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Characterizations of some topological spaces

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Abstract. This paper is concerned with the concepts of some topological spaces. Firstly, we introduce the notions of $\delta s(\Lambda, p)$-open sets. Some properties concerning $\delta s(\Lambda, p)$-open sets are discussed. Secondly, the concept of $s(\Lambda, p)$-connected spaces is introduced. Moreover, we give several characterizations of $s(\Lambda, p)$-connected spaces by utilizing $\delta s(\Lambda, p)$-open sets. Thirdly, we apply the notion of $s(\Lambda, p)$-open sets to present and study new classes of spaces called $s(\Lambda, p)$-regular spaces and $s(\Lambda, p)$-normal spaces. Especially, some characterizations of $s(\Lambda, p)$-regular spaces and $s(\Lambda, p)$-normal spaces are established. Fourthly, we introduce and investigate the concepts of $s(\Lambda, p)$-T_2 spaces and $s(\Lambda, p)$-Urysohn spaces. Finally, the notion of $S(\Lambda, p)$-closed spaces is studied. Basic properties and characterizations of $S(\Lambda, p)$-closed spaces are considered.

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Key Words and Phrases: $\delta s(\Lambda, p)$-open set, $s(\Lambda, p)$-connected space, $s(\Lambda, p)$-regular space, $s(\Lambda, p)$-normal space, $s(\Lambda, p)$-T_2 space, $s(\Lambda, p)$-Urysohn space, $S(\Lambda, p)$-closed space

1. Introduction

In 1968, Veličko [14] introduced δ-open sets, which are stronger than open sets. In 1982, Mashhour et al. [9] introduced and investigated the notion of preopen sets which is weaker than the notion of open sets in topological spaces. In 1993, Raychaudhuri and Mukherjee [11] introduced and studied the notions of δ-preopen sets and δ-closures. The class of δ-preopen sets is larger than that of preopen sets. In 1996, Raychaudhuri and Mukherjee [12] introduced and investigated the concept of δ_p-closed spaces. In 2005, Caldas et al. [4] introduced some weak separation axioms by utilizing the notions of δ-preopen sets and the δ-preclosure operator. Caldas et al. [4] showed that (δ, p)-T_1 spaces, (δ, p)-R_0 spaces and (δ, p)-symmetric spaces are all equivalent. Moreover, Caldas et al. [6] investigated some weak separation axioms by utilizing δ-semiopen sets and the δ-semiclosure operator. Caldas et al. [5] investigated the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a δ-semiclosed set. In 2011, Buadong et al. [1] introduced and investigated some separation axioms in generalized topology and minimal structure

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spaces. Dungthaisong et al. [7] studied some properties of pairwise μ-$T_{\frac{1}{2}}$-spaces. Torton et al. [13] introduced and investigated the notions of $\mu_{(m,n)}$-regular spaces and $\mu_{(m,n)}$-normal spaces. In [3], the present authors introduced the notions of (Λ, p)-open sets and (Λ, p)-closed sets which are defined by utilizing the notions of Λ_p-sets and preclosed sets. This paper is organized as follows: in Section 2 is devoted to basic definitions and preliminaries. In Section 3, we introduce the notions of $\delta s(\Lambda, p)$-open sets and $\delta s(\Lambda, p)$-closed sets in topological spaces. Moreover, some characterizations of $\delta s(\Lambda, p)$-T_0 spaces, $\delta s(\Lambda, p)$-T_1 spaces and $\delta s(\Lambda, p)$-symmetric spaces are investigated. In Section 4, the notion of $s(\Lambda, p)$-connected spaces is introduced. Several characterizations of $s(\Lambda, p)$-connected spaces are obtained. In Section 5, we introduce the concepts of $s(\Lambda, p)$-regular spaces and $s(\Lambda, p)$-normal spaces. Furthermore, we give some characterizations of $s(\Lambda, p)$-regular spaces and $s(\Lambda, p)$-normal spaces by utilizing $\delta s(\Lambda, p)$-open sets. Basic properties and characterizations of $s(\Lambda, p)$-T_2 spaces and $s(\Lambda, p)$-Urysohn spaces are discussed in Section 6. In the last Section 7, we define the notion of $S(\Lambda, p)$-closed spaces. Characterizations and properties concerning $S(\Lambda, p)$-closed spaces are considered.

2. Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [9] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [8] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p-*set* [3] (*pre*-Λ-*set* [8]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)-*closed* [3] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)-*open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)-*cluster point* [3] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)-*closure* [3] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)-*interior* [3] of A and is denoted by $A_{(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be $\alpha(\Lambda, p)$-*open* (resp. $p(\Lambda, p)$-*open*, $s(\Lambda, p)$-*open*, $\beta(\Lambda, p)$-*open*, $r(\Lambda, p)$-*open* [3]) if $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$ (resp. $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$, $A \subseteq [[A^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}$, $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$). The family of all $\alpha(\Lambda, p)$-open (resp. $p(\Lambda, p)$-open, $s(\Lambda, p)$-open, $\beta(\Lambda, p)$-open, $r(\Lambda, p)$-open) sets in a topological space (X, τ) is denoted by $\alpha(\Lambda, p)O(X, \tau)$ (resp. $p(\Lambda, p)O(X, \tau)$, $s(\Lambda, p)O(X, \tau)$, $\beta(\Lambda, p)O(X, \tau)$, $r(\Lambda, p)O(X, \tau)$). The complement of a $p(\Lambda, p)$-open (resp. $s(\Lambda, p)$-open, $\alpha(\Lambda, p)$-open, $\beta(\Lambda, p)$-open, $r(\Lambda, p)$-open) set is said to be $p(\Lambda, p)$-*closed* (resp. $s(\Lambda, p)$-*closed*, $\alpha(\Lambda, p)$-*closed*, $\beta(\Lambda, p)$-*closed*, $r(\Lambda, p)$-*closed*). Let A be a subset of a topological space (X, τ). The intersection of all $s(\Lambda, p)$-closed sets of X containing A is called the $s(\Lambda, p)$-*closure* of A and is denoted by $A^{s(\Lambda,p)}$. A point x of X is called a $\delta(\Lambda,p)$-*cluster point* [2] of A if $A \cap [V^{(\Lambda,p)}]_{(\Lambda,p)} \neq \emptyset$ for every (Λ,p)-open set V of X containing x. The set of all $\delta(\Lambda,p)$-cluster points of A is called the $\delta(\Lambda,p)$-*closure* [2] of A and is denoted by $A^{\delta(\Lambda,p)}$. If $A = A^{\delta(\Lambda,p)}$, then A is said to be $\delta(\Lambda,p)$-*closed* [2]. The complement of a $\delta(\Lambda,p)$-closed set is said to be $\delta(\Lambda,p)$-*open*. The union of all $\delta(\Lambda,p)$-open sets of X contained in A is called the $\delta(\Lambda,p)$-*interior* [2] of A and is denoted by $A_{\delta(\Lambda,p)}$.

3. $\delta s(\Lambda,p)$-open sets

In this section, we introduce the notion of $\delta s(\Lambda,p)$-open sets. Moreover, some characterizations of $\delta s(\Lambda,p)$-T_0 spaces, $\delta s(\Lambda,p)$-T_1 spaces and $\delta s(\Lambda,p)$-symmetric spaces are discussed.

Definition 1. *A subset A of a topological space (X,τ) is said to be $\delta s(\Lambda,p)$-open if $A \subseteq [A_{(\Lambda,p)}]^{\delta(\Lambda,p)}$. The complement of a $\delta s(\Lambda,p)$-open set is said to be $\delta s(\Lambda,p)$-closed.*

The family of all $\delta s(\Lambda,p)$-open (resp. $\delta s(\Lambda,p)$-closed) sets in a topological space (X,τ) is denoted by $\delta s(\Lambda,p)O(X,\tau)$ (resp. $\delta s(\Lambda,p)C(X,\tau)$).

Definition 2. *Let A be a subset of a topological space (X,τ). A point x of X is called a $\delta s(\Lambda,p)$-cluster point of A if $A \cap U \neq \emptyset$ for every $\delta s(\Lambda,s)$-open set U of X containing x. The set of all $\delta s(\Lambda,p)$-cluster points of A is called the $\delta s(\Lambda,p)$-closure of A and is denoted by $A^{\delta s(\Lambda,p)}$.*

Lemma 1. *The intersection of arbitrary collection of $\delta s(\Lambda,s)$-closed sets in (X,τ) is $\delta s(\Lambda,p)$-closed.*

Corollary 1. *Let A be a subset of a topological space (X,τ). Then,*

$$A^{\delta s(\Lambda,p)} = \cap\{F \in \delta s(\Lambda,p)C(X,\tau) \mid A \subseteq F\}.$$

Lemma 2. *For the $\delta s(\Lambda,p)$-closure of subsets A, B in a topological space (X,τ), the following properties hold:*

(1) A is $\delta s(\Lambda,p)$-closed in (X,τ) if and only if $A = A^{\delta s(\Lambda,p)}$.

(2) If $A \subseteq B$, then $A^{\delta s(\Lambda,p)} \subseteq B^{\delta s(\Lambda,p)}$.

(3) $A^{\delta s(\Lambda,p)}$ is $\delta s(\Lambda,p)$-closed, that is, $A^{\delta s(\Lambda,p)} = [A^{\delta s(\Lambda,p)}]^{\delta s(\Lambda,p)}$.

Lemma 3. *For a family $\{A_\gamma \mid \gamma \in \nabla\}$ of a topological space (X,τ), the following properties hold:*

(1) $[\cap\{A_\gamma \mid \gamma \in \nabla\}]^{\delta s(\Lambda,p)} \subseteq \cap\{A_\gamma^{\delta s(\Lambda,p)} \mid \gamma \in \nabla\}$.

(2) $[\cup\{A_\gamma \mid \gamma \in \nabla\}]^{\delta s(\Lambda,p)} \supseteq \cup\{A_\gamma^{\delta s(\Lambda,p)} \mid \gamma \in \nabla\}$.

Definition 3. *A subset A of a topological space (X, τ) is called $s(\Lambda, p)$-regular if A is $s(\Lambda, p)$-open and $s(\Lambda, p)$-closed.*

The family of all $s(\Lambda, p)$-regular sets in a topological space (X, τ) is denoted by $s(\Lambda, p)r(X, \tau)$.

Lemma 4. *For a subset A of a topological space (X, τ), the following properties hold:*

(1) If A is a $s(\Lambda, p)$-regular set, then A is $\delta s(\Lambda, p)$-open.

(2) If A is a $\delta s(\Lambda, p)$-open set, then A is $s(\Lambda, p)$-open.

(3) If A is a $s(\Lambda, p)$-open set, then $A^{s(\Lambda,p)}$ is $s(\Lambda, p)$-regular.

Definition 4. *Let A be a subset of a topological space (X, τ). A point x of X is called a $\theta s(\Lambda, p)$-cluster point of A if $A \cap U^{s(\Lambda,p)} \neq \emptyset$ for every $s(\Lambda, p)$-open set U of X containing x. The set of all $\theta s(\Lambda, p)$-cluster points of A is called the $\theta s(\Lambda, p)$-closure of A, denoted by $A^{\theta s(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be $\theta s(\Lambda, p)$-closed if $A = A^{\theta s(\Lambda,p)}$. The complement of a $\theta s(\Lambda, p)$-closed set is said to be $\theta s(\Lambda, p)$-open.*

Lemma 5. *Let (X, τ) be a topological space. Then, $V^{\theta s(\Lambda,p)} = V^{\delta s(\Lambda,p)} = V^{s(\Lambda,p)}$ for each $V \in s(\Lambda, p)O(X, \tau)$.*

Definition 5. *A topological space (X, τ) is called $\delta s(\Lambda, p)$-T_0 if, for any distinct pair of points in X, there exists a $\delta s(\Lambda, p)$-open set containing one of the points but not the other.*

Theorem 1. *A topological space (X, τ) is $\delta s(\Lambda, p)$-T_0 if and only if for each point of distinct points x, y of X, $\{x\}^{\delta s(\Lambda,p)} \neq \{y\}^{\delta s(\Lambda,p)}$.*

Proof. Suppose that $x, y \in X$, $x \neq y$ and $\{x\}^{\delta s(\Lambda,p)} \neq \{y\}^{\delta s(\Lambda,p)}$. Let z be a point of X such that $z \in \{x\}^{\delta s(\Lambda,p)}$ but $z \notin \{y\}^{\delta s(\Lambda,p)}$. We claim that $x \notin \{y\}^{\delta s(\Lambda,p)}$. For, if $x \in \{y\}^{\delta s(\Lambda,p)}$, then $\{x\}^{\delta s(\Lambda,p)} \subseteq \{y\}^{\delta s(\Lambda,p)}$ and this contradicts the fact that $z \notin \{y\}^{\delta s(\Lambda,p)}$. Thus, x belongs to the $\delta s(\Lambda, p)$-open set $X - \{y\}^{\delta s(\Lambda,p)}$ to which y does not belong.

Conversely, let (X, τ) be a $\delta s(\Lambda, p)$-T_0 space and x, y be any two distinct points of X. Then, there exists a $\delta s(\Lambda, p)$-open set U containing x or y, say x but not y. Then, $X - U$ is a $\delta s(\Lambda, p)$-closed set which does not contain x but contains y. Thus, $\{y\}^{\delta s(\Lambda,p)} \subseteq X - U$ and hence $x \notin \{y\}^{\delta s(\Lambda,p)}$. This shows that $\{x\}^{\delta s(\Lambda,p)} \neq \{y\}^{\delta s(\Lambda,p)}$.

Definition 6. *A topological space (X, τ) is called $\delta s(\Lambda, p)$-T_1 if, for any distinct pair of points x and y in X, there exist a $\delta s(\Lambda, p)$-open set U of X containing x but not y and a $\delta s(\Lambda, p)$-open set V of X containing y but not x.*

Theorem 2. *A topological space (X, τ) is $\delta s(\Lambda, p)$-T_1 if and only if the singletons are $\delta s(\Lambda, p)$-closed sets.*

Proof. Suppose that (X, τ) is $\delta s(\Lambda, p)$-T_1 and x be any point of X. Let $y \in X - \{x\}$. Then, $x \neq y$ and so there exists a $\delta s(\Lambda, p)$-open set V_y such that $y \in V_y$ but $x \notin V_y$. Therefore, $y \in V_y \subseteq X - \{x\}$. Thus, $X - \{x\} = \cup\{V_y \mid y \in (X - \{x\})\}$ which is $\delta s(\Lambda, p)$-open.

Conversely, suppose that $\{z\}$ is $\delta s(\Lambda, p)$-closed for each $z \in X$. Let $x, y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in X - \{x\}$. Thus, $X - \{x\}$ is a $\delta s(\Lambda, p)$-open set containing y but not containing x. Similarly, $X - \{y\}$ is a $\delta s(\Lambda, p)$-open set containing x but not containing y. This shows that (X, τ) is a $\delta s(\Lambda, p)$-T_1 space.

Definition 7. *A topological space (X, τ) is called $\delta s(\Lambda, p)$-symmetric if, for each x and y in X, $x \in \{y\}^{\delta s(\Lambda,p)}$ implies $y \in \{y\}^{\delta s(\Lambda,p)}$.*

Lemma 6. *Let (X, τ) be a topological space. For each point $x \in X$, $\{x\}$ is $s(\Lambda, p)$-open or $s(\Lambda, p)$-closed.*

Theorem 3. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $\delta s(\Lambda, p)$-symmetric.

(2) For each $x \in X$, $\{x\}$ is $\delta s(\Lambda, p)$-closed.

(3) (X, τ) is $\delta s(\Lambda, p)$-T_1.

Proof. (1) $\Rightarrow$ (2): Suppose that (X, τ) is $\delta s(\Lambda, p)$-symmetric. Let x be any point of X and y be any distinct point from x. By Lemma 6, $\{y\}$ is $s(\Lambda, p)$-open or $s(\Lambda, p)$-closed in (X, τ). (i) In case $\{y\}$ is $s(\Lambda, p)$-open, put $V_y = \{y\}$, then $V_y \in \delta s(\Lambda, p)O(X, \tau)$. (ii) In case $\{y\}$ is $s(\Lambda, p)$-closed, $x \notin \{y\} = \{y\}^{s(\Lambda,p)}$ and $x \notin \{y\}^{\delta s(\Lambda,p)}$. By (1), $y \notin \{x\}^{\delta s(\Lambda,p)}$. Now put $V_y = X - \{x\}^{\delta s(\Lambda,p)}$. Then, $x \notin V_y$, $y \in V_y$ and $V_y \in \delta s(\Lambda, p)O(X, \tau)$. Thus, $X - \{x\} = \underset{y \in X - \{x\}}{\cup} V_y \in \delta s(\Lambda, p)O(X, \tau)$ and hence $\{x\}$ is $\delta s(\Lambda, p)$-closed.

(2) $\Rightarrow$ (3): Suppose that $\{z\}$ is $\delta s(\Lambda, p)$-closed for each $z \in X$. Let $x, y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in X - \{x\}$. Thus, $X - \{x\}$ is a $\delta s(\Lambda, p)$-open set containing y but not containing x. Similarly, we have $X - \{y\}$ is a $\delta s(\Lambda, p)$-open set containing x but not containing y. This shows that (X, τ) is $\delta s(\Lambda, p)$-T_1.

(3) $\Rightarrow$ (1): Suppose that $y \notin \{x\}^{\delta s(\Lambda,p)}$. Then, since $x \neq y$, by (3) there exists a $\delta s(\Lambda, p)$-open set U containing x such that $y \notin U$ and hence $x \notin \{y\}^{\delta s(\Lambda,p)}$. This shows that $x \in \{y\}^{\delta s(\Lambda,p)}$ implies $y \in \{x\}^{\delta s(\Lambda,p)}$. Thus, (X, τ) is $\delta s(\Lambda, p)$-symmetric.

Definition 8. *A subset A of a topological space (X, τ) is called generalized $\delta s(\Lambda, p)$-closed (briefly g-$\delta s(\Lambda, p)$-closed) if $A^{\delta s(\Lambda,p)} \subseteq U$ whenever $A \subseteq U$ and U is $\delta s(\Lambda, p)$-open in (X, τ).*

Theorem 4. *A subset A of a topological space (X, τ) is g-$\delta s(\Lambda, p)$-closed if and only if $A^{\delta s(\Lambda,p)} - A$ contains no nonempty $\delta s(\Lambda, p)$-closed set.*

Proof. Let F be a $\delta s(\Lambda,p)$-closed subset of $A^{\delta s(\Lambda,p)} - A$. Since $A \subseteq X - F$ and A is g-$\delta s(\Lambda,p)$-closed, $A^{\delta s(\Lambda,p)} \subseteq X - F$ and hence $F \subseteq X - A^{\delta s(\Lambda,p)}$. Thus,

$$F \subseteq A^{\delta s(\Lambda,p)} \cap [X - A^{\delta s(\Lambda,p)}] = \emptyset$$

and F is empty.

Conversely, suppose that $A \subseteq U$ and U is $\delta s(\Lambda,p)$-open. If $A^{\delta s(\Lambda,p)} \not\subseteq U$, then

$$A^{\delta s(\Lambda,p)} \cap (X - U)$$

is a nonempty $\delta s(\Lambda,p)$-closed subset of $A^{\delta s(\Lambda,p)} - A$.

Theorem 5. *A subset A of a topological space (X,τ) is g-$\delta s(\Lambda,p)$-closed if and only if $F \cap A^{\delta s(\Lambda,p)} = \emptyset$ whenever $A \cap F = \emptyset$ and F is $\delta s(\Lambda,p)$-closed.*

Proof. Suppose that A is a $\delta s(\Lambda,p)$-closed set. Let F be a $\delta s(\Lambda,p)$-closed set and $A \cap F = \emptyset$. Then, $A \subseteq X - F \in \delta s(\Lambda,p)O(X,\tau)$ and $A^{\delta s(\Lambda,p)} \subseteq X - F$. Thus,

$$F \cap A^{\delta s(\Lambda,p)} = \emptyset.$$

Conversely, let $A \subseteq U$ and $U \in \delta s(\Lambda,p)O(X,\tau)$. Then, $A \cap (X - U) = \emptyset$ and $X - U$ is $\delta s(\Lambda,p)$-closed. By the hypothesis, $(X - U) \cap A^{\delta s(\Lambda,p)} = \emptyset$ and hence $A^{\delta s(\Lambda,p)} \subseteq U$. Thus, A is g-$\delta s(\Lambda,p)$-closed.

Theorem 6. *A subset A of a topological space (X,τ) is g-$\delta s(\Lambda,p)$-closed if and only if $A \cap \{x\}^{\delta s(\Lambda,p)} \neq \emptyset$ for every $x \in A^{\delta s(\Lambda,p)}$.*

Proof. Let A be a g-$\delta s(\Lambda,p)$-closed set and suppose that there exists $x \in A^{\delta s(\Lambda,p)}$ such that $A \cap \{x\}^{\delta s(\Lambda,p)} = \emptyset$. Thus, $A \subseteq X - \{x\}^{\delta s(\Lambda,p)}$ and hence $A^{\delta s(\Lambda,p)} \subseteq X - \{x\}^{\delta s(\Lambda,p)}$. Therefore, $x \notin A^{\delta s(\Lambda,p)}$, which is a contradiction.

Conversely, suppose that the condition of the theorem holds and let U be any $\delta s(\Lambda,p)$-open set containing A. Let $x \in A^{\delta s(\Lambda,p)}$. By the hypothesis, $A \cap A^{\delta s(\Lambda,p)} \neq \emptyset$, so there exists $y \in A \cap \{x\}^{\delta s(\Lambda,p)}$ and hence $y \in A \subseteq U$. Thus, $\{x\} \cap U \neq \emptyset$. Therefore, $x \in U$, which implies that $A^{\delta s(\Lambda,p)} \subseteq U$. This shows that A is g-$\delta s(\Lambda,p)$-closed.

Theorem 7. *A topological space (X,τ) is $\delta s(\Lambda,p)$-symmetric if and only if $\{x\}$ is g-$\delta s(\Lambda,p)$-closed for each $x \in X$.*

Proof. Suppose that $x \in \{y\}^{\delta s(\Lambda,p)}$ but $y \in \{x\}^{\delta s(\Lambda,p)}$. This means that the complement of $\{x\}^{\delta s(\Lambda,p)}$ contains y. Thus, the set $\{y\}$ is a subset of the complement of $\{x\}^{\delta s(\Lambda,p)}$. This implies that $\{y\}^{\delta s(\Lambda,p)}$ is a subset of the complement of $\{x\}^{\delta s(\Lambda,p)}$. Now the complement of $\{x\}^{\delta s(\Lambda,p)}$ contains x which is a contradiction.

Conversely, suppose that $\{x\} \subseteq U \in \delta s(\Lambda,p)O(X,\tau)$, but $\{x\}^{\delta s(\Lambda,p)}$ is not a subset of U. This means that $\{x\}^{\delta s(\Lambda,p)}$ and the complement of U are not disjoint. Let y belongs to their intersection. Now we have $x \in \{y\}^{\delta s(\Lambda,p)}$ which is a subset of the complement of U and $x \notin U$. This is a contradiction.

4. Characterizations of $s(\Lambda, p)$-connected spaces

We begin this section by introducing the concept of $s(\Lambda, p)$-connected spaces.

Definition 9. *A topological space (X, τ) is called $s(\Lambda, p)$-connected if X cannot be expressed by the disjoint union of two nonempty $s(\Lambda, p)$-open sets.*

Theorem 8. *For a topological space (X, τ), the following properties are equivalent:*

(1) $V^{(\Lambda,p)} = X$ for every nonempty (Λ, p)-open set V of X;

(2) (X, τ) is $s(\Lambda, p)$-connected;

(3) X cannot be expressed by the disjoint union of two nonempty $\delta s(\Lambda, p)$-open sets;

(4) $V^{\delta s(\Lambda,p)} = X$ for every nonempty $\delta s(\Lambda, p)$-open set V of X.

Proof. (1) ⇔ (2): The proof follows from Theorem 4.3 of [10].

(2) ⇒ (3): Suppose that there exist two nonempty $\delta s(\Lambda, p)$-open sets V_1, V_2 such that $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = X$. Since $\delta s(\Lambda, p)O(X, \tau) \subseteq s(\Lambda, p)O(X, \tau)$, this shows that (X, τ) is not $s(\Lambda, p)$-connected.

(3) ⇒ (4): Suppose that $V^{\delta s(\Lambda,p)} \neq X$ for some nonempty $\delta s(\Lambda, p)$-open set V of X. Then, $X - V^{\delta s(\Lambda,p)} \neq \emptyset$ and $X = (X - V^{\delta s(\Lambda,p)}) \cup V^{\delta s(\Lambda,p)}$. Since

$$\delta s(\Lambda, p)O(X, \tau) \subseteq s(\Lambda, p)r(X, \tau),$$

by Lemma 4 and 5, $V^{\delta s(\Lambda,p)} = V^{s(\Lambda,p)} \in s(\Lambda, p)r(X, \tau)$. Moreover, since $s(\Lambda, p)r(X, \tau) \subseteq \delta s(\Lambda, p)O(X, \tau)$, $(X - V^{\delta s(\Lambda,p)})$ and $V^{\delta s(\Lambda,p)}$ are $\delta s(\Lambda, p)$-open.

(4) ⇒ (1): Let V be any nonempty (Λ, p)-open set of X. Then, $V^{(\Lambda,p)}$ is $r(\Lambda, p)$-closed and hence $s(\Lambda, p)$-regular. Thus, $V^{(\Lambda,p)}$ is $\delta s(\Lambda, p)$-open and

$$X = [V^{(\Lambda,p)}]^{\delta s(\Lambda,p)} = [V^{(\Lambda,p)}]^{s(\Lambda,p)} = V^{(\Lambda,p)}.$$

Theorem 9. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $s(\Lambda, p)$-connected;

(2) $V^{\delta s(\Lambda,p)} = X$ for every nonempty $V \in \beta(\Lambda, p)O(X, \tau)$;

(3) $V^{\delta s(\Lambda,p)} = X$ for every nonempty $V \in s(\Lambda, p)O(X, \tau)$;

(4) $V^{\delta s(\Lambda,p)} = X$ for every nonempty $V \in p(\Lambda, p)O(X, \tau)$;

(5) $V^{\delta s(\Lambda,p)} = X$ for every nonempty $V \in \alpha(\Lambda, p)O(X, \tau)$;

(6) $V^{\delta s(\Lambda,p)} = X$ for every nonempty $V \in \Lambda_p O(X, \tau)$.

Proof. (1) $\Rightarrow$ (2): Let V be any nonempty $\beta(\Lambda,p)$-open set and U be any nonempty $\delta s(\Lambda,p)$-open set. Then, $[V^{(\Lambda,p)}]_{(\Lambda,p)} \neq \emptyset$ and $U_{(\Lambda,p)} \neq \emptyset$. Thus, by Theorem 8,

$$\begin{aligned}\emptyset \neq U_{(\Lambda,p)} \cap [V^{(\Lambda,p)}]_{(\Lambda,p)} &\subseteq U \cap [V^{(\Lambda,p)}]_{(\Lambda,p)} \\ &\subseteq U \cap (V \cup [V^{(\Lambda,p)}]_{(\Lambda,p)}) = U \cap V^{s(\Lambda,p)} \subseteq U \cap V^{\delta s(\Lambda,p)}.\end{aligned}$$

Since $U \in \delta s(\Lambda,p)O(X,\tau)$, $U \cap V \neq \emptyset$. This shows that $V^{\delta s(\Lambda,p)} = X$.

(6) $\Rightarrow$ (1): Let U, V be any nonempty $\delta s(\Lambda,p)$-open sets. Since

$$\delta s(\Lambda,p)O(X,\tau) \subseteq s(\Lambda,p)O(X,\tau)$$

and $V_{(\Lambda,p)} \neq \emptyset$, we have $\emptyset \neq U \cap V_{(\Lambda,p)} \subseteq U \cap V$. This shows that $V^{\delta s(\Lambda,p)} = X$ for every nonempty $V \in \delta s(\Lambda,p)O(X,\tau)$. Thus, by Theorem 8, (X,τ) is $s(\Lambda,p)$-connected.

Other implications are obvious since

$$\Lambda_p O(X,\tau) \subseteq \alpha(\Lambda,p)O(X,\tau) \subseteq s(\Lambda,p)O(X,\tau) \cap p(\Lambda,p)O(X,\tau)$$

and $s(\Lambda,p)O(X,\tau) \cup p(\Lambda,p)O(X,\tau) \subseteq \beta(\Lambda,p)O(X,\tau)$.

Corollary 2. *For a topological space (X,τ), the following properties are equivalent:*

(1) (X,τ) *is* $s(\Lambda,p)$*-connected;*

(2) $U \cap V \neq \emptyset$ *for every nonempty sets* $U \in \beta(\Lambda,p)O(X,\tau)$ *and* $V \in \delta s(\Lambda,p)O(X,\tau)$*;*

(3) $U \cap V \neq \emptyset$ *for every nonempty sets* $U \in p(\Lambda,p)O(X,\tau)$ *and* $V \in \delta s(\Lambda,p)O(X,\tau)$*;*

(4) $U \cap V \neq \emptyset$ *for every nonempty sets* $U \in s(\Lambda,p)O(X,\tau)$ *and* $V \in \delta s(\Lambda,p)O(X,\tau)$*;*

(5) $U \cap V \neq \emptyset$ *for every nonempty sets* $U \in \alpha(\Lambda,p)O(X,\tau)$ *and* $V \in \delta s(\Lambda,p)O(X,\tau)$*;*

(6) $U \cap V \neq \emptyset$ *for every nonempty sets* $U \in \Lambda_p O(X,\tau)$ *and* $V \in \delta s(\Lambda,p)O(X,\tau)$*;*

(7) $U \cap V \neq \emptyset$ *for every nonempty sets* $U \in \delta s(\Lambda,p)O(X,\tau)$ *and* $V \in \delta s(\Lambda,p)O(X,\tau)$*.*

Proof. This is immediate consequence of Theorem 8 and 9.

5. Characterizations of $s(\Lambda, p)$-regular spaces and $s(\Lambda, p)$-normal spaces

In this section, we introduce the notions of $s(\Lambda, p)$-regular spaces and $s(\Lambda, p)$-normal spaces. Moreover, several characterizations of $s(\Lambda, p)$-regular spaces and $s(\Lambda, p)$-normal spaces are discussed.

Definition 10. *A topological space (X, τ) is said to be $s(\Lambda, p)$-regular if, for each $s(\Lambda, p)$-closed set F of X and each point $x \notin F$, there exist $U, V \in s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.*

Theorem 10. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $s(\Lambda, p)$-regular.

(2) For each $s(\Lambda, p)$-closed set F and each point $x \notin F$, there exist $U, V \in \delta s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.

(3) For each point $x \in X$ and each $s(\Lambda, p)$-open set V containing x, there exists

$$U \in \delta s(\Lambda, p)O(X, \tau)$$

such that $x \in U \subseteq U^{\delta s(\Lambda,p)} \subseteq V$.

Proof. (1) $\Rightarrow$ (2): Let F be a $s(\Lambda, p)$-closed set and $x \notin F$. Then, there exist $G, H \in s(\Lambda, p)O(X, \tau)$ such that $x \in G$, $F \subseteq H$ and $G \cap H = \emptyset$. By Lemma 4, $G^{s(\Lambda,p)}$ is $s(\Lambda, p)$-regular and $G^{s(\Lambda,p)} \cap H = \emptyset$. Thus, $G^{s(\Lambda,p)} \cap H^{s(\Lambda,p)} = \emptyset$. Now, we put $U = G^{s(\Lambda,p)}$ and $V = H^{s(\Lambda,p)}$, then U and V are $\delta s(\Lambda, p)$-open sets such that $x \in U$, $F \subseteq V$ and $U \cap V = \emptyset$.

(2) $\Rightarrow$ (3): Let $x \in X$ and V be any $s(\Lambda, p)$-open set containing x. Since $x \notin X - V$, there exist $U, G \in \delta s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $X - V \subseteq G$ and $U \cap G = \emptyset$. Since $X - G$ is $\delta s(\Lambda, p)$-closed and $U \subseteq X - G$, $x \in U \subseteq U^{\delta s(\Lambda,p)} \subseteq X - G \subseteq V$.

(3) $\Rightarrow$ (1): Let F be a $s(\Lambda, p)$-closed set and $x \notin F$. Then, $X - F$ is $s(\Lambda, p)$-open set containing x. By (3), there exists $U \in \delta s(\Lambda, p)O(X, \tau)$ such that $x \in U \subseteq U^{\delta s(\Lambda,p)} \subseteq X - F$. Thus, $x \in U$, $F \subseteq X - U^{\delta s(\Lambda,p)}$ and $U \cap (X - U^{\delta s(\Lambda,p)}) = \emptyset$. Since

$$\delta s(\Lambda, p)O(X, \tau) \subseteq s(\Lambda, p)O(X, \tau),$$

(X, τ) is $s(\Lambda, p)$-regular.

Definition 11. *A topological space (X, τ) is said to be $s(\Lambda, p)$-normal if, for each disjoint $s(\Lambda, p)$-closed sets F and K of X, there exist $U, V \in s(\Lambda, p)O(X, \tau)$ such that $F \subseteq U$, $K \subseteq V$ and $U \cap V = \emptyset$.*

Theorem 11. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $s(\Lambda, p)$-normal.

(2) For each disjoint $s(\Lambda, p)$-closed sets F and K of X, there exist $U, V \in \delta s(\Lambda, p)O(X, \tau)$ such that $F \subseteq U$, $K \subseteq V$ and $U \cap V = \emptyset$.

(3) For each $s(\Lambda, p)$-closed set F and each $s(\Lambda, p)$-open set V containing F, there exists $U \in \delta s(\Lambda, p)O(X, \tau)$ such that $F \subseteq U \subseteq U^{\delta s(\Lambda,p)} \subseteq V$.

Proof. The proof is analogous to that of Theorem 10 and is omitted.

6. Characterizations of $s(\Lambda, p)$-T_2 spaces and $s(\Lambda, p)$-Urysohn spaces

In this section, we introduce the notions of $s(\Lambda, p)$-T_2 spaces and $s(\Lambda, p)$-Urysohn spaces. Furthermore, some characterizations of $s(\Lambda, p)$-T_2 spaces and $s(\Lambda, p)$-Urysohn spaces are investigated.

Definition 12. *A topological space (X, τ) is said to be $s(\Lambda, p)$-T_2 if, for each pair of distinct points $x, y \in X$, there exist $U, V \in s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

Theorem 12. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $s(\Lambda, p)$-T_2.

(2) For each pair of distinct points $x, y \in X$, there exist $U, V \in s(\Lambda, p)r(X, \tau)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

(3) For each pair of distinct points $x, y \in X$, there exist $U, V \in \delta s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{\delta s(\Lambda,p)} \cap V^{\delta s(\Lambda,p)} = \emptyset$.

(4) For each pair of distinct points $x, y \in X$, there exist $U, V \in \delta s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{s(\Lambda,p)} \cap V^{s(\Lambda,p)} = \emptyset$.

(5) For each pair of distinct points $x, y \in X$, there exist $U, V \in \delta s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Proof. (1) $\Rightarrow$ (2): Suppose that (X, τ) is $s(\Lambda, p)$-T_2. Then, for each pair of distinct points $x, y \in X$, there exist $G, H \in s(\Lambda, p)O(X, \tau)$ such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$. Thus, $G^{s(\Lambda,p)} \cap H = \emptyset$. By Lemma 4, we have $G^{s(\Lambda,p)} \in s(\Lambda, p)r(X, \tau)$ and

$$G^{s(\Lambda,p)} \cap H^{s(\Lambda,p)} = \emptyset.$$

Now set $U = G^{s(\Lambda,p)}$ and $V = H^{s(\Lambda,p)}$. Then, U and V are $s(\Lambda, p)$-regular sets such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

(2) $\Rightarrow$ (3): This is follows from the facts that $s(\Lambda, p)r(X, \tau) \subseteq \delta s(\Lambda, p)O(X, \tau)$ and $U^{\delta s(\Lambda,p)} = U^{s(\Lambda,p)} = U$ for every $U \in s(\Lambda, p)r(X, \tau)$.

(3) $\Rightarrow$ (4): This follows from the fact that $U^{\delta s(\Lambda,p)} = U^{s(\Lambda,p)}$ for every

$$U \in \delta s(\Lambda, p)O(X, \tau).$$

(4) $\Rightarrow$ (5): This is obvious.
(5) $\Rightarrow$ (1): This is obvious since $\delta s(\Lambda, p)O(X, \tau) \subseteq s(\Lambda, p)O(X, \tau)$.

Definition 13. *A topological space (X, τ) is said to be $s(\Lambda, p)$-Urysohn if, for each pair of distinct points $x, y \in X$, there exist $U, V \in s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{(\Lambda,p)} \cap V^{(\Lambda,p)} = \emptyset$.*

Theorem 13. *A topological space (X, τ) is $s(\Lambda, p)$-Urysohn if and only if for each pair of distinct points x, y of X, there exist $U, V \in \delta s(\Lambda, p)O(X, \tau)$ such that $x \in U$, $y \in V$ and $U^{(\Lambda,p)} \cap V^{(\Lambda,p)} = \emptyset$.*

Proof. Suppose that (X, τ) is $s(\Lambda, p)$-Urysohn. Then, for each pair of distinct points x, y of X, there exist $U, V \in s(\Lambda, p)O(X, \tau)$ such that $x \in U, y \in V$ and

$$U^{(\Lambda,p)} \cap V^{(\Lambda,p)} = \emptyset.$$

Since $U \in s(\Lambda, p)O(X, \tau)$, $U^{(\Lambda,p)} = [U_{(\Lambda,p)}]^{(\Lambda,p)}$ and $U^{(\Lambda,p)}$ is $r(\Lambda, p)$-closed. Thus,

$$U^{(\Lambda,p)}, V^{(\Lambda,p)} \in s(\Lambda, p)r(X, \tau) \subseteq \delta s(\Lambda, p)O(X, \tau).$$

It is obvious that $x \in U^{(\Lambda,p)}$, $y \in V^{(\Lambda,p)}$ and $[U^{(\Lambda,p)}]^{(\Lambda,p)} \cap [V^{(\Lambda,p)}]^{(\Lambda,p)} = U^{(\Lambda,p)} \cap V^{(\Lambda,p)} = \emptyset$.
Conversely, the proof is obvious since $\delta s(\Lambda, p)O(X, \tau) \subseteq s(\Lambda, p)O(X, \tau)$.

7. Characterizations of $S(\Lambda, p)$-closed spaces

In this section, we introduce the notion of $S(\Lambda, p)$-closed spaces. In particular, several characterizations of $S(\Lambda, p)$-closed spaces are discussed.

Definition 14. *A topological space (X, τ) is said to be $S(\Lambda, p)$-closed if, for every cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X by $s(\Lambda, p)$-open sets of X, there exists a finite subset ∇_0 of ∇ such that $X = \underset{\gamma \in \nabla_0}{\cup} V_\gamma^{s(\Lambda,p)}$.*

Theorem 14. *For a topological space (X, τ), the following properties are equivalent:*

(1) (X, τ) is $S(\Lambda, p)$-closed.

(2) For every $\delta s(\Lambda, p)$-open cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X, there exists a finite subset ∇_0 of ∇ such that $X = \underset{\gamma \in \nabla_0}{\cup} V_\gamma^{s(\Lambda,p)}$.

(3) For every $\delta s(\Lambda, p)$-open cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X, there exists a finite subset ∇_0 of ∇ such that $X = \underset{\gamma \in \nabla_0}{\cup} V_\gamma^{\delta s(\Lambda,p)}$.

Proof. (1) $\Rightarrow$ (2): Suppose that (X,τ) is $S(\Lambda,p)$-closed. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $\delta s(\Lambda,p)$-open cover of X. By Lemma 4, $\delta s(\Lambda,p)O(X,\tau) \subseteq s(\Lambda,p)O(X,\tau)$ and there exists a finite subset ∇_0 of ∇ such that $X = \underset{\gamma\in\nabla_0}{\cup} V_\gamma^{s(\Lambda,p)}$.

(2) $\Rightarrow$ (3): Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $\delta s(\Lambda,p)$-open cover of X. By Lemma 4,

$$\delta s(\Lambda,p)O(X,\tau) \subseteq s(\Lambda,p)O(X,\tau)$$

and it follows from Lemma 5 that $V_\gamma^{\delta s(\Lambda,p)} = V_\gamma^{s(\Lambda,p)}$ for each $\gamma \in \nabla$.

(3) $\Rightarrow$ (1): Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $s(\Lambda,p)$-open cover of X. Then, $X = \underset{\gamma\in\nabla_0}{\cup} V_\gamma^{s(\Lambda,p)}$. By Lemma 4, $V_\gamma^{s(\Lambda,p)} \in s(\Lambda,p)r(X,\tau) \subseteq \delta s(\Lambda,p)O(X,\tau)$ and there exists a finite subset ∇_0 of ∇ such that $X = \underset{\gamma\in\nabla_0}{\cup} [V_\gamma^{s(\Lambda,p)}]^{\delta s(\Lambda,p)}$. By Lemma 5,

$$[V_\gamma^{s(\Lambda,p)}]^{\delta s(\Lambda,p)} = [V_\gamma^{s(\Lambda,p)}]^{s(\Lambda,p)} = V_\gamma^{s(\Lambda,p)}$$

and hence $X = \underset{\gamma\in\nabla_0}{\cup} V_\gamma^{s(\Lambda,p)}$. Thus, (X,τ) is $S(\Lambda,p)$-closed.

Theorem 15. *A topological space (X,τ) is $S(\Lambda,p)$-closed if and only if for every $\theta s(\Lambda,p)$-open cover $\{V_\gamma \mid \gamma \in \nabla\}$ of X, there exists a finite subset ∇_0 of ∇ such that $X = \underset{\gamma\in\nabla_0}{\cup} V_\gamma$.*

Proof. Let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $\theta s(\Lambda,p)$-open cover of X. For each $x \in X$, there exists $\gamma(x) \in \nabla$ such that $x \in V_{\gamma(x)}$. Since $V_{\gamma(x)}$ is $\theta s(\Lambda,p)$-open, there exists

$$G_{\gamma(x)} \in s(\Lambda,p)O(X,\tau)$$

such that $x \in G_{\gamma(x)} \subseteq G_{\gamma(x)}^{s(\Lambda,p)} \subseteq V_{\gamma(x)}$. Since $\{G_{\gamma(x)} \mid x \in X\}$ is a $s(\Lambda,p)$-open cover of X, there exist finite points, say, $x_1, x_2, ..., x_n$ such that $X = \overset{n}{\underset{i=1}{\cup}} G_{\gamma(x_i)}^{s(\Lambda,p)}$. Thus, $X = \overset{n}{\underset{i=1}{\cup}} V_{\gamma(x_i)}$.

Conversely, let $\{V_\gamma \mid \gamma \in \nabla\}$ be a $s(\Lambda,p)$-open cover of X. By Lemma 4,

$$\{V_\gamma^{s(\Lambda,p)} \mid \gamma \in \nabla\}$$

is a $s(\Lambda,p)$-regular cover of X and hence a $\theta s(\Lambda,p)$-open cover of X. Thus, there exists a finite subset ∇_0 of ∇ such that $X = \underset{\gamma\in\nabla_0}{\cup} V_\gamma^{s(\Lambda,p)}$. This shows that (X,τ) is $S(\Lambda,p)$-closed.

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