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A Numerical Simulation of Convection and Conduction Heat Transfer for a Fluid in a Porous Medium

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Abstract. Building a computational formula which is expressed by a system of nonlinear partial differential equations in two components was done in this job. The matter of energy transfer by free convection of a dissipative liquid running in a horizontal channel with a porous medium was examined in this study, and we describe the behavior of fluid flow through the channel and the temperature distribution within it. Numerical methods were used as the ODE45 method, one of the Rang-Kutta methods for processing the resulting differential equations. The effect of the Prandtl number, the Schmidt multitude, and the Gratshof amount were also studied.

2020 Mathematics Subject Classifications: 65Lxx, 65Mxx, 80Mxx

Key Words and Phrases: Heat Transfer, Prandtl number, Schmidt number, Gratshof number, ODE45 Method.

1. Introduction

The flow of reactive viscous fluids via porous media is a theoretically tricky issue in many scientific, technological, and engineering fields. Food drying, extraction of geothermal energy, removal of nuclear waste, thermal and liquid flow inside of internal tissue, constructing insulation, water supply flow, the output of oil and gas, oxidizer astrophysical, Metal extraction and exfoliation, magneto hydrodynamic (MHD) pumps and turbines, aircraft thrusters and ship propellers, and vehicle destroy devices are all examples of extreme flow systems. Such flow systems naturally contain exothermic reactions. As a result, we examine the thermal effects and stability requirements for reactive varying non-Newtonian fluids flowing instability through saturated porous substrates . Since they make up the majority of chemical fluids, this study focuses on non-Newtonian fluids. Differential fluids, such as third-grade fluid, will be given particular care. [13]. Several researchers have made

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contributions to this field.

Wright et al. [17] looked at the effect of on specified surface temperature flux by convective on a nanofluid flow in a horizontal sheet . A study of the effect of a continuous horizontal mass modulus on the thermal conductivity of a conical duct with porous walls by Yih [18]. Gupta and Gupta [6] explored the movement of mass and heat along a sliding plate with suction or blow acts. Andersson et al. [3] then generalized Wang's problem to the context of radiant heat. Abel et al. [1] have used the magnetic properties effect to examine the influence of variable viscosity on unsettled radiant heat and flow in a fluid flick above a boundary layer flow. Noor and Hashim [12] looked into the effects of magnetization on an unreliable mutable stretching plate in a tinny ocean segment. Hasanuzzaman et al. [9] explored the Dufour and thermal permeability impacts on the occasional free magnetic characteristics of heat flow passing via an unbounded vertical permeable layer. Infrared energy's affects on irregular energy transmission and stream in a thin liquid film across a straining slab in a saturated medium were examined by Khader and Megahed [10].

Mehta et al. [15] explored cyclical heat and flow transmission through a plastic structure between steel plates, as well as heat exchange flux and boiler, in the proximity of oblique ferrites. Hammodat A. et al.,[7], deal with this problem of convection and radiation heat transfer of a fluid in a porous channel under influence of a magnetic field using the opposite direction implicitly technique. We observed that shifting the temperature of the fluid within the tube had an important influence on the measurements. In the same year, Hammodat A. et al. [8] analyse the flow of fluids in a bridge under the influence of an electromagnetic force (EMF) and actually fix the partial differential equations that describe the situation using the strategy of lines. Besides that, we aspire to demonstrate the behavior of temperature within the cross-section as well as the effects of physical quantities.

Tarammim et al.[14] examenid instable MHD free convection stream over a perpendicular dish in a two-dimensional coordinate system and examined the stability conditions using an explicit finite difference method. They examined the governing equations of unstable free convection heat transfer flow in the presence of radiant energy in 2021 (see [16]). Nagarju [11] studied the behavior of fluid velocity and temperature by utilizing the finite difference approach, and they evaluated the impact of misaligned magnetic force and temperature radiation on unstable MHD flow on a sloped plate. Bordoloi [4] They tested the free convection of a fluid flowing in a narrow, almost infinite porous plate. The results showed the effect of temperature and velocity in the matter and under the presence of thermal radiation and chemical interaction. The goal of this study is to examine numerically transient combined mass and heat exchange by mixed convection flow in a moveable parallel rectangular layer. The basic equations of the issue are a group of partial differential equations that are transformed into nondimensional equations. To solve the problem numerically, the ODE45 method was used. The results of this investigation will be presented and graphically displayed for well-known parameters with varying parameters.

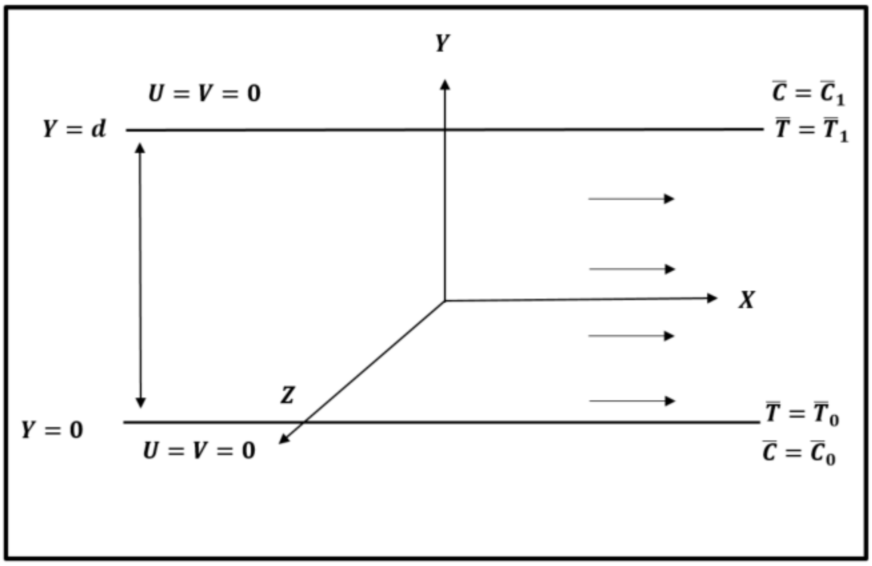

Figure 1: Physical geometry.

2. Basic Equations and Problem Formulation

In a power generation system, the simultaneous mass and heat transmission in mixed convection rivers on a rising horizontal porous material with thermal diffusion is regarded. Let the $X - axis$ be chosen along the porous plate in the flow path and the $Y - axis$ is vertical to the channel. Consider the temperatures at the two horizontal walls to be constant and $\bar{T}_1$ represents the temperature range of the horizontally, $\bar{T}_0$ unchanged where $\bar{T}_1 > \bar{T}_0$.

As well as let U, V be the components of velocity in the channel's bottom (top) walls given X and Y directions, respectively, and that all components in the Z direction have disappeared. Figure 1 depicts a drawing of the structure and coordinate system.

According to the aforementioned premises, the governing equations for continuity, momentum, energy, and concentration can be written as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

$$\frac{\partial}{\partial T}\left[\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}\right]+\left[\frac{\partial}{\partial X}\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)-\frac{\partial}{\partial Y}\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)\right]$$
$$=\upsilon\nabla^2\left[\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}\right]+\frac{\upsilon}{\kappa}\left[\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}\right], \quad (2)$$

$$\frac{\partial\bar{T}}{\partial T}+U\frac{\partial\bar{T}}{\partial X}+V\frac{\partial\bar{T}}{\partial Y}=\frac{\kappa^*}{\rho C_p}\left[\frac{\partial^2\bar{T}}{\partial X^2}+\frac{\partial^2\bar{T}}{\partial Y^2}\right]+\frac{\upsilon}{C_p}\left[\left(\frac{\partial U}{\partial Y}\right)^2\right], \quad (3)$$

$$\frac{\partial\bar{C}}{\partial T}+U\frac{\partial\bar{C}}{\partial X}+V\frac{\partial\bar{C}}{\partial Y}=D\left[\frac{\partial^2\bar{C}}{\partial X^2}+\frac{\partial^2\bar{C}}{\partial Y^2}\right], \quad (4)$$

T is the occurrence and $\bar{T},\bar{C},\rho,\upsilon,p,k,k^*,C_p$ are the aspects to evaluate which are temperature, concentrations, density, kinematic viscosity, pressures, media porosity, thermal transfer, and specific heat at constant pressure. The incremental boundary conditions accompanying (1)-(4) are:

$$\left.\begin{array}{l} U=V=0,\\ \bar{T}=\bar{T}_0,\bar{T}_1,\\ \bar{C}=\bar{C}_0,\bar{C}_1, \end{array}\right\} \quad \text{at} \quad Y=0,d. \quad (5)$$

In order to remove dimensions from the governing equations under the initial and boundary conditions, the following variables are introduced [5]

$$\left.\begin{array}{l} \tilde{x}=\frac{X}{d},\quad \tilde{y}=\frac{Y}{d},\quad \tilde{u}=\frac{Ud}{\upsilon\sqrt{Gr}},\quad \tilde{v}=\frac{Vd}{\upsilon\sqrt{Gr}},\\ \tilde{t}=\frac{T\upsilon\sqrt{Gr}}{d^2},\quad \theta=\frac{\bar{T}-\bar{T}_0}{\bar{T}_1-\bar{T}_0},\quad \tilde{\phi}=\frac{\bar{C}-\bar{C}_0}{\bar{C}_1-\bar{C}_0}, \end{array}\right\}. \quad (6)$$

Dimensionless quantities involved:

$$\left.\begin{array}{l} \alpha=\frac{\kappa^*}{\rho C_p},\quad N=\frac{\upsilon}{d^2\Delta T},\quad Sc=\frac{\upsilon}{d},\quad Pr=\frac{\upsilon}{\alpha},\\ Gr=\frac{g\beta d^3(\bar{T}_1-\bar{T}_0)}{\upsilon^2},\quad \epsilon=\frac{\upsilon}{C_p}, \end{array}\right\}, \quad (7)$$

where α is referred to as the thermal diffusion, N is a newly discovered physical quantity, Sc Schmidt's number, Pr that Prandtl number, Gr is the Gratshof number for heat transmission, ϵ is the parameter for dispersion. By employing the above variables and parameters and aftar the necessary anayisis, the equations (1)-(5) can reduce to;
Continuity equation

$$\frac{\partial\tilde{u}}{\partial\tilde{x}}+\frac{\partial\tilde{v}}{\partial\tilde{y}}=0. \quad (8)$$

Momentum equation

$$\frac{\partial}{\partial\tilde{t}}\left[\frac{\partial\tilde{v}}{\partial\tilde{x}}-\frac{\partial\tilde{u}}{\partial\tilde{y}}\right]+\left[\frac{\partial}{\partial\tilde{x}}\left(\tilde{u}\frac{\partial\tilde{v}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{v}}{\partial\tilde{y}}\right)-\frac{\partial}{\partial\tilde{y}}\left(\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{y}}\right)\right]$$

$$= \frac{1}{\sqrt{Gr}} \nabla^2 \left[\frac{\partial \tilde{v}}{\partial \tilde{x}} - \frac{\partial \tilde{u}}{\partial \tilde{y}} \right] + \frac{1}{\sqrt{Gr}} \left[\frac{\partial \tilde{v}}{\partial \tilde{x}} - \frac{\partial \tilde{u}}{\partial \tilde{y}} \right]. \tag{9}$$

But $\tilde{u} = \frac{\partial \Psi}{\partial \tilde{y}}$ and $\tilde{v} = \frac{\partial \Psi}{\partial \tilde{x}}$ is stream function[12]. Put $\xi = -\nabla \Psi$, then equation (9) become:

$$\frac{\partial \xi}{\partial \tilde{t}} = \frac{1}{\sqrt{Gr}} \nabla^2 \xi + \frac{1}{\sqrt{Gr}} \xi, \tag{10}$$

Energy equation

$$\frac{\partial \theta}{\partial \tilde{t}} + \tilde{u} \frac{\partial \theta}{\partial \tilde{x}} + \tilde{v} \frac{\partial \theta}{\partial \tilde{y}} = \frac{1}{Pr} \frac{1}{\sqrt{Gr}} \left[\frac{\partial^2 \theta}{\partial \tilde{x}^2} \right] + \epsilon N \sqrt{Gr} \left(\frac{\partial \tilde{u}}{\partial \tilde{y}} \right)^2. \tag{11}$$

Diffusion equation

$$\frac{\partial \tilde{\phi}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{\phi}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{\phi}}{\partial \tilde{y}} = \frac{1}{Sc} \frac{1}{\sqrt{Gr}} \left[\frac{\partial^2 \tilde{\phi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\phi}}{\partial \tilde{y}^2} \right]. \tag{12}$$

The boundary conditions (5) take on the non-dimensional form as follows:

$$\left. \begin{aligned} &\tilde{u} = \tilde{v} = 0, \\ &\theta = 0, 1 \\ &\tilde{\phi} = 0, 1, \end{aligned} \right\} \quad \text{at} \quad \tilde{y} = 0, 1. \tag{13}$$

3. Results: Solution Approach

The concept behind this approach is based on keeping the time derivative continuous and discretizing the other terms using the ODE45 solver (Runge-Kutta formula) with recurrent iterations in time. To solve the two-dimensional motion equation (10) and energy equation (11) diffusion equation (12) and the boundary conditions (13), we try to use the numerical simulation for $-L \leq \tilde{x} \leq L$, and $-H \leq \tilde{y} \leq H$, where L, H are the arbitrary length of the computational domain between $\tilde{x}$ and $\tilde{y}$-direaction. The main equation of the motion, energy, and diffusion equations (11-13) are solved as mentioned above using the ODE45 solver. To discretize the domain above in to $N + 1$ points in X-direction and $M + 1$ in Y-direction, we use $\tilde{x}_i = -L + (i - 1)\Delta\tilde{x}, i = 1, \ldots, N + 1$ and $\tilde{y}_i = -H + (i - 1)\Delta\tilde{y}, j = 1, \ldots, M + 1$, Where $\Delta\tilde{x} = \frac{2L}{N}$, and $\Delta\tilde{y} = \frac{2H}{M}$. Consider the forward and backward finite difference methods which are used to discretize the main equation in this paper [6].

$$\xi_{\tilde{x},ij} = \frac{\xi_{i+1j} - \xi_{ij}}{\Delta\tilde{x}}, \quad \xi_{\tilde{x},ij} = \frac{\xi_{ij} - \xi_{i-1j}}{\Delta\tilde{x}}, \tag{14}$$

$$\xi_{\tilde{t},ij} = \frac{1}{\sqrt{Gr}} \left[\left[\left(\frac{\xi_{i+1j} - 2\xi_{ij} + \xi_{i-1j}}{\Delta\tilde{x}^2} \right) + \left(\frac{\xi_{ij+1} - 2\xi_{ij} + \xi_{ij-1}}{\Delta\tilde{y}^2} \right) \right] - \xi_{ij} \right],$$

$$i = 2, ..., N, \quad j = 2, ..., M, \tag{15}$$

$$\theta_{\tilde{t},ij} = -\left(\tilde{u}_{ij}\frac{\theta_{i+1j} - \theta_{i-1j}}{2\Delta\tilde{x}} + \tilde{v}_{ij}\frac{\theta_{ij+1} - \theta_{ij-1}}{2\Delta\tilde{y}}\right) + \frac{1}{Pr}\frac{1}{\sqrt{Gr}}\left(\frac{\theta_{i+1j} - 2\theta_{ij} + \theta_{i-1j}}{\Delta\tilde{x}^2}\right) + \epsilon N\sqrt{Gr}\left(\frac{\tilde{u}_{ij+1} - u_{i,j-1}}{2\Delta\tilde{y}}\right)^2, \quad i = 2, ..., N, \quad j = 2, ..., M. \tag{16}$$

The below is the diffusion equation:

$$\tilde{\phi}_{\tilde{t},ij} = \left(\tilde{u}_{ij}\frac{\tilde{\phi}_{i+1j} - \phi_{i-1j}}{2\Delta\tilde{x}} + \tilde{v}_{ij}\frac{\tilde{\phi}_{ij+1} - \tilde{\phi}_{ij-1}}{2\Delta\tilde{y}}\right) + \frac{1}{Sc}\frac{1}{\sqrt{Gr}}\left[\left(\frac{\tilde{\phi}_{i+1j} - 2\tilde{\phi}_{ij} + \tilde{\phi}_{i-1j}}{\Delta\tilde{x}^2}\right) + \left(\frac{\tilde{\phi}_{ij+1} - 2\phi_{ij} + \phi_{ij-1}}{\Delta\tilde{y}^2}\right)\right], \quad i = 2, ..., N, \quad j = 2, ..., M. \tag{17}$$

4. Discussion and Results

In this study, the system of coupled dimensionless value problem equations for velocity, diffusion, and temperature with the boundary conditions have been solved numerically and carried out for various parameters of Schmidt number (Sc), Gratshof number (Gr), and Prandtl number (Pr). The topic of heat transmission by free convection of a dissipative fluid moving in a horizontal tube with porous walls was looked into in this study. The final dimensionless PDEs with the boundary conditions are solved using the ODE45 MATLAB software [2]. In Fig. 2, we introduce a cartesian coordinate system (X, Y, Z), with the X, Y plane describing the location of points on the mesh, i.e., the spots in the $T*$-matrix resulting from the energy equation simulation results, and the Z-axis normal to the plane indicating temperature readings at any of those sites.

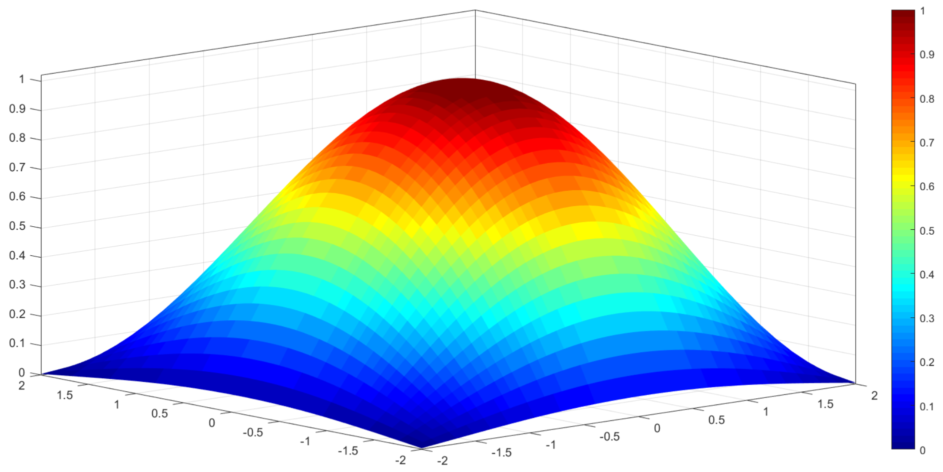

Figure 2: Temperature Behavior inside the Channel

4.1. Temperature variations for various parameter/number ratios

Figs. 3, 4, and 5 analyze the effects of temperature using a variety of Schmidt values, the Prandtl number, and the Gratshof number, respectively. In Figure 3, the temperature transfer is plotted for different values of the (Sc) number, it is clear that the change in temperature profile is not noticeable when increasing the Schmidt number, otherwise in figs. 4 and 5 the effect of the (Pr) number and (Gr) number on temperature is significant respectively. it is observed that when increasing Gr gives a rise in temperature and goes beyond the stability. Furthermore, figure 4 displays that for small values of (Gr), the temperature field falls down and gives rise to higher values. However, the temperature profile increase for any value of (Pr) sees fig. 5.

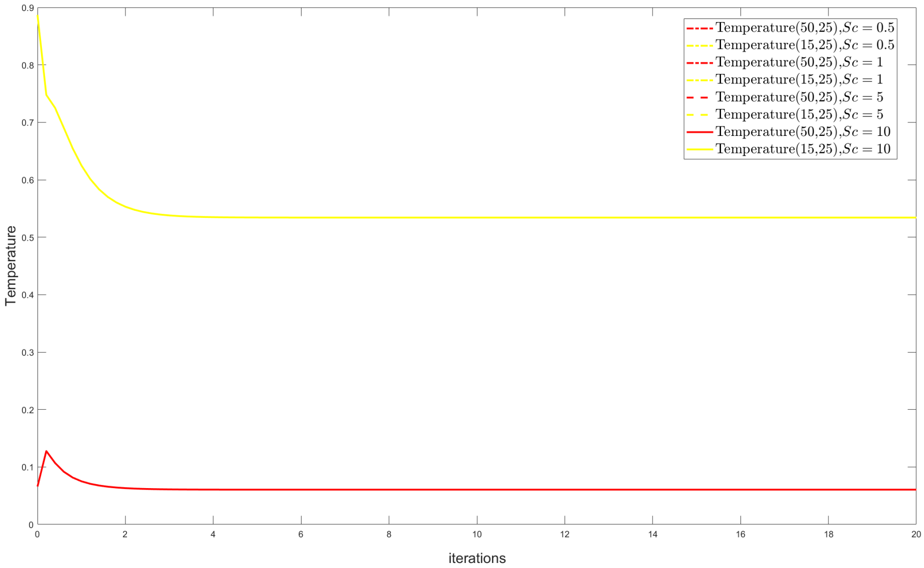

Figure 3: Temperature Distribution for Schimdt number (Sc).

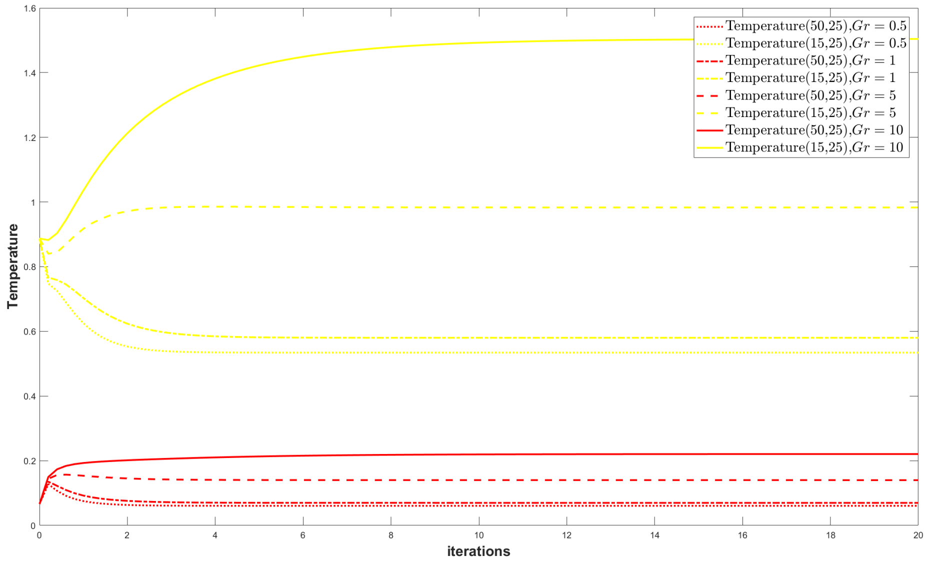

Figure 4: Computation temperature distribution against Gratshof number (Gr).

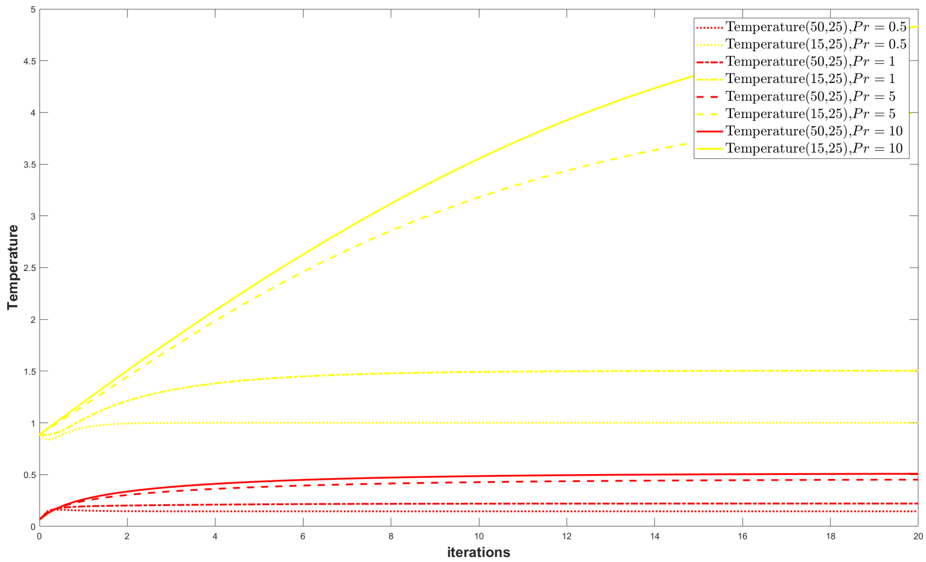

Figure 5: Computation temperature distribution against Prandtl number (Pr).

4.2. Diffusion variations for various parameter/number ratios

Figures 6 and 7 show the effect of (Sc) number as well as (Gr) number on the diffusion equation; these numbers have no discernible effect, and the diffusion profile remains stable. However, for higher values of these parameters, the diffusion profile takes a long time to reach stability.

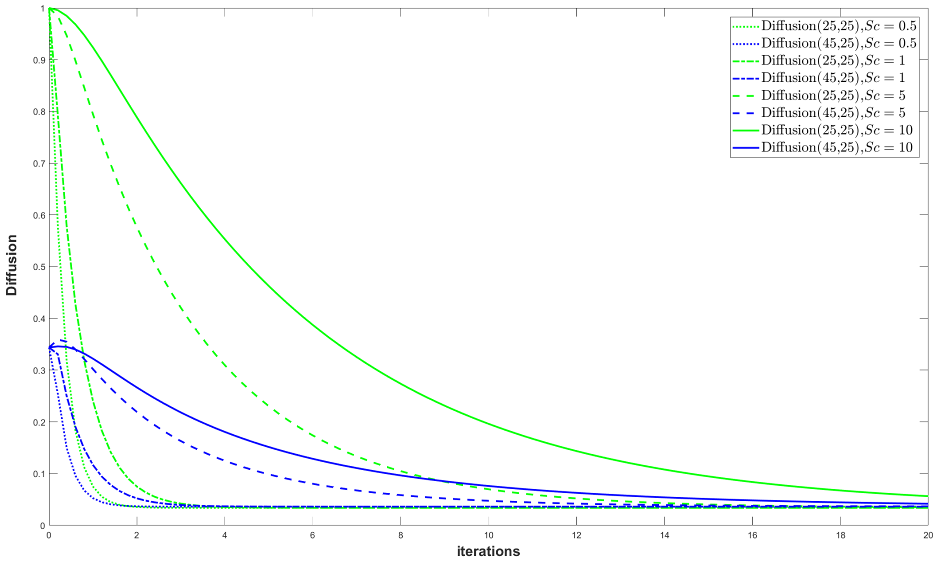

Figure 6: Impact of Schmidt number (Sc) diffusion distribution.

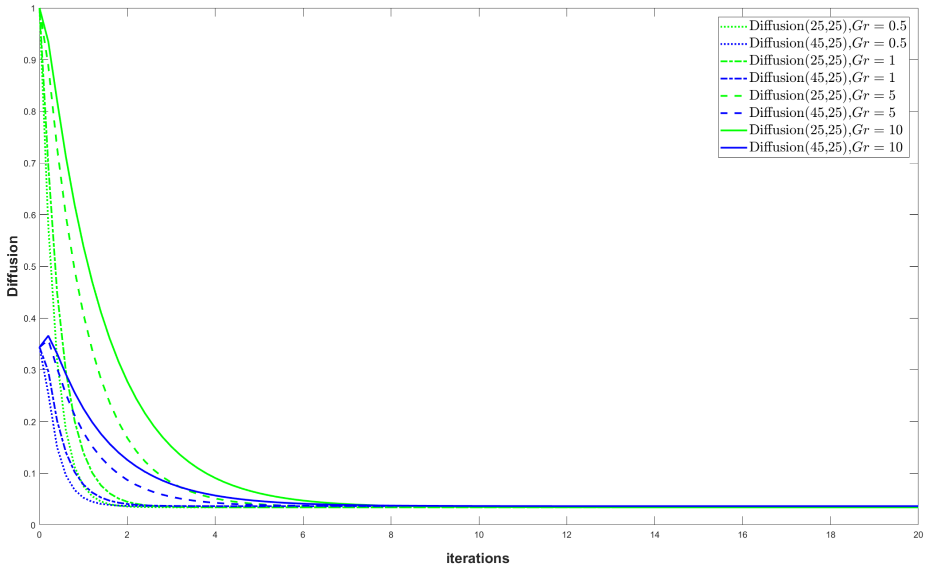

Figure 7: Impact of Gratshof number (Gr) on Diffusion distribution.

4.3. Velocity variations for various parameter/number ratios

For the velocity case Figs. 8, 9, the same stability behaviour (see Fig. 6) is illustrated for various values of the (Gr) number in the motion equations. It is worth noticing that the velocity profile increase with rising values of (Gr) for a very short time and declines with time increasing. The motion distribution is plotted in the Fig. 9 for various (Pr) numbers. we find that the influence of increasing the (Pr) number is unremarkable.

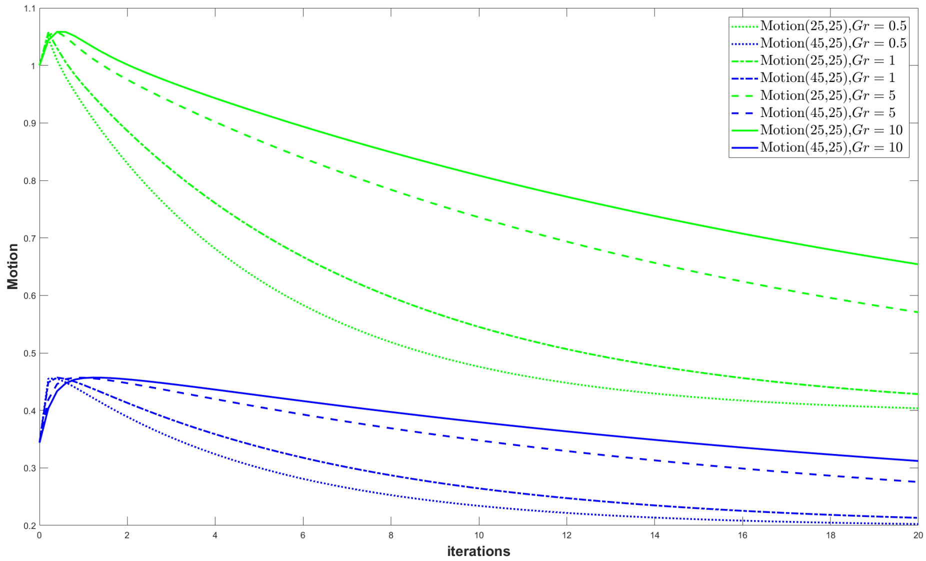

Figure 8: Distribution of the motion equation for Gartshof number (Gr).

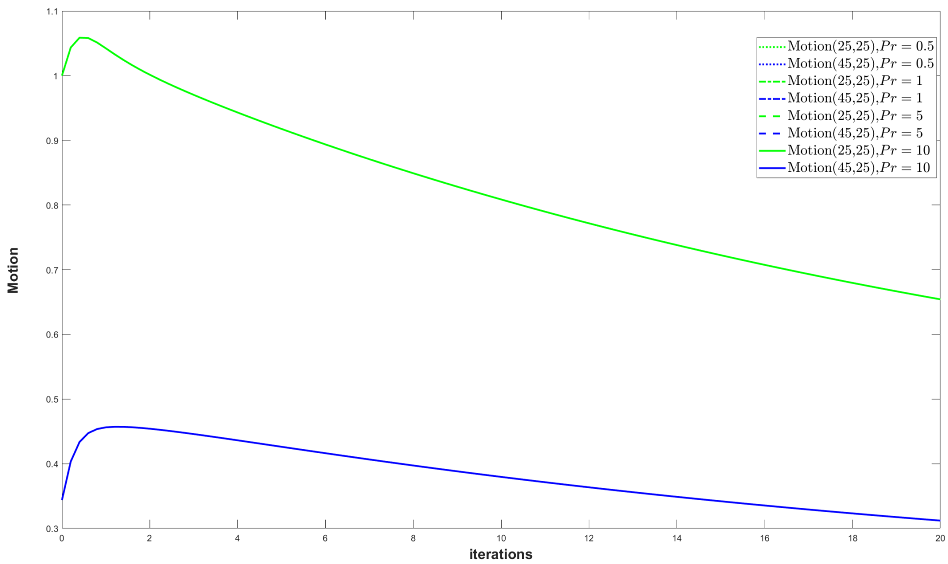

Figure 9: Effect of Prandtl number (Pr) on Motion distribution.

5. Conclusion

The influence of mass and heat transfer on mixed convection fluid flow past a horizontal channel are investigated in the present study. The data are plotted to show when velocity, temperature, and concentration change as a function of various parameters. The following are some of the investigation's key findings.

(i) The influence of temperature profile for different values of Schmidt number, Prandtl number, and Gratshof number is shown in Fig. 3 and Fig. 4. It is seen that increasing the Schmidt number and Prandtl number leads to stability;

(ii) As shown in Fig. 5, increasing the Gratshof number causes a shift away from stability.

(iii) The effect of Schmidt's number and Gratshof number in the diffusion equation is revealed in Figs. 6, 7, where it is noticed that for higher values of these parameters, the results take a long time to be stability.

(iv) In Figs. 8, 9 the effect of the Gratshof number and Prandtl number on the diffusion equation is depicted, where it is noticed that the higher the Gratshof number, the diffusion profile takes a long time to be stable and the influence of increasing the Prandtl numberin Fig. 9 is not remarked.

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