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Results of Semigroup of Linear Operators Generating a General Class of Semilinear Initial Value Problems

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Abstract. This paper present results of ω-order preserving partial contraction mapping generating a general class of semilinear initial value problems. We consider the use of fractional powers of unbounded linear operators for its application by starting with some results concerning such fractional powers. We assume A to be the infinitesimal generator of an analytic semigroup in a Banach space X, $0 \in \rho(A)$ and defined the fractional powers of A for $0 < \alpha \leq 1$. We also show that A^α is a closed linear operator whose domain $D(A^\alpha) \supset D(A)$ is dense in X. Finally we established that the operator is bounded, continuous and Holder continuous.

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1. Front Matter

Assume $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$ and let

$$A(x, D) = \sum_{|\alpha| \leq 2m} a_\alpha(x) D^\alpha \tag{1}$$

be a strongly elliptic differential operator in Ω. For $1 < p < \infty$ we associate with $A(x, D)$ and operator A_p in $L^p(\Omega)$ by

$$D(A_p) = W^{2m,p}(\Omega) \cap W_0^{m,p}(\Omega) \tag{2}$$

and

$$A_p u = A(x, D)u \tag{3}$$

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for $u \in D(A_p)$ and $A \in \omega - OCP_n$. Suppose A_p is the infinitesimal generator of an analytic semigroup on $L^p(\Omega)$. By adding to $A(x, D)$, and hence to A_p, a positive multiple of identity, we obtain an infinitesimal generator $-(A_p + KI)$ of an analytic semigroup, which is invertible. In the sequel we will tactically assume that this has been done and thus assume directly that A_p itself is invertible. Let A be a strongly elliptic operator of order $2m$ on a bounded domain Ω with smooth boundary $\partial\Omega$ in $\mathbb{R}^n$ and let $1 < p < \infty$. There exists a constant C such that

$$\|u\|_{2m,p} \leq C(\|Au\|_{0,p} + \|u\|_{0,p}) \tag{4}$$

for every $u \in D(A_p)$ and $A_p \in \omega - OCP_n$.

Since we assume now that A_p is invertible in $L^p(\Omega)$ it follows readily that $C\|u\|_{0,p} \leq \|A_p u\|_{0,p}$ for some constant $C > 0$ and therefore we have

$$\|u\|_{2m,p} \leq C\|A_p u\|_{0,p} \qquad for \quad u \in D(A_p). \tag{5}$$

Suppose X is a Banach space, $X_n \subseteq X$ is a finite set, $\omega - OCP_n$ the ω-order preserving partial contraction mapping, M_m be a matrix, $L(X)$ be a bounded linear operator on X, P_n a partial transformation semigroup, $\rho(A)$ a resolvent set, $\sigma(A)$ a spectrum of A. This paper consist of results of ω-order preserving partial contraction mapping generating a general class of semilinear initial value problems. Akinyele *et al.* [1], characterized ω-order reversing partial contraction mapping as a compact semigroup of linear operator and also in [2], Akinyele *et al.*, obtained differentiable and analytic results on ω-order preserving partial contraction mapping in semigroup of linear operator. Balakrishnan [3], presented an operator calculus for infinitesimal generators of semigroup. Banach [4], established and introduced the concept of Banach spaces. Brezis and Gallouet [5], generated nonlinear Schrödinger evolution equation. Chill and Tomilov [6], presented some resolvent approach to stability operator semigroup. Davies [7], obtained linear operators and their spectra. Engel and Nagel [8], introduced one-parameter semigroup for linear evolution equations. Omosowon *et al.* [13], generated some analytic results of semigroup of linear operator with dynamic boundary conditions, and also in [11], Omosowon *et al.*, introduced dual Properties of ω-order Reversing Partial Contraction Mapping in Semigroup of Linear Operator. Omosowon *et al.* [10], established a regular weak*-continuous semigroup of linear operators, and also in [9], Omosowon *et al.*, obtained a quasilinear equations of evolution on semigroup of linear operator.reversing partial contraction mapping generating a differential operator. Omosowon *et al.* [12], deduced results of semigroup of linear equation generating a wave equation. Pazy [14], presented asymptotic behavior of the solution of an abstract evolution and some applications and also in [15], obtained a class of semi-linear equations of evolution. Rauf and Akinyele [16], introduced ω-order preserving partial contraction mapping and obtained its properties, also in [17], Rauf *et al.*, established some results of stability and spectra properties on semigroup of linear operator. Vrabie [18], proved some results of C_0-semigroup and its applications. Yosida [19], deduced some results on differentiability and representation of one-parameter semigroup of linear operators.

2. Preliminaries

Definition 2.2 (C_0-Semigroup) [18]
A C_0-Semigroup is a strongly continuous one parameter semigroup of bounded linear operator on Banach space.

Definition 2.3 (ω-OCP_n) [16]
A transformation $\alpha \in P_n$ is called ω-order preserving partial contraction mapping if $\forall x, y \in$ Domα : $x \leq y \implies \alpha x \leq \alpha y$ and at least one of its transformation must satisfy $\alpha y = y$ such that $T(t+s) = T(t)T(s)$ whenever $t, s > 0$ and otherwise for $T(0) = I$.
Definition 2.4 (Evolution Equation) [14]
An evolution equation is an equation that can be interpreted as the differential law of the development (evolution) in time of a system. The class of evolution equations includes, first of all, ordinary differential equations and systems of the form

$$u = f(t, u), u = f(t, u, u),$$

etc., in the case where u(t) can be regarded naturally as the solution of the Cauchy problem; these equations describe the evolution of systems with finitely many degrees of freedom.
Definition 2.5 (Mild Solution) [15]
A continuous solution u of the integral equation.

$$u(t) = T(t - t_0)u_0 + \int_{t_0}^{t} T(t - s)f(s, u(s))ds$$

will be called a mild solution of the initial value problem

$$\begin{cases} \frac{du(t)}{dt} + Au(t) = f(t, u(t)),\ t > t_0 \\ u(t_0) = u_0 \end{cases}$$

if the solution is a Lipschitz continuous function.

Definition 2.6(Analytic Semigroup) [18]
We say that a C_0-semigroup $\{T(t); t \geq 0\}$ is analytic if there exists $0 < \theta \leq \pi$, and a mapping $S : \bar{\mathbb{C}}_\theta \to L(X)$ such that:
(i) $T(t) = S(t)$ for each $t \geq 0$;
(ii) $S(z_1 + z_2) = S(z_1)S(z_2)$ for $z_1, z_2 \in \bar{\mathbb{C}}_\theta$;
(iii) $lim_{z_1 \in \bar{\mathbb{C}}_\theta, z_1 \to 0} S(z_1)x = x$ for $x \in X$; and
(iv) the mapping $z_1 \to S(z_1)$ is analytic from $\bar{\mathbb{C}}_\theta$ to $L(X)$. In addition, for each $0 < \delta < \theta$, the mapping $z_1 \to S(z_1)$ is bounded from $\mathbb{C}_\delta$ to $L(X)$, then the C_0-Semigroup $\{T(t); t \geq 0\}$ is called analytic and uniformly bounded.
Definition 2.7(Strongly Elliptic) [15]
The operator $A(x, D)$ is strongly elliptic if there exists a constant $C > 0$ such that

$$Re(-1)^m A^1(x, \xi) \geq C|\xi|^{2m}$$

for all $x \in \overline{\Omega}$ and $\xi \in \mathbb{R}^n$.

Example 1

For every 2×2 matrix in $[M_m(\mathbb{R}^n)]$.
Suppose

$$A = \begin{pmatrix} 2 & 0 \\ \Delta & 2 \end{pmatrix}$$

and let $T(t) = e^{tA}$, then we have

$$e^{tA} = \begin{pmatrix} e^{2t} & I \\ e^{\Delta t} & e^{2t} \end{pmatrix}.$$

Example 2
For every 3×3 matrix in $[M_m(\mathbb{C})]$, we have
for each $\lambda > 0$ such that $\lambda \in \rho(A)$ where $\rho(A)$ is a resolvent set on X.
Suppose we have

$$A = \begin{pmatrix} 2 & 2 & I \\ 2 & 2 & 2 \\ \Delta & 2 & 2 \end{pmatrix}$$

and let $T(t) = e^{tA_\lambda}$, then we have

$$e^{tA_\lambda} = \begin{pmatrix} e^{2t\lambda} & e^{2t\lambda} & I \\ e^{2t\lambda} & e^{2t\lambda} & e^{2t\lambda} \\ e^{\Delta t\lambda} & e^{2t\lambda} & e^{2t\lambda} \end{pmatrix}.$$

Example 3
Let $X = C_{ub}(\mathbb{N} \cup \{0\})$ be the space of all bounded and uniformly continuous function from $\mathbb{N} \cup \{0\}$ to $\mathbb{R}$, endowed with the sup-norm $\| \cdot \|_\infty$ and let $\{T(t); t \in \mathbb{R}_+\} \subseteq L(X)$ be defined by

$$[T(t)f](s) = f(t + s)$$

For each $f \in X$ and each $t, s \in \mathbb{R}_+$, one may easily verify that $\{T(t); t \in \mathbb{R}_+\}$ satisfies Examples 1 and 2 above.

Lemma 2.1
Let Ω be a bounded domain in $\mathbb{R}^n$ with boundary $\partial\Omega$ of class C^m and let $u \in W^{m,r}(\Omega) \cap L^q(\Omega)$ where $1 \le r$, $q \le \infty$. For any integer j, $0 \le j < m$ and any $\frac{j}{m} \le \vartheta \le 1$ we have

$$\|D^j u\|_{0,p} \le C \|u\|_{m,r}^{\vartheta} \|u\|_{0,q}^{1-\vartheta} \tag{6}$$

provided that

$$\frac{1}{p} = \frac{j}{n} + \vartheta \left(\frac{1}{r} - \frac{m}{n} \right) + (1 - \vartheta) \frac{1}{q} \tag{7}$$

and $m - j - \frac{n}{r}$ is not a nonnegative integer, the (6) holds with $\vartheta = \frac{j}{m}$.

Theorem 2.2 [Sobolev]

Let Ω be a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$ (e.g. $\partial\Omega$ is of class C^m), then

$$W^{k,p} \subset L^{np/(n-kp)}(\Omega) \quad for \quad kp < n \tag{8}$$

and

$$W^{k,p}(\Omega) \subset C^m(\overline{\Omega}) \quad for \quad 0 \le m < k - \frac{n}{p}. \tag{9}$$

Moreover, there exists a constant C_1 and C_2 such that for any $u \in W^{m,p}(\Omega)$

$$\|u\|_{0,n,p/(n-kp)} \le C_1 \|u\|_{k,p} \quad for \quad kp < n \tag{10}$$

and

$$\sup(|D^\alpha_\alpha u(x)| : |\alpha| \le m, x \in \overline{\Omega}) \le C_2 \|u\|_{k,p} \quad for \quad 0 \le m < k - \frac{n}{p}. \tag{11}$$

3. Main Results

This section present results of semigroup of linear operator by using ω-OCP_n to generates a general class of semilinear initial value problems:

Theorem 3.1

Suppose $A_p : D(A_p) \subseteq X \to X$ is the infinitesimal generator of an analytic semigroup $\{T(t);\ t \ge 0\}$. Assume $1 < p < \infty$ and let A_p be the operator defined in Lemma 2.1. For any multi-index β, $|\beta| = j < 2m$ and any $j/2m < \alpha \le 1$ we have

$$\|D^\beta A_p^{-\alpha} u\|_{0,p} \le C\|u\|_{0,p} \tag{12}$$

for $u \in D(A_p)$ and $A_p \in \omega - OCP_n$.

Proof:

Set $B = D^\beta$. Since $|\beta| < 2m$, it is clear that $D(B) \supset D(A_p)$ for all $A, B \in \omega - OCP_n$. From Lemma 2.1, we have

$$\|D^\beta u\|_{0,p} \le C\|u\|^{j/2m}_{2m,p}\|u\|^{1-j/2m}_{0,p}. \tag{13}$$

Polarization of (13) together with estimate (5) yields

$$\|D^\beta u\|_{0,p} \le C(\rho^{-1+j/2m}\|A_p u\|_{0,p} + \rho^{j/2m}\|u\|_{0,p}) \tag{14}$$

for $p > 0$, $u \in D(A_p)$ and $A_p \in \omega - OCP_n$. Suppose B is a closed linear operator satisfiying $D(B) \supset D(A)$. If for some γ, $0 < \gamma < 1$, and every $\rho \ge \rho_0 > 0$ we have

$$\|Bx\| \le C(\rho^\gamma \|x\| + \rho^{\gamma-1}\|Ax\|) \tag{15}$$

for $x \in D(A)$ and $A \in \omega - OCP_n$, then

$$D(B) \supset D(A^{\alpha}) \quad \text{for every} \quad \gamma \leq 1. \tag{16}$$

It follows now that $D(B) \supset D(A_p^{\alpha})$ for $j/2m < \alpha \leq 1$, that, is $BA_p^{-\alpha}$ is bounded for these values of α and this achieved the proof.

Theorem 3.2

Assume $A_p : D(A_p) \subseteq L^p(\Omega) \to L^p(\Omega)$ is the infinitesimal generator of a semigroup $\{T(t);\ t \geq\}$. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$ such that $A \in \omega - OCP_n$. If $0 \leq \alpha \leq 1$, then

$$X_{\alpha} \subset W^{k,q}(\Omega) \quad for \quad k - \frac{n}{q} < 2m\alpha - \frac{n}{p}, \quad q \geq p \tag{17}$$

$$X_{\alpha} \subset C^{p}(\overline{\Omega}) \quad for \quad 0 \leq v < 2m\alpha - \frac{n}{p}, \tag{18}$$

and the embeddings are continuous.

Proof:

From Theorem 3.1 it follows readily that $X_{\alpha} \subset W^{j,p}(\Omega)$ provided that $j < 2m\alpha$ and the imbedding is continuous. Since Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, assume $\partial\Omega$ is of class C' and let $1 \leq r, p < \infty$. If j, m are integers such that $0 \leq r, j < m$ and

$$\frac{1}{p} > \frac{1}{r} + \frac{j}{n} - \frac{m}{n} \tag{19}$$

then $W^{m,r}(\Omega) \supset W^{j,p}(\Omega)$ and the imbedding is compact. It follows that $W^{j,p}(\Omega)$ is continuously imbedded in $W^{k,q}(\Omega)$ provided that $k - n/q < j - n/p$ and (17) follows. From Theorem 2.2 (Sobolev), it follows that $W^{j,p}(\Omega)$ is continuously imbedded in $C^{v}(\overline{\Omega})$ for $0 \leq v < j - n/p$ and (18) follows. Hence, the proof is completed.

Theorem 3.3

Let $A(x, D)$ be a strongly elliptic operator given by

$$A(x, D) = -\sum_{k,l=1}^{3} \frac{\partial}{\partial x_k} a_{k,l}(x) \frac{\partial}{\partial x_l}.$$

Let Ω be a bounded domain in $\mathbb{R}^3$ with smooth boundary $\partial\Omega$ such that $A \in \omega - OCP_n$ where $a_{k,l}(x) = a_{l,k}(x)$ are real valued and continuously differentiable in $\overline{\Omega}$. Let $f(t, x, u, p), p \in \mathbb{R}^3$, be locally Lipschitz continuous function of all its arguments and assume further that there is a continuous function $\rho(t, r) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+$ and a real constant $1 \leq r < 3$ such that

$$|f(t, x, u, p)| \leq \rho(t, |u|)(1 + |p|^{\gamma}) \tag{20}$$

$$|f(t, x, u, p) - f(t, x, u, q)| \leq \rho(t, |u|)(1 + |p|^{\gamma-1} + |q|^{\gamma-1})|p - q| \tag{21}$$

$$|f(t,x,u,p) - f(t,x,v,p)| \leq \rho(t,|u|+|v|)(1+|p|^{\gamma})|u-v|. \tag{22}$$

Then for every $u_0 \in H^2(\Omega) \cap H_0^1(\Omega)$, the initial value problem

$$\begin{cases} \frac{\partial u}{\partial t} = A(x,D)u + f(t,x,u,grad\ u) & in \quad \Omega \\ u(t,x) = 0 & on \quad \partial\Omega \\ u(0,x) = u_0(x) & in \quad \Omega \end{cases} \tag{23}$$

has a unique local strong solution in $L^2(\Omega)$.

Proof:

We recall that with the strongly elliptic operator $A(x,D)$, we associate an operator A in $L^2(\Omega)$ by

$$D(A) = H^2(\Omega) \cap H_0^1(\Omega)$$

and

$$Au = A(x,D)u \quad for \quad u \in D(A)\ and\ A \in \omega - OCP_n.$$

Let $1 < p < \infty$, the operator A is the infinitesimal generator of an analytic semigroup of contractions on $L^p(\Omega)$, then it follows that A is the infinitesimal generator of an analytic semigroup on $L^2(\Omega)$. From the strong ellipticity together with Poincare's inequality it follows readily that A is also invertible. From Theorem 3.2 it follows that if $\alpha > 3/4$, then $X_\alpha \subset L^\infty(\Omega)$ and if also $1/q > (5-4\alpha)/6$, then $X_\alpha \subset W^{1,q}(\Omega)$. Thus for $\max(3/4,(5\gamma - 3)/4\gamma) < a < 1$, we have

$$X_\alpha \subset W^{1,2\gamma}(\Omega) \cap L^\infty(\Omega). \tag{24}$$

In order to show that the initial value problem has a unique local solution, we have to show that the mapping

$$F(t,u)(x) = f(t,x,x(x),\nabla u(x)),\ x \in \Omega \tag{25}$$

is well defined on $\mathbb{R}^+ \times X_\alpha$ and satisfies a local Holder condition. From (20) and (24), we have for every $u \in X_\alpha$

$$\|F(t,u)\|_{0,2} \leq 2p(t,\|u\|_{0,\infty})(M^{1/2} + \|u\|_{1,2\gamma}^{\gamma})$$

where M is the measure of Ω. Therefore F is well defined on $\mathbb{R}^+ \times X_\alpha$. To show that F satisfies a local Hölder condition we note that

$$\|F(t,u) - F(t,v)\|_{0,2}^2 \leq 2\int_0 |f(t,x,u,\nabla u) - f(t,x,u,\nabla v)|^2 dx$$
$$+ 2\int_\Omega |f(t,x,u,\nabla v) - f(t,x,v,\nabla v)|^2 dx \tag{26}$$

and estimate each of the two terms on the right of (26) separately. From (21) and (23) we have

$$\int_\Omega |f(t,x,u,\nabla u) - f(t,x,u,\nabla v)|^2 dx$$

$$\leq C \cdot \rho(t, \|u\|_{0,\infty})^2 \int_\Omega (1 + |\nabla u|^{2\gamma-2} + |\nabla v|^{2\gamma-2})|\nabla(u-v)|^2 dx$$
$$\leq C \cdot \rho(t, \|u\|_{0,\infty})^2 (M_1 + \|\nabla u\|_{0,2\gamma}^{2\gamma-2} + \|\nabla v\|_{0,2\gamma}^{2\gamma-2})\|\nabla(u-v)\|_{0,2\gamma}^2 dx$$
$$\leq L(\|u\|_\alpha, \|v\|_\alpha)\|u-v\|_{1,2\gamma}^2 \leq L(\|u\|_\alpha, \|v\|_\alpha)\|u-v\|_\alpha^2 \tag{27}$$

where $\| \quad \|_\alpha$ denotes the norm in X_α and L is a constant depending on $\|u\|_\alpha$ and $\|v\|_\alpha$. To obtain the second inequality we used Hölder's inequality. The last inequality (27) is a consequence of the continuous imbedding of X_α in $W^{1,2\gamma}(\Omega)$. Similarly for the second term we have (22) and (24), then we have

$$\int_\Omega |f(t,x,u,\nabla v) - f(t,x,v,\nabla v)|^2 dx$$
$$\leq C\rho(t, \|u\|_{0,\infty} + \|v\|_{0,\infty})^2 \int_\Omega (1 + |\nabla u|^{2\gamma})|u-v|^2 dx$$
$$\leq C\rho(t, \|u\|_{0,\infty} + \|v\|_{0,\infty})^2 \|u-v\|_{0,\infty}^2 (1 + \|v\|_{1,2\gamma}^{2\gamma})$$
$$\leq L(\|u\|_\alpha, \|v\|_\alpha)\|u-v\|_\alpha^2 \tag{28}$$

and therefore,

$$\|F(t,u) - F(t,v)\|_{0,2} \leq L(\|u\|_\alpha, \|v\|_\alpha)\|u-v\| \tag{29}$$

and the existence of the strong local solution (23) is a direct consequence that the initial value problem (23) has a unique local solution u. Hence the proof is completed.

Theorem 3.4
Assume $A : D(A) \subseteq H^2(\Omega) \to H^2(\Omega)$ is the infinitesimal generator of a C_0-semigroup $\{T(t)_{t\geq 0}\}$. Let

$$f(u) = \sum_{i=1}^{3} u \frac{\partial u}{\partial x_i}. \tag{30}$$

If $\gamma > \frac{3}{4}$, $u \in D(A)$ and $A \in \omega - OCP_n$, then $f(u)$ is well defined and

$$\|f(u)\| \leq C\|A^\gamma u\|\|A^{\frac{1}{2}}u\|. \tag{31}$$

If $u, v \in D(A)$ and $A \in \omega - OCP_n$, then

$$\|f(u) - f(v)\| \leq C(\|A^\gamma u\|\|A^{\frac{1}{2}}u - A^{\frac{1}{2}}v\| + \|A^{\frac{1}{2}}v\|\|A^\gamma u - A^\gamma v\|). \tag{32}$$

Proof:
Since $D(A) \subset H^2(\Omega)$, then it follows from Sobolev's theorem which states that if Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$ of class C^m, then

$$W^{k,p}(\Omega) \subset \mathcal{L}^{np/(n-kp)}(\Omega) \quad for \quad kp < n \tag{33}$$

and

$$W^{k,p}(\Omega) \subset C^m(\overline{\Omega}) \quad for \quad 0 \leq m < K - \frac{n}{p}. \tag{34}$$

Moreover, there exist constants C_1 and C_2 such that for any $u \in W^{m,p}(\Omega)$,

$$\|u\|_{0,np/(n-kp)} \leq C_1 \|u\|_{k,p} \quad for \quad kp < n \tag{35}$$

and

$$\sup\{|D^\alpha u(x)| : |\alpha| \leq m, \ x \in \overline{\Omega} \leq C_2 \|u\|_{k,p}\} \tag{36}$$

for $0 \leq m < K - \frac{n}{p}$ and it follows that $u \in \mathcal{L}^\infty(\Omega)$ and therefore $f(u) \in \mathcal{L}^2(\Omega)$ is thus well-defined. Moreover, from Theorem 3.3 we have

$$\|f(u)\| \leq \|u\|_{0,\infty} \|\nabla u\| \leq C \|A^\gamma u\| \|\nabla u\| = C \|A^\gamma u\| \|A^{\frac{1}{2}} u\|.$$

Also

$$\begin{aligned} \|f(u) - f(v)\| &\leq \|u\|_{0,\infty} \|\nabla(u - v)\| + \|u - v\|_{0,\infty} \|\nabla u\| \\ &\leq C(\|A^\gamma u\| \|A^{\frac{1}{2}} u - A^{\frac{1}{2}^v} v\| + \|A^{\frac{1}{2}} v\| \|A^\gamma u - A^\gamma v\|). \end{aligned} \tag{37}$$

Hence the proof in completed.

4. Conclusion

In this paper, it has been established that ω-order preserving partial contraction mapping generates some results of a general class of semilinear initial value problems.

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