(t) + ST^{-1}[&\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \\
&\times ST(-M_1 y_{1(n)} + M_3 y_{2(n)} y_{3(n)})], \\
H[y_{2(n+1)}(t)] = y_{2(n+1)}(t) = y_{2(n)}(0) + ST^{-1}[&\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \\
&\times ST(M_1 y_{1(n)} - M_2 y_{2(n)}^2 - M_3 y_{2(n)} y_{3(n)})], \\
H[y_{3(n+1)}(t)] = y_{3(n+1)}(t) = y_{3(n)}(0) + ST^{-1}[&\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \\
&\times ST(M_2 y_{2(n)}^2)].
\end{aligned} \tag{34}$$

Proof. By using norm properties, then the iteration is H-Stable

$$\begin{aligned}
\|H[y_{1(n)}(t)] - H[y_{1(m)}(t)]\| \leq \|y_{1(n)}(t) - y_{1(m)}(t)\| + ST^{-1}[&\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \\
&\times ST(-M_1\|y_{1(n)} - y_{1(m)}\| + M_3\|y_{2(n)}y_{3(n)} - y_{2(m)}y_{3(m)}\|)], \\
\|H[y_{2(n)}(t)] - H[y_{2(m)}(t)]\| \leq \|y_{2(n)}(t) - y_{2(m)}(t)\| + ST^{-1}[&\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \\
&\times ST(M_1\|y_{1(n)} - y_{1(m)}\| - M_2\|y_{2(n)}^2 - y_{2(m)}^2\| - M_3\|y_{2(n)}y_{3(n)} - y_{2(m)}y_{3(m)}\|)], \\
\|H[y_{3(n)}(t)] - H[y_{3(m)}(t)]\| \leq \|y_{3(n)}(t) - y_{3(n)}(t)\| + ST^{-1}[&\frac{1-\alpha}{B(\alpha)\alpha\Gamma(\alpha+1)E_\alpha(-\frac{1}{(1-\alpha)}w^\alpha)} \\
&\times ST(M_2\|y_{2(n)}^2 - y_{2(m)}^2\|)].
\end{aligned} \tag{35}$$

Its satisfied in theorem 3, when

$$\begin{aligned}
\theta = (0,0,0),\ \Theta = (&\|y_{1(n)}(t) - y_{1(m)}(t)\| \times \| - (y_{1(n)}(t) + y_{1(m)}(t))\| - M_1\|y_{1(n)} - y_{1(m)}\| + \\
&M_3\|y_{2(n)}y_{3(n)} - y_{2(m)}y_{3(m)}\| \times \|y_{2(n)}(t) - y_{2(m)}(t)\| \times \| - (y_{2(n)}(t) + y_{2(m)}(t))\| + \\
&M_1\|y_{1(n)} - y_{1(m)}\| - M_2\|y_{2(n)}^2 - y_{2(m)}^2\| - M_3\|y_{2(n)}y_{3(n)} - y_{2(m)}y_{3(m)}\| \times \\
&\|y_{3(n)}(t) - y_{3(m)}(t)\| \times \| - (y_{3(n)}(t) + y_{3(m)}(t))\| + M_2\|y_{2(n)}^2 - y_{2(m)}^2\|)
\end{aligned} \tag{36}$$

Theorem 5. *The special solution of system* (28) *using the iteration method is unique singular solution.*

Proof. By using Hilbert space $H = L^2((p,q)\times(0,r))$ which can be described as
$h:(p,q)\times(0,T)\to\mathbb{R}$
Considering $\theta=(0,0,0)$, $\Theta=(-M_1y_1+M_3y_2y_3,\ M_1y_1-M_2y_2^2-M_3y_2y_3,\ M_2y_2^2)$. We have

$$T((y_{1(11)}(t)-y_{1(12)}(t), y_{2(21)}(t)-y_{2(22)}(t), y_{3(31)}(t)-y_{3(32)}(t)),(V_1,V_2,V_3)). \tag{37}$$

we get

$$\begin{aligned}
&(-M_1(y_{1(11)}-y_{1(12)})+M_3(y_{2(21)}-y_{2(22)})(y_{3(31)}-y_{3(32)}),V_1)\le M_1\|y_{1(11)}-y_{1(12)}\|\\
&\|V_1\|+M_3\|y_{2(21)}-y_{2(22)}\|\|y_{3(31)}-y_{3(32)}\|\|V_1\|,\\
&(M_1(y_{1(11)}-y_{1(12)})-M_2(y^2_{2(21)}-y^2_{2(22)})-M_3(y_{2(21)}-y_{2(22)})(y_{3(31)}-y_{3(32)}),V_2)\\
&\le M_1\|y_{1(11)}-y_{1(12)}\|\|V_2\|+M_2\|(y^2_{2(21)}-y^2_{2(22)})\|\\
&\|V_2\|+\|+M_3\|y_{2(21)}-y_{2(22)}\|\|y_{3(31)}-y_{3(32)}\|\|V_2\|,\\
&(M_2(y^2_{2(21)}-y^2_{2(22)}),V_3)\le M_2\|(y^2_{2(21)}-y^2_{2(22)})\|\|V_3\|.
\end{aligned} \tag{38}$$

By using conditions, we get

$$\begin{aligned}
\|y_1-y_{1(11)}\|,\|y_1-y_{1(12)}\| &\le \frac{\chi e1}{\omega},\\
\|y_2-y_{2(21)}\|,\|y_2-y_{2(22)}\| &\le \frac{\chi e2}{\varsigma},\\
\|y_3-y_{3(31)}\|,\|y_3-y_{3(32)}\| &\le \frac{\chi e3}{\upsilon}.
\end{aligned} \tag{39}$$

where

$$\begin{aligned}
&\omega = 3(M_1\|y_{1(11)}-y_{1(12)}\|+M_3\|y_{2(21)}-y_{2(22)}\|\|y_{3(31)}-y_{3(32)}\|)\|V_1\|,\\
&\varsigma = 3(M_1\|y_{1(11)}-y_{1(12)}\|+M_2\|(y^2_{2(21)}-y^2_{2(22)})\|+M_3\|y_{2(21)}-y_{2(22)}\|\\
&\quad\|y_{3(31)}-y_{3(32)}\|)\|V_2\|,\\
&\upsilon = 3(M_2\|(y^2_{2(21)}-y^2_{2(22)})\|)\|V_3\|.
\end{aligned} \tag{40}$$

But, it is obvious that

$$\begin{gathered}
(M_1\|y_{1(11)}-y_{1(12)}\|+M_3\|y_{2(21)}-y_{2(22)}\|\|y_{3(31)}-y_{3(32)}\|)\ne 0\\
(M_1\|y_{1(11)}-y_{1(12)}\|+M_2\|(y^2_{2(21)}-y^2_{2(22)})\|+M_3\|y_{2(21)}-y_{2(22)}\|\|y_{3(31)}-y_{3(32)}\|)\ne 0\\
(M_2\|(y^2_{2(21)}-y^2_{2(22)})\|)\ne 0\\
Where\|V_1\|,\|V_2\|,\|V_3\|\ne 0
\end{gathered} \tag{41}$$

Therefore, we have

$$\|y_{1(11)} - y_{1(12)}\| = 0, \;\; \|y_{2(21)} - y_{2(22)}\| = 0, \;\; \|y_{3(31)} - y_{3(32)}\| = 0, \tag{42}$$

Which yields that

$$y_{1(11)} = y_{1(12)}, \;\; y_{2(21)} = y_{2(22)}, \;\; y_{3(31)} = y_{3(32)} \tag{43}$$

This completes the proof of uniqueness.

5. Results and Discussion

The mathematical analysis of chemistry kinetics model with non-linear occurrence has been offered. Some convergence of theoretical results with some numerical method is also given in [11]. In Figures 1, 2, 3, 4, 5, 6 the memory effect of fractional order technique for Robertson problem has been demonstrated. Figures 1, 2, 3 represent the results by using Caputo-Fabrizio derivative and Figures 4, 5, 6 are obtained with ABC derivative. We can get better concentration of the components by using the fractional derivative which are very important for chemical problem to check the actual behavior of the concentration of the chemical with smallest changes in derivative with respect to time.

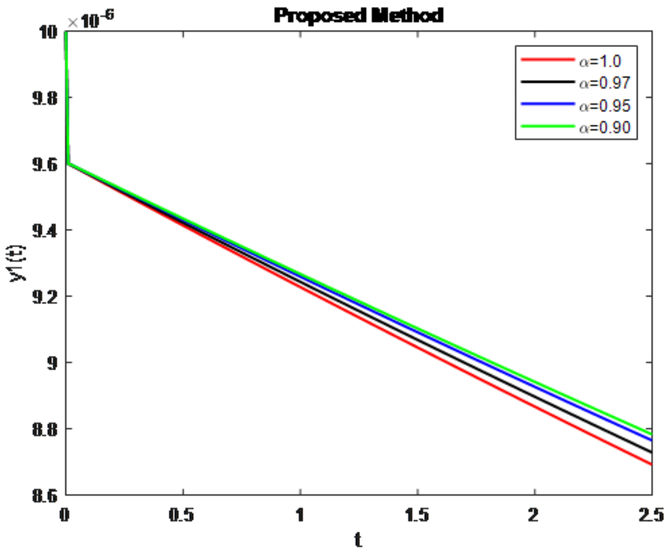

Figure 1: Concentration results of $y_1(t)$ with CF operator at different fractional order .

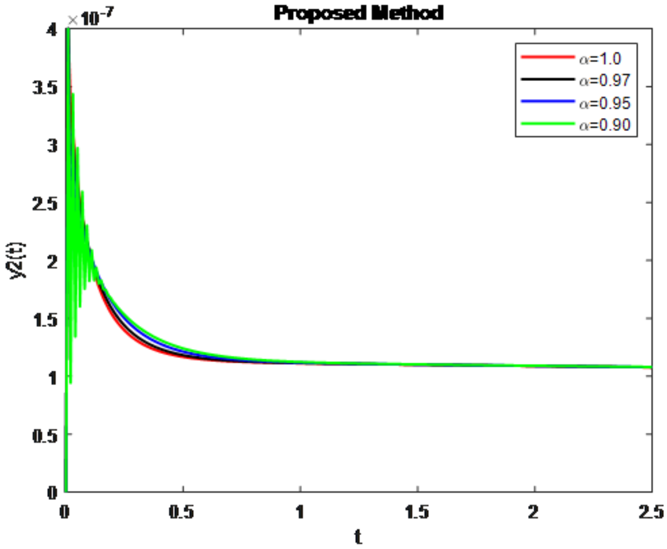

Figure 2: Concentration results of $y_2(t)$ with CF operator at different fractional order .

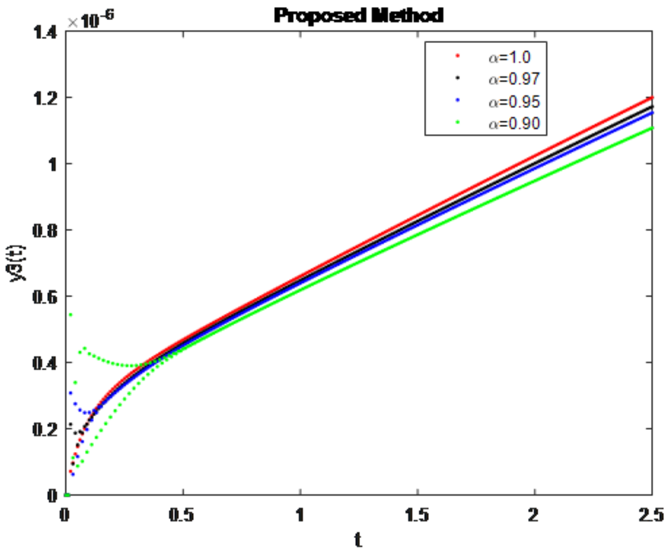

Figure 3: Concentration results of $y_3(t)$ with CF operator at different fractional order .

6. Conclusion

In this paper, we considered the stiff systems of nonlinear ordinary equations which are depend on time t with given initial conditions. The new fractional operator has been implemented to several initial value problems arising from chemical reactions composed of large systems of stiff ordinary differential equations. By using the fixed point theory results, stability and uniqueness of the chemistry kinetic model have been researched. The arbitrary derivative of fractional order has been taken in the Caputo-Fabrizio sense

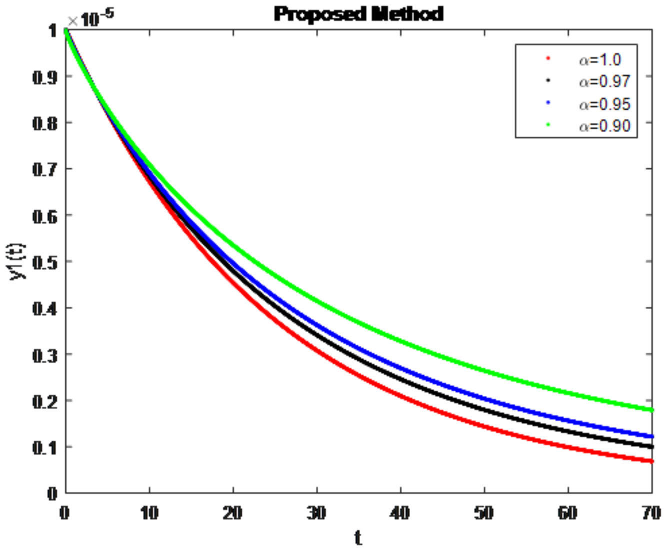

Figure 4: Concentration results of $y_1(t)$ with ABC operator at different fractional order .

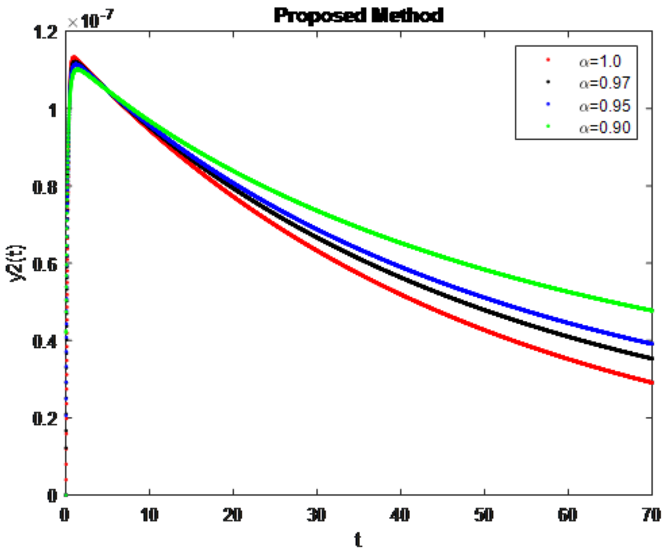

Figure 5: Concentration results of $y_2(t)$ with ABC operator at different fractional order .

with no singular kernel and Atangana-Baleanu in caputo sense with Mittag-Leffler kernel respectively. Sumudu transform is used to obtain the results for proposed schemes. These concepts are very important to use real life problems like Brine tank cascade, Recycled Brine tank cascade, pond pollution, home heating and biomass transfer problem.

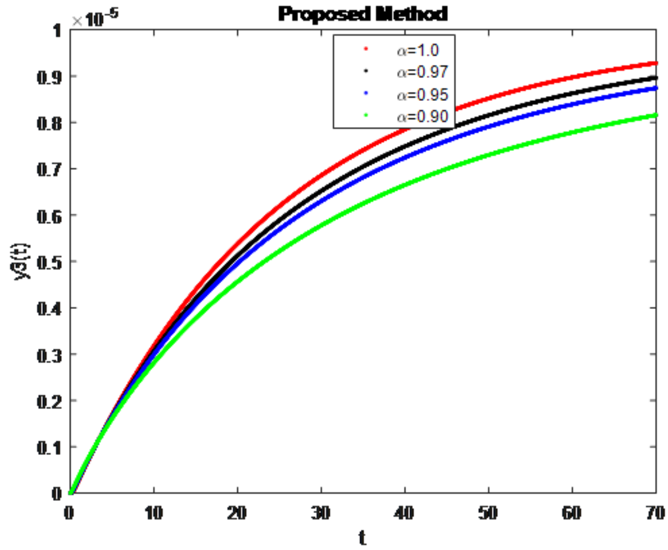

Figure 6: Concentration results of $y_3(t)$ with ABC operator at different fractional order.

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