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On 1-Movable Strong Resolving Hop Domination in Graphs

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Abstract. A set S is a 1-*movable strong resolving hop dominating set* of G if for every $v \in S$, either $S\setminus\{v\}$ is a strong resolving hop dominating set or there exists a vertex $u \in (V(G)\setminus S)\cap N_G(v)$ such that $(S \setminus \{v\}) \cap \{u\}$ is a strong resolving hop dominating set of G. The minimum cardinality of a 1-movable strong resolving hop dominating set of G is denoted by $\gamma^1_{msRh}(G)$. In this paper, we obtained the corresponding parameter in graphs resulting from the join, corona and lexicographic product of two graphs. Specifically, we characterize the 1-movable strong resolving hop dominating sets in these types of graphs and determine the bounds or exact values of their 1-movable strong resolving hop domination numbers.

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Key Words and Phrases: 1-movable strong resolving hop dominating set, 1-movable strong resolving hop domination number, join, corona, lexicographic product

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1. Introduction

The study of domination can be traced way back 1960. Since then numerous authors contribute several interesting domination parameters to nurture the growth of this research area. In 1977, E.J Cockayne and S.T Hedetniemi introduced the notation $\gamma(G)$ for the domination number of graph G. Until the initiation of the concept of 2-step domination number by Chartrand et al. [1] in 1995, which is closely related to hop domination number. Subsequently, Natarajan and Ayyaswamy (2015) introduced the hop domination concept. Some variation of domination can be seen in these papers [7], [6].

Blair et al. [3] introduced and investigated a new variant of the standard domination parameter called 1-movable domination. In 2011, they established results on the 1-movable

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dominating sets of some graphs and identified bounds on the 1-movable domination number for certain classes of graphs.

The concept of 1-movable dominating set was discussed in the paper of Hinampas and Canoy [4]. Their paper also presented some characterizations involving the concept and investigated the 1-movable dominating sets in the join and corona of graphs.

Inspired by the above works, this present study investigates the concepts of restrained strong resolving hop dominating and 1- movable strong resolving hop dominating sets of some graphs.

In this study, we only consider graphs that are finite, simple, undirected and connected. Readers are referred to [2] for elementary Graph Theory concepts.

Let G be a connected graph. A set $S \subseteq V(G)$ is a *hop dominating set* of G if for every $v \in V(G)\backslash S$, there exists $u \in S$ such that $d_G(u,v) = 2$. The minimum cardinality of a hop dominating set of G, denoted by $\gamma_h(G)$, is called the *hop domination number* of G. Any hop dominating set with cardinality equal to $\gamma_h(G)$ is called a γ_h-set.

A set $C \subseteq V(G)$ is called a *superclique* in G if $\langle C\rangle$ is a clique and for every pair of distinct vertices $u, v \in C$, there exists $w \in V(G) \setminus C$ such that $w \in N_G(u) \setminus N_G(v)$ or $w \in N_G(v) \setminus N_G(u)$. A superclique C is *maximum* in G if $|C| \geq |C^*|$ for all supercliques C^* in G. The superclique number of G, denoted by $\omega_S(G)$, is the cardinality of a maximum superclique in G.

A superclique C in G is called a *hop dominated superclique* if for every $v \in C$ there exists $u \in V(G)\backslash C$ such that $d_G(u,v) = 2$. A hop dominated superclique C is *maximum* in G if $|C| \geq |C^*|$ for all hop dominated supercliques C^* in G. The *hop dominated superclique number* denoted by $\omega_{hS}(G)$, of G is the cardinality of a maximum hop dominated superclique in G.

A superclique $C \subseteq V(G)$ is called a *point-wise non-dominated superclique* of G if for every $x \in C$ there exists $y \in V(G) \setminus C$ such that $y \notin N_G(x)$. A maximum cardinality of a point-wise non-dominated superclique in G is denoted by $\omega_{pndS}(G)$.

A vertex u of G is *maximally distant* from vertex v of G, $u \neq v$, if for every vertex $w \in N_G(u)$, $d_G(v,w) \leq d_G(u,v)$. If u is maximally distant from v and v is maximally distant from u, then we say that u and v are *mutually maximally distant*, denoted by uMMDv.

A vertex x of a connected graph G is said to *resolve vertices* u and v of G if $d_G(x,u) \neq d_G(x,v)$. For an ordered set $W = \{x_1, \dots, x_k\} \subseteq V(G)$ and a vertex v in G, the *k-vector*

$$r_G(v/W) = (d_G(v,x_1), d_G(v,x_2), \dots d_G(v,x_k))$$

is called the *representation* of v with respect to W. The set W is a *resolving set* for G if and only if no two vertices of G have the same representation with respect to W. The *metric dimension* of G, denoted by $dim(G)$, is the minimum cardinality over all resolving sets of G. A resolving set of cardinality $dim(G)$ is called a *basis.*

For two vertices $u, v \in V(G)$, the interval $I_G[u,v]$ between u and v is the collection of all vertices that belong to some shortest u-v path. A vertex w *strongly resolves* two vertices u and v if $v \in I_G[u,w]$ or if $u \in I_G[v,w]$. A set W of vertices in G is a *strong* *resolving set* of G if every two vertices of G are strongly resolved by some vertex of W. The smallest cardinality of a strong resolving set of G is called the *strong metric dimension* of G and is denoted by $sdim(G)$. A strong resolving set of cardinality $sdim(G)$ is called a *strong metric basis* of G.

A subset $S \subseteq V(G)$ is a *strong resolving hop dominating set* of G if S is both a strong resolving set and a hop dominating set. The minimum cardinality of a strong resolving hop dominating set of G, denoted by $\gamma_{sRh}(G)$, is called the *strong resolving hop domination number* of G. Any resolving hop dominating set with cardinality equal to $\gamma_{sRh}(G)$ is called a γ_{sRh}-*set*.

A strong resolving hop dominating set S is a 1-*movable strong resolving hop dominating set* of G if for every $v \in S$, either $S \setminus \{v\}$ is a strong resolving hop dominating set or there exists a vertex $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cap \{u\}$ is a strong resolving hop dominating set of G. The minimum cardinality of a 1-movable strong resolving hop dominating set of G is denoted by $\gamma^1_{msRh}(G)$.

2. Some Known Results

The following known results are taken from [5].

Theorem 1. Let G and H be nontrivial connected graphs of orders m and n, respectively. A proper subset S of $V(G+H)$ is a strong resolving set of $G+H$ if and only if at least one of the following is satisfied:

(i) $S = V(G+H) \setminus C_G$ where C_G is a superclique in G.

(ii) $S = V(G+H) \setminus C_H$ where C_H is a superclique in H.

(iii) If $\gamma(G) \neq 1$ or $\gamma(H) \neq 1$,

$$S = V(G+H) \setminus (C_G \cup C_H) = (V_(G) \setminus C_G) \cup (V(H) \setminus C_H),$$

where C_G and C_H are supercliques in G and H respectively.

Lemma 1. Let G be a nontrivial connected graph with $diam(G) \leq 2$. Then $S = V(G) \setminus C$ is a strong resolving set of G if and only if $C = \varnothing$ or C is a superclique in G. In particular, $sdim(G) = |V(G)| - \omega_S(G)$.

Theorem 2. Let G be a nontrivial connected graph and H a connected graph. A proper subset S of $V(G \circ H)$ is a strong resolving set of $G \circ H$ if and only if one of the following holds:

(i) $S = A \cup (\underset{u \in V(G)}{\cup} V(H^u))$ where $A \subseteq V(G)$.

(ii) $S = \cup(\bigcup_{u \in V(G)\setminus\{v\}} V(H^u)) \cup B_v$ for a unique v in $V(G)$,
where $A \subseteq V(G)$ and B^v is a strong resolving set of H^v if $\gamma(H) = 1$ or B_v is a resolving set of $\{v\} + H^v$ if $\gamma(H) \neq 1$.

Remark 1. Any superset of a strong resolving set is a strong resolving set.

Theorem 3. Let $G = K_n$ for $n > 1$ and H a nontrivial connected graph with $\gamma(H) \neq 1$. A subset S of $V(G[H])$ is a strong resolving set of $G[H]$ if and only $S = V(G[H]) \setminus (A \times C)$, where A is a subset of $V(G)$ and $C = \varnothing$ or C is a superclique in H.

3. Preliminary Results

This section introduces the 1-movable strong resolving hop domination in some graphs. It also characterizes some graphs in terms of its 1-movable strong resolving hop domination number.

Remark 2. Every 1-movable strong resolving hop dominating set of a connected graph G is a strong resolving hop dominating set in G. Hence, $\gamma_{sRh}(G) \leq \gamma^1_{msRh}(G)$.

Remark 3. The converse of Remark 2 does not hold. To see this, the set $S = \{v_1, v_2, v_3\}$ of the path $P_4 = [v_1, v_2, v_3, v_4]$ is a strong resolving dominating set of P_4 but it is not a 1-movable strong resolving hop dominating set since $S \setminus \{v_1\}$ is not a strong resolving set of P_4.

Proposition 1. Any superset of a 1-movable strong resolving hop dominating set is a 1-movable strong resolving dominating set.

Proof: Let S be a 1-movable strong resolving hop dominating set of G and $S \subseteq S'$. Then S is a strong resolving set. By Remark 1, S' is a strong resolving set of G. We show that S' is a 1-movable strong resolving hop dominating set of G. Let $x \in S'$. If $x \in S$ then $S \setminus \{x\} \subseteq S' \setminus \{x\}$. Since S is a 1-movable strong resolving hop dominating set of G either $S \setminus \{x\}$ is strong resolving hop dominating set of G or $\exists y \in (V(G) \setminus S) \cap N_G(x)$ such that $(S \setminus \{x\}) \cup \{y\}$ is a strong resolving hop dominating set of G. If $S \setminus \{x\}$ is a strong resolving hop dominating set of G, then $S' \setminus \{x\}$ is also a strong resolving set of G by Remark 1. If there exists $y \in (V(G) \setminus S) \cap N_G(x)$ such that $(S \setminus \{x\}) \cup \{y\}$ is a strong resolving hop dominating set of G, then

$$(S \setminus \{x\}) \cup \{y\} \subseteq (S' \setminus \{x\}) \cup \{y\}.$$

It follows that $(S' \setminus \{x\}) \cup \{y\}$ is strong resolving set of G. It can be verified that every superset of hop dominating set is hop dominating. Therefore, S' is a 1-movable strong resolving hop dominating set of G. □

Proposition 2. Let $P_n = [v_1, v_2, \ldots, v_n]$ where $n \geq 1$. If a set $S \subseteq V(P_n)$ is a 1-movable strong resolving hop dominating set of P_n, then S contains the vertices v_1 and v_n.

Proof: Suppose S is a 1-movable strong resolving hop dominating set of P_n and suppose that S does not contain v_1 or v_n, say v_1. Since v_1MMDv_n, $S \cap \{v_1, v_n\} \neq \varnothing$. Hence, $v_n \in S$. This implies that $S \setminus \{v_n\}$ and $(S \setminus \{v_n\}) \cup \{v_{n-1}\}$ are not strong resolving sets of P_n, a contradiction. Therefore, S contains v_1 and v_n. □

Proposition 3. Let G be a nontrival connected graph with $diam(G) \leq 2$ and $\gamma(G) \neq 1$. Then $S = V(G) \setminus C$ is a 1-movable strong resolving hop dominating set of G if and only if $C = \varnothing$ or C is a hop dominated superclique in G and either for each $x \in S$, $C \cup \{x\}$ is a hop dominated superclique or there exists $y \in [C \cap N_G(x)]$ such that $(C \setminus \{y\}) \cup \{x\}$ is a hop dominated superclique in G.

Proof: Suppose $S = V(G) \setminus C$ is a 1-movable strong resolving hop dominating set of G. Then S is strong resolving set in G. By Lemma 1, $C = \varnothing$ or C is a dominated superclique in G. We claim that C is a hop dominated superclique. Let $z \in C$. Then $z \notin S$. Since S is hop dominating, there exists $y \in (S \setminus C)$ such that $d_G(z, y) = 2$. Hence, C is a hop dominated superclique. Let $x \in S$. Since S is a 1-movable strong resolving hop dominating set, either $S \setminus \{x\}$ is strong resolving hop dominating or there exists $y \in [(V(G) \setminus S) \cap N_G(x)]$ such that $(S \setminus \{x\}) \cup \{y\}$ is a strong resolving hop dominating set of G. Since $S \setminus \{x\} = V(G) \setminus (C \cup \{x\})$ and $(S \setminus \{x\}) \cup \{y\} = [V(G) \setminus C \setminus \{x\}] \cup \{y\}$, $C \cup \{x\}$ is a hop dominated superclique or $(C \setminus y) \cup \{x\}$ is a hop dominated superclique in G.

For the converse, suppose $C = \varnothing$. Then $S = V(G)$ is a strong resolving hop dominating set of G. Thus, $S \setminus \{x\} = V(G) \setminus \{x\}$ is a strong resolving hop dominating since $\{x\}$ is a superclique for each $x \in V(G)$. Since $\gamma(G) \neq 1$, a vertex $y \in S \setminus \{x\}$ exists such that $d_G(x, y) = 2$. Hence, S is a hop dominating. So, suppose C is a hop dominated superclique in G and for each $x \in S$ either $C \cup \{x\}$ is a hop dominated superclique or there exists $y \in [C \cap N_G(x)]$ such that $(C \setminus \{y\}) \cup \{x\}$ is a hop dominated superclique. Hence, for each $x \in S$.

$$(S \setminus \{x\}) \cup \{y\} = [V(G) \setminus (C \setminus \{y\})] \cup \{x\}$$

is a strong resolving hop dominating set of G. Therefore, S is a 1-movable strong resolving hop dominating set of G. □

4. Join of Graphs

Theorem 4. Let G be a connected graph of order n and $\gamma(G) = 1$. Then a 1-movable strong resolving hop dominating set of G does not exist.

Proof: Suppose G has a 1-movable strong resolving hop dominating set S.
Let $D = \{x \in V(G) : deg_G(x) = n - 1\}$. Since S is hop dominating, $D \subseteq S$. Let $x \in D$. Then $S \setminus \{x\}$ is not hop dominating and for each $y \in (V(G) \setminus S) \cap N_G(x)$, $(S \setminus \{x\}) \cup \{y\}$ is also not hop dominating. Hence, S is not a 1-movable strong resolving hop dominating. □

As a consequence of Theorem 4 the next result follows.

Corollary 1. Let G be a graph, then the 1-movable strong resolving hop dominating set of $K_1 + G$ does not exist.

Theorem 5. Let G and H be graphs where $\gamma(G) \neq 1$ and $\gamma(H) \neq 1$. A proper subset S of $V(G + H)$ is a 1-movable strong resolving hop dominating set of $G + H$ if and only if at least one of the following is satisfied.

(i) $S = V(G + H) \setminus C_G$ where C_G and $C_G \cup \{x\}$ or $(C_G \cup \{x\}) \setminus \{y\}$ are point-wise non-dominated superclique in G for each $x \in S$.

(ii) $S = V(G + H) \setminus C_H$ where C_H and $C_H \cup \{z\}$ or $(C_H \cup \{z\}) \setminus \{w\}$ are point-wise non-dominated superclique in H for each $z \in S$ and $w \in (V(H) \setminus S_H) \cap N_H(z)$.

(iii) $S = V(G + H) \setminus (C_H \cup C_G)$ where $C_H, C_G, C_H \cup \{x\}, (C_H \cup \{x\}) \setminus \{y\}$, $C_G \cup \{z\}, (C_G \cup \{z\}) \setminus \{w\}$ are point-wise non-dominated supercliques in H and G, respectively for all $x \in S_H$.

Proof: Suppose $S \subseteq V(G + H)$ is a 1-movable strong resolving hop dominating set of G. Then S is a strong resolving set of $G + H$. By Theorem 1, at least one of the following is satisfied:

(a) $S = V(G + H) \setminus C_G$ where C_G is a superclique in G.

(b) $S = V(G + H) \setminus C_H$ where C_H is a superclique in H.

(c) If $\gamma(G) \neq 1$ or $\gamma(H) \neq 1$,

$$S = V(G + H) \setminus (C_G \cup C_H) = (V_(G) \setminus C_G) \cup (V(H) \setminus C_H).$$

We claim that C_G is a point-wise non-dominated set of G. Let $x \in C_G$. Since S is hop dominating and $x \in V(G + H) \setminus S$, there exists $y \in S$ such that $d_{G+H}(x, y) = 2$. By Definition of point-wise non-dominated superclique, $y \in V(G) \setminus C_G$ and $y \notin N_G(x)$. Thus, C_G is a point-wise non-dominated superclique of G. Let $x \in S$. Since S is a 1-movable strong resolving hop dominating set of $G + H$, either $S \setminus \{x\}$ or $(S \setminus \{x\}) \cup \{y\}$ is a strong resolving hop dominating set of $G + H$ where $y \in [V(G + H) \setminus S] \cap N_{G+H}(x)$. Since $S = V(G + H) \setminus C_G$, $S \setminus \{x\} = V(G + H) \setminus (C_G \cup \{x\})$ and $(S \setminus \{x\}) \cup \{y\} = [V(G + H)] \setminus [(C_G \cup \{x\}) \setminus \{y\}]$
by Lemma 1, $C_G \cup \{x\}$ or $(C_G \cup \{x\}) \setminus \{y\}$ is a superclique. By similar argument above, $C_G \cup \{x\}$ or $(C_G \cup \{x\}) \setminus \{y\}$ is a point-wise non-dominated superclique in G. This proves (i). Statements (i) and (iii) are proved similarly.

For the converse, suppose (i) holds. By Theorem 1, S is a strong resolving set. Let $u \in V(G + H) \setminus S$. Then $u \in C_G$. Since C_G is a point-wise non-dominated superclique of G, there exists $v \in V(G) \setminus C_G$ such that $v \notin N_G(u)$. Hence, $v \in S$ and $d_{G+H}(u, v) = 2$. Let $x \in S$. Since $S \setminus \{x\} = V(G + H) \setminus (C_G \cup \{x\})$, by (i) of Theorem 1 and the Definition of point-wise non-dominated superclique, S is a 1-movable strong resolving hop dominating set of $G+H$. Similarly if (ii) and (iii) holds, S is a 1-movable strong resolving hop dominating set of $G+H$. □

The next result follows from Theorem 5.

Corollary 2. Let G and H be nontrivial connected graphs of orders m and n, respectively. Then,

$$\gamma^1_{msRh}(G+H) = m - \omega_{pndS}\}(G) + n - \omega_{pndS}(H).$$

5. Corona of Graphs

This section gives characterization of the 1-movable strong resolving hop dominating sets in the corona of graphs as well as its 1-movable strong resolving hop domination number.

Theorem 6. Let G be a nontrivial connected graph and H a connected graph with $\gamma(H) \neq 1$. A proper subset S of $V(G \circ H)$ is a 1-movable strong resolving hop dominating set of $G \circ H$ if and only if $S = A \bigcup \left(\bigcup_{u \in V(G)} V(H^u) \right)$ where $A \subseteq V(G)$.

Proof: Suppose that a proper subset S of $V(G \circ H)$ is a 1-movable strong resolving hop dominating set of $G \circ H$. Since S is strong resolving set of $G \circ H$, (i) or (ii) of Theorem 2 holds. If (i) holds, then $S = A \bigcup \left(\bigcup_{u \in V(G)} V(H^u) \right)$, where $A \subseteq V(G)$. Suppose (ii) holds. Let $x \in V(H^w)$ for some $w \in V(G)$ with $w \neq v$. Then

$$S \setminus \{x\} = A \bigcup \left(\bigcup_{u \in V(G) \setminus \{w,v\}} V(H^u) \right) \bigcup \left(V(H^w) \setminus \{x\} \right) \bigcup B_v$$

is not a strong resolving set by Theorem 2. Hence, $S = A \bigcup \left(\bigcup_{u \in V(G)} V(H^u) \right)$ where $A \subseteq V(G)$.

For the converse, suppose $S = A \bigcup \left(\bigcup_{u \in V(G)} V(H^u) \right)$ where $A \subseteq V(G)$. By Theorem 2, S is a strong resolving set of $G \circ H$. It can be seen that S is a hop dominating set also. Let $p \in S$. If $p \in A$, then

$$S \setminus \{p\} = (A \setminus \{p\}) \cup \left(\bigcup_{u \in V(G)} V(H^u) \right)$$

is a strong resolving hop dominating set. If $p \in V(H^u)$ for each $u \in V(G)$, then $S \setminus \{p\} = A \cup \left(\bigcup_{u \in V(G)} V(H^u) \setminus \{p\} \right) \cup \left(\bigcup_{v \in V(G) \setminus \{u\}} V(H^v) \right)$ is a strong resolving set by Theorem 2 and hop dominating since $\gamma(H) \neq 1$.

Accordingly S is a 1-movable strong resolving hop dominating set in $G \circ H$. □

Corollary 3. Let G be a connected graph of order $m > 1$ and H be any graph of order n with $\gamma(H) \neq 1$. Then

$$\gamma^1_{msRh}(G \circ H) = mn.$$

Proof: Let S be a γ^1_{msRh}-set of $G \circ H$. Then by Theorem 6, $S = A \bigcup \left(\bigcup_{u \in V(G)} V(H^u) \right)$ where $A \subseteq V(G)$. Thus,

$$\begin{aligned} \gamma^1_{msRh}(G \circ H) &= |S| \\ &= |A| + \left| \bigcup_{u \in V(G)} V(H^u) \right| \\ &\geq |V(G)||V(H)| \\ &= mn. \end{aligned}$$

Let $A = \varnothing$. Then $S^* = A \bigcup \left(\bigcup_{u \in V(G)} V(H^u) \right)$ is a 1-movable strong resolving hop dominating set of $G \circ H$ by Theorem 6. Hence,

$$\begin{aligned} \gamma^1_{msRh}(G \circ H) &\leq |S^*| \\ &= \left| \bigcup_{u \in V(G)} V(H^u) \right| \\ &= mn. \end{aligned}$$

Therefore, $\gamma^1_{msRh}(G \circ H) = mn$. □

Example 1. *For the graph of $P_3 \circ P_4$, the minimum 1-movable strong resolving hop dominating set is $\gamma^1_{msRh}(P_3 \circ P_4) = 3(4) = 12$.*

6. Lexicographic of Graphs

This section gives characterization of a 1-movable strong resolving hop dominating sets in the lexicographic product of graphs as well as its 1-movable strong resolving hop domination number.

Theorem 7. Let $G = K_n$ for $n > 1$ and H is a connected graph with $\gamma(H) \neq 1$. A subset S of $V(G[H])$ is a 1-movable strong resolving hop dominating set of $G[H]$ if and only if $S = V(G[H]) \setminus (A \times C)$, where A is a subset of $V(G)$ and $C = \varnothing$.

Proof: Suppose S is a 1-movable strong resolving hop dominating set $G[H]$. By Theorem 3, $S = V(G[H]) \setminus (A \times C)$ where $A \subseteq V(G)$ and $C = \varnothing$ or C is a superclique in H. Suppose $C \neq \varnothing$ and C is a superclique in H.
Let $(x, y) \in S$. Then $y \notin C$. Hence, $S \setminus \{x, y\} = V(G[H]) \setminus ((A \times C) \cup \{x, y\})$. Since $\gamma(H) \neq 1$, $C \cup \{y\}$ is not a superclique in H. Therefore $A \subseteq V(G)$and $C = \varnothing$.

The converse follows immediately from Theorem 3. □

As a consequence of Theorem 7 the next result follows.

Corollary 4. Let $G = K_n$ for $n > 1$ and H is a connected graph of order m and $\gamma(H) \neq 1$. Then

$$\gamma^1_{msRh}(G[H]) = mn.$$

Example 2. *The sets of shaded vertices in* $K_4[P_4]$ *and* $K_4[P_3]$ *in Figure 1 represent* 1-*movable strong resolving hop dominating sets.*

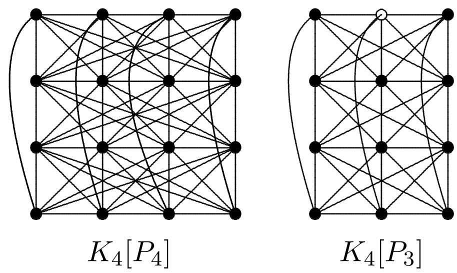

Figure 1: A 1-movable strong resolving hop dominating sets of $K_4[P_4]$ and $K_4[P_3]$

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