EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 15, No. 4, 2022, 1738-1749
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Solving conformable fractional differential equations with "EJS" software and visualization of sub-diffusion process

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Abstract. In this work, we numerically solve ordinary and conformable fractional differential equations using Easy Java Simulations software. Their solutions, including homogeneous and non-homogeneous parts, are compared in various time intervals. Using software's visualization and simulation features, we may better examine, compare, and evaluate solutions of analytical and numerical fractional differential equations. A kind of oscillatory behavior is seen in long enough times. In simulation of diffusion and sub-diffusion processes, two intriguing events have been observed.

2020 Mathematics Subject Classifications: 26A33, 34A08

Key Words and Phrases: Conformable fractional derivative, easy java simulations (EJS), fractional differential equation, sub-diffusion.

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1. Introduction

Computer modeling and simulation are tightly intertwined. Modeling is the technique through which we build them. A model is a conceptual representation of a physical system and its features. A computer simulation is a model implementation that allows us to test the model in various scenarios to understand its behavior better. Easy Java/Javascript Simulations is a modeling application that enables non-computer scientists to develop simulations in two programming languages. Easy Java/JavaScript Simulations (EJS) is a free, open-source application with over a thousand simulations accessible in the ComPADRE digital library [4, 13]. EJS automates operations like animation and solving ordinary differential equations numerically. Easy Java/Javascript Simulations has three modeling

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v15i4.4547

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work panels. We utilize the application's sequence of work panels to develop the model and its graphical user interface [16].

As a result of advancements in computer hardware and software, industry, business, and research have successfully undergone technology-driven changes. All three industries have long made use of computer simulations. Programs were first created to make tasks simpler or quicker. Simulations work well for processes that take a very long period or happen very fast. Processes that are challenging, risky, or expensive are also excellent candidates. Computer simulations can help students comprehend science's hidden mental realms through animation, resulting in a more abstract comprehension of scientific ideas. Students can manipulate and visualize quantitative data to form a qualitative mental image[23].

Involving students in creating physical models to describe, explain, and predict events has been proven to minimize deficiencies in traditional teaching. Although the modeling technique may be used without a computer, employing one allows students to research difficult and time-consuming issues, illustrate their results, and communicate their discoveries. Computer modeling, theory, and experiment may provide insights and information that cannot be achieved with a single method alone[12].

By providing a central Website with computer modeling tools, simulations, educational resources like lesson plans, and a computational physics textbook that explains the algorithms used in the academic simulations, the Open Source Physics (OSP) project, located at www.compadre.org/osp/, aims to improve computational physics education (1). Our teaching materials are built around simple single-concept simulations with source codes that may be read, changed, recompiled, and disseminated. These interactive simulations will teach students to critically analyze and examine the premises and results of simulations [14].

The Open Source Physics (OSP) project at www.compadre.org/osp/ aims to improve computational physics education by providing a central Web site with computer modeling tools, simulations, curricular resources like lesson plans, and a computational physics textbook that explains the algorithms of the scientific simulations. Our tools are based on short, single-idea simulations with source code that can be looked at, changed, recompiled, and shared freely. These simulations are used to teach important computer skills. All levels of students will benefit from these interactive simulations because they will learn to question and evaluate the simulation's assumptions and results [14].

Every technical task necessitates the use of the appropriate instrument. Easy Java Simulations is a Java authoring tool that was created exclusively for the construction of interactive simulations. Though it's vital to distinguish between the finished output and the tool used to create it, theoretically, any existing programming language could be used to create the simulations we'll be making with Ejs. This tool stems from the specific expertise accumulated over several years of experience in the creation of computer simulations and will thus be very useful to simplify our task, both technically and conceptually [16].

The proper instrument is necessary for every technical task. Easy Java Simulations is an authoring tool created exclusively for developing Java-based interactive simulations. Although it's important to distinguish between the final product and the tool used to make it, the simulations we'll build with Ejs could theoretically also be constructed using any modern computer programming language. Consequently, this tool comes from the specific expertise developed over many years of experience in creating computer simulations. It will be of great assistance to simplify our task from both the technical and conceptual points of view [3].

Using conformable fractional derivative, in this paper, we will solve some fractional differential equations numerically through Easy Java Simulations (EJSs) software.

2. Fractional calculus and Fractional differential equations

Calculus came before fractional derivatives. L'Hospital pondered the meaning of $\frac{d^n f}{dx^n}$ if $n = 1/2$ in 1695. Since then, other academics have worked to clarify what a fractional derivative is. Different forms of fractional derivatives exist (for example, see [26, 27, 29, 31, 32]). Famous mathematicians including Riemann, Liouville, Letnikov, Sonin, Weyl, Riesz, Erdelyi, Kober, and others proposed fractional derivatives. In the natural sciences, processes and systems with spatial and temporal non-locality are often explained using fractional derivatives of non-integer orders (the non-locality in time is usually called memory). In general, integro-differential operators are a subclass of fractional derivatives of non-integer orders.

In recent years, fractional differential equations (FDEs) in the sense of Riemann-Liouville, Caputo, and Grunwald-Letnikov have played an essential role in modeling various real-world issues. FDEs have been used to describe real-world events in many fields, such as diffusion and dynamics in biology, fluid mechanics, fluid flow, signal processing, and others [15, 20, 22, 25, 30, 33]. An alternate formulation of the diffusion equation is presented to enhance anomalous diffusion modeling by employing a new derivative with fractional order known as the conformable derivative. The analytical solutions to the conformable derivative model are provided in terms of the Gauss kernel and the error function. The conformable diffusion model's power law of the mean square displacement is examined using the time-dependent Gauss kernel. The Levenberg-Marquardt approach is used to calculate the parameters of the conformable derivative model from the experimental data of chloride ion transport in reinforced concrete. Fitting the data shows that the conformable derivative model and the experimental data match up better than the usual diffusion equation [36].

Mathematical modeling is one of the viable ways of solving real-world issues. Modeling may be done in a variety of ways, including statistical approaches that can predict several occurrences. Health is now one of the most important fields of study in the world [17]. Over the last three years, the world has been threatened by a new fatal disease known as COVID-19, caused mainly by the Coronavirus. This virus was discovered for the first time in Wuhan, China, and rapidly spread to the rest of the world. Numerous researchers have better developed mathematical models to comprehend the Coronavirus's dynamics and intricacy. For the last three years, the globe has been threatened by a new lethal illness known as COVID-19, which is mostly caused by the Coronavirus. This virus was first detected in Wuhan, China, and quickly spread to the rest of the globe. Many researchers have produced mathematical models to better understand the dynamics and complexity of the Coronavirus [18, 19]. Using chaotic attractors, the conformable and fractal derivatives appear to better describe anomalous diffusion and even the flow of water inside a fracture system than the classical derivative [2].

We provide a quick overview of the "local formulations" in this subsection. Given that they are, at most, a multiplicative factor of the derivative of order one, these formulations shouldn't bear the moniker "fractional," according to a recent study [32]. Despite this, since 2010, similar plans, which first surfaced in the late 1990s of the 20th century, have become quite common. Chen introduced the local operator in 2006 in order to describe the turbulence [11] and anomalous diffusion [10] phenomena.

In recent decades, fractional differential equations (FDEs) have become an important tool in mathematical modeling and studying the dynamics of many natural processes, such as viscoelastic materials, chaotic systems, optimal control problems, and financial markets. To solve these equations, several numerical and analytical methods are used. Among them, Chatibi et al. mention the continuous and discrete symmetry methods and efficient techniques to furnish various solutions for FDEs. The essential concept behind these approaches is the construction of transformations that preserve the form of the examined FDE [7]. The Lie group approach is used in [8] to derive the Lie symmetry algebra accepted by the time fractional Black-Scholes equation. The built symmetry generators are researched in order to build a family of precise solutions and conservation laws for the analyzed problem. Simultaneously, the family of solutions is expanded by using the invariant subspace approach. In [5], Chatibi et al. established the essential optimality requirements for variations problems of the Euler-Lagrange type, where the variational functional is dependent on the Atangana-Baleanu derivative. Examples are provided to show the outcomes that were produced. They provide a correct prolongation formula for conformable derivatives of the traditional prolongation formula of point transformations. The construction of a symmetry group accepted by conformable ordinary and partial differential equations uses this approach, which is shown. They also provide an exact solution to the conformable heat equation using Lie symmetry analysis [6]. The study [9] builds discrete symmetries for a variety of ordinary, partial, and fractional differential equations using the Hydon technique to find discrete symmetries for a differential equation. It is shown how to use those discrete symmetries to create new solutions out of existing ones.

Fractional Taylor power series were recently introduced, and a nice theory was established. However, no work has been done on fractional Fourier series, while some work has been done on fractional Fourier transform. Conformable fractional Fourier series will be introduced in this study. For instance, we solve some fractional partial differential equations using fractional Fourier series [24, 28].

The conformable fractional derivative, which is based on the derivative's fundamental limit formulation in [21], is a novel fractional derivative that has just been presented. Following that, [1] creates fractional versions of chain laws, exponential functions, Gronwall's inequality, integration by parts, and Taylor power series expansions. In [34] the conformable fractional differential transform technique is described, and it is shown how it may be used to conformable fractional differential equations.

A strong and efficient technique for modeling nonlinear systems is fractional calculus. In order to explain the physical universe, Zhao developed a new class of fractional derivatives called general conformable fractional derivative (GCFD). From Khalil's notion of conformable fractional derivative (CFD), the GCFD is generalized. We draw attention to the fact that the word "t1" used in the definition of CFD is only a type of "fractional conformable function" and is not necessary. We also provide physical and geometrical interpretations of GCFD, indicating possible engineering and physics applications. Since it is simple to show that CFD is a particular instance of GCFD, Zhao first discusses its physical and geometrical meanings [35]. Recently, the term conformable fractional derivative —a novel, straightforward, well-behaved formulation of the fractional derivative — was presented by the authors Khalil et al. [21].

Definition 1. *Given a function $f : [0, \infty] \to \mathbb{R}$. Then, the "conformable fractional derivative" of f of order α is defined by*

$$f^{(\alpha)}(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \tag{1}$$

for all $t > 0, \alpha \in (0, 1)$. If f is α-differentiable in some $(0, a), a > 0$, and $\lim_{t \to 0^+} f^{(\alpha)}(t)$ exists then define
$f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t)$. It can be easily shown that $f^{(\alpha)}(t)$ satisfies all the properties in the following theorem.

Theorem 1. *Let $\alpha \in (0, 1]$ and g, h be α-differentiable at a point $t > 0$ then.*

(i) $f^{(\alpha)}(ag + bh) = af^{(\alpha)}g + bf^{(\alpha)}h$, *for all* $a, b \in \mathbb{R}$.

(ii) $f^{(\alpha)}(t^p) = pt^{p-\alpha}$ *for all* $p \in \mathbb{R}$.

(iii) $f^{(\alpha)}(\lambda) = 0$, *for all constant functions* $f(t) = \lambda$.

(iv) $f^{(\alpha)}(gh) = gf^{(\alpha)}(h) + hf^{(\alpha)}(g)$.

(v) $f^{(\alpha)}(\frac{g}{h}) = \frac{gf^{(\alpha)}(h) - hf^{(\alpha)}(g)}{h^2}$.

(vi) If, in addition, f is differentiable, then $f^{(\alpha)}(g(t)) = t^{1-\alpha}\frac{dg}{dt}(t)$.

The conformable fractional derivative of some elementary functions is as follows.

- $f^{(\alpha)}(t^p) = pt^{p-\alpha}$ for all $p \in \mathbb{R}$.
- $f^{(\alpha)}(1) = 0$.
- $f^{(\alpha)}(e^{cx}) = cx^{1-\alpha}e^{cx}, c \in \mathbb{R}$.
- $f^{(\alpha)}(\sin bx) = bx^{1-\alpha}\cos bx, b \in \mathbb{R}$.
- $f^{(\alpha)}(\cos bx) = -bx^{1-\alpha}\sin bx, b \in \mathbb{R}$.
- $f^{(\alpha)}(\frac{1}{\alpha}t^{\alpha})$=1.

One should notice that a function could be α-differentiable at a point but not differentiable.

Example 1. *Let* $g(t) = 2\sqrt{t}$
$f^{(\frac{1}{2})}(g)(0) = \lim_{t\to 0^+} f^{(\frac{1}{2})}(g)(t) = 1$, *where* $f^{(\frac{1}{2})}(g)(t) = 1$ *for* $t > 0$. *However,* $f^{(1)}(g)(0)$ *does not exist.*

Using conformable fractional derivative, we will now solve some fractional differential equations numerically through Easy Java Simulations (EJSs) software.

3. Solving conformable fractional differential equations by using EJS

Easy Java simulation software can be used to visualize mathematical physics problems. To utilize this program to solve the conformable fractional differential equation, we first use the definition of the conformable fractional derivative.

To begin, we draw both the analytical and numerical solutions of the equation at the same time to see if they coincide (Fig. 1, and Fig. 2 which is in fact the data table). We first solve the conformable fractional differential equations according to the definition.

$$y^{(\frac{1}{2})} + y = t^2 + 2t^{\frac{3}{2}}, \quad y(0^+) = 1. \tag{2}$$

Using $f^{(\alpha)}(t) = t^{1-\alpha}\frac{df}{dt}$, and EJS software we solve this equation numerically through following steps (see Fig. 1), The index "h" refers to the solution of the homogeneous equation:

- $y_{1_h}^{(\frac{1}{2})} + y_{1_h} = 0, \qquad y_3^{(\frac{1}{2})} + y_3 = t^2 + 2t^{\frac{3}{2}}$.
- $y_{2_h}^{(1)} + y_{2_h} = 0, \qquad y_4^{(1)} + y_4 = t^2 + 2t^{\frac{3}{2}}$.

Comparing the solutions of an ordinary differential equation with its fractional counterpart shows that from *one point* onwards, the fractional solution *exceeds* the ordinary solution (see both Fig.1).

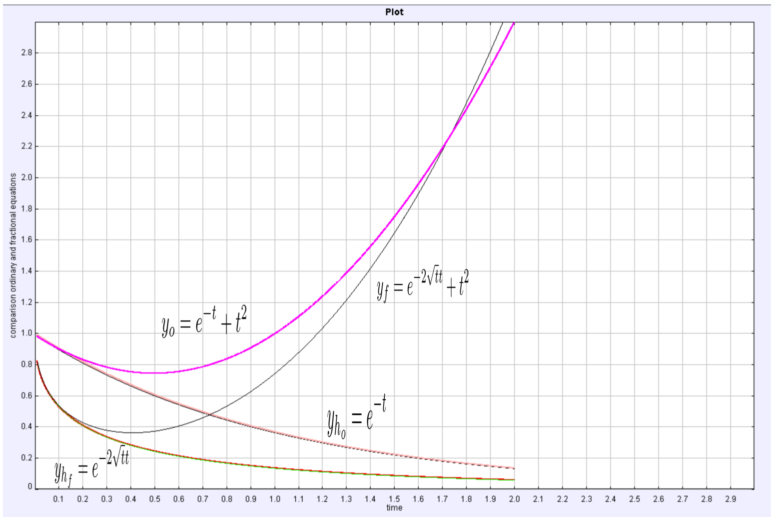

Figure 1: Two homogeneous solutions and two general solutions of ordinary and fractional differential equations.

4. Conformable sub-diffusion equation in EJS

The standard diffusion equation established by Fick is written as

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial^2 C(x,t)}{\partial x^2}, \tag{3}$$

where $C(x,t)$ is the concentration function, and units are mass percentage(%), kg/m^3, or mol/L. D stands for diffusivity or diffusion coefficient (m^2/s). This model is ineffective in characterizing the more intricate time-dependent diffusion processes, such as sub-diffusion. Zhou et al. [36] suggest an effective model illustrating the time-dependent sub-diffusion to get beyond this restriction. The conformable derivative model for sub-diffusion is derived by considering one-dimensional diffusion.

$$\frac{\partial^\alpha C(x,t)}{\partial t^\alpha} = D_\alpha \frac{\partial^2 C(x,t)}{\partial x^2}, \tag{4}$$

where $0 < \alpha \leq 1$, and D_α is the generalized diffusion coefficinet (m^2/s^α).

We may attempt to obtain an analytical solution to the given conformable diffusion equation using the Gaussian kernel if we assume that the diffusion process happens in infinite media and corresponds with the initial condition $C(x,0) = \delta(x)$, where $\delta(x)$ indicates the Dirac delta function. Applying the Fourier transform to both sides of the fractional diffusion equation, Eq. (4), gives

$$\frac{\partial^\alpha \hat{C}(\xi,t)}{\partial t^\alpha} = D_\alpha (2\pi i \xi)^2 \hat{C}(\xi,t), \tag{5}$$

or

$$f^{(\alpha)}(\hat{C}(\xi,t)) = -4\pi^2 D_\alpha \xi^2 \hat{C}(\xi,t), \tag{6}$$

TableFrame

row#	t	y1	y2	y3	y4
0	0.010	0.820	0.980	0.820	0.980
1	0.011	0.816	0.980	0.816	0.980
2	0.011	0.812	0.979	0.812	0.979
3	0.012	0.808	0.979	0.808	0.979
4	0.012	0.804	0.978	0.805	0.978
5	0.013	0.801	0.978	0.801	0.978
6	0.013	0.797	0.977	0.797	0.977
7	0.014	0.794	0.977	0.794	0.977
8	0.014	0.790	0.976	0.791	0.976
9	0.015	0.787	0.976	0.787	0.976
10	0.015	0.784	0.975	0.784	0.975
11	0.016	0.781	0.975	0.781	0.975
12	0.016	0.778	0.974	0.778	0.974
13	0.017	0.775	0.974	0.775	0.974
14	0.017	0.772	0.973	0.772	0.973
15	0.018	0.769	0.973	0.769	0.973
16	0.018	0.766	0.972	0.766	0.972
17	0.019	0.763	0.972	0.763	0.972
18	0.019	0.760	0.971	0.760	0.971
19	0.020	0.757	0.971	0.758	0.971
20	0.020	0.755	0.970	0.755	0.970

Figure 2: From view elements in EJS software, we add "Data table" element to "tree element"

where the initial condition is $\hat{C}(\xi, 0) = 1$. Using EJS, in this paper, we solve the following equation numerically (see Fig. 3).

$$\frac{d}{dt}\hat{C}(\xi, t) + 4\pi^2 D_\alpha \xi^2 t^{\alpha-1}\hat{C}(\xi, t) = 0. \tag{7}$$

It is possible to write the general solution of the ordinary differential equation, Eq. (7), as

$$\hat{C}(\xi, t) = c_0 \exp \frac{-4D_\alpha \pi^2 \xi^2}{\alpha} t^\alpha. \tag{8}$$

Changing the general solution to include the initial condition $\hat{C}(\xi, 0) = 1$ on Eq. (8) produces

$$\hat{C}(\xi, t) = \exp \frac{-4D_\alpha \pi^2 \xi^2}{\alpha} t^\alpha. \tag{9}$$

The analytical solution of the conformable diffusion equation, Eq. (5), is shown to be a time-dependent Gaussian distribution resulting from the inverse Fourier transform. Obviously, the conventional Gauss kernel model, which is the normal diffusion with $\alpha = 1$, and has an analytical solution, is an extension of the conformable derivative Gauss kernel model that is being described. The analytical solution of the conformable diffusion equation, Eq. (9), is shown to be a time-dependent Gaussian distribution resulting from the inverse Fourier transform.

$$C(x, t) = \sqrt{\frac{\alpha}{4\pi D_\alpha t^\alpha}} \exp(-\frac{\alpha}{4D_\alpha t^\alpha} x^2). \tag{10}$$

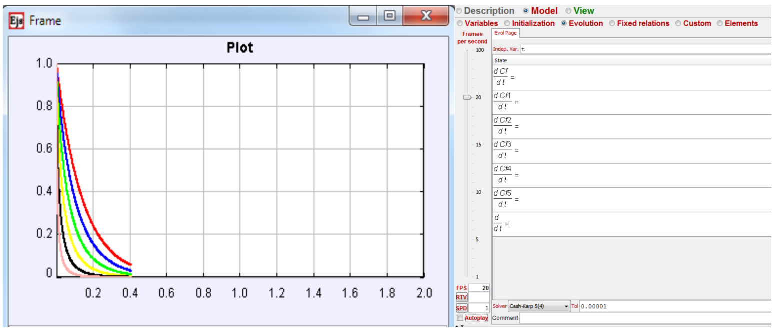

Figure 3: (a) Solving conformable differential equations Eq. (1.6) numerically (b) for different $\alpha = 1$ (red), 0.9 (blue), 0.8 (green), 0.7 (yellow), 0.6 (black), and 0.5 (pink)-$Cf \equiv \hat{C}(\xi, t)$.

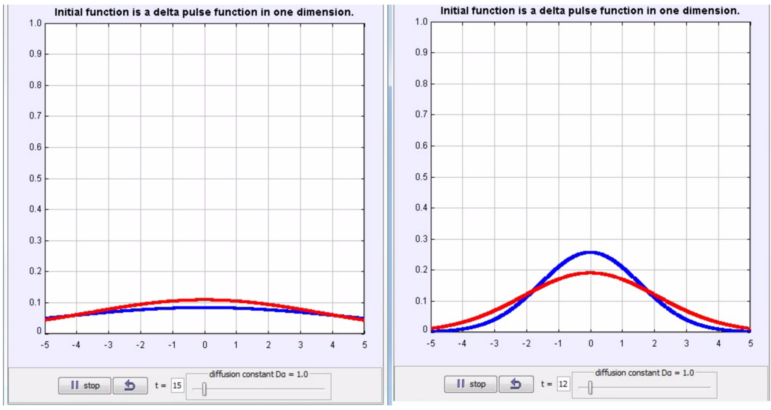

Figure 4: (a) Comparing diffusion process (blue curve for $\alpha = 1$) with (b) sub-diffusion process (red curve for $\alpha = 0.5$).

Using a Gaussian of similar narrowness, the computation begins at $t = 0.0001s$ to prevent the delta function's singularity at time $t = 0$. It appears as a blue line before the commencement (see Fig. 4). After clicking the Run key, the maximum amplitude decreases (observe the shifting scale). Still, the width of the distribution rises proportionately.

5. Conclusion

The method used in this article differs from the other methods. At the same time as clicking the execute button to solve the differential equation numerically, the solution is also displayed as an animation diagram. In addition, it also provides the relevant data table in real time. With Easy Java simulations (EJSs) software, you may verify the analytical solution derived from another method with the numerical results of EJS software. This program has both educational and research purposes. Different algorithms for solving differential equations and adjusting the step size are available inside this application.

Acknowledgements

The authors are sincerely grateful to the editor and anonymous referees for careful reading of the original manuscript and useful comments.

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