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Semi-Total Point Graph of Neighbourhood Edge Corona Graph OF G and H

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Abstract. A topological index is a function having a set of graphs as its domain and a set of real numbers as its range. Here we concentrated on topological indices involving the number of vertices, the number of edges and the maximum and minimum vertex degree. The aim of this paper is to compute the lower and upper bounds of the second Zagreb index, third Zagreb index, Hyper Zagreb index, Harmonic index, Redefined first Zagreb index, First reformulated Zagreb index, Forgotten topological index, square F-index, Sum-connectivity index, Randic index, Reciprocal Randic index, Gourava index, Sombar index, Nirmala index, Geometric-Arithmetic index and lower bonds of Atom bond connectivity index, Redefined second Zagreb.

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1. Introduction

A graph invariant that correlates the physico-chemical properties of a molecular graph with a number is called a topological index. The first topological index was introduced by Wiener, a chemist, in 1947 to calculate the boiling points of paraffins [22]. Applications of molecular structure descriptors are a standard procedure in the study of structure - property relations nowadays, especially in the field of $QSPR/QSAR$ study [13],[27],[24], and [16]. During the last century, theoretical chemists started working on the use of topological indices to obtain information of various properties of organic substances which depend upon their molecular structure. For this purpose, numerous topological indices were found and studied in the chemical literature. Throughout the paper, we only consider

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simple graphs without isolated vertex. The study on topological indices and its properties are growing vastly, thus many results can be found, for instance in [10],[21],[26], and [20].

Now we recall some well known topological indices. For a graph G, Gutman *et al.* [12] defined the second Zagreb index as $M_2(G) = \sum_{uv\in E(G)}[d_u \cdot d_v]$. Fath-Taber *et al.* [9] proposed the third Zagreb index and defined it as $ZG_3(G) = \sum_{uv\in E(G)} |d_u - d_v|$. The Hyper Zagreb index is defined in [23] as $M[G] = \sum_{uv\in E[G]}[d_u + d_v]^2$. The Harmonic index is defined in [14] as $H[G] = \sum_{uv\in E[G]} \frac{2}{d_u+d_v}$. The Redefined first Zagreb index is defined in [4] as $ReZG_1[G] = \sum_{uv\in E[G]} \left[\frac{d_u+d_v}{d_u\cdot d_v}\right]$. The First Reformulated Zagreb index is defined in [19] as $EM_1[G] = \sum_{uv\in E[G]}[d_u + d_v - 2]^2$

Furthermore, for a graph G, Furtula *et al.*, [8] proposed the definition of the Forgotten topological index as $F(G) = \sum_{uv\in E(G)}[d_u^2 + d_v^2]$. The extension works which have to be done on this topological indices are recommended in [17][28].The square F-index of a graph G is defined in [30] as $QF[G] = \sum_{uv\in E[G]}[d_u^2 - d_v^2]^2$. The Sum-connectivity index is defined in [2] as

$$SC[G] = \sum_{uv\in E[G]} \frac{2}{\sqrt{d_u + d_v}}$$

The Randic index is defined in [18] as

$$R[G] = \sum_{uv\in E[G]} \frac{1}{\sqrt{d_u d_v}}$$

Moreover, for a graph G, the Reciprocal Randic index is defined in [3] as $RR[G] = \sum_{uv\in E[G]} \sqrt{d_u.d_v}$. Gourava index of graph G is defined in [29] as $GO(G) = \sum_{uv\in E(G)}[d_u + d_v + d_u d_v]$. The Atom bond connectivity index is defined in [7] as

$$ABC[G] = \sum_{uv\in E[G]} \sqrt{\frac{d_u + d_v - 2}{d_u.d_v}}$$

The Redefined second Zagreb index is defined in [5] as

$$ReZG_2[G] = \sum_{uv\in E[G]} \frac{d_u \cdot d_v}{d_u + d_v}$$

The Geometric-Arithmetic index is defined in [6] as

$$GA[G] = \sum_{uv\in E[G]} \frac{2\sqrt{d_u.d_v}}{d_u + d_v}$$

The Sombor index is defined in [11] as $SO[G] = \sum_{uv\in E[G]} \sqrt{d_u^2 + d_v^2}$. Finally, the Nirmala index is defined in [31] as $N[G] = \sum_{uv\in E[G]} \sqrt{d_u + d_v}$.

In the following, we will recall the two definitions which are important in this paper.

Definition 1. *[15] The semi-total point graph of neighbourhood edge corona graph of G and H is a connected graph, denoted by $G \ominus_{nR} H = \psi$.*

Definition 2. *[25] By $G \ominus_{nR} H = \psi$, we mean a graph obtained from one copy of $R(G)$ and m_1 copies of H and joining every vertex of i^{th} copy of H to the vertices which are incident to the edge $e_i \in E(G)$, $[1 \leq i \leq m_1]$.*

Throughout the paper, we utilize the finite simple connected graphs. Let G and H be graphs with vertex sets $V(G)$, $V(H)$ and edge sets $E(G)$, $E(H)$, respectively. The degree of vertex v is the number of vertices adjacent to v. Let $\{V(G) \cap V(H) = \emptyset / g \in V(G), h \in V(H)\}$. The number of vertices and number of edges in the graphs G and H are represented by v_1 , v_2 and e_1 , e_2, respectively. By this definition, we have $\Delta_G \geq deg_G(g)$, and $\delta_G \leq deg_G(g)$ The bounds for different topological indices are obtained by many researchers for graphs.

Now we will define a new class of graph operator , namely semi-total point graph of neighbourhood edge corona graph of G and H as (ψ graph), see [1].

Definition 3. *By $G \ominus_{nR} H = \psi$, we mean a graph obtained from one copy of graph G and e_1 copies of H and joining a vertex of $V(G)$, that is, on the i^{th} vertex in G is adjacent to every vertex of i^{th} copy of H.*

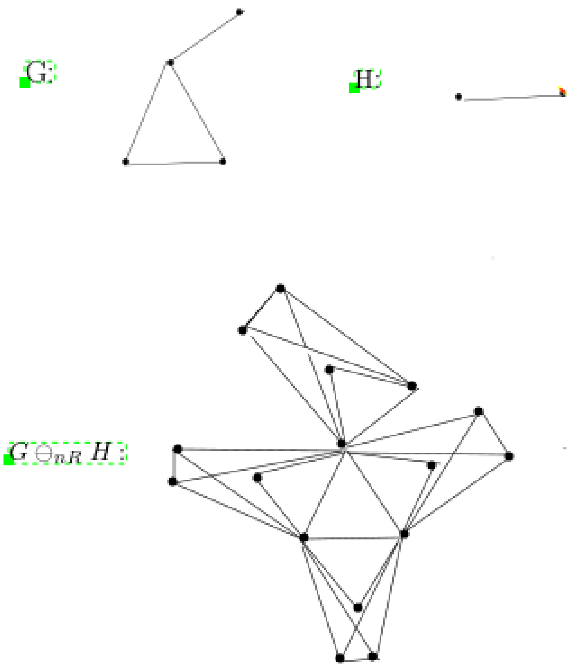

Table 1. Edge partition of ψ graph

$Edge$	$d_G(2+v_2), d_G(2+v_2)$	$(2, d_G(2+v_2)$	(d_H+2, d_H+2)	$(d_H+2, d_G(2+v_2)$
$frequency$	e_1	$2e_1$	e_1e_2	$2v_2e_1$

From now on, we will ready to describe our new results related to bounds for defined class of graphs using recalled topological indices.

2. Bounds on various topological indices of ψ graph

In this section, we formulate the bounds on the M_2, ZG_3, HM, H, $ReZG_1$, EM_1, F, QF, SC, R, RR, GO, ABC, $ReZG_2$, GA, SO and N indices of ψ graph.

Theorem 1. *Let G and H be two simple connected graphs, then*

$$ZG_3[\psi] \leq 2e_1|2-2\Delta_G-\Delta_G v_2| + 2v_2e_1|\Delta_G+2-2\Delta_G-v_2\Delta_G|$$

and

$$ZG_3[\psi] \geq 2e_1|2-2\delta_G-\delta_G v_2| + 2v_2e_1|\delta_G+2-2\delta_G-v_2\delta_G|.$$

Proof. Using Table 1 and the definition of third Zagreb index, we have the following

$$\begin{aligned}
ZG_3(\psi) &= \sum_{uv\in E(G)} |d_u - d_v| \\
&= e_1|d_G(2+v_2) - d_G(2+v_2)| + 2e_1|2-d_G(2+v_2)| + e_1e_2|(d_H+2)-(d_H+2)| \\
&\quad + 2v_2e_1|(d_H+2)-d_G(2+v_2)| \\
&= e_1|0| + 2e_1|2-2d_G+d_Gv_2| + e_1e_2|0| + 2v_2e_1|d_H+2-2d_G+d_Gv_2| \\
&= 2e_1|2-2d_G-d_Gv_2| + 2v_2e_1|d_G+2-2d_G-v_2d_G| \\
Z_3[\psi] &\leq 2e_1|2-2\Delta_G-\Delta_Gv_2| + 2v_2e_1|\Delta_G+2-2\Delta_G-v_2\Delta_G|.
\end{aligned}$$

Similarly, we have $Z_3[\psi] \geq 2e_1|2-2\delta_G-\delta_Gv_2| + 2v_2e_1|\delta_G+2-2\delta_G-v_2\delta_G|$.

Theorem 2. *Let G and H be two simple connected graphs. We have*

$$F[\psi] \leq e_1|2\Delta_G^2(2+v_2)^2| + 2e_1|4+\Delta_G^2(2+v_2)^2| + e_1e_2|2(\Delta_H+2)^2| + 2v_2e_1|(\Delta_H+2)^2+\Delta_G^2(2+v_2)^2$$

and

$$F[\psi] \geq e_1|2\delta_G^2(2+v_2)^2| + 2e_1|4+\delta_G^2(2+v_2)^2| + e_1e_2|2(\delta_H+2)^2| + 2v_2e_1|(\delta_H+2)^2+\delta_G^2(2+v_2)^2|.$$

Proof. Using Table 1 and the definition of forgotten topological index, we have

$$\begin{aligned}
F(\psi) &= \sum_{uv\in E(G)} |d_u^2 + d_v^2| \\
&= e_1|d_G^2(2+v_2)^2 + d_G^2(2+v_2)^2| + 2e_1|2^2 + d_G^2(2+v_2)^2| + e_1e_2|(d_H+2)^2 + (d_H+2)^2|
\end{aligned}$$

$$\begin{aligned}
&+ 2v_2e_1|(d_H + 2)^2 + d_G^2(2 + v_2)^2| \\
&= e_1|2d_G^2(2 + v_2)^2| + 2e_1|4 + d_G^2(2 + v_2)^2| + e_1e_2|2(d_H + 2)^2| \\
&+ 2v_2e_1|(d_H + 2)^2 + d_G^2(2 + v_2)^2| \\
F(\psi) &\leq e_1|2\Delta_G^2(2 + v_2)^2| + 2e_1|4 + \Delta_G^2(2 + v_2)^2| + e_1e_2|2(\Delta_H + 2)^2| \\
&+ 2v_2e_1|(\Delta_H + 2)^2 + \Delta_G^2(2 + v_2)^2|
\end{aligned}$$

Similarly,

$$F[\psi] \geq e_1|2\delta_G^2(2+v_2)^2|+2e_1|4+\delta_G^2(2+v_2)^2|+e_1e_2|2(\delta_H+2)^2|+2v_2e_1|(\delta_H+2)^2+\delta_G^2(2+v_2)^2|.$$

Theorem 3. *Let G and H be two simple connected graphs, then*

$$\begin{aligned}
GO[\psi] &\leq e_1[4\Delta_G + 2v_2\Delta_G + \Delta_G^2(2 + v_2)^2] + 2e_1[6\Delta_G + 3v_2\Delta_G + 2] + e_1e_2[2\Delta_H + 4 \\
&+ (\Delta_H + 2)^2] + 2v_2e_1[2\Delta_G + \Delta_H + \Delta_Gv_2 + 2 + \Delta_G(\Delta_H + 2)(2\Delta_G + v_2)]
\end{aligned}$$

and

$$\begin{aligned}
GO[\psi] &\geq e_1[4\delta_G + 2v_2\delta_G + \delta_G^2(2 + v_2)^2] + 2e_1[6\delta_G + 3v_2\delta_G + 2] + e_1e_2[2\delta_H + 4 \\
&+ (\delta_H + 2)^2] + 2v_2e_1[2\delta_G + \delta_H + \delta_Gv_2 + 2 + \delta_G(\delta_H + 2)(2\delta_G + v_2)].
\end{aligned}$$

Proof. Using Table 1 and definition of Gourava index, we have

$$\begin{aligned}
GO(\psi) &= \sum_{uv\in E(G)} [d_u + d_v + d_ud_v] \\
&= e_1[d_G(2 + v_2) + d_G(2 + v_2) + d_G^2(2 + v_2)^2] + 2e_1[2 + d_G(2 + v_2) + 2d_G(2 + v_2)] \\
&+ e_1e_2[(d_H + 2) + (d_H + 2) + (d_H + 2)^2] + 2v_2e_1[(d_H + 2) + d_G(2 + v_2) \\
&+ d_G(d_H + 2)(2d_G + v_2)] \\
GO(\psi) &\leq e_1[4\Delta_G + 2v_2\Delta_G + \Delta_G^2(2 + v_2)^2] + 2e_1[6\Delta_G + 3v_2\Delta_G + 2] + e_1e_2[2\Delta_H + 4 \\
&+ (\Delta_H + 2)^2] + 2v_2e_1[2\Delta_G + \Delta_H + \Delta_Gv_2 + 2 + \Delta_G(\Delta_H + 2)(2\Delta_G + v_2)].
\end{aligned}$$

Similarly,

$$\begin{aligned}
GO[\psi] &\geq e_1[4\delta_G + 2v_2\delta_G + \delta_G^2(2 + v_2)^2] + 2e_1[6\delta_G + 3v_2\delta_G + 2] + e_1e_2[2\delta_H + 4 + (\delta_H + 2)^2] \\
&+ 2v_2e_1[2\delta_G + \delta_H + \delta_Gv_2 + 2 + \delta_G(\delta_H + 2)(2\delta_G + v_2)].
\end{aligned}$$

Theorem 4. *Let G and H be two simple connected graphs, then*

$$M_2[\psi] \leq e_1[\Delta_G^2(2+v_2)^2]+4e_1[\Delta_G(2+v_2)]+e_1e_2[(\Delta_H+2)^2]+2v_2e_1[(\Delta_H+2)\Delta_G(2+v_2)]$$

and

$$M_2[\psi] \geq e_1[\delta_G^2(2 + v_2)^2] + 4e_1[\delta_G(2 + v_2)] + e_1e_2[(\delta_H + 2)^2] + 2v_2e_1[(\delta_H + 2)\delta_G(2 + v_2)].$$

Proof. Using table 1 and definition of second zagreb index, we have

$$
\begin{aligned}
M_2(\psi) &= \sum_{uv\in E(G)} [d_u d_v] \\
&= e_1[(d_G(2+v_2))(d_G(2+v_2))] + 2e_1[2d_G(2+v_2)] + e_1e_2[(d_H+2)(d_H+2)] \\
&\quad + 2v_2e_1[(d_H+2)d_G(2+v_2)] \\
&= e_1[d_G^2(2+v_2)^2] + 4e_1[d_G(2+v_2)] + e_1e_2[(d_H+2)^2] + 2v_2e_1[(d_H+2)d_G(2+v_2)] \\
M_2[\psi] &\leq e_1[\Delta_G^2(2+v_2)^2] + 4e_1[\Delta_G(2+v_2)] + e_1e_2[(\Delta_H+2)^2] + 2v_2e_1[(\Delta_H+2)\Delta_G(2+v_2)].
\end{aligned}
$$

similarly, $M_2[\psi] \geq e_1[\delta_G^2(2+v_2)^2] + 4e_1[\delta_G(2+v_2)] + e_1e_2[(\delta_H+2)^2] + 2v_2e_1[(\delta_H+2)\delta_G(2+v_2)]$.

Theorem 5. *Let G and H be two simple connected graphs, then*

$$QF(\psi) \leq 2e_1[4 - \Delta_G^2(2+v_2)^2] + 2v_2e_1[(\Delta_H+2)^2 - \Delta_G^2(2+v_2)^2]$$

and

$$QF(\psi) \geq 2e_1[4 - \delta_G^2(2+v_2)^2] + 2v_2e_1[(\delta_H+2)^2 - \delta_G^2(2+v_2)^2].$$

Proof. Using Table 1 and definition of square F-index, we have

$$
\begin{aligned}
QF[\psi] &= \sum_{uv\in E[G]} [d_u^2 - d_v^2]^2 \\
&= e_1[d_G^2(2+v_2)^2 - d_G^2(2+v_2)^2] + 2e_1[2^2 - d_G^2(2+v_2)^2] + e_1e_2[(d_H+2)^2 - (d_H+2)^2] \\
&\quad + 2v_2e_1[(d_H+2)^2 - d_G^2(2+v_2)^2] \\
&= 2e_1[4 - d_G^2(2+v_2)^2] + 2v_2e_1[(d_H+2)^2 - d_G^2(2+v_2)^2] \\
QF[\psi] &\leq 2e_1[4 - \Delta_G^2(2+v_2)^2] + 2v_2e_1[(\Delta_H+2)^2 - \Delta_G^2(2+v_2)^2].
\end{aligned}
$$

Similarly, $QF(\psi) \geq 2e_1[4 - \delta_G^2(2+v_2)^2] + 2v_2e_1[(\delta_H+2)^2 - \delta_G^2(2+v_2)^2]$.

Theorem 6. *Let G and H be two simple connected graphs, then*

$$EM_1(\psi) \leq e_1[2\Delta_G(2+v_2)-2]^2 + 2e_1[\Delta_G(2+v_2)]^2 + e_1e_2[2\Delta_H+2]^2 + 2v_2e_1[\Delta_H+\Delta_G(2+v_2)]^2$$

and

$$EM_1(\psi) \geq e_1[2\delta_G(2+v_2)-2]^2 + 2e_1[\delta_G(2+v_2)]^2 + e_1e_2[2\delta_H+2]^2 + 2v_2e_1[\delta_H+\delta_G(2+v_2)]^2$$

Proof. Using Table 1 and the definition of first reformulated zagreb index, we have

$$
\begin{aligned}
EM_1[\psi] &= \sum_{uv\in E[G]} [d_u + d_v - 2]^2 \\
&= e_1[d_G(2+v_2) + d_G(2+v_2) - 2]^2 + 2e_1[2 + d_G(2+v_2) - 2]^2
\end{aligned}
$$

$$
\begin{aligned}
&+ e_1e_2[d_H + 2 + d_H + 2 - 2]^2 + 2v_2e_1[d_H + 2 + d_G(2 + v_2) - 2]^2 \\
&+ 2v_2e_1[(d_H + 2)^2 - d_G^2(2 + v_2)^2] \\
EM_1(\psi) &\leq e_1[2\Delta_G(2 + v_2) - 2]^2 + 2e_1[\Delta_G(2 + v_2)]^2 + e_1e_2[2\Delta_H + 2]^2 \\
&+ 2v_2e_1[\Delta_H + \Delta_G(2 + v_2)]^2
\end{aligned}
$$

similarly,

$$
\begin{aligned}
EM_1(\psi) \geq\ & e_1[2\delta_G(2 + v_2) - 2]^2 + 2e_1[\delta_G(2 + v_2)]^2 + e_1e_2[2\delta_H + 2]^2 + 2v_2e_1[\delta_H \\
&+ \delta_G(2 + v_2)]^2.
\end{aligned}
$$

Theorem 7. *Let G and H be two simple connected graphs, then*

$$
\begin{aligned}
HM[\psi] &\leq e_1[2\Delta_G(2 + v_2)]^2 + 2e_1[2 + \Delta_G(2 + v_2)]^2 + e_1e_2[(\Delta_H + 2)^2] \\
&+ 2v_2e_1[\Delta_H + 2 + \Delta_G(2 + v_2)]^2 \\
&\textit{and} \\
HM[\psi] &\geq e_1[2\delta_G(2 + v_2)]^2 + 2e_1[2 + \delta_G(2 + v_2)]^2 + e_1e_2[(\delta_H + 2)^2] \\
&+ 2v_2e_1[\delta_H + 2 + \delta_G(2 + v_2)]^2.
\end{aligned}
$$

Proof. Using Table 1 and definition of Hyper zagreb index, we have

$$
\begin{aligned}
HM(\psi) &= \sum_{uv\in E(G)} [d_u + d_v]^2 \\
&= e_1[d_G(2 + v_2) + d_G(2 + v_2)]^2 + 2e_1[2 + d_G(2 + v_2)]^2 + e_1e_2[(d_H + 2) \\
&\ + (d_H + 2)]^2 + 2v_2e_1[(d_H + 2) + d_G(2 + v_2)]^2 \\
&= e_1[2d_G(2 + v_2)]^2 + 2e_1[2 + d_G(2 + v_2)]^2 + e_1e_2[2(d_H + 2)]^2 \\
&+ 2v_2e_1[(d_H + 2) + d_G(2 + v_2)]^2 \\
HM_2[\psi] &\leq e_1[2\Delta_G(2 + v_2)]^2 + 2e_1[2 + \Delta_G(2 + v_2)]^2 + e_1e_2[(\Delta_H + 2)^2] \\
&+ 2v_2e_1[\Delta_H + 2 + \Delta_G(2 + v_2)]^2
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
HM[\psi] &\geq e_1[2\delta_G(2 + v_2)]^2 + 2e_1[2 + \delta_G(2 + v_2)]^2 + e_1e_2[(\delta_H + 2)^2] \\
&+ 2v_2e_1[\delta_H + 2 + \delta_G(2 + v_2)]^2.
\end{aligned}
$$

Theorem 8. *Let G and H be two simple connected graphs, then*

$$
\begin{aligned}
SO[\psi] &\leq e_1\sqrt{2{\Delta_G}^2(2 + v_2)^2} + 2e_1\sqrt{4 + {\Delta_G}^2(2 + v_2)^2} + e_1e_2\sqrt{2(\Delta_H + 2)^2} \\
&+ 2v_2e_1\sqrt{(\Delta_H + 2)^2 + {\Delta_G}^2(2 + v_2)^2}
\end{aligned}
$$

and

$$SO[\psi] \geq e_1\sqrt{2{\delta_G}^2(2+v_2)^2} + 2e_1\sqrt{4+{\delta_G}^2(2+v_2)^2} + e_1e_2\sqrt{2(\delta_H+2)^2}$$
$$+ 2v_2e_1\sqrt{(\delta_H+2)^2+{\delta_G}^2(2+v_2)^2}.$$

Proof. Using Table1 and definition of sombor index, we have

$$\begin{aligned}
SO[\psi] &= \sum_{uv\in E[G]} \sqrt{d_u^2+d_v^2} \\
&= e_1\sqrt{{d_G}^2(2+v_2)^2+{d_G}^2(2+v_2)^2} + 2e_1\sqrt{2^2+{d_G}^2(2+v_2)^2} \\
&\quad + e_1e_2\sqrt{(d_H+2)^2+(d_H+2)^2} + 2v_2e_1\sqrt{(d_H+2)^2+{d_G}^2(2+v_2)^2} \\
&= e_1\sqrt{2{d_G}^2(2+v_2)^2} + 2e_1\sqrt{4+{d_G}^2(2+v_2)^2} + e_1e_2\sqrt{2(d_H+2)^2} \\
&\quad + 2v_2e_1\sqrt{(d_H+2)^2+{d_G}^2(2+v_2)^2} \\
SO[\psi] &\leq e_1\sqrt{2{\Delta_G}^2(2+v_2)^2} + 2e_1\sqrt{4+{\Delta_G}^2(2+v_2)^2} + e_1e_2\sqrt{2(\Delta_H+2)^2} \\
&\quad + 2v_2e_1\sqrt{(\Delta_H+2)^2+{\Delta_G}^2(2+v_2)^2}.
\end{aligned}$$

Similarly,

$$SO[\psi] \geq e_1\sqrt{2{\delta_G}^2(2+v_2)^2} + 2e_1\sqrt{4+{\delta_G}^2(2+v_2)^2} + e_1e_2\sqrt{2(\delta_H+2)^2}$$
$$+ 2v_2e_1\sqrt{(\delta_H+2)^2+{\delta_G}^2(2+v_2)^2}.$$

Theorem 9. *Let G and H be two simple connected graphs, then*

$$RR[\psi] \leq e_1[\Delta_G(2+v_2)] + 2e_1\sqrt{2\Delta_G(2+v_2)} + e_1e_2[\Delta_H+2] + 2v_2e_1\sqrt{(\Delta_H+2)\Delta_G(2+v_2)}$$

and

$$RR[\psi] \geq e_1[\delta_G(2+v_2)] + 2e_1\sqrt{2\delta_G(2+v_2)} + e_1e_2[\delta_H+2] + 2v_2e_1\sqrt{(\delta_H+2)\delta_G(2+v_2)}.$$

Proof. Using Table 1 and the definition of Reciprocal Randic index, we have

$$\begin{aligned}
RR[\psi] &= \sum_{uv\in E[G]} \sqrt{d_u.d_v} \\
&= e_1\sqrt{d_G(2+v_2)d_G(2+v_2)} + 2e_1\sqrt{2d_G(2+v_2)} + e_1e_2\sqrt{(d_H+2)(d_H+2)} \\
&\quad + 2v_2e_1\sqrt{(d_H+2)d_G(2+v_2)} \\
&= e_1[d_G(2+v_2)] + 2e_1\sqrt{2d_G(2+v_2)} + e_1e_2[d_H+2] + 2v_2e_1\sqrt{(d_H+2)d_G(2+v_2)} \\
RR[\psi] &\leq e_1[\Delta_G(2+v_2)] + 2e_1\sqrt{2\Delta_G(2+v_2)} + e_1e_2[\Delta_H+2] + 2v_2e_1\sqrt{(\Delta_H+2)\Delta_G(2+v_2)}
\end{aligned}$$

similarly,

$$RR[\psi] \geq e_1[\delta_G(2+v_2)] + 2e_1\sqrt{2\delta_G(2+v_2)} + e_1e_2[\delta_H+2] + 2v_2e_1\sqrt{(\delta_H+2)\delta_G(2+v_2)}.$$

Theorem 10. *Let G and H be two simple connected graphs, then*

$$\begin{aligned} N[\psi] \leq\ & e_1[\sqrt{2\Delta_G(2+v_2)}] + 2e_1\sqrt{2+\Delta_G(2+v_2)} + e_1e_2[\sqrt{2(\Delta_H+2)}] \\ & + 2v_2e_1\sqrt{(\Delta_H+2)+\Delta_G(2+v_2)} \end{aligned}$$

and

$$\begin{aligned} N[\psi] \geq\ & e_1[\sqrt{2\delta_G(2+v_2)}] + 2e_1\sqrt{2+\delta_G(2+v_2)} + e_1e_2[\sqrt{2(\delta_H+2)}] \\ & + 2v_2e_1\sqrt{(\delta_H+2)+\delta_G(2+v_2)}. \end{aligned}$$

Proof. Using table 1 and definition of Nirmala index, we have

$$\begin{aligned} N[\psi] &= \sum_{uv\in E[G]} \sqrt{d_u+d_v} \\ &= e_1\sqrt{d_G(2+v_2)+d_G(2+v_2)} + 2e_1\sqrt{2+d_G(2+v_2)} \\ &\quad + e_1e_2\sqrt{(d_H+2)+(d_H+2)} + 2v_2e_1\sqrt{(d_H+2)+d_G(2+v_2)} \\ &= e_1[2d_G(2+v_2)] + 2e_1\sqrt{2+d_G(2+v_2)} \\ &\quad + e_1e_2[2(d_H+2)] + 2v_2e_1\sqrt{(d_H+2)+d_G(2+v_2)} \\ N[\psi] &\leq e_1[\sqrt{2\Delta_G(2+v_2)}] + 2e_1\sqrt{2+\Delta_G(2+v_2)} + e_1e_2[\sqrt{2(\Delta_H+2)}] \\ &\quad + 2v_2e_1\sqrt{(\Delta_H+2)+\Delta_G(2+v_2)} \end{aligned}$$

similarly,

$$\begin{aligned} N[\psi] \geq\ & e_1[\sqrt{2\delta_G(2+v_2)}] + 2e_1\sqrt{2+\delta_G(2+v_2)} + e_1e_2[\sqrt{2(\delta_H+2)}] \\ & + 2v_2e_1\sqrt{(\delta_H+2)+\delta_G(2+v_2)}. \end{aligned}$$

Theorem 11. *Let G and H be two simple connected graphs, then*

$$\begin{aligned} ABC[\psi] \geq\ & e_1\left[\sqrt{\frac{\delta_G(2_+v_2)+\delta_G(2+v_2)-2}{\delta_G(2+v_2)\delta_G(2+v_2)}}\right] + 2e_1\left[\sqrt{\frac{2+\delta_G(2+v_2)-2}{2\delta_G(2+v_2)}}\right] \\ & + e_1e_2\left[\sqrt{\frac{(\delta_H+2)+(\delta_H+2)-2}{(\delta_H+2)+(\delta_H+2)}}\right] + 2v_2e_1\left[\sqrt{\frac{(\delta_H+2)+\delta_G(2+v_2)-2}{(\delta_H+2)+\delta_G(2+v_2)}}\right]. \end{aligned}$$

Proof. Using table 1 and definition of Atombond connectivity index, we have

$$ABC[\psi] = \sum_{uv\in E[G]} \sqrt{\frac{d_u+d_v-2}{d_u.d_v}}$$

$$= e_1\left[\sqrt{\frac{d_G(2_+v_2)+d_G(2+v_2)-2}{d_G(2+v_2)d_G(2+v_2)}}\right]+2e_1\left[\sqrt{\frac{2+d_G(2+v_2)-2}{2d_G(2+v_2)}}\right]$$
$$+e_1e_2\left[\sqrt{\frac{(d_H+2)+(d_H+2)-2}{(d_H+2)+(d_H+2)}}\right]+2v_2e_1\left[\sqrt{\frac{(d_H+2)+d_G(2+v_2)-2}{(d_H+2)+d_G(2+v_2)}}\right]$$
$$\geq e_1\left[\sqrt{\frac{\delta_G(2_+v_2)+\delta_G(2+v_2)-2}{\delta_G(2+v_2)\delta_G(2+v_2)}}\right]+2e_1\left[\sqrt{\frac{2+\delta_G(2+v_2)-2}{2\delta_G(2+v_2)}}\right]$$
$$+e_1e_2\left[\sqrt{\frac{(\delta_H+2)+(\delta_H+2)-2}{(\delta_H+2)+(\delta_H+2)}}\right]+2v_2e_1\left[\sqrt{\frac{(\delta_H+2)+\delta_G(2+v_2)-2}{(\delta_H+2)+\delta_G(2+v_2)}}\right].$$

Theorem 12. *Let G and H be two simple connected graphs, then*

$$ReZG_2[\psi]\geq e_1\left[\frac{\delta_G^2(2+v_2)^2}{2\delta_G(2+v_2)}\right]+2e_1\left[\frac{2\delta_G(2+v_2)}{2+\delta_G(2+v_2)}\right]+e_1e_2\left[\frac{(\delta_H+2)^2}{2(\delta_H+2)}\right]$$
$$+2v_2e_1\left[\frac{(\delta_H+2)\delta_G(2+v_2)}{(\delta_H+2)+\delta_G(2+v_2)}\right].$$

Proof. Using table 1 and definition of Redefined second Zagreb index, we have

$$ReZG_2[\psi]=\sum_{uv\in E[G]}\frac{d_u.d_v}{d_u+d_v}$$
$$=e_1\left[\frac{d_G^2(2+v_2)^2}{2d_G(2+v_2)}\right]+2e_1\left[\frac{2d_G(2+v_2)}{2+d_G(2+v_2)}\right]+e_1e_2\left[\frac{(d_H+2)^2}{2(d_H+2)}\right]$$
$$+2v_2e_1\left[\frac{(d_H+2)d_G(2+v_2)}{(d_H+2)+d_G(2+v_2)}\right]$$
$$ReZG_2\geq e_1\left[\frac{\delta_G^2(2+v_2)^2}{2\delta_G(2+v_2)}\right]+2e_1\left[\frac{2\delta_G(2+v_2)}{2+\delta_G(2+v_2)}\right]+e_1e_2\left[\frac{(\delta_H+2)^2}{2(\delta_H+2)}\right]$$
$$+2v_2e_1\left[\frac{(\delta_H+2)\delta_G(2+v_2)}{(\delta_H+2)+\delta_G(2+v_2)}\right].$$

Observation: Inequality can change when the value of numerator of topological indices is less than the value of denominatorof topological indices.
i.e.,

$$\Delta_G\leq deg_G(g),\qquad \delta_G\geq deg_G(g)$$

Theorem 13. *Let G and H be two simple connected graphs, then*

$$GA[\psi] \geq e_1\left[\frac{2\sqrt{\Delta_G^2(2+v_2)^2}}{2\Delta_G(2+v_2)}\right] + 2e_1\left[\frac{2\sqrt{2\Delta_G(2+v_2)}}{2+\Delta_G(2+v_2)}\right] + e_1e_2\left[\frac{2\sqrt{(\Delta_H+2)^2}}{2(\Delta_H+2)}\right] + 2v_2e_1\left[\frac{2\sqrt{(\Delta_H+2)\Delta_G(2+v_2)}}{(\Delta_H+2)+\Delta_G(2+v_2)}\right]$$

and

$$GA[\psi] \leq e_1\left[\frac{2\sqrt{\delta_G^2(2+v_2)^2}}{2\delta_G(2+v_2)}\right] + 2e_1\left[\frac{2\sqrt{2\delta_G(2+v_2)}}{2+\delta_G(2+v_2)}\right] + e_1e_2\left[\frac{2\sqrt{(\delta_H+2)^2}}{2(\delta_H+2)}\right] + 2v_2e_1\left[\frac{2\sqrt{(\delta_H+2)\delta_G(2+v_2)}}{(\delta_H+2)+\delta_G(2+v_2)}\right].$$

Proof. Using Table 1 and the definition of Geometric-Arithmetic index, we have

$$\begin{aligned}
GA[\psi] &= \sum_{uv\in E[G]} \frac{2\sqrt{d_u.d_v}}{d_u+d_v}\\
&= e_1\left[\frac{2\sqrt{d_G(2+v_2).d_G(2+v_2)}}{d_G(2+v_2)+d_G(2+v_2)}\right] + 2e_1\left[\frac{2\sqrt{2d_G(2+v_2)}}{2+d_G(2+v_2)}\right]\\
&\quad + e_1e_2\left[\frac{2\sqrt{(d_H+2).(d_H+2)}}{(d_H+2)+(d_H+2)}\right] + 2v_2e_1\left[\frac{2\sqrt{(d_H+2)d_G(2+v_2)}}{(d_H+2)+d_G(2+v_2)}\right]\\
&= e_1\left[\frac{2\sqrt{d_G^2(2+v_2)^2}}{2d_G(2+v_2)}\right] + 2e_1\left[\frac{2\sqrt{2d_G(2+v_2)}}{2+d_G(2+v_2)}\right] + e_1e_2\left[\frac{2\sqrt{(d_H+2)^2}}{2(d_H+2)}\right]\\
&\quad + 2v_2e_1\left[\frac{2\sqrt{(d_H+2)d_G(2+v_2)}}{(d_H+2)+d_G(2+v_2)}\right]\\
GA[\psi] &\geq e_1\left[\frac{2\sqrt{\Delta_G^2(2+v_2)^2}}{2\Delta_G(2+v_2)}\right] + 2e_1\left[\frac{2\sqrt{2\Delta_G(2+v_2)}}{2+\Delta_G(2+v_2)}\right] + e_1e_2\left[\frac{2\sqrt{(\Delta_H+2)^2}}{2(\Delta_H+2)}\right]\\
&\quad + 2v_2e_1\left[\frac{2\sqrt{(\Delta_H+2)\Delta_G(2+v_2)}}{(\Delta_H+2)+\Delta_G(2+v_2)}\right]
\end{aligned}$$

Similarly,

$$GA[\psi] \leq e_1\left[\frac{2\sqrt{\delta_G^2(2+v_2)^2}}{2\delta_G(2+v_2)}\right] + 2e_1\left[\frac{2\sqrt{2\delta_G(2+v_2)}}{2+\delta_G(2+v_2)}\right] + e_1e_2\left[\frac{2\sqrt{(\delta_H+2)^2}}{2(\delta_H+2)}\right] + 2v_2e_1\left[\frac{2\sqrt{(\delta_H+2)\delta_G(2+v_2)}}{(\delta_H+2)+\delta_G(2+v_2)}\right].$$

Theorem 14. *Let G and H be two simple connected graphs, then*

$$H[\psi] \geq e_1\left[\frac{e_1}{\Delta_G(2+v_2)}\right] + \left[\frac{4e_1}{2+\Delta_G(2+v2)}\right] + \left[\frac{e_1e_2}{(\Delta_H+2)}\right] + \left[\frac{4v_2e_1}{\Delta_H+2+\Delta_G(2+v_2)}\right]$$

and

$$H[\psi] \leq e_1\left[\frac{e_1}{\delta_G(2+v_2)}\right] + \left[\frac{4e_1}{2+\delta_G(2+v2)}\right] + \left[\frac{e_1e_2}{(\delta_H+2)}\right] + \left[\frac{4v_2e_1}{\delta_H+2+\delta_G(2+v_2)}\right].$$

Proof. Using table 1 and definition of Harmonic index, we have

$$\begin{aligned}
H[\psi] &= \sum_{uv\in E[G]} \frac{2}{d_u+d_v} \\
&= e_1\left[\frac{2}{d_G(2+v_2)+d_G(2+v_2)}\right] + 2e_1\left[\frac{2}{2+d_G(2+v_2)}\right] \\
&+ e_1e_2\left[\frac{2}{(d_H+2)+(d_H+2)}\right] + 2e_1v_2\left[\frac{2}{d_H+2+d_G(2+v_2)}\right] \\
H[\psi] &\geq e_1\left[\frac{e_1}{\Delta_G(2+v_2)}\right] + \left[\frac{4e_1}{2+\Delta_G(2+v2)}\right] + \left[\frac{e_1e_2}{(\Delta_H+2)}\right] \\
&+ \left[\frac{4v_2e_1}{\Delta_H+2+\Delta_G(2+v_2)}\right]
\end{aligned}$$

Similarly,

$$\begin{aligned}
H[\psi] &\leq e_1\left[\frac{e_1}{\delta_G(2+v_2)}\right] + \left[\frac{4e_1}{2+\delta_G(2+v2)}\right] + \left[\frac{e_1e_2}{(\delta_H+2)}\right] \\
&+ \left[\frac{4v_2e_1}{\delta_H+2+\delta_G(2+v_2)}\right].
\end{aligned}$$

Theorem 15. *Let G and H be two simple connected graphs, then*

$$\begin{aligned}
SC[\psi] &\geq \left[\frac{2e_1}{\sqrt{2\Delta_G(2+v_2)}}\right] + \left[\frac{4e_1}{\sqrt{2+\Delta_G(2+v2)}}\right] + \left[\frac{2e_1e_2}{\sqrt{2(\Delta_H+2)}}\right] \\
&+ \left[\frac{4v_2e_1}{\sqrt{(\Delta_H+2)+\Delta_G(2+v_2)}}\right]
\end{aligned}$$

and

$$\begin{aligned}
SC[\psi] &\leq \left[\frac{2e_1}{\sqrt{2\delta_G(2+v_2)}}\right] + \left[\frac{4e_1}{\sqrt{2+\delta_G(2+v2)}}\right] + \left[\frac{2e_1e_2}{\sqrt{2(\delta_H+2)}}\right] \\
&+ \left[\frac{4v_2e_1}{\sqrt{(\delta_H+2)+\delta_G(2+v_2)}}\right].
\end{aligned}$$

Proof. Using Table 1 and the definition of Sum-connectivity index, we have

$$\begin{aligned}
SC[\psi] &= \sum_{uv\in E[G]} \frac{2}{\sqrt{d_u+d_v}} \\
&= e_1\left[\frac{2}{\sqrt{d_G(2+v_2)+d_G(2+v_2)}}\right] + 2e_1\left[\frac{2}{\sqrt{2+d_G(2+v_2)}}\right]
\end{aligned}$$

$$+ e_1e_2\left[\frac{2}{\sqrt{(d_H+2)+(d_H+2)}}\right] + 2e_1v_2\left[\frac{2}{\sqrt{d_H+2+d_G(2+v_2)}}\right]$$

$$Sc[\psi] \geq \left[\frac{2e_1}{\sqrt{2\Delta_G(2+v_2)}}\right] + \left[\frac{4e_1}{\sqrt{2+\Delta_G(2+v2)}}\right] + \left[\frac{2e_1e_2}{\sqrt{2(\Delta_H+2)}}\right]$$
$$+ \left[\frac{4v_2e_1}{\sqrt{(\Delta_H+2)+\Delta_G(2+v_2)}}\right]$$

Similarly,

$$SC[\psi] \leq \left[\frac{2e_1}{\sqrt{2\delta_G(2+v_2)}}\right] + \left[\frac{4e_1}{\sqrt{2+\delta_G(2+v2)}}\right] + \left[\frac{2e_1e_2}{\sqrt{2(\delta_H+2)}}\right]$$
$$+ \left[\frac{4v_2e_1}{\sqrt{(\delta_H+2)+\delta_G(2+v_2)}}\right].$$

Theorem 16. *Let G and H be two simple connected graphs, then*

$$ReZG_1[\psi] \geq e_1\left[\frac{2e_1}{\Delta_G(2+v_2)}\right] + e_1\left[\frac{2+\Delta_G(2+v_2)}{\Delta_G(2+v_2)}\right] + \left[\frac{2e_1e_2}{(\Delta_H+2)}\right]$$
$$+ 2v_2e_1\left[\frac{(\Delta_H+2)\Delta_G(2+v_2)}{(\Delta_H+2)\Delta_G(2+v_2)}\right].$$

and

$$ReZG_1[\psi] \leq e_1\left[\frac{2e_1}{\delta_G(2+v_2)}\right] + e_1\left[\frac{2+\delta_G(2+v_2)}{\delta_G(2+v_2)}\right] + \left[\frac{2e_1e_2}{(\delta_H+2)}\right]$$
$$+ 2v_2e_1\left[\frac{(\delta_H+2)\delta_G(2+v_2)}{(\delta_H+2)\delta_G(2+v_2)}\right].$$

Proof. Using Table 1 and the definition of Redefined first Zagreb index, we have

$$ReZG_1[\psi] = \sum_{uv\in E[G]}\left[\frac{d_u+d_v}{d_u.d_v}\right]$$
$$= e_1\left[\frac{2d_G(2+v_2)}{d_G^2(2+v_2)^2}\right] + 2e_1\left[\frac{2+d_G(2+v_2)}{2d_G(2+v_2)}\right] + e_1e_2\left[\frac{2(d_H+2)}{(d_H+2)^2}\right]$$
$$+ 2v_2e_1\left[\frac{(d_H+2)+d_G(2+v_2)}{(d_H+2)d_G(2+v_2)}\right]$$
$$ReZG_1[\psi] \geq e_1\left[\frac{2e_1}{\Delta_G(2+v_2)}\right] + e_1\left[\frac{2+\Delta_G(2+v_2)}{\Delta_G(2+v_2)}\right] + \left[\frac{2e_1e_2}{(\Delta_H+2)}\right]$$
$$+ 2v_2e_1\left[\frac{(\Delta_H+2)\Delta_G(2+v_2)}{(\Delta_H+2)\Delta_G(2+v_2)}\right].$$

Similarly,

$$ReZG_1[\psi] \leq e_1\left[\frac{2e_1}{\delta_G(2+v_2)}\right] + e_1\left[\frac{2+\delta_G(2+v_2)}{\delta_G(2+v_2)}\right] + \left[\frac{2e_1e_2}{(\delta_H+2)}\right]$$

$$+ 2v_2e_1\left[\frac{(\delta_H + 2)\delta_G(2 + v_2)}{(\delta_H + 2)\delta_G(2 + v_2)}\right].$$

3. Conclusion

In this work, we have considered ψ graph and concentrated a few important topological indices and determine their bounds. In the same way, we can examine the different classes of topological indices and determine their corresponding bounds for ψ graph.

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