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Weak forms of (Λ, b)-open sets and weak (Λ, b)-continuity

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Abstract. This paper is concerned with the concepts of $s(\Lambda, b)$-open sets, $p(\Lambda, b)$-open sets, $\alpha(\Lambda, b)$-open sets, $\beta(\Lambda, b)$-open sets and $b(\Lambda, b)$-open sets. Some properties of $s(\Lambda, b)$-open sets, $p(\Lambda, b)$-open sets, $\alpha(\Lambda, b)$-open sets, $\beta(\Lambda, b)$-open sets and $b(\Lambda, b)$-open sets are investigated. Moreover, several characterizations of weakly (Λ, b)-continuous functions are established.

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1. Introduction

The concepts of openness and continuity are fundamental with respect to the investigation of topological spaces. The notion of continuity is one of the most important tools in Mathematics and many different forms of generalizations of continuity have been introduced and investigated. The concept of weakly continuous functions was introduced by Levine [10]. Rose [13] introduced the notion of subweakly continuous functions and investigated the relationships between subweak continuity and weak continuity. In [4] has obtained further properties of subweakly continuous functions. In 1970, Willard [16] introduced the concept of extremally disconnected spaces. Extremally disconnected spaces play a prominent role in set-theoretical topology. Due to their peculiar properties extremally disconnected spaces provide crucial applications in the theory of Boolean algebra, in axiomatic set theory and in some branches of functional analysis. Sivaraj [14] investigated some characterizations of extremally disconnected spaces by utilizing semi-open sets due to Levine [11]. The concept of submaximality of generalized topological spaces was introduced by Hewitt [9]. He discovered a general way of constructing maximal topologies. The first systematic study of submaximal spaces was undertaken in the paper of Arhangel'skiĭ and Collins [3]. They gave various necessary and suffient conditions for a space to be submaximal and showed that every submaximal space is left-separated. In 1996, Andrijević [2] introduced a new class of generalized open sets called b-open sets into the

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field of topology. This class is a subset of the class of semi-preopen sets [1], i.e. a subset of a topological space which is contained in the closure of the interior of its closure. Also the class of b-open sets in a superset of the class of semi-open sets [11], i.e. a set which is contained in the closure of its interior and the class of locally dense set [8] or preopen sets [12], i.e. a set which is contained in the interior of its closure. Moreover, Andrijević studied several fundamental and interesting properties of b-open sets. Caldas et al. [7] introduced the concept of Λ_b-sets which is the intersection of b-open sets and investigated the fundamental properties of Λ_b-sets and V_b-sets. In [5], the present authors introduced and studied the notions of (Λ, b)-closed sets and (Λ, b)-open sets. The notions of (Λ, b)-extremally disconnected spaces, (Λ, b)-hyperconnected spaces and (Λ, b)-submaximal spaces were introduced by Viriyapong and Boonpok [15]. Recently, Boonpok and Viriyapong [6] introduced and studied the notion of weakly (Λ, p)-continuous functions. In this paper, we introduce new classes of sets called $s(\Lambda, b)$-open sets, $p(\Lambda, b)$-open sets, $\alpha(\Lambda, b)$-open sets, $\beta(\Lambda, b)$-open sets and $b(\Lambda, b)$-open sets. We also investigate some of their fundamental properties. Furthermore, we investigate several characterizations of (Λ, b)-submaximal spaces and (Λ, b)-extremally disconnected spaces by utilizing the notion of (Λ, b)-open sets. In particular, some characterizations of weakly (Λ, b)-continuous functions are discussed.

2. Preliminaries

Throughout this paper, the spaces (X, τ) and (X, σ) (or simply X and Y) always means topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of X, the closure of A and the interior of A in (X, τ) are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is called *b-open* [2] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A)) \cup \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a b-open set is called *b-closed*. By $BO(X, \tau)$, we denote the collection of all b-open sets of a topological space (X, τ). Let A be a subset of a topological space (X, τ). A subset A^{Λ_b} [7] is defined to be the set $\cap\{U \in BO(X, \tau) | A \subseteq U\}$. A subset A of a topological space (X, τ) is called a *Λ_b-set* [7] if $A = A^{\Lambda_b}$. A subset A of a topological space (X, τ) is called *(Λ, b)-closed* [5] if $A = T \cap C$, where T is a Λ_b-set and C is a b-closed set. The complement of a (Λ, b)-closed set is called *(Λ, b)-open*. The collection of all (Λ, b)-closed (resp. (Λ, b)-open) sets in a topological space (X, τ) is denoted by $\Lambda_b C(X, \tau)$ (resp. $\Lambda_b O(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, b)-cluster point* [5] of A if for every (Λ, b)-open set U of X containing x we have $A \cap U \neq \emptyset$. The set of all (Λ, b)-cluster points of A is called the *(Λ, b)-closure* [5] of A and is denoted by $A^{(\Lambda,b)}$. The union of all (Λ, b)-open sets contained in A is called the *(Λ, b)-interior* [5] of A and is denoted by $A_{(\Lambda,b)}$. Let A be a subset of a topological space (X, τ). A subset $\Lambda_{(\Lambda,b)}(A)$ [5] is defined as follows: $\Lambda_{(\Lambda,b)}(A) = \cap\{U \in \Lambda_b O(X, \tau) \mid A \subseteq U\}$.

Lemma 1. [5] *Let A and B be subsets of a topological space (X, τ). For the (Λ, b)-closure, the following properties hold:*

(1) $A \subseteq A^{(\Lambda,b)}$ and $[A^{(\Lambda,b)}]^{(\Lambda,p)} = A^{(\Lambda,b)}$.

(2) *If* $A \subseteq B$, *then* $A^{(\Lambda,b)} \subseteq B^{(\Lambda,b)}$.

(3) $A^{(\Lambda,b)} = \cap\{F | A \subseteq F$ *and* F *is* (Λ, b)*-closed*$\}$.

(4) $A^{(\Lambda,b)}$ *is* (Λ, b)*-closed.*

(5) A *is* (Λ, b)*-closed if and only if* $A = A^{(\Lambda,b)}$.

Lemma 2. [5] *Let* A *and* B *be subsets of a topological space* (X, τ). *For the* (Λ, b)*-interior, the following properties hold:*

(1) $A_{(\Lambda,b)} \subseteq A$ *and* $[A_{(\Lambda,b)}]_{(\Lambda,b)} = A_{(\Lambda,b)}$.

(2) *If* $A \subseteq B$, *then* $A_{(\Lambda,b)} \subseteq B_{(\Lambda,b)}$.

(3) $A_{(\Lambda,b)}$ *is* (Λ, b)*-open.*

(4) A *is* (Λ, b)*-open if and only if* $A_{(\Lambda,b)} = A$.

Lemma 3. [5] *For subsets* A, B *of a topological space* (X, τ), *the following properties hold:*

(1) $A \subseteq \Lambda_{(\Lambda,b)}(A)$.

(2) *If* $A \subseteq B$, *then* $\Lambda_{(\Lambda,b)}(A) \subseteq \Lambda_{(\Lambda,b)}(B)$.

(3) $\Lambda_{(\Lambda,b)}[\Lambda_{(\Lambda,b)}(A)] = \Lambda_{(\Lambda,b)}(A)$.

(4) *If* A *is* (Λ, b)*-open, then* $\Lambda_{(\Lambda,b)}(A) = A$.

3. Some properties of generalized (Λ, b)-open sets

In this section, we introduce new classes of sets called $s(\Lambda, b)$-open sets, $p(\Lambda, b)$-open sets, $\alpha(\Lambda, b)$-open sets, $\beta(\Lambda, b)$-open sets and $b(\Lambda, b)$-open sets. We also investigate some of their fundamental properties.

Definition 1. *A subset* A *of a topological space* (X, τ) *is said to be:*

(i) $s(\Lambda, b)$*-open if* $A \subseteq [A_{(\Lambda,b)}]^{(\Lambda,b)}$;

(ii) $p(\Lambda, b)$*-open if* $A \subseteq [A^{(\Lambda,b)}]_{(\Lambda,b)}$;

(iii) $\alpha(\Lambda, b)$*-open if* $A \subseteq [[A_{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)}$;

(iv) $\beta(\Lambda, b)$*-open if* $A \subseteq [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$.

Example 1. *Let* $X = \{-1, 0, 1\}$ *with the topology* $\tau = \{\emptyset, \{-1\}, \{0, 1\}, X\}$. *Let* $A = \{-1\}$. *Then,* A *is* $s(\Lambda, b)$*-open and* $p(\Lambda, b)$*-open. On the other hand, let* $B = \{0, 1\}$. *Then,* B *is* $\alpha(\Lambda, b)$*-open and* $\beta(\Lambda, b)$*-open.*

The family of all $s(\Lambda, b)$-open (resp. $p(\Lambda, b)$-open, $\alpha(\Lambda, b)$-open, $\beta(\Lambda, b)$-open) sets in a topological space (X, τ) is denoted by $s\Lambda_b O(X, \tau)$ (resp. $p\Lambda_b O(X, \tau)$, $\alpha\Lambda_b O(X, \tau)$, $\beta\Lambda_b O(X, \tau)$). The complement of a $s(\Lambda, b)$-open (resp. $p(\Lambda, b)$-open, $\alpha(\Lambda, b)$-open, $\beta(\Lambda, b)$-open) set is said to be $s(\Lambda, b)$-*closed* (resp. $p(\Lambda, b)$-*closed*, $\alpha(\Lambda, b)$-*closed*, $\beta(\Lambda, b)$-*closed*). The family of all $s(\Lambda, b)$-closed (resp. $p(\Lambda, b)$-closed, $\alpha(\Lambda, b)$-closed, $\beta(\Lambda, b)$-closed) sets in a topological space (X, τ) is denoted by $s\Lambda_b C(X, \tau)$ (resp. $p\Lambda_b C(X, \tau)$, $\alpha\Lambda_b C(X, \tau)$, $\beta\Lambda_b O(X, \tau)$).

Proposition 1. *For a topological space (X, τ), the following properties hold:*

(1) $\Lambda_b O(X, \tau) \subseteq \alpha\Lambda_b O(X, \tau) \subseteq s\Lambda_b O(X, \tau) \subseteq \beta\Lambda_b O(X, \tau)$.

(2) $\alpha\Lambda_b O(X, \tau) \subseteq p\Lambda_b O(X, \tau) \subseteq \beta\Lambda_b O(X, \tau)$.

(3) $\alpha\Lambda_b O(X, \tau) = s\Lambda_b O(X, \tau) \cap p\Lambda_b O(X, \tau)$.

A subset A of a topological space (X, τ) is called $r(\Lambda, b)$-*open* [15] if $A = [A^{(\Lambda,b)}]_{(\Lambda,b)}$. The complement of a $r(\Lambda, b)$-open set is called $r(\Lambda, b)$-*closed*. The family of all $r(\Lambda, b)$-open (resp. $r(\Lambda, b)$-closed) sets in a topological space (X, τ) is denoted by $r\Lambda_b O(X, \tau)$ (resp. $r\Lambda_b C(X, \tau)$).

Proposition 2. *For a subset A of a topological space (X, τ), the following properties are equivalent:*

(1) A is $r(\Lambda, b)$-open.

(2) A is (Λ, b)-open and $s(\Lambda, b)$-closed.

(3) A is $\alpha(\Lambda, b)$-open and $s(\Lambda, b)$-closed.

(4) A is $p(\Lambda, b)$-open and $s(\Lambda, b)$-closed.

(5) A is (Λ, b)-open and $\beta(\Lambda, b)$-closed.

(6) A is $\alpha(\Lambda, b)$-open and $\beta(\Lambda, b)$-closed.

Proof. (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4): Obvious.

(4) $\Rightarrow$ (5): Let A be (Λ, b)-open and $s(\Lambda, b)$-closed. Then, $A \subseteq [A^{(\Lambda,b)}]_{(\Lambda,b)}$ and $[A^{(\Lambda,b)}]_{(\Lambda,b)} \subseteq A$. This implies that $A = [A^{(\Lambda,b)}]_{(\Lambda,b)}$. Thus, A is $r(\Lambda, b)$-open and hence A is (Λ, b)-open. Since every $s(\Lambda, b)$-closed set is $\beta(\Lambda, b)$-closed. This shows that A is (Λ, b)-open and $\beta(\Lambda, b)$-closed.

(5) $\Rightarrow$ (6): Obvious.

(6) $\Rightarrow$ (1): Let A be $\alpha(\Lambda, b)$-open and $\beta(\Lambda, b)$-closed. Then, $A \subseteq [[A_{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)}$ and $[[A_{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)} \subseteq A$. This shows that $A = [[A_{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)}$. Thus, $A_{(\Lambda,b)} = [[A_{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)} = A$ and hence $[A^{(\Lambda,b)}]_{(\Lambda,b)} = [[A_{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)} = A$. Therefore, A is $r(\Lambda, b)$-open.

Corollary 1. *For a subset A of a topological space (X,τ), the following properties are equivalent:*

(1) A is $r(\Lambda,b)$-closed.

(2) A is (Λ,b)-closed and $s(\Lambda,b)$-open.

(3) A is $\alpha(\Lambda,b)$-closed and $s(\Lambda,b)$-open.

(4) A is $p(\Lambda,b)$-closed and $s(\Lambda,b)$-open.

(5) A is (Λ,b)-closed and $\beta(\Lambda,b)$-open.

(6) A is $\alpha(\Lambda,b)$-closed and $\beta(\Lambda,b)$-open.

Definition 2. *A subset A of a topological space (X,τ) is said to be $b(\Lambda,b)$-open if $A\subseteq [A_{(\Lambda,b)}]^{(\Lambda,b)}\cup[A^{(\Lambda,b)}]_{(\Lambda,b)}$. The complement of a $b(\Lambda,b)$-open set is said to be $b(\Lambda,b)$-closed.*

Example 2. *Let $X=\{-2,-1,0,1,2\}$ with the topology $\tau=\{\emptyset,\{-2,-1\},\{0,1,2\},X\}$. Let $C=\{-2,-1\}$. Then, C is $b(\Lambda,b)$-open.*

The family of all $b(\Lambda,b)$-open (resp. $b(\Lambda,b)$-closed) sets in a topological space (X,τ) is denoted by $b\Lambda_bO(X,\tau)$ (resp. $b\Lambda_bC(X,\tau)$).

Proposition 3. *Let A be a subset of a topological space (X,τ). If A is $b(\Lambda,b)$-open, then $A^{(\Lambda,b)}$ is $r(\Lambda,b)$-closed.*

Proof. Suppose that A is $b(\Lambda,b)$-open. Then, $A\subseteq [A_{(\Lambda,b)}]^{(\Lambda,b)}\cup[A^{(\Lambda,b)}]_{(\Lambda,b)}$. Thus,

$$\begin{aligned}A^{(\Lambda,b)}&\subseteq [[A_{(\Lambda,b)}]^{(\Lambda,b)}\cup[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}\\&\subseteq [[A_{(\Lambda,b)}]^{(\Lambda,b)}]^{(\Lambda,b)}\cup[[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}=[[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}\subseteq A^{(\Lambda,b)}\end{aligned}$$

and hence $A^{(\Lambda,b)}=[[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$. Therefore, $A^{(\Lambda,b)}$ is $r(\Lambda,b)$-closed.

Corollary 2. *For a subset A of a topological space (X,τ), the following properties hold:*

(1) If A is $s(\Lambda,b)$-open, then $A^{(\Lambda,b)}$ is $r(\Lambda,b)$-closed.

(2) If A is $p(\Lambda,b)$-open, then $A^{(\Lambda,b)}$ is $r(\Lambda,b)$-closed.

(3) If A is $\alpha(\Lambda,b)$-open, then $A^{(\Lambda,b)}$ is $r(\Lambda,b)$-closed.

Proposition 4. *For a subset A of a topological space (X,τ), the following properties are equivalent:*

(1) $A\in\beta\Lambda_bO(X,\tau)$.

(2) $A^{(\Lambda,b)}\in r\Lambda_bC(X,\tau)$.

(3) $A^{(\Lambda,b)}\in\beta\Lambda_bO(X,\tau)$.

(4) $A^{(\Lambda,b)} \in s\Lambda_b O(X,\tau)$.

(5) $A^{(\Lambda,b)} \in b\Lambda_b O(X,\tau)$.

Proof. (1) $\Rightarrow$ (2): Let $A \in \beta\Lambda_b O(X,\tau)$. Then, $A \subseteq [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$. Thus, $A^{(\Lambda,b)} \subseteq [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)} \subseteq A^{(\Lambda,b)}$ and hence $A^{(\Lambda,b)} = [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$. Therefore, $A^{(\Lambda,b)} \in r\Lambda_b C(X,\tau)$.

(2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5): Obvious.

(5) $\Rightarrow$ (1): Let $A^{(\Lambda,b)} \in b\Lambda_b O(X,\tau)$. Then, $A^{(\Lambda,b)} \subseteq [[A^{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)} \cup [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)} = [A^{(\Lambda,b)}]_{(\Lambda,b)} \cup [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)} = [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$. Thus, $A \subseteq [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$ and hence $A \in \beta\Lambda_b O(X,\tau)$.

Definition 3. *Let (X,τ) be a topological space and $x \in X$. A subset $\langle x\rangle_b$ is defined as follows: $\langle x\rangle_b = \Lambda_{(\Lambda,b)}(\{x\}) \cap \{x\}^{(\Lambda,b)}$.*

Example 3. *Let $X = \{-1, 1\}$ with the topology $\tau = \{\emptyset, \{-1\}, X\}$. Then, $\langle 1\rangle_b = \{1\}$.*

Recall that a subset N of a topological space (X,τ) is said to be a (Λ,b)*-neighbourhood* [5] of a point $x \in X$ if there exists a (Λ,b)-open set U of X such that $x \in U \subseteq N$.

Theorem 1. *Let (X,τ) be a topological space. Then, the following properties hold:*

(1) $\Lambda_{(\Lambda,b)}(A) = \{x \in X \mid A \cap \{x\}^{(\Lambda,b)} \neq \emptyset\}$ *for each subset A of X.*

(2) For each $x \in X$, $\Lambda_{(\Lambda,b)}(\langle x\rangle_b) = \Lambda_{(\Lambda,b)}(\{x\})$.

(3) For each $x \in X$, $(\langle x\rangle_b)^{(\Lambda,b)} = \{x\}^{(\Lambda,b)}$.

(4) If U is (Λ,b)-open in (X,τ) and $x \in U$, then $\langle x\rangle_b \subseteq U$.

(5) If F is (Λ,b)-closed in (X,τ) and $x \in F$, then $\langle x\rangle_b \subseteq F$.

Proof. (1) Suppose that $A \cap \{x\}^{(\Lambda,b)} = \emptyset$. Then, we have $x \notin X - \{x\}^{(\Lambda,b)}$ which is a (Λ,b)-open set containing A. Therefore, $x \notin \Lambda_{(\Lambda,b)}(A)$. Consequently, we obtain $\Lambda_{(\Lambda,b)}(A) \subseteq \{x \in X \mid A \cap \{x\}^{(\Lambda,b)} \neq \emptyset\}$. Next, let $x \in X$ such that $A \cap \{x\}^{(\Lambda,b)} \neq \emptyset$ and suppose that $x \notin \Lambda_{(\Lambda,b)}(A)$. Then, there exists a (Λ,b)-open set U containing A and $x \notin U$. Let $y \in A \cap \{x\}^{(\Lambda,b)}$. Thus, U is a (Λ,b)-neighbourhood of y which does not contain x. By this contradiction $x \in \Lambda_{(\Lambda,b)}(A)$.

(2) Let $x \in X$, Then, we have $\{x\} \subseteq \{x\}^{(\Lambda,b)} \cap \Lambda_{(\Lambda,b)}(\{x\}) = \langle x\rangle_b$. By Lemma 3, $\Lambda_{(\Lambda,b)}(\{x\}) \subseteq \Lambda_{(\Lambda,b)}(\langle x\rangle_b)$. Next, we show the opposite implication. Suppose that $y \notin \Lambda_{(\Lambda,b)}(\{x\})$. There exists a (Λ,b)-open set V such that $x \in V$ and $y \notin V$. Since $\langle x\rangle_b \subseteq \Lambda_{(\Lambda,b)}(\{x\}) \subseteq \Lambda_{(\Lambda,b)}(V) = V$, we have $\Lambda_{(\Lambda,b)}(\langle x\rangle_b) \subseteq V$. Since $y \notin V, y \notin \Lambda_{(\Lambda,b)}(\langle x\rangle_b)$. Thus, $\Lambda_{(\Lambda,b)}(\langle x\rangle_b) \subseteq \Lambda_{(\Lambda,b)}(\{x\})$ and hence $\Lambda_{(\Lambda,b)}(\{x\}) = \Lambda_{(\Lambda,b)}(\langle x\rangle_b)$.

(3) By the definition of $\langle x\rangle_b$, we have $\{x\} \subseteq \langle x\rangle_b$ and $\{x\}^{(\Lambda,b)} \subseteq (\langle x\rangle_b)^{(\Lambda,b)}$ by Lemma 1. On the other hand, we have $\langle x\rangle_b \subseteq \{x\}^{(\Lambda,b)}$ and $(\langle x\rangle_b)^{(\Lambda,b)} \subseteq (\{x\}^{(\Lambda,b)})^{(\Lambda,b)} = \{x\}^{(\Lambda,b)}$. Thus, $(\langle x\rangle_b)^{(\Lambda,b)} \subseteq \{x\}^{(\Lambda,b)}$.

(4) Since $x \in U$ and U is (Λ, b)-open, we have $\Lambda_{(\Lambda,b)}(\{x\}) \subseteq U$. Thus, $\langle x \rangle_b \subseteq U$.

(5) Since $x \in F$ and F is (Λ, b)-closed, $\langle x \rangle_b = \{x\}^{(\Lambda,b)} \cap \Lambda_{(\Lambda,b)}(\{x\}) \subseteq \{x\}^{(\Lambda,b)} \subseteq F^{(\Lambda,b)} = F$.

Definition 4. [15] *A subset A of a topological space (X, τ) is said to be:*

(i) (Λ, b)-dense if $A^{(\Lambda,b)} = X$;

(ii) (Λ, b)-codense if its complement is (Λ, b)-dense.

Recall that a topological space (X, τ) is called (Λ, b)-*submaximal* [15] if each (Λ, b)-dense subset of X is (Λ, b)-open.

Proposition 5. *Let (X, τ) be a topological space. If each $p(\Lambda, b)$-open set is $s(\Lambda, b)$-open and each $\alpha(\Lambda, b)$-open set is (Λ, b)-open, then (X, τ) is (Λ, b)-submaximal.*

Proof. Let D be (Λ, b)-dense subset of X. Since $D^{(\Lambda,b)} = X$, we have D is $p(\Lambda, b)$-open. Thus, D is $s(\Lambda, b)$-open and hence D is $\alpha(\Lambda, b)$-open. Since each $\alpha(\Lambda, b)$-open set is (Λ, b)-open, D is (Λ, b)-open. This shows that (X, τ) is (Λ, b)-submaximal.

Proposition 6. *Let (X, τ) be a topological space. If each $p(\Lambda, b)$-open set is (Λ, b)-open, then (X, τ) is (Λ, b)-submaximal.*

Proof. Suppose that each $p(\Lambda, b)$-open set is (Λ, b)-open. It follows that every $p(\Lambda, b)$-open set is $s(\Lambda, b)$-open. Since each $\alpha(\Lambda, b)$-open set is $p(\Lambda, b)$-open, then each $\alpha(\Lambda, b)$-open set is (Λ, b)-open. Thus, by Proposition 5, (X, τ) is (Λ, b)-submaximal.

Proposition 7. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, b)-submaximal.

(2) Each (Λ, b)-codense subset of X is (Λ, b)-closed.

Recall that a topological space (X, τ) is called (Λ, b)-*extremally disconnected* [15] if $U^{(\Lambda,b)}$ is (Λ, b)-open in X for every (Λ, b)-open set U of X.

Lemma 4. *Let A be a subset of a topological space (X, τ). If $U \in \Lambda_b O(X, \tau)$ and $U \cap A = \emptyset$, then $U \cap A^{(\Lambda,b)} = \emptyset$.*

Theorem 2. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, b)-extremally disconnected.

(2) $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$ for all (Λ, b)-open subsets U and V of X with $U \cap V = \emptyset$.

Proof. (1) ⇒ (2): Suppose that U and V are (Λ, b)-open sets of X such that $U \cap V = \emptyset$. Thus, by Lemma 4, $U^{(\Lambda,b)} \cap V = \emptyset$ which implies that $[U^{(\Lambda,b)}]_{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$ and hence $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$.

(2) ⇒ (1): Let U be any (Λ, b)-open set of X. Then, $X - U$ is (Λ, b)-closed and $[X-U]_{(\Lambda,b)}$ is (Λ, b)-open such that $U \cap [X-U]_{(\Lambda,b)} = \emptyset$. Thus, $U^{(\Lambda,b)} \cap [[X-U]_{(\Lambda,b)}]^{(\Lambda,b)} = \emptyset$ which implies that $U^{(\Lambda,b)} \cap [X - [U^{(\Lambda,b)}]_{(\Lambda,b)} = \emptyset$. Therefore, $U^{(\Lambda,b)} \subseteq [U^{(\Lambda,b)}]_{(\Lambda,b)}$ and hence $U^{(\Lambda,b)} = [U^{(\Lambda,b)}]_{(\Lambda,b)}$. This shows that $U^{(\Lambda,b)}$ is (Λ, b)-open. Thus, (X, τ) is (Λ, b)-extremally disconnected.

Lemma 5. *Let A be a subset of a topological space (X, τ). If $U \in \Lambda_b O(X, \tau)$, then $U^{(\Lambda,b)} \cap A \subseteq [U \cap A]^{(\Lambda,b)}$.*

Lemma 6. *For a subset A of a topological space (X, τ), the following properties hold:*

(1) $[X - A]^{(\Lambda,b)} = X - A_{(\Lambda,b)}$.

(2) $[X - A]_{(\Lambda,b)} = X - A^{(\Lambda,b)}$.

Theorem 3. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, b)-extremally disconnected.

(2) $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = [U \cap V]^{(\Lambda,b)}$ for all (Λ, b)-open sets U and V of X.

(3) $E_{(\Lambda,b)} \cup F_{(\Lambda,b)} = [E \cup F]_{(\Lambda,b)}$ for all (Λ, b)-closed sets E and F of X.

Proof. (1) ⇒ (2): Let U and V be (Λ, b)-open sets of X. Thus, by Lemma 5,

$$U^{(\Lambda,b)} \cap V^{(\Lambda,b)} \subseteq [U \cap V^{(\Lambda,b)}]^{(\Lambda,b)} \subseteq [[U \cap V]^{(\Lambda,b)}]^{(\Lambda,b)} = [U \cap V]^{(\Lambda,b)}$$

and hence $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = [U \cap V]^{(\Lambda,b)}$.

(2) ⇒ (3): Let E and F be (Λ, b)-closed sets of X. Then, $X - E$ and $X - F$ are (Λ, b)-open. By (2) and Lemma 6, we have $E_{(\Lambda,b)} \cup F_{(\Lambda,b)} = X - [X - [E_{(\Lambda,b)} \cup F_{(\Lambda,b)}]] = X - [[X - E_{(\Lambda,b)}] \cap [X - F_{(\Lambda,b)}]] = X - [X - [E \cup F]_{(\Lambda,b)}] = [E \cup F]_{(\Lambda,b)}$.

(3) ⇒ (2): The proof is obvious.

(2) ⇒ (1): Let U be any (Λ, b)-open set of X. Then, $X - U$ is (Λ, b)-closed and $[X - U]_{(\Lambda,b)}$ is (Λ, b)-open. By (2), $U^{(\Lambda,b)} \cap [[X - U]_{(\Lambda,b)}]^{(\Lambda,b)} = [U \cap [X - U]_{(\Lambda,b)}]^{(\Lambda,b)}$ which implies that $[X - [U^{(\Lambda,b)}]_{(\Lambda,b)}] \cap U^{(\Lambda,b)} = \emptyset^{(\Lambda,b)} = \emptyset$. Thus, $U^{(\Lambda,b)} \subseteq [U^{(\Lambda,b)}]_{(\Lambda,b)}$ and hence $U^{(\Lambda,b)} = [U^{(\Lambda,b)}]_{(\Lambda,b)}$. Therefore, $U^{(\Lambda,b)}$ is (Λ, b)-open. This shows that (X, τ) is (Λ, b)-extremally disconnected.

Theorem 4. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is (Λ, b)-extremally disconnected.

(2) $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = [U \cap V]^{(\Lambda,b)}$ for all (Λ, b)-open sets U and V of X.

(3) $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$ *for all* (Λ, b)*-open sets* U *and* V *of* X *with* $U \cap V = \emptyset$.

Proof. This follows from Theorem 2 and Theorem 3.

Theorem 5. *For a topological space* (X, τ)*, the following properties are equivalent:*

(1) (X, τ) *is* (Λ, b)*-extremally disconnected.*

(2) The (Λ, b)*-closure of every* $\beta(\Lambda, b)$*-open set of* X *is* (Λ, b)*-open.*

(3) The (Λ, b)*-closure of every* $p(\Lambda, b)$*-open set of* X *is* (Λ, b)*-open.*

Proof. This follows immediately since $A^{(\Lambda,b)} = [[A^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$.

Theorem 6. *For a topological space* (X, τ)*, the following properties are equivalent:*

(1) (X, τ) *is* (Λ, b)*-extremally disconnected.*

(2) If $U \in \beta\Lambda_b O(X, \tau)$, $V \in s\Lambda_b O(X, \tau)$ *and* $U \cap V = \emptyset$*, then* $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$.

(3) If $U \in b\Lambda_b O(X, \tau)$, $V \in s\Lambda_b O(X, \tau)$ *and* $U \cap V = \emptyset$*, then* $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$.

(4) If $U \in p\Lambda_b O(X, \tau)$, $V \in s\Lambda_b O(X, \tau)$ *and* $U \cap V = \emptyset$*, then* $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$.

(5) If $U \in r\Lambda_b O(X, \tau)$, $V \in s\Lambda_b O(X, \tau)$ *and* $U \cap V = \emptyset$*, then* $U^{(\Lambda,b)} \cap V^{(\Lambda,\alpha)} = \emptyset$.

(6) If $U, V \in r\Lambda_b O(X, \tau)$ *and* $U \cap V = \emptyset$*, then* $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$.

Proof. (1) $\Rightarrow$ (2): Suppose that $U \in \beta\Lambda_b O(X, \tau)$, $V \in s\Lambda_b O(X, \tau)$ and $U \cap V = \emptyset$. Thus, $U^{(\Lambda,b)} \cap V_{(\Lambda,b)} = \emptyset$, by Theorem 5, $U^{(\Lambda,b)}$ is (Λ, b)-open and hence $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = U^{(\Lambda,p)} \cap [V_{(\Lambda,b)}]^{(\Lambda,b)} = \emptyset$.

(2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (6): Obvious.

(6) $\Rightarrow$ (1): Let U and V be (Λ, b)-open sets of X. Then, $U^{(\Lambda,b)}, V^{(\Lambda,b)} \in r\Lambda_b O(X, \tau)$ and $U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$. By (6), $[U^{(\Lambda,b)}]^{(\Lambda,b)} \cap [V^{(\Lambda,b)}]^{(\Lambda,b)} = U^{(\Lambda,b)} \cap V^{(\Lambda,b)} = \emptyset$. Thus, (X, τ) is (Λ, b)-extremally disconnected.

4. Characterizations of weakly (Λ, b)-continuous functions

In this section, we introduce the notion of weakly (Λ, b)-continuous functions. Moreover, several characterizations of weakly (Λ, b)-continuous functions are discussed.

Definition 5. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called weakly* (Λ, b)*-continuous at a point* $x \in X$ *if, for each* (Λ, b)*-open set* V *of* Y *containing* $f(x)$*, there exists a* (Λ, b)*-open set* U *of* X *containing* x *such that* $f(U) \subseteq V^{(\Lambda,b)}$*. A function* $f : (X, \tau) \to (Y, \sigma)$ *is called* (Λ, b)*-continuous if* f *has this property at each point* $x \in X$.

Example 4. *Let* $X = \{a, b, c\}$ *with the topology* $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$*. Let* $Y = \{-1, 0, 1\}$ *with the topology* $\sigma = \{\emptyset, \{-1\}, \{0, 1\}, Y\}$*. Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function defined as follows:* $f(a) = -1$*,* $f(b) = 0$ *and* $f(c) = 1$*. Then,* f *is weakly* (Λ, b)*-continuous.*

Theorem 7. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is weakly* (Λ, b)*-continuous at* $x \in X$ *if and only if for each* (Λ, b)*-open set* V *of* Y *containing* $f(x)$*,* $x \in [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$*.*

Proof. Let V be any (Λ, b)-open set of Y containing $f(x)$. Then, there exists a (Λ, b)-open set U of X containing x such that $f(U) \subseteq V^{(\Lambda,b)}$. Thus, $x \in U \subseteq f^{-1}(V^{(\Lambda,b)})$ and hence $x \in [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$.

Conversely, let V be any (Λ, b)-open set of Y containing $f(x)$. Then, $x \in [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$ and there exists a (Λ, b)-open set U of X such that $x \in U \subseteq f^{-1}(V^{(\Lambda,b)})$; hence $f(U) \subseteq V^{(\Lambda,b)}$. Thus, f is weakly (Λ, b)-continuous at $x \in X$.

Theorem 8. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is weakly* (Λ, b)*-continuous if and only if* $f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$ *for every* (Λ, b)*-open set* V *of* Y*.*

Proof. Let V be any (Λ, b)-open set of Y and $x \in f^{-1}(V)$. Then, $f(x) \in V$. Since f is weakly (Λ, b)-continuous at x and by Theorem 7, $x \in [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$. Therefore, $f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$.

Conversely, let $x \in X$ and V be any (Λ, b)-open set of Y containing $f(x)$. Then, $x \in f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$ and hence $x \in [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$. By Theorem 7, f is weakly (Λ, b)-continuous.

Theorem 9. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is weakly* (Λ, b)*-continuous if and only if* $[f^{-1}(V)]^{(\Lambda,b)} \subseteq f^{-1}(V^{(\Lambda,b)})$ *for every* (Λ, b)*-open set* V *of* Y*.*

Proof. This is obvious from Theorem 8.

Conversely, let V be any (Λ, b)-open set of Y containing $f(x)$. Since $V \cap (Y - V^{(\Lambda,b)}) = \emptyset$, $f(x) \notin [Y - V^{(\Lambda,b)}]^{(\Lambda,b)}$. Since $Y - V^{(\Lambda,b)}$ is (Λ, b)-open and $x \in f^{-1}([Y - V^{(\Lambda,b)}]^{(\Lambda,b)})$. By the hypothesis, $x \notin [f^{-1}([Y - V^{(\Lambda,b)}]^{(\Lambda,b)})]^{(\Lambda,b)} = [X - f^{-1}(V^{(\Lambda,b)})]^{(\Lambda,b)}$ and there exists a (Λ, b)-open set U of X containing x such that $U \cap (X - f^{-1}(V^{(\Lambda,b)})) = \emptyset$. This shows that $f(U) \subseteq V^{(\Lambda,b)}$. Thus, f is is weakly (Λ, b)-continuous.

Theorem 10. *For a function* $f : (X, \tau) \to (Y, \sigma)$*, the following properties are equivalent:*

(1) f *is weakly* (Λ, b)*-continuous;*

(2) $f^{-1}(U) \subseteq [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$ *for every* (Λ, b)*-open set* U *of* Y*;*

(3) $[f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(F)$ *for every* (Λ, b)*-closed set* F *of* Y*;*

(4) $[f^{-1}([A^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(A^{(\Lambda,b)})$ *for every subset* A *of* Y*;*

(5) $f^{-1}(A_{(\Lambda,b)}) \subseteq [f^{-1}([A_{(\Lambda,b)}]^{(\Lambda,b)})]_{(\Lambda,b)}$ *for every subset* A *of* Y*;*

(6) $[f^{-1}(U)]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$ *for every* (Λ, b)*-open set* U *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let U be any (Λ, b)-open set of Y and $x \in f^{-1}(U)$. Then, there exists a (Λ, b)-open set V of X containing x such that $f(V) \subseteq U^{(\Lambda,b)}$. Since $x \in V \subseteq f^{-1}(U^{(\Lambda,b)})$, $x \in [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$. Thus, $f^{-1}(U) \subseteq [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$.

(2) $\Rightarrow$ (3): Let F be any (Λ, b)-closed set of Y. Then, $Y - F$ is (Λ, b)-open and by (2), $f^{-1}(X - F) \subseteq [f^{-1}([X - F]^{(\Lambda,b)})]_{(\Lambda,b)} = [f^{-1}(Y - F_{(\Lambda,b)})]_{(\Lambda,b)} = X - [f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)}$. Therefore, $[f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(F)$.

(3) $\Rightarrow$ (4): Let A be any subset of Y. Since $A^{(\Lambda,b)}$ is (Λ, b)-closed in Y and by (3), $[f^{-1}([A^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(A^{(\Lambda,b)})$.

(4) $\Rightarrow$ (5): Let A be any subset of Y. By (4), $f^{-1}(A_{(\Lambda,b)}) = X - f^{-1}([Y - A]^{(\Lambda,b)}) \subseteq X - [f^{-1}([[Y - A]^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} = [f^{-1}([A_{(\Lambda,b)}]^{(\Lambda,b)})]_{(\Lambda,b)}$.

(5) $\Rightarrow$ (6): Let U be any (Λ, b)-open set of Y. Suppose that $x \notin f^{-1}(U^{(\Lambda,b)})$. Then, $f(x) \notin U^{(\Lambda,b)}$ and so there exists a (Λ, b)-open set V of Y containing $f(x)$ such that $U \cap V = \emptyset$ and hence $U \cap V^{(\Lambda,b)} = \emptyset$. By (5), we have $x \in f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$. There exists a (Λ, b)-open set W containing x such that $x \in W \subseteq f^{-1}(V^{(\Lambda,b)})$. Since $U \cap V^{(\Lambda,b)} = \emptyset$ and $f(W) \subseteq V^{(\Lambda,b)}$, we have $W \cap f^{-1}(U) = \emptyset$. Thus, $x \notin [f^{-1}(U)]^{(\Lambda,b)}$ and hence $[f^{-1}(U)]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$.

(6) $\Rightarrow$ (1): Let $x \in X$ and U be any (Λ, b)-open set of Y containing $f(x)$. Then, we have $U = U_{(\Lambda,b)} \subseteq [U^{(\Lambda,b)}]_{(\Lambda,b)}$. Thus, by (6), $x \in f^{-1}(U) \subseteq f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)}) = X - f^{-1}([Y - U^{(\Lambda,b)}]^{(\Lambda,b)}) \subseteq X - [f^{-1}(Y - U^{(\Lambda,b)})]^{(\Lambda,b)} = [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$. Then, there exists a (Λ, b)-open set V of X containing x such that $V \subseteq f^{-1}(U^{(\Lambda,b)})$. This shows that f is weakly (Λ, b)-continuous.

Theorem 11. *For a function* $f : (X, \tau) \to (Y, \sigma)$*, the following properties are equivalent:*

(1) f *is weakly* (Λ, b)*-continuous;*

(2) $[f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(F)$ *for every* $r(\Lambda, b)$*-closed set* F *of* Y*;*

(3) $[f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$ *for every* $\beta(\Lambda, b)$*-open set* U *of* Y*;*

(4) $[f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$ *for every* $s(\Lambda, b)$*-open set* U *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let F be any $r(\Lambda, b)$-closed set of Y. Then, $F_{(\Lambda,b)}$ is (Λ, b)-open and by Theorem 10, $[f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}([F_{(\Lambda,b)}]^{(\Lambda,b)})$. Since F is $r(\Lambda, b)$-closed, we have $[f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}([F_{(\Lambda,b)}]^{(\Lambda,b)}) \subseteq f^{-1}(F)$.

(2) $\Rightarrow$ (3): Let U be any $\beta(\Lambda, b)$-open set of Y. Then, $U^{(\Lambda,b)} \subseteq [[U^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)} \subseteq U^{(\Lambda,b)}$ and hence $U^{(\Lambda,b)}$ is $r(\Lambda, b)$-closed. By (2), $[f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$.

(3) $\Rightarrow$ (4): The proof is obvious.

(4) $\Rightarrow$ (1): Let U be any (Λ, b)-open set of Y. Thus, by (4),

$$[f^{-1}(U)]^{(\Lambda,b)} \subseteq [f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$$

and by Theorem 10, f is weakly (Λ, b)-continuous.

Theorem 12. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is weakly (Λ, b)-continuous;

(2) $[f^{-1}([U_{(\Lambda,b)}]^{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$ for every $p(\Lambda, b)$-open set U of Y;

(3) $[f^{-1}(U)]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$ for every $p(\Lambda, b)$-open set U of Y;

(4) $f^{-1}(U) \subseteq [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$ for every $p(\Lambda, b)$-open set U of Y.

Proof. (1) ⇒ (2): Let U be any $p(\Lambda, b)$-open set of Y. Then, $U^{(\Lambda,b)} = [[U^{(\Lambda,b)}]_{(\Lambda,b)}]^{(\Lambda,b)}$ and hence $U^{(\Lambda,b)}$ is $r(\Lambda, b)$-closed. By Theorem 11, $[f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$.

(2) ⇒ (3): Let U be any $p(\Lambda, b)$-open set of Y. Then, $U \subseteq [U^{(\Lambda,b)}]_{(\Lambda,b)}$ and by (2), we have $[f^{-1}(U)]^{(\Lambda,b)} \subseteq [f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$.

(3) ⇒ (4): Let U be any $p(\Lambda, b)$-open set of Y. By (3), $f^{-1}(U) \subseteq f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)}) = X - f^{-1}([Y - U^{(\Lambda,b)}]^{(\Lambda,b)}) = X - [f^{-1}(Y - U^{(\Lambda,b)})]^{(\Lambda,b)} = [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$.

(4) ⇒ (1): Since every (Λ, b)-open set is $p(\Lambda, b)$-open, by (4) and Theorem 10, f is weakly (Λ, b)-continuous.

Theorem 13. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) f is weakly (Λ, b)-continuous;

(2) $[f^{-1}([A^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(A^{(\Lambda,b)})$ for every subset A of Y;

(3) $[f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(F)$ for every $r(\Lambda, b)$-closed set F of Y;

(4) $[f^{-1}(U)]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$ for every (Λ, b)-open set U of Y;

(5) $f^{-1}(U) \subseteq [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$ for every (Λ, b)-open set U of Y;

(6) $[f^{-1}(U)]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$ for every $p(\Lambda, b)$-open set U of Y;

(7) $f^{-1}(U) \subseteq [f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$ for every $p(\Lambda, b)$-open set U of Y.

Proof. (1) ⇒ (2): Let A be any subset of Y and $x \in X - f^{-1}(A^{(\Lambda,b)})$. Then, $f(x) \in Y - A^{(\Lambda,b)}$ and there exists a (Λ, b)-open set U of Y containing $f(x)$ such that $U \cap A = \emptyset$. Thus, $U^{(\Lambda,b)} \cap [A^{(\Lambda,b)}]_{(\Lambda,b)} = \emptyset$. Since f is weakly (Λ, b)-continuous, there exists a (Λ, b)-open set W containing x such that $f(W) \subseteq U^{(\Lambda,b)}$. Then, $W \cap f^{-1}([A^{(\Lambda,b)}]_{(\Lambda,b)}) = \emptyset$ and hence $x \in X - [f^{-1}([A^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)}$. Therefore, $[f^{-1}([A^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(A^{(\Lambda,b)})$.

(2) ⇒ (3): Let F be any $r(\Lambda, b)$-closed set of Y. By (2), we have

$$[f^{-1}(F_{(\Lambda,b)})]^{(\Lambda,b)} = [f^{-1}([[F_{(\Lambda,b)}]^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}([F_{(\Lambda,b)}]^{(\Lambda,b)}) = f^{-1}(F).$$

(3) ⇒ (4): Let U be any (Λ, b)-open set of Y. Since $U^{(\Lambda,b)}$ is $r(\Lambda, b)$-closed, we have $[f^{-1}(U)]^{(\Lambda,b)} \subseteq [f^{-1}([U^{(\Lambda,b)}]_{(\Lambda,b)})]^{(\Lambda,b)} \subseteq f^{-1}(U^{(\Lambda,b)})$.

(4) ⇒ (5): Let U be any (Λ,b)-open set of Y. Since $Y-U^{(\Lambda,b)}$ is (Λ,b)-open and by (4), $X-[f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}=[f^{-1}(Y-U^{(\Lambda,b)})]^{(\Lambda,b)}\subseteq f^{-1}([Y-U^{(\Lambda,b)}]^{(\Lambda,b)})\subseteq X-f^{-1}(U)$. Thus, $f^{-1}(U)\subseteq[f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$.

(5) ⇒ (1): Let $x\in X$ and U be any (Λ,b)-open set of Y containing $f(x)$. Then, $x\in f^{-1}(U)\subseteq[f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$. Put $W=[f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$. Then, W is a (Λ,b)-open set of X containing x such that $f(W)\subseteq U^{(\Lambda,b)}$. Thus, f is weakly (Λ,b)-continuous at x. This shows that f is weakly (Λ,b)-continuous.

(1) ⇒ (6): Let U be any $p(\Lambda,b)$-open set of Y and $x\in X-f^{-1}(U^{(\Lambda,b)})$. There exists a (Λ,b)-open set V of Y containing $f(x)$ such that $V\cap U=\emptyset$. Thus, $[V\cap U]^{(\Lambda,b)}=\emptyset$. Since U is (Λ,b)-open, $U\cap V^{(\Lambda,b)}\subseteq[U\cap V]^{(\Lambda,b)}=\emptyset$. Since f is weakly (Λ,b)-continuous and V is a (Λ,b)-open set of Y containing $f(x)$, there exists a (Λ,b)-open set W of X containing x such that $f(W)\subseteq V^{(\Lambda,b)}$. Therefore, $f(W)\cap U=\emptyset$. Thus, $W\cap f^{-1}(U)=\emptyset$ and hence $x\in X-[f^{-1}(U)]^{(\Lambda,b)}$. This shows that $[f^{-1}(U)]^{(\Lambda,b)}\subseteq f^{-1}(U^{(\Lambda,b)})$.

(6) ⇒ (7): Let U be any $p(\Lambda,b)$-open set of Y. Since $Y-U^{(\Lambda,b)}$ is (Λ,b)-open, we have $X-[f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}=[f^{-1}(Y-U^{(\Lambda,b)})]^{(\Lambda,b)}\subseteq f^{-1}([Y-U^{(\Lambda,b)}]^{(\Lambda,b)})\subseteq X-f^{-1}(U)$ and hence $f^{-1}(U)\subseteq[f^{-1}(U^{(\Lambda,b)})]_{(\Lambda,b)}$.

(7) ⇒ (1): Let $x\in X$ and V be any (Λ,b)-open set of Y containing $f(x)$. Then, we have $x\in f^{-1}(V)\subseteq[f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$. Put $U=[f^{-1}(V^{(\Lambda,b)})]^{(\Lambda,b)}$. Then, U is a (Λ,b)-open set of X containing x such that $f(U)\subseteq V^{(\Lambda,b)}$. Thus, f is weakly (Λ,b)-continuous at x and hence f is weakly (Λ,b)-continuous.

Definition 6. *A topological space (X,τ) is called (Λ,b)-connected if X cannot be written as a disjoint union of two nonempty (Λ,b)-open sets.*

Example 5. *Let $X=\{a,b\}$ with the topology $\tau=\{\emptyset,\{a\},X\}$. Then, (X,τ) is (Λ,b)-connected.*

Lemma 7. *For a topological space (X,τ), the following properties are equivalent:*

(1) (X,τ) is (Λ,b)-connected.

(2) The only subsets of X, which are both (Λ,b)-open and (Λ,b)-closed are $\emptyset$ and X.

Theorem 14. *Let $f:(X,\tau)\to(Y,\sigma)$ is a weakly (Λ,b)-continuous surjection. If (X,τ) is (Λ,b)-connected, then (Y,σ) is (Λ,b)-connected.*

Proof. Assume that (Y,σ) is not (Λ,b)-connected. Then, there exist nonempty (Λ,b)-open sets V_1,V_2 such that $V_1\cap V_2=\emptyset$ and $V_1\cup V_2=Y$. By Theorem 10, $f^{-1}(V_i)\subseteq[f^{-1}[V^{(\Lambda,b)}]]_{(\Lambda,b)}$ for $i=1,2$. Since V_i is (Λ,b)-closed in Y for each $i=1,2$. Therefore, $f^{-1}(V_i)\subseteq[f^{-1}(V_i)]_{(\Lambda,b)}$ and by Lemma 2, $f^{-1}(V_i)$ is (Λ,b)-open for each $i=1,2$. Moreover, X is union of nonempty disjoint sets $f^{-1}(V_1)$ and $f^{-1}(V_2)$. This shows that (X,τ) is not (Λ,b)-connected. This is contrary to the hypothesis that (X,τ) is (Λ,b)-connected. Thus, (Y,σ) is (Λ,b)-connected.

Definition 7. *A subset K of a topological spaces (X,τ) is said to be Λ_b-closed (resp. Λ_b-compact) relative to (X,τ) if for any cover $\{V_i : i \in I\}$ of K by (Λ, b)-open sets of X, there exist a finite subset I_0 of I such that $K \subseteq \cup\{V_i^{(\Lambda,b)} \mid i \in I_0\}$ (resp. $K \subseteq \cup\{V_i \mid i \in I_0\}$).*

If X is Λ_b-closed (resp. Λ_b-compact) relative to (X,τ), then (X,τ) is said to be Λ_b-closed (resp. Λ_b-compact).

Theorem 15. *If $f : (X,\tau) \to (Y,\sigma)$ is weakly (Λ, b)-continuous and K is Λ_b-compact, then $f(K)$ is Λ_b-closed relative to (Y,σ).*

Proof. Let $\{V_\alpha : \alpha \in \nabla\}$ be any cover of $f(K)$ by (Λ, b)-open sets of Y. For each $x \in K$, there exists $i(x) \in I$ such that $f(x) \in V_i(x)$. Since f is weakly (Λ, b)-continuous, there exists a (Λ, b)-open set $U(x)$ containing x such that $f(U(x)) \subseteq [V_{i(x)}]^{(\Lambda,b)}$. The family $\{U(x) : x \in K\}$ is a cover of K by (Λ, b)-open sets of X. Since K is Λ_b-compact, there exist a finite number of points, say, $x_1, x_2, ..., x_n$ in K such that

$$K \subseteq \cup\{U(x_k) : x_k \in K, 1 \leq k \leq n\}.$$

Thus, $f(K) \subseteq \cup\{f(U(x_k)) : x_k \in K, 1 \leq k \leq n\} \subseteq \cup\{[V_{i(x_k)}]^{(\Lambda,b)} : x_k \in K, 1 \leq k \leq n\}$. This shows that $f(K)$ is Λ_b-closed relative to (Y,σ).

Corollary 3. *If $f : (X,\tau) \to (Y,\sigma)$ is a weakly (Λ, b)-continuous surjection and (X,τ) is Λ_b-compact, then (Y,σ) is Λ_b-closed.*

Definition 8. [5] *Let A be a subset of a topological space (X,τ). The (Λ, b)-frontier of A, $\Lambda_bFr(A)$, is defined as follows: $\Lambda_bFr(A) = A^{(\Lambda,b)} \cap [X - A]^{(\Lambda,b)}$.*

Theorem 16. *The set of all points $x \in X$ at which a function $f : (X,\tau) \to (Y,\sigma)$ is not weakly (Λ, b)-continuous is identical with the union of the (Λ, b)-frontiers of the inverse images of the (Λ, b)-closure of the (Λ, b)-open sets of Y containing $f(x)$.*

Proof. Suppose that f is not weakly (Λ, b)-continuous at $x \in X$. There exists a (Λ, b)-open set V of Y containing $f(x)$ such that $f(U)$ is not contained in $V^{(\Lambda,b)}$ for every (Λ, b)-open set U containing x. Then, $U \cap (X - f^{-1}(V^{(\Lambda,b)})) \neq \emptyset$ for every (Λ, b)-open set U of X containing x and hence $x \in [X - f^{-1}(V^{(\Lambda,b)})]^{(\Lambda,b)}$. On the other hand, we have $x \in f^{-1}(V) \subseteq [f^{-1}(V^{(\Lambda,b)})]^{(\Lambda,b)}$. Thus, $x \in \Lambda_bFr(f^{-1}(V^{(\Lambda,b)}))$.

Conversely, suppose that f is weakly (Λ, b)-continuous at $x \in X$ and let V be any (Λ, b)-open set of Y containing $f(x)$. Thus, by Theorem 7, we have $x \in [f^{-1}(V^{(\Lambda,b)})]_{(\Lambda,b)}$. This shows that $x \notin \Lambda_bFr(f^{-1}(V^{(\Lambda,b)}))$ for each (Λ, b)-open set V of Y containing $f(x)$. This completes the proof.

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