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# Optimal Control of COVID-19 Model with Partial Comorbidity Sub-populations and Two Isolation Treatments in Indonesia

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**Abstract.** We applied sensitivity analysis and optimum control to the COVID-19 model in this research. In addition, the basic reproduction number calculated as 1.57 indicates that this illness is widespread across Indonesia. The most important factor in this model is the contact rate with infected people, with or without comorbidity. Optimal control will minimize the number of infected populations without and with comorbidity, and costs. Numerical experiments will be carried out to describe and compare the graphical models of the spread of COVID-19 with and without controls. From the numerical results and cost-effectiveness analysis on the optimal control problem, it is found that applying a combination of controls can give the best results compared to a single control.

**2020 Mathematics Subject Classifications**: 92D30, 93E20

**Key Words and Phrases**: COVID-19, comorbidity, sensitivity analysis, optimal control

## 1. Introduction

COVID-19 symptoms are usually mild and appear gradually. COVID-19 symptoms include fever, a dry cough, and tiredness. Other symptoms include chest pain and tenderness, nasal congestion, headache, conjunctivitis, diarrhea, loss of taste or smell, skin rash, etc [22, 24]. People who have had diabetes, lung, or heart disease in the past are more likely to get a severe disease with stronger COVID-19 symptoms than people who don't have comorbidity [9, 25]. COVID-19 comorbidity in Indonesia has recorded 12 different diseases, listed from most dangerous to least dangerous: high blood pressure, diabetes, heart disease, pregnancy, lung, kidney, immune disorders, cancer, other respiratory disorders, asthma, tuberculosis, and liver disease [15, 21].

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On March 2, 2020, President Jokowi Widodo directly reported the first case in Indonesia [1]. According to data from the web *worldometer* [23], On October 2, 2020, Indonesia ranked 23 out of 215 nations known to be afflicted [16].

The studies on the COVID-19 model are presented as follows: Das *et al.* [7] adds a sub-population of people infected with comorbidity. This makes the population into five sub-populations. The general congenital disease is the comorbidity of this study. While research from Omame *et al.* [13] also proposed a comorbidity model of COVID-19 with co-morbidity (especially diabetes mellitus). So, Omame *et al.* construct a model by dividing the population into eight sub-populations. Another study, by Rois *et al.*[19], incorporates quarantine and isolation sub-populations; hence, the model splits the population into seven sub-populations. The model is also based on the most recent WHO data, which says that susceptible people must be quarantined first to stop the disease from spreading further. Research on COVID-19 was also conducted by Prathumwan *et al.* [14] by adding quarantine and isolation sub-populations so that the constructed model has six sub-populations.

Managing the developed mathematical model is necessary to lower COVID-19 infections. Researchers discussing control issues include Deressa & Duressa [8], Olaniyi *et al.* [12], and Rois *et al.* [18]. Deressa & Duressa propose three controls: public education, protecting yourself from COVID-19 infection (such as wearing masks, washing hands, and keeping a safe distance), and treating COVID-19 patients in hospitals. While Olaniyi *et al.* and Rois *et al.* provide two controls: public education, and individual care management in hospitals. There are many other studies related to COVID-19 besides the ones listed above. For example, see the following literature [2–4, 6, 11, 17].

The COVID-19 model will be built in this study by combining the research of Das *et al.* [7], Rois *et al.* [19], and Prathumwan *et al.* [14] with two controls: 1) public education ($u_1$), and 2) individual treatment efforts for infected ($u_2$). Model formulation, model validation, sensitivity analysis, effect parameters, and optimum control are discussed. In addition, a numerical simulation of the model is provided. The final topic is cost-effectiveness.

## 2. Results and Discussion

### 2.1. Model formulation and validation

The COVID-19 model consist of eight sub-populations: susceptible ($S$), exposed without comorbidity ($E_N$), exposed with comorbidity ($E_C$), infected without comorbidity ($I_N$), infected with comorbidity ($I_C$), isolated with treatment ($H_T$), isolated without treatment ($H_N$), and recovered ($R$). Furthermore, the COVID-19 model can be presented in a compartment diagram in Figure 1.

The following system of differential equations is derived from the compartmental diagram in Figure 1:

$$\frac{dS}{dt} = \pi - \frac{\beta_1 S I_N}{N} - \frac{\beta_2 S I_C}{N} - \mu S,$$

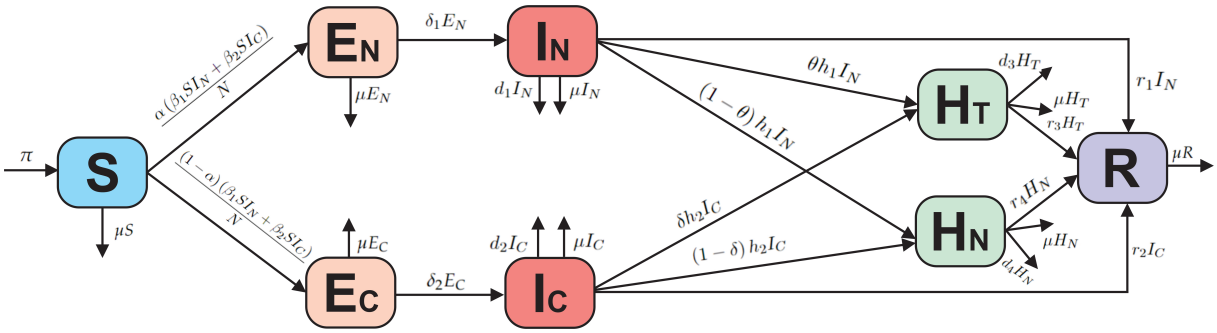


Figure 1: COVID-19 Model with Partial Comorbidity Sub-populations and Two Isolation Treatments

$$
\begin{aligned}
\frac{dE_N}{dt} &= \frac{\alpha\left(\beta_1 SI_N + \beta_2 SI_C\right)}{N} - \delta_1 E_N - \mu E_N,\\
\frac{dE_C}{dt} &= \frac{(1-\alpha)\left(\beta_1 SI_N + \beta_2 SI_C\right)}{N} - \delta_2 E_C - \mu E_C,\\
\frac{dI_N}{dt} &= \delta_1 E_N - h_1 I_N - r_1 I_N - d_1 I_N - \mu I_N,\\
\frac{dI_C}{dt} &= \delta_2 E_C - h_2 I_C - r_2 I_C - d_2 I_C - \mu I_C,\\
\frac{dH_T}{dt} &= \theta h_1 I_N + \delta h_2 I_C - r_3 H_T - d_3 H_T - \mu H_T,\\
\frac{dH_N}{dt} &= (1-\theta)\, h_1 I_N + (1-\delta)\, h_2 I_C - r_4 H_N - d_4 H_N - \mu H_N,\\
\frac{dR}{dt} &= r_1 I_N + r_2 I_C + r_3 H_T + r_4 H_N - \mu R.
\end{aligned}
\tag{1}
$$

Based on Indonesian data, it can be concluded that the MAPE (Mean Absolute Percentage Error) for model validation that we present in Figure 2 using the *lsqcurvefit* command is 0.028579. This time period spans from November 1, 2020 to May 19, 2021. Furthermore, the new parameter values and descriptions are shown in Table 1.

## 2.2. Sensitivity analysis

The basic reproduction number ($R_0$) has a sensitivity index that is differentiation for each of its parameters [5, 20]. $R_0$ for system (1) is found.

$$R_0 = \rho\left(M\right) = R_I + R_C,$$

with $R_I = \frac{\beta_1\delta_1\alpha}{a_1a_3}$ and $R_C = \frac{\beta_2\delta_2(1-\alpha)}{a_2a_4}$.

Here is the parameter sensitivity index $R_0$, with $Z = \beta_1\delta_1 a_2a_4\alpha + \beta_2\delta_2 a_1a_3\left(1-\alpha\right)$.

$$
\begin{aligned}
S_{\beta_1}^{R_0} &= \frac{\delta_1\beta_1\alpha a_2a_4}{Z}, S_{\beta_2}^{R_0} = \frac{\delta_2\beta_2 a_1a_3\left(1-\alpha\right)}{Z}, S_{d_1}^{R_0} = -\frac{\delta_1\beta_1 d_1 a_2a_4\alpha}{a_3Z},\\
S_{\alpha}^{R_0} &= \frac{\left(\delta_1\beta_1 a_2a_4 - \delta_2\beta_2 a_1a_3\right)\alpha}{Z}, S_{\delta_1}^{R_0} = \frac{\beta_1\delta_1 a_2a_4\alpha}{Z}\left(1 - \frac{\delta_1}{a_1}\right),
\end{aligned}
$$

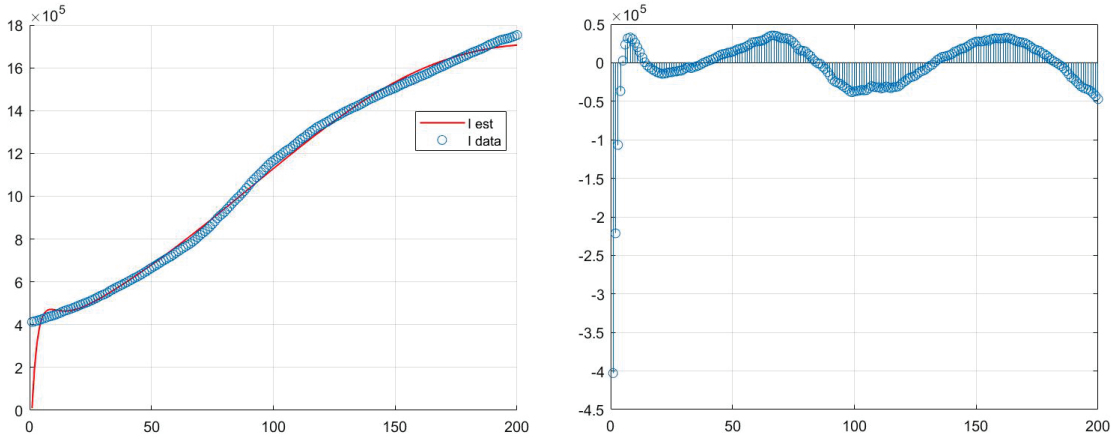


Figure 2: Model validation

$$S_{\delta_2}^{R_0} = \frac{\beta_2\delta_2 a_1 a_3\,(1-\alpha)}{Z}\left(1-\frac{\delta_2}{a_2}\right), S_{h_1}^{R_0} = -\frac{\delta_1\beta_1 h_1 a_1 a_2 a_4\alpha}{a_3 Z},$$
$$S_{r_1}^{R_0} = -\frac{\delta_1\beta_1 r_1 a_2 a_4\alpha}{a_3 Z}, S_{d_2}^{R_0} = -\frac{\delta_2\beta_2 d_2 a_1 a_3\,(1-\alpha)}{a_4 Z},$$
$$S_{h_2}^{R_0} = -\frac{\delta_2\beta_2 h_2 a_1 a_3\,(1-\alpha)}{a_4 Z}, S_{r_2}^{R_0} = -\frac{\delta_2\beta_2 r_2 a_1 a_3\,(1-\alpha)}{a_4 Z}, \text{ and}$$
$$S_{\mu}^{R_0} = -\frac{\delta_1\beta_1 a_2 a_4\alpha\mu\left(\frac{1}{a_1}+\frac{1}{a_3}\right)+\delta_2\beta_2 a_1 a_3\mu\,(1-\alpha)\left(\frac{1}{a_2}+\frac{1}{a_4}\right)}{Z}.$$

Table 2 displays the parameter sensitivity index, which indicates that $\beta_2$ and $\beta_1$ are the most sensitive parameters.

## 2.3. Effect parameters

Using contour plots, the effect of parameters on $R_0$ was investigated. We pick three significant parameters ($\beta_1$, $\beta_2$, and $h_2$) and plot them as a function of $R_0$. Figure 3 investigates the impact of some $R_0$ parameters further and shows that increasing parameters $\beta_1$ and $\beta_2$ as parameters with a positive index can increase the value of $R_0$. This implies that more personal interaction will accelerate the transmission of COVID-19. Meanwhile, increasing the parameter with a negative index, namely the parameter $h_2$, can reduce the value of $R_0$ and implies that increased isolation will reduce the spread of COVID-19. As a result, It is essential to improve education and isolation to stop the development of COVID-19.

Table 1: The new parameter values and descriptions

| Parameter | Value | Descriptions |
|---|---|---|
| $\pi$ | 3783175.865 | recruitment or birth rate |
| $\beta_1$ | 0.5524 | contact rate (without comorbidity) |
| $\beta_2$ | 0.55348 | contact rate (with comorbidity) |
| $\alpha$ | 0.49383 | contact proportions |
| $\delta_1$ | 0.028911 | progressions rate from exposed to infection (without comorbidity) |
| $\delta_2$ | 0.22241 | progressions rate from exposed to infection (with comorbidity) |
| $\delta$ | 0.2349 | proportion of isolation from infection (with comorbidity) |
| $\theta$ | 0.25353 | proportion of isolation from infection (without comorbidity) |
| $h_1$ | 0.11791 | isolation rate from infection (without comorbidity) |
| $h_2$ | 0.11161 | isolation rate from infection (with comorbidity) |
| $r_1$ | 0.087527 | recovery rate |
| $r_2$ | $3.987 \times 10^{-5}$ | recovery rate |
| $r_3$ | 0.54385 | recovery rate |
| $r_4$ | 0.37245 | recovery rate |
| $d_1$ | 0.036233 | COVID-19 death rate |
| $d_2$ | 0.18549 | COVID-19 death rate |
| $d_3$ | 0.28641 | COVID-19 death rate |
| $d_4$ | 0.34042 | COVID-19 death rate |
| $\mu$ | 0.0138 | natural death rate |

## 2.4. Optimal control

The COVID-19 model by incorporate control variables $u_1$ (public education) and $u_2$ (individual treatment efforts for infected) is given by

$$\begin{aligned}
\frac{dS}{dt} &= \pi - \frac{(1-u_1)(\beta_1 SI_N + \beta_2 SI_C)}{N} - \mu S,\\
\frac{dE_N}{dt} &= \frac{\alpha(1-u_1)(\beta_1 SI_N + \beta_2 SI_C)}{N} - \delta_1 E_N - \mu E_N,\\
\frac{dE_C}{dt} &= \frac{(1-\alpha)(1-u_1)(\beta_1 SI_N + \beta_2 SI_C)}{N} - \delta_2 E_C - \mu E_C,\\
\frac{dI_N}{dt} &= \delta_1 E_N - (h_1+u_2) I_N - r_1 I_N - d_1 I_N - \mu I_N,\\
\frac{dI_C}{dt} &= \delta_2 E_C - (h_2+u_2) I_C - r_2 I_C - d_2 I_C - \mu I_C,\\
\frac{dH_T}{dt} &= (\theta h_1 + u_2) I_N + (\delta h_2 + u_2) I_C - r_3 H_T - d_3 H_T - \mu H_T,\\
\frac{dH_N}{dt} &= ((1-\theta) h_1 + u_2) I_N + ((1-\delta) h_2 + u_2) I_C - r_4 H_N - d_4 H_N - \mu H_N,\\
\frac{dR}{dt} &= r_1 I_N + r_2 I_C + r_3 H_T + r_4 H_N - \mu R.
\end{aligned} \tag{2}$$

Table 2: Sensitivity analysis.

| Parameter | $\beta_2$ | $\beta_1$ | $d_2$ | $\mu$ | $h_1$ | $h_2$ |
|---|---|---|---|---|---|---|
| Index | 0.53996 | 0.46004 | −0.32211 | −0.229 | −0.21233 | −0.19382 |
| Parameter | $r_1$ | $\delta_1$ | $\alpha$ | $d_1$ | $\delta_2$ | $r_2$ |
| Index | −0.15762 | 0.14864 | −0.06675 | −0.06525 | 0.03155 | −0.000069 |

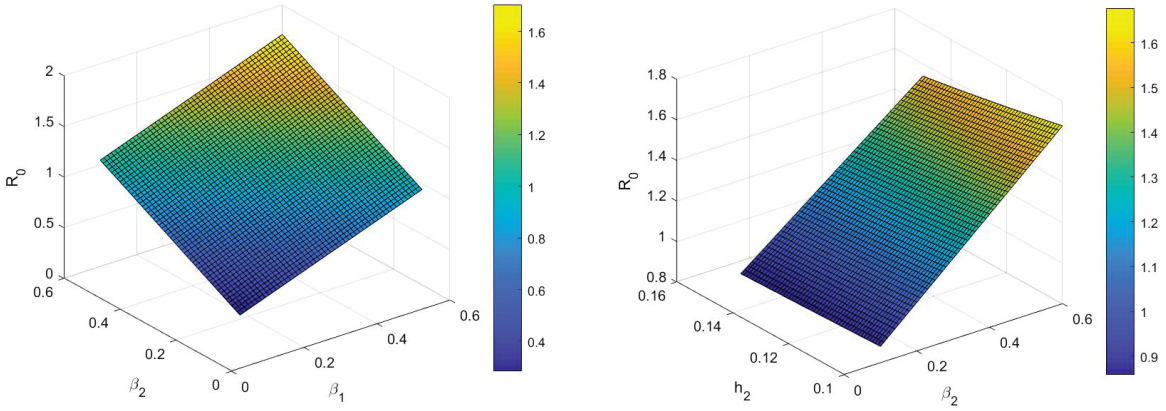


Figure 3: Effect parameters of $R_0$.

Over a time range of $[0, T]$, the function that minimizes the number of infected cases without and with comorbidity can be written as

$$F(u_1, u_2) = \int_0^T v\left(t, \overrightarrow{x}, \overrightarrow{u}\right) dt = \int_0^T \left(I_N + I_C + \frac{1}{2}\left(K_1 u_1^2 + K_2 u_2^2\right)\right) dt, , \tag{3}$$

where $K_1$ and $K_2$ are the relative cost associated with the controls $u_1$ and $u_2$, and $T$ is the final time. The objective of the control is to reduce the cost function.

$$F(u_1^*, u_2^*) = \min F(u_1, u_2),$$

subject to the system (2), where $0 \leq (u_1, u_2) \leq 1$ and $t \in (0, T)$.

## 2.5. Optimal Control Analysis

The Hamilton function can be defined as follows

$$\begin{aligned}\mathcal{H} = {} & I_N + I_C + \frac{1}{2}\left(K_1 u_1^2 + K_2 u_2^2\right) + \tau_1 \left(\pi - \frac{(1-u_1)\left(\beta_1 S I_N + \beta_2 S I_C\right)}{N} - \mu S\right) \\ & + \tau_2 \left(\frac{\alpha\,(1-u_1)\left(\beta_1 S I_N + \beta_2 S I_C\right)}{N} - \delta_1 E_N - \mu E_N\right) \\ & + \tau_3 \left(\frac{(1-\alpha)\,(1-u_1)\left(\beta_1 S I_N + \beta_2 S I_C\right)}{N} - \delta_2 E_C - \mu E_C\right)\end{aligned}$$

$$
\begin{aligned}
&+\tau_4\left(\delta_1 E_N-\left(h_1+u_2\right)I_N-r_1 I_N-d_1 I_N-\mu I_N\right)+\tau_5\left(\delta_2 E_C-\left(h_2+u_2\right)I_C-r_2 I_C-d_2 I_C-\mu I_C\right)\\
&+\tau_6\left(\left(\theta h_1+u_2\right)I_N+\left(\delta h_2+u_2\right)I_C-r_3 H_T-d_3 HT-\mu H_T\right)\\
&+\tau_7\left(\left(\left(1-\theta\right)h_1+u_2\right)I_N+\left(\left(1-\delta\right)h_2+u_2\right)I_C-r_4 H_N-d_4 H_N-\mu H_N\right)\\
&+\tau_8\left(r_1 I_N+r_2 I_C+r_3 H_T+r_4 H_N-\mu R\right).
\end{aligned}
\tag{4}
$$

Deriving the Hamilton function (4) for each co-state variable as (2) yields the equation state. The next step is to realize that the negative value of the Hamilton function (4) derivative for each state variable is the co-state, which is represented by the following equation.

$$
\begin{aligned}
\frac{d\tau_1}{dt}&=-\frac{\partial H}{\partial S}=\left(\tau_1-\alpha\tau_2-\left(1-\alpha\right)\tau_3\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 I_N+\beta_2 I_C\right)}{N}\right)\\
&\quad+\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\mu\tau_1,\\
\frac{d\tau_2}{dt}&=-\frac{\partial H}{\partial E_N}=\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\left(\tau_2-\tau_4\right)\delta_1+\mu\tau_2,\\
\frac{d\tau_3}{dt}&=-\frac{\partial H}{\partial E_C}=\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\left(\tau_3-\tau_5\right)\delta_2+\mu\tau_3,\\
\frac{d\tau_4}{dt}&=-\frac{\partial H}{\partial I_N}=\left(\tau_1-\alpha\tau_2-\left(1-\alpha\right)\tau_3\right)\left(\frac{\left(1-u_1\right)\beta_1 S}{N}\right)\\
&\quad+\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\tau_4\left(d_1+\mu\right)\\
&\quad+h_1\left(\tau_4-\theta\tau_6-\left(1-\theta\right)\tau_7\right)+u_2\left(\tau_4-\tau_6-\tau_7\right)+r_1\left(\tau_4-\tau_8\right)-1,\\
\frac{d\tau_5}{dt}&=-\frac{\partial H}{\partial I_C}=\left(\tau_1-\alpha\tau_2-\left(1-\alpha\right)\tau_3\right)\left(\frac{\left(1-u_1\right)\beta_2 S}{N}\right)\\
&\quad+\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\lambda_5\left(d_2+\mu\right)\\
&\quad+h_2\left(\tau_5-\delta\tau_6-\left(1-\delta\right)\tau_7\right)+u_2\left(\tau_5-\tau_6-\tau_7\right)+r_2\left(\tau_5-\tau_8\right)-1,\\
\frac{d\tau_6}{dt}&=-\frac{\partial H_T}{\partial H}=\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\tau_6\left(d_3+\mu\right)+r_3\left(\tau_6-\tau_8\right),\\
\frac{d\tau_7}{dt}&=-\frac{\partial H}{\partial H_N}=\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\tau_7\left(d_4+\mu\right)+r_4\left(\tau_7-\tau_8\right),\\
\frac{d\tau_8}{dt}&=-\frac{\partial H}{\partial R}=\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)\left(\frac{\left(1-u_1\right)\left(\beta_1 S I_N+\beta_2 S I_C\right)}{N^2}\right)+\mu\tau_8.
\end{aligned}
\tag{5}
$$

with transverse condition

$\tau_1(T)=\tau_2(T)=\tau_3(T)=\tau_4(T)=\tau_5(T)=\tau_6(T)=\tau_7(T)=\tau_8(T)=0.$

So, the optimal control of $u_1^*$ and $u_2^*$ can be written as

$$
u_1^*=\frac{\left(\beta_1 S^* I_N^*+\beta_2 S^* I_C^*\right)\left(\alpha\tau_2+\left(1-\alpha\right)\tau_3-\tau_1\right)}{NK_1},u_2^*\ =\frac{I_N^*\left(\tau_4-\tau_6-\tau_7\right)+I_C^*\left(\tau_5-\tau_6-\tau_7\right)}{K_2}
$$

## 3. Simulation

The forward-backward sweep method is used to solve this optimal control problem [10]. In this numerical simulation, the values of the parameters used are shown in Table 1. These values are based on the COVID-19 case in Indonesia, and the final time ($T$) is 100 days. Next, the initial values given are as follows $S(0) = 270911990$, $E_N(0) = 1000000$, $E_C(0) = 10000$, $I_N(0) = 412784$, $I_C(0) = 500000$, $H_T(0) = 56899$, $H_N(0) = 200000$, and $R(0) = 341942$, with simulation intervals $t \in [0, 100]$. Following are the findings of the simulation of optimum numerical control:

### 3.1. Strategy 1 ($u_1 \neq 0$ and $u_2 = 0$)

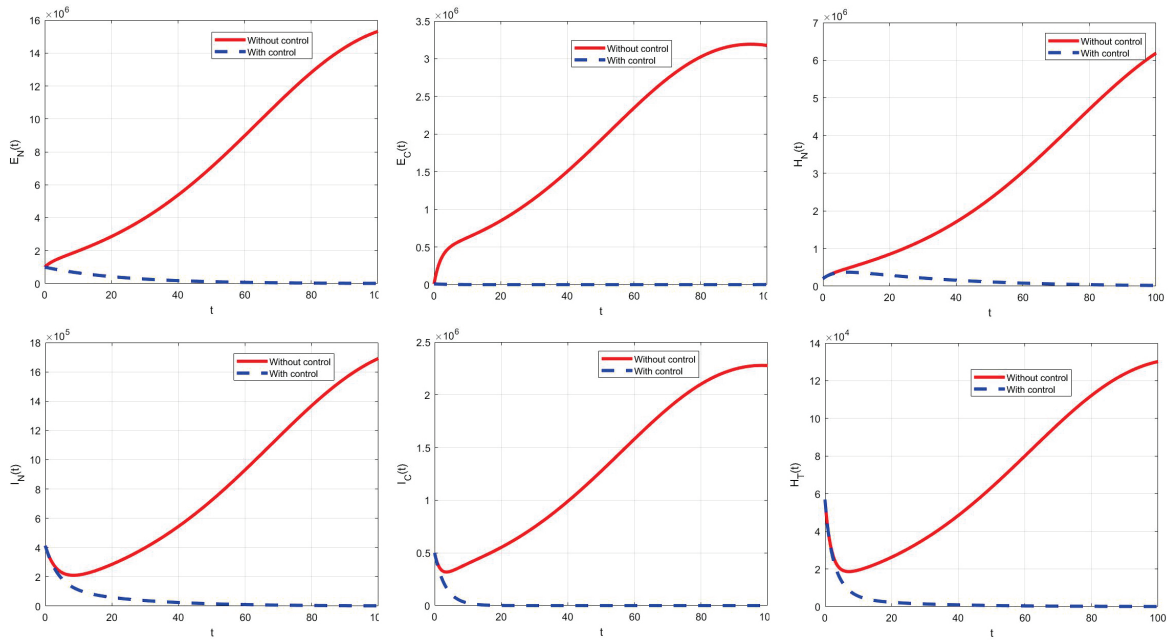


Figure 4: Optimal control simulation results with $u_1 \neq 0$.

Figure 4 shows the control strategy, which is $u_1 \neq 0$ and $u_2 = 0$. This result of education gives people a constant sense of caution when interacting with others outside the home. By using this strategy, the number of people who are exposed (with and without comorbidity), infected (with and without comorbidity), and isolated (with and without treatment) is significantly reduced. Furthermore, Figure 5 shows the control strategy profiles $u_1 \neq 0$ and $u_2 = 0$ for reducing the number of COVID-19 cases during $t = 100$. The control strategy $u_1 \neq 0$ and $u_2 = 0$ is given by the maximum from the beginning of the period to $t = 94$. At the end of the period, $u_1 \neq 0$ and $u_2 = 0$ decrease by a large amount to reach zero. Control is ended at the end of the period, which means no more control is given.

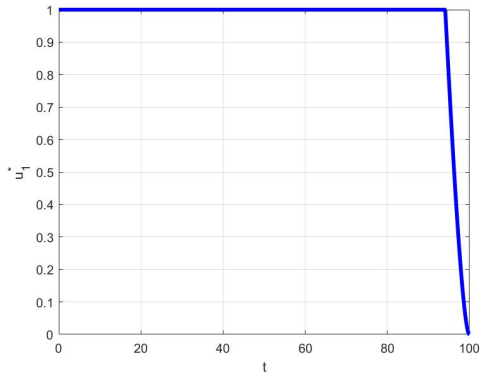


Figure 5: Optimal control profile with $u_1 \neq 0$.

## 3.2. Strategy 2 ($u_1 = 0$ and $u_2 \neq 0$)

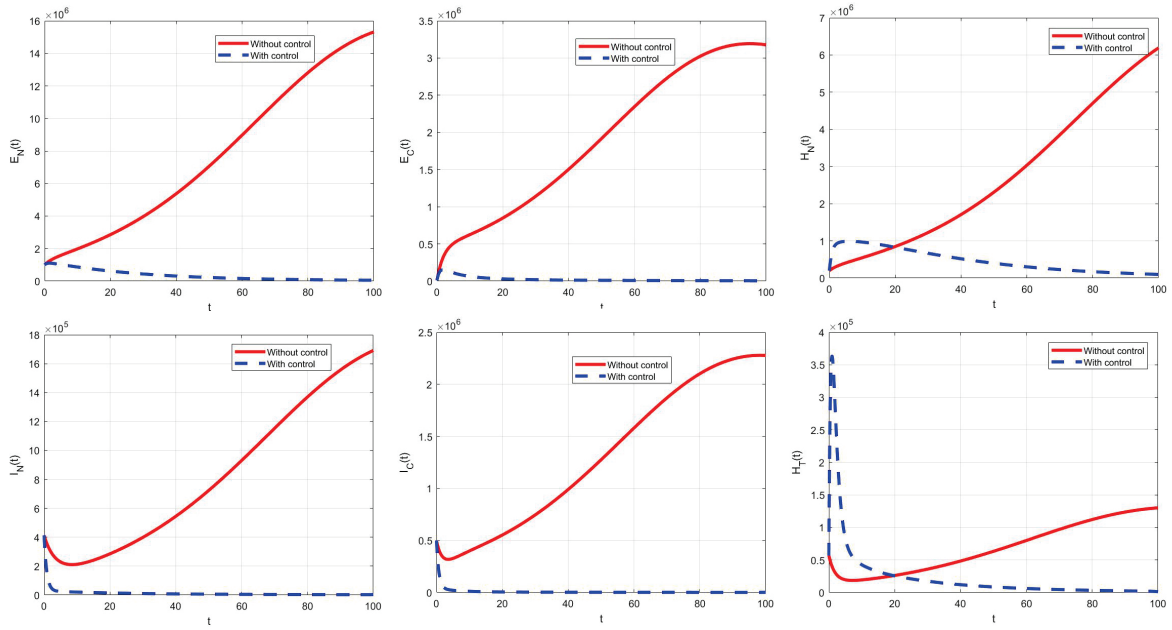


Figure 6: Optimal control simulation results with $u_2 \neq 0$.

Figure 6 shows that $u_1 = 0$ and $u_2 \neq 0$ are the control strategy. Because there is more care for infected people, this strategy can reduce the number of people infected with or without comorbidity. By using this strategy, the number of people who are exposed (with or without comorbidity), infected (with or without comorbidity), and isolated (with or without treatment) is reduced by a large amount. Figure 7 depicts the profiles of $u_1 = 0$ and $u_2 \neq 0$ control strategies for reducing the number of COVID-19 instances for $t = 100$. Furthermore, the management strategy $u_1 = 0$ and $u_2 \neq 0$ are given by maximum from the

beginning to $t = 31.4$ and then decline gradually until $t = 100$ approaches zero, indicating that control is terminated at the end of the period.

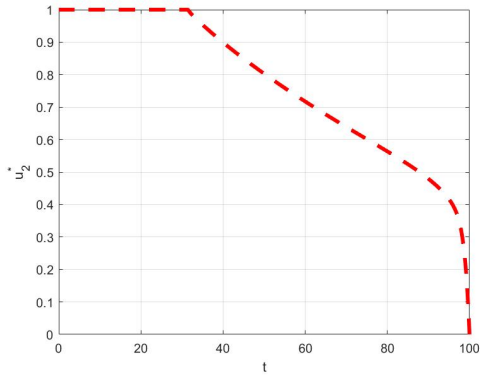


Figure 7: Optimal control profile with $u_2 \neq 0$.

## 3.3. Strategy 3 ($u_1 \neq 0$ and $u_2 \neq 0$)

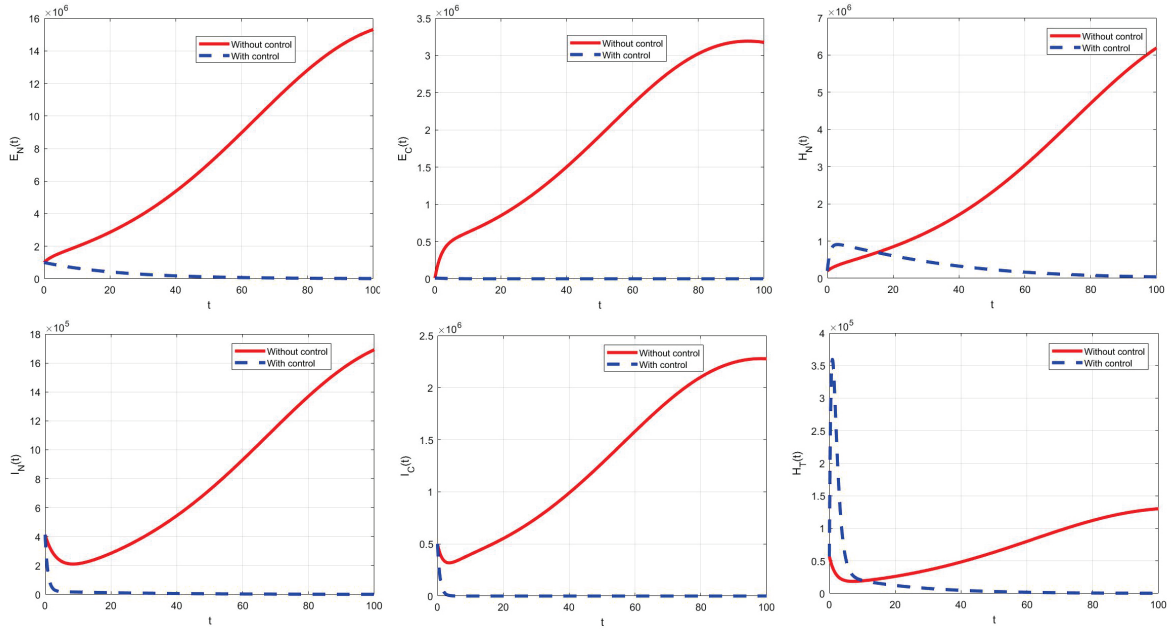


Figure 8: Combined optimal control simulation results.

The combination control approach is depicted in Figure 8. This is the outcome of education urging people to use caution at all times (such as while dealing with others outside the home) and increased care for affected individuals. Combined control tactics

can greatly minimize or limit deployment. In addition, Figure 9 depicts the combined control plan profile for reducing the number of COVID-19 cases during $t = 100$. The combined control technique incorporates two controls. Control $u_1$ has a maximum value of $t = 84.3$ and progressively approaches 0 as time progresses. Then, $u_2$ is set to a maximum value of $t = 5.4$ before declining till $t = 100$ reaches zero gradually. Both controls expire at the end of time, rendering them powerless over $u_1$ and $u_2$.

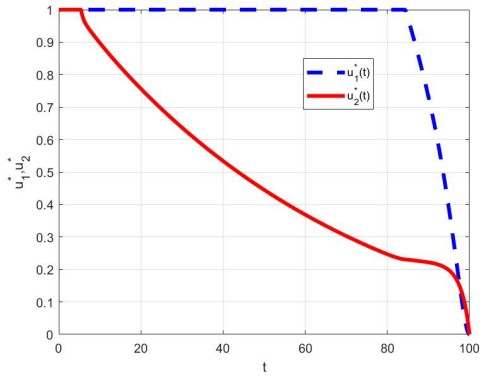


Figure 9: Combined control profile.

### 3.4. Total infections comparison

Different starting values are given for the sub-populations that were exposed. Figure 10 shows the total number of infected sub-populations for four different initial conditions of the exposed sub-populations: $E_N(0) = 200000$, $E_N(0) = 1000000$, $E_N(0) = 10000000$, and $E_N(0) = 100000000$. Figure 10 shows that when the third strategy is used instead of the other strategies, the number of infected sub-populations goes down. The above example of an initial value is meant to help figure out how a disease will spread by showing several ways to stop it. Based on strategies 1 to 3, we can say that strategy 3 is the best way to reduce the number of people in the community infected with COVID-19.

## 4. Cost evaluation

The objective of the cost analysis is to identify the COVID-19 spread control approach with the lowest cost-effectiveness ratio. This study evaluates costs using ACER (Average Cost-Effectiveness Ratio) and ICER (Incremental Cost-Effectiveness Ratio). ACER is mathematically defined as follows, according to the approach of cost-effectiveness analysis:

$$ACER = \frac{\text{Objective function } (F)}{\text{Total number of infections averted}}$$

The most cost-effective use of ACER is the smallest ACER value. The ACER value is presented in Table 3.

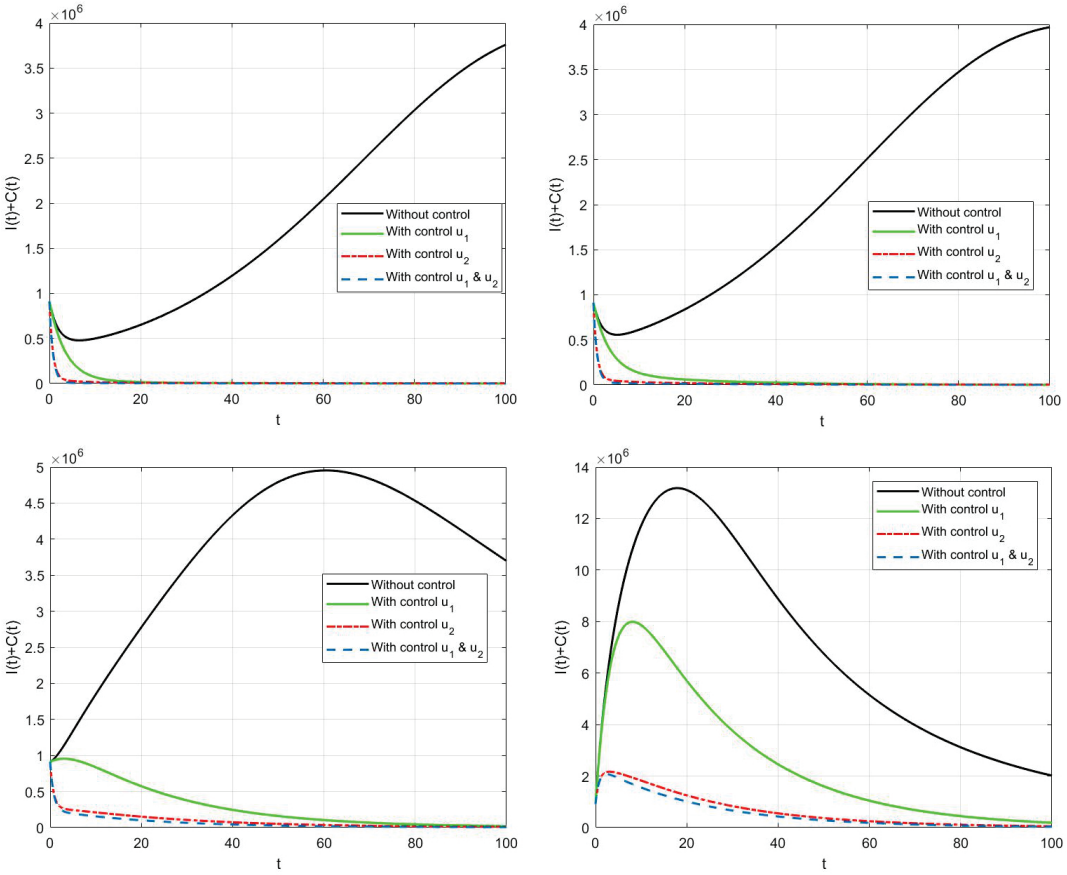


Figure 10: Total subpopulation infected using various strategies.

In addition, ICER analyzes two competing intervention choices for the same scarce resource, follows costs and converts them to health benefits. Considering the p and q methods as two competing control intervention techniques, the ICER is defined as follows:

$$ICER = \frac{\text{Change in total costs for p and q strategies}}{\text{Control benefits in strategies p and q change}}.$$

ICER was computed in order to identify the most cost-effective control strategy among the available options. First, calculate the competition for strategy 1 and strategy 2 using the following formula:

$$ICER\,(1) = \frac{5,918,200 - 0}{53,996,000,000 - 0} = 0.0001096,$$
$$ICER\,(2) = \frac{2,533,500 - 5,918,200}{54,000,000,000 - 53,996,000,000} = -\frac{3,384,700}{4,000,000} = -0.846,$$

ICER results for strategy 1 were greater than those for strategy 2, indicating that educational controls alone were more expensive and ineffective than medical care enhancement

Table 3: Total infections prevented, total costs, and ACER for strategies 1, 2, and 3.

| Strategy | Infections prevented | Total cost (thousands) | ACER |
|---|---|---|---|
| No strategy | 0 | 0 | 0 |
| Strategy 1 | 53,996,000,000 | 5,918,200 | 0.0001096 |
| Strategy 2 | 54,000,000,000 | 2,533,500 | 0.0000469 |
| Strategy 3 | 54,001,000,000 | 1,738,200 | 0.0000322 |

controls. Thus, strategy 1 is eliminated from the list of potential control strategies. The ICER for strategies 2 and 3 is then recalculated as follows:

$$ICER\,(2) = \frac{2,533,500 - 0}{54,000,000,000 - 0} = 0.0000469,$$
$$ICER\,(3) = \frac{1,738,200 - 2,533,500}{54,001,000,000 - 54,000,000,000} = -\frac{795,300}{1,000,000} = -0,795.$$

Strategy 2 has a greater ICER than strategy 3. Due to its cost-effectiveness and ability to prevent the spread of infectious illnesses, strategy 3 (combined control) is the best control plan of all possibilities.

## 5. Conclusions

The COVID-19 model uses public health education and improves the care of infected individuals. The Comorbidity of COVID-19 with other diseases can be controlled by implementing public health education and improving the care of infected individuals. The sensitivity analysis of model parameters in this paper shows that contact rate with comorbidity ($\beta_2 = 0.53996$) and without comorbidity ($\beta_1 = 0.46004$) have the biggest effect on $R_0$. Numerical studies, comparison of total infections, and cost-effectiveness analysis of the optimal control problem show that using controls together can improve performance compared to using one control alone.

## References

[1] W. Adisasmito, A. Suwandono, Trihono, A. Gani, D. N. Aisyah, D. A. Solikha, A. Fitria, M. Kim, C. Jeon, J. Jang, Y. H. Kim, B. J. Kim, J. Seo, and S. Lee. *Studi Komparasi Pembelajaran Penanganan COVID-19 Indonesia-Korea Selatan.* Direktorat Kesehatan dan Gizi Masyarakat Kementerian PPN/ BAPPENAS, 2021.

[2] I. A. Baba, B. A. Nasidi, and D. Baleanu. Optimal control model for the transmission of novel covid-19. *Computers, Materials & Continua*, 66:3089–3106, 2021.

[3] E. Bonyah, M. Juga, and Fatmawati. Fractional dynamics of coronavirus with comorbidity via caputo-fabrizio derivative. *Communications in Mathematical Biology and Neuroscience*, 2022:12, 2022.

[4] I. O. Onwubuya C. E. Madubueze, S. Dachollom. Controlling the spread of covid-19: Optimal control analysis. *Computational and Mathematical Methods in Medicine*, 2020:6862516, 2020.

[5] N. Chitnis, J. M. Hyman, and J. M. Cushing. Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. *Bulletin of Mathematical Biology*, 70:1272, 2008.

[6] P. Das, S.S. Nadim, S. Das, and P. Das. Dynamics of covid-19 transmission with comorbidity: a data driven modelling based approach. *Nonlinear Dynamics*, 106:1197–1211, 2021a.

[7] P. Das, R. K. Upadhyay, A. K. Misra, F. A. Rihan, P. Das, and D. Ghosh. Mathematical model of covid-19 with comorbidity and controlling using non-pharmaceutical interventions and vaccination. *Nonlinear Dynamics*, 106:1213–1227, 2021b.

[8] C. T. Deressa and G. F. Duressa. Modeling and optimal control analysis of transmission dynamics of covid-19: The case of ethiopia. *Alexandria Engineering Journal*, 60:719–732, 2020.

[9] W. Guan, W. Liang, Y. Zhao, H. Liang, Z. Chen, Y. Li, X. Liu, R. Chen, C. Tang, T. Wang, C. Ou, L. Li, P. Chen, L. Sang, W. Wang, J. Li, C. Li, L. Ou, B. Cheng, S. Xiong, Z. Ni, J. Xiang, Y. Hu, L. Liu, H. Shan, C. Lei, Y. Peng, L. Wei, Y. Liu, Y. Hu, P. Peng, J. Wang, J. Liu, Z. Chen, Z., G. Li, Z. Zheng, S. Qiu, J. Luo, C. Ye, S. Zhu, L. Cheng, F. Ye, S. Li, J. Zheng, N. Zhang, N. Zhon, and J. He. Comorbidityity and its impact on 1590 patients with covid-19 in china: a nationwide analysis. 55:2000547, 2020.

[10] S. Lenhart and J. T. Workman. *Optimal Control Applied to Biological Models*. Chapman and Hall/CRC, New York, 2007.

[11] S. S. Musa, S. Qureshi, S. Zhao, A. Yusuf, U. T. Mustapha, and D. He. Mathematical modeling of covid-19 epidemic with effect of awareness programs. *Infectious Disease Modelling*, 6:448–460, 2021.

[12] S. Olaniyi, O. S. Obabiyi, K. O. Okosun, A. T. Oladipo, and S. O. Adewale. Mathematical modelling and optimal cost-effective control of covid-19 transmission dynamics. *The European Physical Journal Plus*, 135:938, 2020.

[13] A. Omame, N. Sene, I. Nometa, C. I. Nwakanma, E. U. Nwafor, N. O. Iheonu, and D. Okuonghae. Analysis of covid-19 and comorbidity co-infection model with optimal control. *Optimal Control Applications and Methods*, 42:1568–1590, 2021.

[14] D. Prathumwan, K. Trachoo, and I. Chaiya. Mathematical modeling for prediction dynamics of the coronavirus disease 2019 (covid-19) pandemic, quarantine control measures. *Symmetry*, 12:1404, 2020.

[15] M A Rois, Fatmawati, and C Alfiniyah. Mathematical modeling of covid-19 with partial comorbid subpopulations and two isolation treatments in indonesia. *International Journal of Mathematics and Computer Science*, 18:233–242, 2023.

[16] M A Rois, Fatmawati, C Alfiniyah, and C W Chukwu. Dynamic analysis and optimal control of covid-19 with comorbidity: A modeling study of indonesia. *Front. Appl. Math. Stat*, 8, 2023.

[17] M A Rois, M Tafrikan, Y Norasia, I Anggriani, and M. Ghani. Seihr model on spread of covid-19 and its simulation. *Telematika*, 15, 2022.

[18] M. A. Rois, Trisilowati, and U. Habibah. Optimal control of mathematical model for covid-19 with quarantine and isolation. *International Journal of Engineering Trends and Technology*, 69:154–160, 2021.

[19] M. A. Rois, T. Trisilowati, and U. Habibah. Dynamic analysis of covid-19 model with quarantine and isolation. *JTAM (Jurnal Teori dan Aplikasi Matematika)*, 5:418–433, 2021.

[20] M. A. Rois, T. Trisilowati, and U. Habibah. Local sensitivity analysis of covid-19 epidemic with quarantine and isolation using normalized index. *Telematika*, 14:13–24, 2021.

[21] Satgas. Peta sebaran covid-19. 2021.

[22] WHO. Novel coronavirus. 2020.

[23] worldometers. Report coronavirus cases. 2021.

[24] Y. C. Wu, C. S. Chen, and Y. J. Chan. The outbreak of covid-19: An overview. *Journal of the Chinese Medical Association*, 83:217–220, 2020.

[25] J. Yang, Y. Zheng, X. Gou, K. Pu, Z. Chen, and Q. Guo. Prevalence of comorbidities and its effects in patients infected with sars-cov-2: a systematic review and meta-analysis. *International Journal of Infectious Diseases Journal*, 94:91–95, 2020.