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Application of Bipolar Fuzzy Set to a Novel of Fuzzy Ideal in Γ-Semigroups

P. Khamrot[1], T. Gaketem[2,*]

[1] *Department of Mathematics, Faculty of Science and Agricultural Technology, Rajamangala University of Technology Lanna of Phitsanulok, Phitsanulok, Thailand*
[2] *Fuzzy Algebras and Decision-Making Problems Research Unit, Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand*

Abstract. In this paper we define new types of bipolar fuzzy ideals, bipolar fuzzy almost ideals in Γ-semigroups. We discussed properties of bipolar fuzzy ideals, bipolar fuzzy almost ideals, minimal bipolar fuzzy ideal, minimal bipolar almost ideal in Γ-semigroups. Moreover, we prove connection between almost ideals and bipolar fuzzy almost ideals of Γ-semigroups.

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1. Introduction

As a theory that deals with uncertainty, the fuzzy set theory was studied by Zadeh in 1965 [12]. It has been applied to many areas, such as medical science, robotics, computer science, information science, control engineering, measure theory, logic, set theory, topology and others. In 1994, Zhang [13] introduced the concept of bipolar fuzzy sets. In 2000, Lee [8] extended a fuzzy set to theory of a bipolar fuzzy set whose function ranges from the interval $[-1, 0] \cup [0, 1]$. As application of the bipolr set theory affects and effectiveness and efficiency of decision making. Therefore, it is used to solve problem related to decision-making, organization problems, economic problems, and evaluation, risk management, environmental and social impact assessments. Later in 2012, S.K. Majumder [9] studied the bipolar fuzzy set in Γ-semigroups and integration properties of bipolar fuzzy ideals in Γ-semigroups. The ideal theory is an essential structures in semigroups and many researchers have applied the knowledge of ideals in Γ-semigroups studies in a fuzzy semigroup. For example, Chinram et al. [1] studied almost quasi-Γ-ideal and fuzzy almost quasi-Γ-ideals in Γ-semigroup, M. K. R. Marapureddy and PRV S. R. Doradla [7] investigated weak interior ideals of Γ-semigroups, S.K. Majumder and M. Mandal [4] examimed

*Corresponding author.
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Email addresses: thiti.ga@up.ac.th (T. Gaketem), pk_g@rmutl.ac.th (P. Khamrot)

a fuzzy generalized bi-ideal in Γ-semigroups. In 2021, T. Gaketem and P. Khamrot [2] discussed bipolar fuzzy weakly interior ideals in semigroups. In the same year, [11] A. Simuen et al. studied a novel of ideals and fuzzy ideals of Γ-semigroups. Recently, in 2022-2023, T. Gaketem and P. Khamrot [3, 5, 6] investigated propertes of a novel of ideals on intuitionistic fuzzy ideals, cubic ideals and interval valued fuzzy ideals of Γ-semigroups.

In this paper, we extend the new fuzzy ideal to the bipolar fuzzy ideal of Γ-semigroups and investigate their properties. We prove connection between almost ideals and bipolar fuzzy almost ideals of Γ-semigroups.

2. Preliminaries

In this section, we review some basic concepts that are necessary to understand our section.

A *sub*-Γ-*semigroup* of a Γ-semigroup S is a non-empty set K of S such that $K\Gamma K \subseteq K$. A *left (right) ideal* of a Γ-semigroup S is a non-empty set K of S such that $S\Gamma K \subseteq K$ ($K\Gamma S \subseteq K$). By an *ideal* of a Γ-semigroup S, we mean a non-empty set of S which is both a left and a right ideal of S. A *quasi-ideal* of a Γ-semigroup S is a non-empty set K of S such that $K\Gamma S \cap S\Gamma K \subseteq K$. A sub-Γ-semigroup K of a Γ-semigroup S is called a *bi-ideal* of S if $K\Gamma S\Gamma K \subseteq K$.

Definition 1. *[11] Let S be a Γ-semigroup, K be a non-empty subset of S and $\alpha, \beta \in \Gamma$. Then K is said to be*

[(i)]

(i) A left (right) almost ideal *of Γ-semigroup S which is a non-empty set K such that $(s\Gamma K) \cap K \neq \emptyset$ ($(K\Gamma s) \cap K \neq \emptyset$) for all $s \in S$.*

(ii) An almost bi-ideal *of Γ-semigroup S which is a non-empty set K such that $(K\Gamma s\Gamma K) \cap K \neq \emptyset$ for all $s \in S$.*

(iii) An almost quasi-ideal *of Γ-semigroup S which is a non-empty set K such that $(s\Gamma K \cap K\Gamma s) \cap K \neq \emptyset$ for all $s \in S$.*

(iv) A left α-ideal of a Γ-semigroup S which is a non-empty set K such that $S\alpha K \subseteq K$. A right α-ideal of a Γ-semigroup S is a non-empty set K such that $K\beta S \subseteq K$.

(v) An (α, β)-ideal *of a Γ-semigroup S which is a non-empty set K such that it is both a left α-ideal and a right β-ideal of S.*

For any $m_i \in [0, 1]$, $i \in \mathcal{A}$, define

$$\underset{i\in\mathcal{A}}{\vee} m_i := \sup_{i\in\mathcal{A}}\{m_i\} \quad \text{and} \quad \underset{i\in\mathcal{A}}{\wedge} m_i := \inf_{i\in\mathcal{A}}\{m_i\}.$$

We see that for any $m, n \in [0, 1]$, we have

$$m \vee n = \max\{m, n\} \quad \text{and} \quad m \wedge n = \min\{m, n\}.$$

Definition 2. *[12] A* fuzzy set ξ *of a non-empty set* T *is a function* $\xi : T \to [0, 1]$.

For any two fuzzy sets ξ and ς of a non-empty set T, define $\geq, =, \wedge$, and $\vee$ as follows:

[(*i*)]$\xi \geq \varsigma \Leftrightarrow \xi(k) \geq \varsigma(k)$ for all $k \in T$, $\xi = \varsigma \Leftrightarrow \xi \geq \varsigma$ and $\varsigma \geq \xi$, $(\xi \wedge \varsigma)(k) = \min\{\xi(k), \varsigma(k)\} = \xi(k) \wedge \varsigma(k)$ for all $k \in T$, $(\xi \vee \varsigma)(k) = \max\{\xi(k), \varsigma(k)\} = \xi(k) \vee \varsigma(k)$ for all $k \in T$. For the symbol $\xi \leq \varsigma$, we mean $\varsigma \geq \xi$.

For any two fuzzy sets of ξ and ς of a non-empty of T, we define the *support* of ξ instead of $\operatorname{supp}(\xi) = \{k \in T \mid \xi(k) \neq 0\}$, $\xi \subseteq \varsigma$ if $\xi(k) \leq \varsigma(k)$, $(\xi \cup \varsigma)(k) = \max\{\xi(k), \varsigma(k)\}$ and $(\xi \cap \varsigma)(k) = \min\{\xi(k), \varsigma(k)\}$ for all $k \in T$.

For any element k in a semigroup S, define the set F_k by $F_k := \{(y, z) \in S \times S \mid k = yz\}$.

For two fuzzy sets ξ and ς on a semigroup S, define the product $\xi \circ \varsigma$ as follows: for all $k \in S$,

$$(\xi \circ \varsigma)(k) = \begin{cases} \bigvee\limits_{(y,z) \in F_k} \{\xi(y) \wedge \varsigma(z)\} & \text{if } \quad F_k \neq \emptyset, \\ 0 & \text{if } \quad F_k = \emptyset. \end{cases}$$

Definition 3. *Let* I *be a non-empty set of a semigroup* S*. A* ***positive characteristic function and a negative characteristic function*** *are respectively defined by*

$$\lambda_I : S \to [0, 1], k \mapsto \lambda_I(u) := \begin{cases} 1 & k \in I, \\ 0 & k \notin I, \end{cases}$$

The following definitions are types of fuzzy almost ideal on semigroups.

Definition 4. *[11] A fuzzy set* ξ *of a semigroup* S *and* $s \in S$*, is said to be*

[(i)]

(i) a fuzzy almost left (right) ideal of S *if* $\lambda_s \circ \xi \cap \xi \neq 0$ $(\xi \circ \lambda_s \cap \xi \neq 0)$ *for all* $s \in S$*,*

(ii) a fuzzy almost ideal of S *if it is both a fuzzy almost left ideal and a fuzzy almost right ideal of* S *for all* $s \in S$*,*

(iii) a fuzzy almost bi-ideal of S *if* $\xi \circ \lambda_s \circ \xi \cap \xi \neq 0$ *for all* $s \in S$*,*

(iv) a fuzzy almost quasi-ideal of S *if* $\lambda_s \circ \xi \cap \xi \circ \lambda_s \cap \xi \neq 0$*.*

Theorem 1. *[10] Let* ξ *be a nonzero fuzzy set of a semigroup* S*. Then* ξ *is a fuzzy subsemigroup (ideal) of* S *if and only if* $\operatorname{supp}(\xi)$ *is a subsemigroup (ideal) of* S*.*

The following definitions are types of fuzzy subsemigroups on Γ-semigroups.

Definition 5. *[10] A fuzzy set* ξ *of a* Γ*-semigroup* S *is said to be*

[(i)]

(i) A fuzzy subsemigroup of S *if* $\xi(u\gamma v) \geq \xi(u) \wedge \xi(v)$ *for all* $u, v \in S$ *and* $\gamma \in \Gamma$*.*

(ii) *A fuzzy left (right) ideal of* S *if* $\xi(u\gamma v) \geq \xi(v)$ $(\xi(u\gamma v) \geq \xi(u))$ *for all* $u, v \in S$ *and* $\gamma \in \Gamma$.

(iii) *A fuzzy ideal of* S *if it is both a fuzzy left ideal and a fuzzy right ideal of* S.

(iv) *A fuzzy bi-ideal of* S *if* ξ *is a fuzzy subsemigroup of* S *and* $\xi(u\gamma v\beta w) \geq \xi(u) \wedge \xi(w)$ *for all* $u, v, w \in S$ *and* $\gamma, \beta \in \Gamma$.

Now, we review the definition of a bipolar valued fuzzy set and basic properties used in the next section.

Definition 6. *[8] Let* S *be a non-empty set. A* ***bipolar fuzzy set*** *(BF set)* ξ *on* S *is an object having the form*

$$\xi := \{(k, \xi^p(k), \xi^n(k)) \mid k \in S\},$$

where $\xi^p : S \to [0,1]$ *and* $\xi^n : S \to [-1,0]$.

Remark 1. *For the sake of simplicity, we shall use the symbol* $\xi = (S; \xi^p, \xi^n)$ *for the BF set* $\xi = \{(k, \xi^p(k), \xi^n(k)) \mid k \in S\}$.

The following presents an example of a BF set.

Example 1. *Let* $S = \{21, 22, 23...\}$. *Define* $\xi^p : S \to [0,1]$ *as a function*

$$\xi^p(u) = \begin{cases} 0 & \text{if } u \text{ is an old number} \\ 1 & \text{if } u \text{ is an even number} \end{cases}$$

and $\xi^n : S \to [-1,0]$ *as a function*

$$\xi^n(u) = \begin{cases} -1 & \text{if } u \text{ is an old number} \\ 0 & \text{if } u \text{ is an even number.} \end{cases}$$

Then $\xi = (S; \xi^p, \xi^n)$ *is a BF set.*

For bipolar fuzzy sets $\xi = (S; \xi^p, \xi^n)$ and $\varsigma = (S; \varsigma^p, \varsigma^n)$, define the products $\xi^p \circ \varsigma^p$ and $\xi^n \circ \varsigma^n$ as follows: For $u \in S$

$$(\xi^p \circ \varsigma^p)(k) = \begin{cases} \bigvee_{(y,z) \in F_k} \{\xi^p(y) \wedge \varsigma^p(z)\} & \text{if } k = yz \\ 0 & \text{if otherwise.} \end{cases}$$

and

$$(\xi^n \circ \varsigma^n)(k) = \begin{cases} \bigwedge_{(y,z) \in F_k} \{\xi^n(y) \vee \varsigma^n(z)\} & \text{if } k = yz \\ 0 & \text{if otherwise.} \end{cases}$$

Definition 7. *Let* I *be a non-empty set of a semigroup* S. *A* ***positive characteristic function and a negative characteristic function*** *are respectively defined by*

$$\lambda_I^p : S \to [0,1], k \mapsto \lambda_I^p(u) := \begin{cases} 1 & k \in I, \\ 0 & k \notin I, \end{cases}$$

and

$$\lambda_I^n : S \to [-1,0], k \mapsto \lambda_I^n(k) := \begin{cases} -1 & k \in I, \\ 0 & k \notin I. \end{cases}$$

Remark 2. *For the sake of simplicity, we shall use the symbol $\lambda_I = (S; \lambda_I^p, \lambda_I^n)$ for the BF set $\lambda_I := \{(k, \lambda_I^p(k), \lambda_I^n(k)) \mid k \in I\}$.*

For $u \in S$ and $(t, s) \in [0,1] \times [-1,0]$, a bipolar fuzzy point $x_{(t,s)} = (S; x_t^p, x_s^n)$ of a set S is a bipolar set of S defined by

$$x_t^p(u) = \begin{cases} t & \text{if } u = x \\ 0 & \text{if } u \neq x \end{cases}$$

and

$$x_s^n(u) = \begin{cases} s & \text{if } u = x \\ 0 & \text{if } u \neq x. \end{cases}$$

Next, we study the intersection of the BF set as defined.
Let $\xi = (S; \xi^p, \xi^n)$ and $\varsigma = (S; \varsigma^p, \varsigma^n)$ be BF sets of a semigroup S.
Define $\xi \cap \varsigma = (\xi^p \cap \varsigma^p, \xi^n \cap \varsigma^n)$ where $(\xi^p \cap \varsigma^p)(k) = \xi^p(k) \wedge \varsigma^p(k)$ and $(\xi^n \cap \varsigma^n)(k) = \xi^n(k) \vee \varsigma^n(k)$ for all $k \in S$.

Definition 8. *[9] A BF set $\xi = (S; \xi^p, \xi^n)$ on a Γ-semigroup S is called a **BF subsemigroup** on S if it satisfies the following conditions: $\xi^p(u\alpha v) \geq \xi^p(u) \wedge \xi^p(v)$ and $\xi^n(u\alpha v) \leq \xi^n(u) \vee \xi^n(v)$ for all $u, v \in S$ and $\alpha \in \Gamma$.*

The following presents an example of a BF subsemigroup.

Example 2. *Let S be the set of all negative integers and Γ be the set of all non positive even intergers. Then S is a Γ-semigroup by usual multiplication of integers.*
Let $\xi = (S; \xi^p, \xi^n)$ be a BF set of S, as defined follows

$$\xi^p(u) = \begin{cases} 1 & \text{if } u = 0 \\ -\frac{1}{u} & \text{if } u = -1, -2 \\ -\frac{1}{u-2} & \text{if } u < -2 \end{cases} \quad \text{and } \xi^n(u) = \begin{cases} -1 & \text{if } u = 0 \\ \frac{1}{u} & \text{if } u = -1, -2 \\ \frac{1}{u-2} & \text{if } u < -2 \end{cases}$$

Then $\xi = (S; \xi^p, \xi^n)$ is a BF subsemigroup of S.

Definition 9. *[9] A BF set $\xi = (S; \xi^p, \xi^n)$ on a Γ-semigroup S is called a **BF left (right) ideal** on S if it satisfies the following conditions: $\xi^p(u\alpha v) \geq \xi^p(v)$ $(\xi^p(u\alpha v) \geq \xi^p(u))$ and $\xi^n(u\alpha v) \leq \xi^n(v)$ $(\xi^n(u\alpha v) \leq \xi^n(u))$ for all $u, v \in S$ and $\alpha \in \Gamma$.*

3. New Types of Bipolar Valued Fuzzy Ideals

In this section, we define the bipolar fuzzy (α, β)-ideal and study its basic properties.

Definition 10. *Let $\xi = (S; \xi^p, \xi^n)$ be a BF set of a Γ-semigroup S and $\alpha, \beta \in \Gamma$. Then $\xi = (S; \xi^p, \xi^n)$ is called*

[(i)]A BF left α-ideal *of S if $\xi^p(u\alpha v) \geq \xi^p(v)$ and $\xi^n(u\alpha v) \leq \xi^n(v)$ for all $u, v \in S$. A* BF right β-ideal *of S if $\xi^p(u\beta v) \geq \xi(u)$ and $\xi^n(u\beta v) \leq \xi^n(u)$ for all $u, v \in S$. A* BF (α, β)-ideal *of S if it is both a BF left α-ideal and a BF right β-ideal of S. A* BF α-ideal *of S if it is a BF (α, α)-ideal of S.*

Theorem 2. *Let K be a non-empty subset of Γ-semigroup S. Then K is a left α-ideal (right β-ideal, (α, β)-ideal) of S if and only if $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF left α-ideal (right β-ideal, (α, β)-ideal) of S.*

Proof. Suppose that K is a left α-ideal of S and $u, v \in S$.

If $v \in K$, then $u\alpha v \in K$. Thus, $\lambda_K^p(v) = \lambda_K^p(u\alpha v) = 1$ and $\lambda_K^n(v) = \lambda_K^n(u\alpha v) = -1$. Hence, $\lambda_K^p(u\alpha v) \geq \lambda_K^p(v)$ and $\lambda_K^n(u\alpha v) \leq \lambda_K^n(v)$.

If $v \notin K$, then $u\alpha v \in K$. Thus, $\lambda_K^p(v) = \lambda_K^n(v) = 0$ and $\lambda_K^p(u\alpha v) = 1$, $\lambda_K^n(u\alpha v) = -1$. Hence, $\lambda_K^p(u\alpha v) \geq \lambda_K^p(v)$ and $\lambda_K^n(u\alpha v) \leq \lambda_K^n(v)$.

Therefore, $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF left α-ideal of S.

Conversely, assume that $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF left α-ideal of S and $u, v \in S$ with $v \in K$. Then $\lambda_K^p(v) = 1$ and $\lambda_I^p(v) = -1$. By assumption, $\lambda_K^p(u\alpha v) \geq \lambda_K^p(v)$ and $\lambda_K^n(u\alpha v) \leq \lambda_K^n(v)$ Thus, $u\alpha v \in K$. Hence, K is a left α-ideal of S.

Theorem 3. *The positive and negative of the intersection and union of any two BF left α-ideals (right β-ideals, (α, β)-ideals) of a Γ-semigroup S is a BF left α-ideal (right β-ideal, (α, β)-ideal) of S.*

Proof. Let $\xi = (S; \xi^p, \xi^n)$ and $\varsigma = (S; \varsigma^p, \varsigma^n)$ be BF left α-ideals of S and let $u, v \in S$. Then

$$(\xi^p \cap \varsigma^p)(u\alpha v) = \xi^p(u\alpha v) \wedge \varsigma^p(u\alpha v) \geq \xi^p(v) \wedge \varsigma^p(v) = (\xi^p \cap \varsigma^p)(v)$$

and

$$(\xi^n \cap \varsigma^n)(u\alpha v) = \xi^n(u\alpha v) \vee \varsigma^n(u\alpha v) \leq \xi^n(v) \vee \varsigma^n(v) = (\xi^n \cap \varsigma^n)(v).$$

Similarly,

$$(\xi^p \cup \varsigma^p)(u\alpha v) = \xi^p(u\alpha v) \vee \varsigma^p(u\alpha v) \geq \xi^p(v) \vee \varsigma^p(v) = (\xi^p \cup \varsigma^p)(v)$$

and

$$(\xi^n \cup \varsigma^n)(u\alpha v) = \xi^n(u\alpha v) \wedge \varsigma^n(u\alpha v) \leq \xi^n(v) \wedge \varsigma^n(v) = (\xi^n \cap \varsigma^n)(v).$$

Thus, $\xi \cap \varsigma$ and $\xi \cup \varsigma$ are BF left α-ideals of S.

Theorem 4. *Let $\xi = (S;\xi^p,\xi^n)$ be a BF set of a Γ-semigroup S and $\xi_{(l,m)} = (S;\xi^p_l,\xi^n_m)$ be BF point with $\xi^p_l = \{x \in S \mid \xi^p_l(x) \geq l\}$ and $\xi^n_m = \{x \in S \mid \xi^n_m(x) \leq m\}$. Then $\xi = (S;\xi^p,\xi^n)$ is a BF left α-ideal (right β-ideal, (α,β)-ideal) of S if and only if $\xi_{(l,m)} = (S;\xi^p_l,\xi^n_m)$ is a non-empty set and $\xi_{(l,m)}$ is a left α-ideal (right β-ideal, (α,β)-ideal) of S for all $(l,m) \in (0,1] \times [-1,0)$.*

Proof. Suppose that $\xi = (S;\xi^p,\xi^n)$ is a BF left α-ideal of S. Then $\xi^p(u\alpha r) \geq \xi^p(r)$ and $\xi^n(u\alpha r) \leq \xi^n(r)$ for all $u,r \in S$. Let $(l,m) \in (0,1] \times [-1,0)$ be such that $\xi_{(l,m)} \neq \emptyset$. Let $r \in \xi_{(l,m)}$ and $u \in S$. Then $\xi^p(r) \geq l$ and $\xi^n(r) \leq m$. Thus, $\xi^p(u\alpha r) \geq \xi^p(r) \geq l$ and $\xi^p(u\alpha r) \leq \xi^n(r) \leq m$. So, $u\alpha r \in \xi_{(l,m)}$. Hence, $\xi_{(l,m)} = (S;\xi^p_l,\xi^n_m)$ is a left α-ideal of S.

Conversely, assume that $\xi_{(l,m)} = (S;\xi^p_l,\xi^n_m)$ is a left α-ideal of S if $(l,m) \in (0,1] \times [-1,0)$ and $\xi_{(l,m)} \neq \emptyset$. Let $u,v \in S$ and $l = \xi^p(v), m = \xi^n(v)$. By assumption, $\xi^p(v) \geq l$ and $\xi^n(v) \leq m$. Then $v \in \xi_{(l,m)}$. Thus, $\xi_l \neq \emptyset$. Hence, ξ_l is a left α-ideal of S. Since $v \in \xi_{(l,m)}$ and $u \in S$, we have $x\alpha v \in \xi_{(l,m)}$. Thus, $\xi^p(u\alpha v) \geq l = \xi^p(v)$ and $\xi^n(u\alpha v) \leq m = \xi^n(v)$. Hence, $\xi = (S;\xi^p,\xi^n)$ is a BF left α-ideal of S.

Next, we will define the (α,β)-product.

For BF sets $\xi = (S;\xi^p,\xi^n)$ and $\varsigma = (S;\varsigma^p,\varsigma^n)$, define the product $\xi^p \circ_\alpha \varsigma^p$ and $\xi^n \circ_\alpha \varsigma^n$ as follows: For $u \in S$

$$(\xi^p \circ_\alpha \varsigma^p)(u) = \begin{cases} \bigvee\limits_{(y,\alpha,z)\in F_{u_\alpha}} \{\xi^p(y) \wedge \varsigma^p(z)\} & \text{if } u = y\alpha z \\ 0 & \text{if otherwise.} \end{cases}$$

and

$$(\xi^n \circ_\alpha \varsigma^n)(u) = \begin{cases} \bigwedge\limits_{(y,\alpha,z)\in F_{u_\alpha}} \{\xi^n(y) \vee \varsigma^n(z)\} & \text{if } u = y\alpha z \\ 0 & \text{if otherwise,} \end{cases}$$

where $F_{u_\alpha} = \{(y,z) \in S \times \Gamma \times S \mid u = y\alpha z\}$, for $u \in S$ and $\alpha \in \Gamma$.

Next, we define BF (α,β)-bi-ideal and study its basic properties.

Definition 11. *Let $\xi = (S;\xi^p,\xi^n)$ be a BF set of a Γ-semigroup S and $\alpha,\beta \in \Gamma$. Then $\xi = (S;\xi^p,\xi^n)$ is called a* BF (α,β)-bi-ideal *of S if $\xi^p \circ_\alpha \lambda^p_S \circ_\beta \xi^p \geq \xi^p$ and $\xi^n \circ_\alpha \lambda^n_S \circ_\beta \xi^n \leq \xi^n$ where $\lambda_S = (S;\lambda^p_S,\lambda^n_S)$ is a BF set mapping every element of S to $[-1,1]$.*

Theorem 5. *Let K be a non-empty subset of Γ-semigroup S. Then K is an (α,β)-bi-ideal of S if and only if the characteristic function $\lambda_K = (S;\lambda^p_K,\lambda^n_K)$ is a BF (α,β)-bi-ideal of S.*

Proof. Suppose that K is an (α,β)-bi-ideal of S and $K\alpha S\beta K \subseteq K$.

If $u \in K\alpha S\beta K$, then $\lambda^p_K(u) = (\xi^p \circ_\alpha \lambda^p_S \circ_\beta \xi^p)(u) = 1$ and
$\lambda^n_K(u) = (\xi^n \circ_\alpha \lambda^n_S \circ_\beta \xi^n)(u) = -1$.
Hence, $(\xi^p \circ_\alpha \lambda^p_S \circ_\beta \xi^p)(u) \geq \xi^p(u)$ and $(\xi^n \circ_\alpha \lambda^n_S \circ_\beta \xi^n)(u) \leq \xi^n(u)$

If $u \notin K\alpha S\beta K$, then $\lambda^p_K(u) = (\xi^p \circ_\alpha \lambda^p_S \circ_\beta \xi^p)(u) = 0$ and
$\lambda^n_K(u) = (\xi^n \circ_\alpha \lambda^n_S \circ_\beta \xi^n)(u) = 0$.
Hence, $(\xi^p \circ_\alpha \lambda^p_S \circ_\beta \xi^p)(u) \geq \xi^p(u)$ and $(\xi^n \circ_\alpha \lambda^n_S \circ_\beta \xi^n)(u) \leq \xi^n(u)$

Therefore, $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF (α, β)-bi-ideal of S.

Conversely, assume that $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF (α, β)-bi-ideal of S and $u \in K\alpha S\beta K$. Then $(\xi^p \circ_\alpha \lambda_S^p \circ_\beta \xi^p)(u) = 1$ and $(\xi^n \circ_\alpha \lambda_S^n \circ_\beta \xi^n)(u) = -1$. By assumption, $(\xi^p \circ_\alpha \lambda_S^p \circ_\beta \xi^p)(u) \geq \xi^p(u)$ and $(\xi^n \circ_\alpha \lambda_S^n \circ_\beta \xi^n)(u) \leq \xi^n(u)$. Thus, $u \in K$. Hence, K is an (α, β)-bi-ideal of S.

Theorem 6. *The positive and negative of intersection of any two BF (α, β)-bi-ideals of a Γ-semigroup S are a BF (α, β)-bi-ideal of S.*

Proof. Let $\xi = (S; \xi^p, \xi^n)$ and $\varsigma = (S; \varsigma^p, \varsigma^n)$ be BF (α, β)-bi-ideals of S and $u \in S$. Then

$$((\xi^p \cap \varsigma^p) \circ_\alpha \lambda_S^p \circ_\beta (\xi^p \cap \varsigma^p))(u) \geq (\xi^p \circ_\alpha \lambda_S^p \circ_\beta \xi^p)(u) \wedge (\varsigma^p \circ_\alpha \lambda_S^p \circ_\beta \varsigma^p)(u) \geq (\xi^p \cap \varsigma^p)(u)$$

and

$$((\xi^n \cap \varsigma^n) \circ_\alpha \lambda_S^n \circ_\beta (\xi^n \cap \varsigma^n))(u) \leq (\xi^n \circ_\alpha \lambda_S^n \circ_\beta \xi^n)(u) \vee (\varsigma^n \circ_\alpha \lambda_S^n \circ_\beta \varsigma^n)(u) \leq (\xi^n \cap \varsigma^n)(u).$$

Thus, $\xi \cap \varsigma$ is a BF (α, β)-bi-ideal of S.

Theorem 7. *Let $\xi = (S; \xi^p, \xi^n)$ be a BF set of a Γ-semigroup S and $\xi_{(l,m)} = (S; \xi_l^p, \xi_m^n)$ be BF point with $\xi_l^p = \{x \in S \mid \xi_l^p(x) \geq l\}$ and $\xi_m^n = \{x \in S \mid \xi_m^n(x) \leq m\}$. Then $\xi = (S; \xi^p, \xi^n)$ is a BF (α, β)-bi-ideal of S if and only if $\xi_{(l,m)} = (S; \xi_l^p, \xi_m^n)$ is a non-empty set and $\xi_{(l,m)}$ is an (α, β)-bi-ideal of S for all $(l, m) \in (0, 1] \times [-1, 0)$.*

Proof. Suppose that $\xi = (S; \xi^p, \xi^n)$ is a BF (α, β)-bi-ideal of S. Then $\xi^p(u\alpha v\beta w) \geq \xi^p(u) \wedge \xi^p(w)$ and $\xi^n(u\alpha v\beta w) \leq \xi^n(u) \vee \xi^n(w)$ for all $u, v, w \in S$. Let $(l, m) \in (0, 1] \times [-1, 0)$ be such that $\xi_{(l,m)} \neq \emptyset$. Let $u, w \in \xi_{(l,m)}$ and $v \in S$. Then $\xi^p(u) \geq l$, $\xi^p(w)$ and $\xi^n(u) \leq m$, $\xi^n(w) \leq m$. Thus, $\xi^p(u\alpha v\beta w) \geq \xi^p(u) \wedge \xi^p(w) \geq l$ and $\xi^p(u\alpha v\beta w) \leq \xi^n(u) \vee \xi^n(w) \leq m$. So, $u\alpha v\beta w \in \xi_{(l,m)}$. Hence, $\xi_{(l,m)} = (S; \xi_l^p, \xi_m^n)$ is an (α, β)-bi-ideal of S.

Conversely, assume that $\xi_{(l,m)} = (S; \xi_l^p, \xi_m^n)$ is an (α, β)-bi-ideal of S if $(l, m) \in (0, 1] \times [-1, 0)$ and $\xi_{(l,m)} \neq \emptyset$. Let $u, v, w \in S$ and $l = \xi^p(u)$, $l = \xi^p(u)$, $m = \xi^n(u)$, $m = \xi^n(w)$. By assumption, $\xi^p(u) \wedge \xi^p(w) \geq l$ and $\xi^n(u) \vee \xi^n(w) \leq m$. Then $u, w \in \xi_{(l,m)}$. Thus, $\xi_l \neq \emptyset$. Hence, ξ_l is an (α, β)-bi-ideal of S. Since $u, w \in \xi_{(l,m)}$ and $v \in S$, we have $u\alpha v\beta w \in \xi_{(l,m)}$. Thus, $\xi^p(u\alpha v\beta w) \geq l = \xi^p(u) \wedge \xi^(p)(w)$ and $\xi^n(u\alpha v\beta w) \leq m = \xi^n(u) \vee \xi^n(w)$. Hence, $\xi = (S; \xi^p, \xi^n)$ is a BF (α, β)-bi-ideal of S.

Next, we define a BF (α, β)-quasi-ideal and study its basic properties.

Definition 12. *Let $\xi = (S; \xi^p, \xi^n)$ be a BF set of a Γ-semigroup S and $\alpha, \beta \in \Gamma$. Then $\xi = (S; \xi^p, \xi^n)$ is called a* BF (α, β)-quasi-ideal *of S if $\lambda_S^p \circ_\alpha \xi^p \cap \xi^p \circ_\beta \lambda_S^p \subseteq \xi^p$ and $\lambda_S^n \circ_\alpha \xi^n \cup \xi^n \circ_\beta \lambda_S^n \supseteq \xi^n$.*

Theorem 8. *If $\xi = (S; \xi^p, \xi^n)$ and $\varsigma = (S; \varsigma^p, \varsigma^n)$ is a BF left α-ideal and a BF right α-ideal of a Γ-semigroup S, respectively, then $\xi \cap \varsigma$ is a BF α-quasi-ideal of S.*

Proof. Let $\xi = (S; \xi^p, \xi^n)$ and $\varsigma = (S; \varsigma^p, \varsigma^n)$ is a BF left α-ideal and a BF right α-ideal of S, respectively. Then $\varsigma^p \circ_\alpha \xi^p \subseteq \lambda^p_S \circ_\alpha \xi^p \subseteq \xi^p$ and $\varsigma^p \circ_\alpha \xi^p \subseteq \varsigma^p \circ_\alpha \lambda^p_S \subseteq \varsigma^p$. Thus, $\varsigma^p \circ_\alpha \xi^p \subseteq \xi^p \cap \varsigma^p$. So,

$$\lambda^p_S \circ_\alpha (\xi^p \cap \varsigma^p) \cap (\xi^p \cap \varsigma^p) \circ_\alpha \lambda^p_S \subseteq \lambda^p_S \circ_\alpha (\xi^p \cap \varsigma^p) \circ_\alpha \lambda^p_S \subseteq \xi^p \cap \varsigma^p.$$

Thus, $\xi^p \cap \varsigma^p$ is a BF α-quasi-ideal of S. Similarly, we can show that $\xi^n \cap \varsigma^n$ is a BF α-quasi-ideal of S. Hence, $\xi \cap \varsigma$ is a BF α-quasi-ideal of S.

Theorem 9. *Every BF (α, β)-quasi-ideal of Γ-semigroup S is the intersection of a BF left α-ideal and a BF right β-ideal of S*

Proof. Let $\xi = (S; \xi^p, \xi^n)$ be a BF (α, β)-quasi-ideal of S.
Consider $\varsigma^p = \xi^p \cup (\lambda^p_S \circ_\alpha \xi^p)$ and $\varsigma^n = \xi^n \cup (\lambda^n_S \circ_\alpha \xi^n)$ where $\varsigma = (S; \varsigma^p, \varsigma^n)$, $\kappa^p = \xi^p \cup (\xi^p \circ_\beta \lambda^p_S)$ and $\kappa^n = \xi^n \cup (\xi^n \circ_\beta \lambda^n_S)$ where $\kappa = (S; \kappa^p, \kappa^n)$. Then

$$\begin{aligned} \lambda^p_S \circ_\alpha \varsigma^p &= \lambda^p_S \circ_\alpha (\xi^p \cup (\lambda^p_S \circ_\alpha \xi^p)) = (\lambda^p_S \circ_\alpha \xi^p) \cup (\lambda^p_S \circ_\alpha (\lambda^p_S \circ_\alpha \xi^p)) \\ &= (\lambda^p_S \circ_\alpha \xi^p) \cup ((\lambda^p_S \circ_\alpha \lambda^p_S) \circ_\alpha \xi^p) = (\lambda^p_S \circ_\alpha \xi^p) \cup (\lambda^p_S \circ_\alpha \xi^p) \\ &\subseteq \xi^p \cup (\lambda^p_S \circ_\alpha \xi^p) = \varsigma^p. \end{aligned}$$

And

$$\begin{aligned} \kappa^p \circ_\beta \lambda^p_S &= (\xi^p \cup (\xi^p \circ_\beta \lambda^p_S)) \circ_\alpha \lambda^p_S = (\xi^p \circ_\alpha \lambda^p_S) \cup (\xi^p \circ_\beta \lambda^p_S \circ_\alpha \lambda^p_S) \\ &= (\xi^p \circ_\alpha \lambda^p_S) \cup \xi^p \circ_\beta (\lambda^p_S \circ_\alpha \lambda^p_S) = (\xi^p \circ_\alpha \lambda^p_S) \cup (\xi^p \circ_\beta \lambda^p_S) \\ &\subseteq \xi^p \cup (\xi^p \circ_\beta \lambda^p_S) = \kappa^p. \end{aligned}$$

Similarly, we can show that $\lambda^n_S \circ_\alpha \varsigma^n \supseteq \varsigma^n$ and $\kappa^n \circ_\beta \lambda^n_S \supseteq \kappa^n$.
Thus, $\varsigma = (S; \varsigma^p, \varsigma^n)$ and $\kappa = (S; \kappa^p, \kappa^n)$ is a BF left α-ideal and a BF right β-ideal of S. Now,

$$\xi^p \subseteq (\xi^p \cup (\lambda^p_S \circ_\alpha \xi^p)) \cap (\xi^p \cup (\xi^p \circ_\beta \lambda^p_S)) = \varsigma^p \cap \kappa^p$$

and

$$\varsigma^p \cap \kappa^p = (\xi^p \cup (\lambda^p_S \circ_\alpha \xi^p)) \cap (\xi^p \cup (\xi^p \circ_\beta \lambda^p_S)) = \xi^p \cap ((\lambda^p_S \circ_\alpha \xi^p) \cup (\xi^p \circ_\beta \lambda^p_S)) \subseteq \xi^p \cap \xi^p = \xi^p.$$

Hence, $\xi^p = \varsigma^p \cap \kappa^p$. Simlarly, we can show that $\xi^n = \varsigma^n \cap \kappa^n$.

Theorem 10. *Let K be a non-empty subset of Γ-semigroup S. Then K is a (α, β)-quasi-ideal of S if and only if the characteristic function $\lambda_K = (S; \lambda^p_K, \lambda^n_K)$ is a BF (α, β)-quasi-ideal of S.*

Proof. Suppose that K is a (α, β)-quasi-ideal of S and $u \in S$.
If $u \in (S\alpha K) \cap (K\beta S)$, then $u \in K$. Thus, $\lambda^p_K(u) = 1$ and $\lambda^n_K(u) = -1$.
Hence, $((\lambda^p_K \circ_\alpha \lambda^p_S) \cap (\lambda^p_S \circ_\beta \lambda^p_K))(u) \leq \lambda^p_K(u)$ and $((\lambda^n_K \circ_\alpha \lambda^n_S) \cup (\lambda^n_S \circ_\beta \lambda^n_K))(u) \geq \lambda^n_K(u)$
If $u \notin (S\alpha K) \cap (K\beta S)$, then $\lambda^p_K(u) = 0$ and $\lambda^p_K(u) = 0$.
Hence, $((\lambda^p_K \circ_\alpha \lambda^p_S) \cap (\lambda^p_S \circ_\beta \lambda^p_K))(u) \leq \lambda^p_K(u)$ and $((\lambda^n_K \circ_\alpha \lambda^n_S) \cup (\lambda^n_S \circ_\beta \lambda^n_K))(u) \geq \lambda^n_K(u)$.
Therefore, $\lambda_K = (S; \lambda^p_K, \lambda^n_K)$ is a BF (α, β)-quasi-ideal of S.

Conversely, assume that $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF (α, β)-quasi-ideal of S and $u \in (S\alpha K) \cap (K\beta S)$. Then $((\lambda_K^p \circ_\alpha \lambda_S^p) \cap (\lambda_S^p \circ_\beta \lambda_K^p))(u) = 1$ and $((\lambda_K^n \circ_\alpha \lambda_S^n) \cap (\lambda_S^n \circ_\beta \lambda_K^n))(u) = -1$. By assumption, $((\lambda_K^p \circ_\alpha \lambda_S^p) \cap (\lambda_S^p \circ_\beta \lambda_K^p))(u) \leq \lambda_K^p(u)$ and $((\lambda_K^n \circ_\alpha \lambda_S^n) \cup (\lambda_S^n \circ_\beta \lambda_K^n))(u) \geq \lambda_K^n(u)$. Thus, $u \in K$. Hence, K is an (α, β)-quasi-ideal of S.

4. New Types of Bipolar Fuzzy Almost Ideals

Definition 13. *Let $\xi = (S; \xi^p, \xi^n)$ be a BF set of a Γ-semigroup S, and $\alpha, \beta \in \Gamma$ is said to be*

[(i)]

(i) A BF almost left α-ideal of S if $(x_t^p \circ_\alpha \xi^p) \wedge \xi^p \neq 0$ and $(x_s^n \circ_\alpha \xi^n) \vee \xi^n \neq 0$.

(ii) A BF almost right β-ideal of S if $\xi^p \circ_\beta (x_t^p) \wedge \xi^p \neq 0$ and $(\xi^n \circ_\beta x_s^n) \vee \xi^n \neq 0$.

(iii) A BF almost (α, β)-ideal of S if it is both a BF almost left α-ideal and a BF almost right β-ideal of S.

Theorem 11. *If $\xi = (S; \xi^p, \xi^n)$ is a BF almost left α-ideal (right β-ideal, (α, β)-ideal) of a Γ-semigroup S, and $\varsigma = (S; \varsigma^p, \varsigma^n)$ is a BF set of S such that $\xi \subseteq \varsigma$, then $\varsigma = (S; \varsigma^p, \varsigma^n)$ is a BF left almost α-ideal (right β-ideal, (α, β)-ideal) of S.*

Proof. Suppose that $\xi = (S; \xi^p, \xi^n)$ is a BF almost left α-ideal of S, and $\varsigma = (S; \varsigma^p, \varsigma^n)$ is a BF set of S such that $\xi \subseteq \varsigma$. Then $(x_t^p \circ_\alpha \xi^p) \wedge \xi^p \neq 0$ and $(x_s^n \circ_\alpha \xi^n) \vee \xi^n \neq 0$. Thus, $(x_t^p \circ_\alpha \xi^p) \wedge \xi^p \subseteq (x_t^p \circ_\alpha \varsigma^p) \wedge \varsigma^p \neq 0$ and $(x_s^n \circ_\alpha \xi^n) \vee \xi^n \subseteq (x_s^n \circ_\alpha \varsigma^n) \vee \varsigma^n \neq 0$. Hence, $\varsigma = (S; \varsigma^p, \varsigma^n)$ is a BF left almost α-ideal of S.

Theorem 12. *Let K be a non-empty subset of Γ-semigroup S. Then K is an almost left α-ideal (right β-ideal, (α, β)-ideal) of S if and only if the characteristic function $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF almost left α-ideal (right β-ideal, (α, β)-ideal) of S.*

Proof. Suppose that K is an almost left α-ideal of S. Then $u\alpha K \cap K \neq \emptyset$ for all $u \in S$. Thus, there exists $v \in u\alpha K$ and $v \in K$. So, $(x_t^p \circ_\alpha \lambda_K^p)(v) = \lambda_K^p(v) = 1$ and $(x_s^n \circ_\alpha \lambda_K^n)(v) = \lambda_K^n(v) = -1$. Hence, $(x_t^p \circ_\alpha \lambda_K^p) \wedge \lambda_K^p \neq 0$ and $(x_s^n \circ_\alpha \lambda_K^n) \vee \lambda_K^n \neq 0$. Therefore, $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF almost left α-ideal of S.

Conversely, assume that $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF almost left α-ideal of S and $u \in S$. Then $(x_t^p \circ_\alpha \lambda_K^p) \wedge \lambda_K^p \neq 0$ and $(x_s^n \circ_\alpha \lambda_K^n) \vee \lambda_K^n \neq 0$. Thus, there exists $r \in S$ such that $((x_t^p \circ_\alpha \lambda_K^p) \wedge \lambda_K^p)(r) \neq 0$ and $((x_s^n \circ_\alpha \lambda_K^n) \vee \lambda_K^n)(r) \neq 0$. Hence, $r \in u\alpha K \cap K$ implies $u\alpha K \cap K \neq \emptyset$. Therefore, K is an almost left α-ideal of S.

Next, we review the definition of supp(ξ), and we study the properties between supp(ξ) and BF almost left α-ideal (right β-ideal, (α, β)-ideal) of Γ-semigroups.

Let $\xi = (S; \xi^p, \xi^n)$ be a BF set of a non-empty of S. Then the *support* of ξ instead of supp$(\xi) = \{u \in S \mid \xi(u) \neq 0\}$ where $\xi^p(u) \neq 0$ and $\xi^n(u) \neq 0$ for all $u \in S$.

Theorem 13. *Let $\xi = (S; \xi^p, \xi^n)$ be a BF set of a non-empty of a Γ-semigroup S. Then $\xi = (S; \xi^p, \xi^n)$ is a BF almost left α-ideal (right β-ideal, (α, β)-ideal) of S if and only if* $\text{supp}(\xi)$ *is an almost left α-ideal (right β-ideal, (α, β)-ideal) of S.*

Proof. Let $\xi = (S; \xi^p, \xi^n)$ be a BF almost left α-ideal of S and $u \in S$. Then $(x_t^p \circ_\alpha \xi^p) \wedge \xi^p \neq 0$ and $(x_s^n \circ_\alpha \xi^n) \vee \xi^n \neq 0$. Thus, there exists $r \in S$ such that $((x_t^p \circ_\alpha \xi^p) \wedge \xi^p)(r) \neq 0$ and $((x_s^n \circ_\alpha \xi^n) \vee \xi^n)(r) \neq 0$. So, there exists $k \in S$ such that $r = u\alpha k$, $x_t^p(r) \neq 0$, $x_s^n(r) \neq 0$ and $x_t^p(k) \neq 0$, $x_s^n(k) \neq 0$. It implies that $r, k \in \text{supp}(\xi)$. Thus, $(x_t^p \circ_\alpha \lambda^p_{\text{supp}(\xi)})(r) \neq 0$, $(x_s^n \circ_\alpha \lambda^n_{\text{supp}(\xi)})(r) \neq 0$, and $\lambda^p_{\text{supp}(\xi)} \neq 0$, $\lambda^n_{\text{supp}(\xi)} \neq 0$. Hence, $(x_t^p \circ_\alpha \lambda^p_{\text{supp}(\xi)}) \wedge \lambda^p_{\text{supp}(\xi)} \neq 0$ and $(x_s^n \circ_\alpha \lambda^n_{\text{supp}(\xi)}) \vee \lambda^n_{supp(\xi)} \neq 0$. Therefore, $\lambda_{\text{supp}(\xi)}$ is a BF almost left α-ideal of S. This shows that $\text{supp}(\xi)$ is an almost left α-ideal of S.

Conversely, let $\text{supp}(\xi)$ be an almost left α-ideal of S. Then, by Theorem 12, $\lambda_{\text{supp}(\xi)}$ is a BF almost left α-ideal of S. Thus, $(x_t^p \circ_\alpha \lambda^p_{\text{supp}(\xi)}) \wedge \lambda^p_{\text{supp}(\xi)} \neq 0$ and $(x_s^n \circ_\alpha \lambda^n_{\text{supp}(\xi)}) \vee \lambda^n_{\text{supp}(\xi)} \neq 0$. So, there exists $r \in S$ such that $((x_t^p \circ_\alpha \lambda^p_{\text{supp}(\xi)}) \wedge \lambda^p_{\text{supp}(\xi)})(r) \neq 0$ and $((x_s^n \circ_\alpha \lambda^n_{\text{supp}(\xi)}) \vee \lambda^n_{\text{supp}(\xi)})(r) \neq 0$. It implies that $(x_t^p \circ_\alpha \lambda^p_{\text{supp}(\xi)})(r) \neq 0$, $(x_t^n \circ_\alpha \lambda^n_{\text{supp}(\xi)})(r) \neq 0$ and $\lambda^p_K(r) \neq 0$, $\lambda^n_K(r) \neq 0$. Thus, there exists $k \in S$ such that $r = u\alpha k$, $x_t^p(r) \neq 0$, $x_s^n(r) \neq 0$ and $x_t^p(k) \neq 0$, $x_s^n(k) \neq 0$. Hence, $(x_t^p \circ_\alpha \xi^p) \wedge \xi^p \neq 0$ and $(x_s^n \circ_\alpha \xi^n) \vee \xi^n \neq 0$. Therefore, $\xi = (S; \xi^p, \xi^n)$ is a BF almost left α-ideal of S.

Definition 14. *An almost ideal I of a Γ-semigroup S is called* minimal *if for every almost ideal of J of S such that $J \subseteq I$, we have $J = I$.*

Definition 15. *A BF almost left α-ideal (right β-ideal, (α, β)-ideal) $\xi = (S; \xi^p, \xi^n)$ of a Γ-semigroup S is* minimal *if for all BF almost left α-ideal (right β-ideal, (α, β)-ideal) $\varsigma = (S; \varsigma^p, \varsigma^n)$ of S such that $\varsigma \subseteq \xi$, then* $\text{supp}(\varsigma) = \text{supp}(\xi)$.

Theorem 14. *Let K be a non-empty subset of a Γ-semigroup S. Then K is a minimal almost left α-ideal (right β-ideal, (α, β)-ideal) if and only if $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost left α-ideal (right β-ideal, (α, β)-ideal) of S.*

Proof. Suppose that K is a minimal almost left α-ideal of S. Then K is an almost left α-ideal of S. Thus, by Theorem 12, $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF left α-ideal of S. Let $\varsigma = (S; \varsigma^p, \varsigma^n)$ be a BF left α-ideal of S such that $\varsigma \subseteq \xi$. Then, by Theorem 13, $\text{supp}(\varsigma)$ is an almost left α-ideal of S. Thus, $\text{supp}(\varsigma) \subseteq \text{supp}(\lambda_K) = K$. By assumption, $\text{supp}(\varsigma) = K = \text{supp}(\lambda_K)$. Thus, $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost left α-ideal of S.

Conversely, suppose that $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost left α-ideal of S. Then, by Theorem 12, K is an almost left α-ideal of S. Let J be an almost left α-ideal of S such that $J \subseteq K$. Then by Theorem 12, $\lambda_J = (S; \lambda_J^p, \lambda_J^n)$ is a BF left α-ideal of S such that $\lambda_J \subseteq \lambda_K$. Thus, $J = \text{supp}(\lambda_J) = \text{supp}(\lambda_K) = K$. Hence, K is a minimal almost left α-ideal of S.

Corollary 1. *Let S be a Γ-semigroup S. Then S has no proper almost left α-ideal (right β-ideal, (α, β)-ideal) of S if and only if for any BF almost left α-ideal (right β-ideal, (α, β)-ideal) $\xi = (S; \xi^p, \xi^n)$ of S, supp(ξ) $= S$.*

Next, we define the BF almost (α,β)-quasi-ideals, and we study their properties.

Definition 16. *Let $\xi=(S;\xi^p,\xi^n)$ be a BF set of a Γ-semigroup S, and $\alpha,\beta\in\Gamma$ is said to be a* BF almost (α,β)-quasi-ideal *of S if $(\xi\circ_\alpha x_{(t,s)})\cap(x_{(t,s)}\circ_\beta\xi)\neq 0$.*

Theorem 15. *If $\xi=(S;\xi^p,\xi^n)$ is a BF almost (α,β)-quasi-ideal of a Γ-semigroup S, and $\varsigma=(S;\varsigma^p,\varsigma^n)$ is a BF set of S such that $\xi\subseteq\varsigma$, then $\varsigma=(S;\varsigma^p,\varsigma^n)$ is a BF (α,β)-quasi-ideal of S.*

Proof. Suppose that $\xi=(S;\xi^p,\xi^n)$ is a BF almost (α,β)-quasi-ideal of S, and $\varsigma=(S;\varsigma^p,\varsigma^n)$ is a BF set of S such that $\xi\subseteq\varsigma$. Then $(x_t^p\circ_\alpha\xi^p)\wedge(\xi^p\circ_\beta x_t^p)\neq 0$ and $(x_s^n\circ_\alpha\xi^n)\vee(\xi^n\circ_\beta x_s^n)\neq 0$. Thus, $(x_t^p\circ_\alpha\xi^p)\wedge(\xi^p\circ_\beta x_t^p)\subseteq(x_t^p\circ_\alpha\varsigma^p)\wedge(\varsigma^p\circ_\beta x_t^p)\neq 0$, and $(x_s^n\circ_\alpha\xi^n)\vee(\xi^n\circ_\beta x_s^n)\subseteq(x_s^n\circ_\alpha\varsigma^n)\wedge(\varsigma^n\circ_\beta x_s^n)\neq 0$. Hence, $\varsigma=(S;\varsigma^p,\varsigma^n)$ is a BF (α,β)-quasi-ideal of S.

Theorem 16. *Let K be a non-empty subset of Γ-semigroup S. Then K is an almost (α,β)-quasi-ideal of S if and only if the characteristic function $\lambda_K=(S;\lambda_K^p,\lambda_K^n)$ is a BF almost (α,β)-quasi-ideal of S.*

Proof. Suppose that K is an almost (α,β)-quasi-ideal of S. Then $(K\alpha u)\cap(u\beta K)\cap K\neq\emptyset$ for all $u\in S$. Thus, there exists $v\in(K\alpha u)\cap(u\beta K)$ and $v\in K$. So, $((x_t^p\circ_\alpha\lambda_K^p)\wedge(\lambda_K^p\circ_\beta x_t^p))(v)\neq 0$ and $((x_s^n\circ_\alpha\lambda_K^n)\vee(\lambda_K^n\circ_\beta x_s^n))(v)\neq 0$. Hence, $(\lambda_K\circ_\alpha x_{(t,s)})\cap(x_{(t,s)}\circ_\beta\lambda_K)\neq 0$. Therefore, $\lambda_K=(S;\lambda_K^p,\lambda_K^n)$ is a BF almost (α,β)-quasi-ideal of S.

Conversely, assume that $\lambda_K=(S;\lambda_K^p,\lambda_K^n)$ is a BF almost (α,β)-quasi-ideal of S and $u\in S$. Then $(\lambda_K\circ_\alpha x_{(t,s)})\cap(x_{(t,s)}\circ_\beta\lambda_K)\neq 0$. Thus, there exists $r\in S$ such that $((x_t^p\circ_\alpha\lambda_K^p)\wedge(\lambda_K^p\circ_\beta x_t^p))(r)\neq 0$ and $((x_s^n\circ_\alpha\lambda_K^n)\vee(\lambda_K^n\circ_\beta x_s^n))(r)\neq 0$. Hence, $r\in(K\alpha u)\cap(u\beta K)\cap K$ implies $(K\alpha u)\cap(u\beta K)\cap K\neq\emptyset$. Therefore, K is an almost (α,β)-quasi-ideal of S.

Next, we study the properties between $\mathrm{supp}(\xi)$ and a BF almost (α,β)-quasi-ideal of Γ-semigroups.

Theorem 17. *Let $\xi=(S;\xi^p,\xi^n)$ be a BF set of a non-empty of a Γ-semigroup S. Then $\xi=(S;\xi^p,\xi^n)$ is a BF almost (α,β)-quasi-ideal of S if and only if $\mathrm{supp}(\xi)$ is an almost (α,β)-quasi-ideal of S.*

Proof. Let $\xi=(S;\xi^p,\xi^n)$ be a BF almost (α,β)-quasi-ideal of S and $u\in S$. Then $(\lambda_K\circ_\alpha x_{(t,s)})\cap(x_{(t,s)}\circ_\beta\lambda_K)\neq 0$. Thus, there exists $r\in S$ such that $(\xi^p\circ_\alpha x_t)\cap(x_t\circ_\beta\xi^p)\neq 0$ and $(\xi^n\circ_\alpha x_s)\cup(x_s\circ_\beta\xi^n)\neq 0$ So, there exists $k_1,k_2\in S$ such that $r=k_1\alpha u=u\beta k_2$, $x_t^p(r)\neq 0$, $x_s^n(r)\neq 0$ and $x_t^p(k_1)\neq 0$, $x_s^n(k_1)\neq 0$. It implies that $r,k_1,k_2\in\mathrm{supp}(\xi)$. Thus $((\xi^p\circ_\alpha x_t)\cap(x_t\circ_\beta\xi^p))(r)\neq 0$ and $\lambda^p_{\mathrm{supp}(\xi)}\neq 0$. Similalry, $(\xi^n\circ_\alpha x_s)\cup(x_s\circ_\beta\xi^n)(r)\neq 0$ and $\lambda^n_{\mathrm{supp}(\xi)}\neq 0$. Hence, $(\lambda^p_{\mathrm{supp}(\xi)}\circ_\alpha x_t^p)\cap(x_t^p\circ_\beta\lambda^p_{\mathrm{supp}(\xi)})\cap\lambda^p_{\mathrm{supp}(\xi)}\neq 0$ and $(\lambda^n_{\mathrm{supp}(\xi)}\circ_\alpha x_s^n)\cup(x_s^n\circ_\beta\lambda^n_{\mathrm{supp}(\xi)})\cup\lambda^n_{supp(\xi)}\neq 0$. Therefore, $\lambda_{\mathrm{supp}(\xi)}$ is a BF almost (α,β)-quasi-ideal of S. This shows that $\mathrm{supp}(\xi)$ is an almost (α,β)-quasi-ideal of S.

Conversely, let $\text{supp}(\xi)$ be an almost (α,β)-quasi-ideal of S. Then, by Theorem 16, $\chi_{\text{supp}(\xi)}$ is a BF (α,β)-quasi-ideal of S. Thus, $[(\lambda^p_{supp(\xi)} \circ_\alpha x^p_t) \cap (x^p_t \circ_\beta \lambda^p_{supp(\xi)})] \cap \lambda^p_{\text{supp}(\xi)} \neq 0$ and $[(\lambda^n_{\text{supp}(\xi)} \circ_\alpha x^n_s) \cap (x^n_s \circ_\beta \lambda^n_{supp(\xi)})] \cap \lambda^n_{\text{supp}(\xi)} \neq 0$. So, there exists $r \in S$ such that $[(\lambda^p_{supp(\xi)} \circ_\alpha x^p_t) \cap (x^p_t \circ_\beta \lambda^p_{supp(\xi)})] \cap \lambda^p_{\text{supp}(\xi)}(r) \neq 0$ and $([(\lambda^n_{supp(\xi)} \circ_\alpha x^n_s) \cap (x^n_s \circ_\beta \lambda^n_{supp(\xi)})] \cap \lambda^n_{\text{supp}(\xi)})(r) \neq 0$. It implies that $((\lambda_{\text{supp}(\xi)} \circ_\alpha x_t) \cap (x_t \circ_\beta \lambda_{\text{supp}(\xi)}))(r) \neq 0$ and, $\lambda_K(r) \neq 0$. Thus, there exist $k_1, k_2 \in S$ such that $r = k_1 \alpha u = u \beta k_2$, $\xi^p(r) \neq 0$, $\xi^n(r) \neq 0$, and $\xi^p(k) \neq 0$, $\xi^n(k) \neq 0$. Hence, $(\xi^p \circ_\alpha x_t) \cap (x_t \circ_\beta \xi^p) \neq 0$, and $(\xi^n \circ_\alpha x_s) \cup (x_s \circ_\beta \xi^n) \neq 0$. So, $(\xi \circ_\alpha x_{(t,s)}) \cap (x_{(t,s)} \circ_\beta \xi) \neq 0$. Therefore, ξ is a BF almost (α,β)-quasi-ideal of S.

Definition 17. *A BF almost (α,β)-quasi-ideal $\xi = (S;\xi^p,\xi^n)$ of a Γ-semigroup S is* minimal *if for all BF almost (α,β)-quasi-ideal $\varsigma = (S;\varsigma^p,\varsigma^n)$ of S such that $\varsigma \subseteq \xi$, then* $\text{supp}(\varsigma) = \text{supp}(\xi)$.

Theorem 18. *Let K be a non-empty subset of a Γ-semigroup S. Then K is a minimal almost (α,β)-quasi-ideal if and only if $\lambda_K = (S;\lambda^p_K,\lambda^n_K)$ is a minimal BF almost (α,β)-quasi-ideal of S.*

Proof. Suppose that K is a minimal almost (α,β)-quasi-ideal of S. Then K is an almost (α,β)-quasi-ideal of S. Thus, by Theorem 16, $\lambda_K = (S;\lambda^p_K,\lambda^n_K)$ is a BF (α,β)-quasi-ideal of S. Let $\varsigma = (S;\varsigma^p,\varsigma^n)$ be a BF (α,β)-quasi-ideal of S such that $\varsigma \subseteq \lambda_K$. Then, by Theorem 15, $\text{supp}(\varsigma)$ is an almost (α,β)-quasi-ideal of S. Thus, $\text{supp}(\varsigma) \subseteq \text{supp}(\lambda_K) = K$. By assumption, $\text{supp}(\varsigma) = K = \text{supp}(\lambda_K)$. Thus, $\lambda_K = (S;\lambda^p_K,\lambda^n_K)$ is a minimal BF almost (α,β)-quasi-ideal of S.

Conversely, suppose that $\lambda_K = (S;\lambda^p_K,\lambda^n_K)$ is a minimal BF almost (α,β)-quasi-ideal of S. Then, by Theorem 17, K is an almost (α,β)-quasi-ideal of S. Let J be an almost (α,β)-quasi-ideal of S such that $J \subseteq K$. Then, by Theorem 17, $\lambda_J = (S;\lambda^p_J,\lambda^n_J)$ is a BF (α,β)-quasi-ideal of S such that $\lambda_J \subseteq \lambda_K$. Thus, $J = \text{supp}(\lambda_J) = \text{supp}(\lambda_K) = K$. Hence, K is a minimal almost (α,β)-quasi-ideal of S.

Corollary 2. *Let S be a Γ-semigroup. Then S has no proper almost (α,β)-quasi-ideal of S if and only if for any BF almost (α,β)-quasi-ideal $\xi = (S;\xi^p,\xi^n)$ of S, supp$(\xi) = K$.*

Next, we define the BF almost (α,β)-bi-ideals and we study their properties.

Definition 18. *Let $\xi = (S;\xi^p,\xi^n)$ be a BF set of a Γ-semigroup S, and $\alpha,\beta \in \Gamma$ is said to be* BF almost (α,β)-bi-ideal *of S if $(\xi^p \circ_\alpha x^p_t \circ_\beta \xi^p) \wedge \xi^p \neq 0$ and $(\xi^n \circ_\alpha x^n_s \circ_\beta \xi^n) \vee \xi^n \neq 0$.*

Theorem 19. *If $\xi = (S;\xi^p,\xi^n)$ is a BF almost (α,β)-bi-ideal of a Γ-semigroup S, and $\varsigma = (S;\varsigma^p,\varsigma^n)$ is a BF set of S such that $\xi \subseteq \varsigma$, then $\varsigma = (S;\varsigma^p,\varsigma^n)$ is a BF (α,β)-bi-ideal of S.*

Proof. Suppose that $\xi = (S;\xi^p,\xi^n)$ is a BF almost (α,β)-bi-ideal of S, and $\varsigma = (S;\varsigma^p,\varsigma^n)$ is a BF set of S such that $\xi \subseteq \varsigma$. Then $(\xi^p \circ_\alpha x^p_t \circ_\beta \xi^p) \wedge \xi^p \neq 0$, and $(\xi^n \circ_\alpha x^n_s \circ_\beta \xi^n) \vee \xi^n \neq 0$. Thus, $(\xi^p \circ_\alpha x^p_t \circ_\beta \xi^p) \wedge \xi^p \subseteq (\varsigma^p \circ_\alpha x^p_t \circ_\beta \varsigma^p) \wedge \varsigma^p \neq 0$, and $(\xi^n \circ_\alpha x^n_s \circ_\beta \xi^n) \vee \xi^n \subseteq (\varsigma^n \circ_\alpha x^n_s \circ_\beta \varsigma^n) \vee \varsigma^n \neq 0$. Hence, $\varsigma = (S;\varsigma^p,\varsigma^n)$ is a BF (α,β)-bi-ideal of S.

Theorem 20. *Let K be a non-empty subset of Γ-semigroup S. Then K is an almost (α,β)-bi-ideal of S if and only if the characteristic function $\lambda_K = (S;\lambda_K^p,\lambda_K^n)$ is a BF almost (α,β)-bi-ideal of S.*

Proof. Suppose that K is an almost (α,β)-bi-ideal of S. Then $K\alpha u\beta K \cap K \neq \emptyset$ for all $u \in S$. Thus, there exists $v \in K\alpha u\beta K$ and $v \in K$. So, $((\lambda_K^p \circ_\alpha x_t^p \circ_\beta \lambda_K^p))(v) = \lambda_K^p(v) = 1$, and $((\lambda_K^n \circ_\alpha x_s^n \circ_\beta \lambda_K^n))(v) = \lambda_K^n(v) = -1$. Hence, $(\lambda_K^p \circ_\alpha x_t^p \circ_\beta \lambda_K^p) \wedge \lambda_K^p \neq 0$ and $(\lambda_K^n \circ_\alpha x_s^n \circ_\beta \lambda_K^n) \vee \lambda_K^n \neq 0$. Therefore, $\lambda_K = (S;\lambda_K^p,\lambda_K^n)$ is a BF almost (α,β)-bi-ideal of S.

Conversely, assume that $\lambda_K = (S;\lambda_K^p,\lambda_K^n)$ is a BF almost (α,β)-bi-ideal of S, and $u \in S$. Then $(\lambda_K^p \circ_\alpha x_t^p \circ_\beta \lambda_K^p) \wedge \lambda_K^p \neq 0$, and $(\lambda_K^n \circ_\alpha x_s^n \circ_\beta \lambda_K^n) \vee \lambda_K^n \neq 0$. Thus, there exists $r \in S$ such that $((\lambda_K^p \circ_\alpha x_t^p \circ_\beta \lambda_K^p) \wedge \lambda_K^p)(r) \neq 0$, and $((\lambda_K^n \circ_\alpha x_s^n \circ_\beta \lambda_K^n) \vee \lambda_K^n)(r) \neq 0$. Hence, $r \in K\alpha u\beta K \cap K$ implies that $K\alpha u\beta K \cap K \neq \emptyset$. Therefore, K is an almost (α,β)-bi-ideal of S.

Next, we study the properties between $\text{supp}(\xi)$ and a BF almost (α,β)-bi-ideal of Γ-semigroups.

Theorem 21. *Let $\xi = (S;\xi^p,\xi^n)$ be a fuzzy set of a non-empty of a Γ-semigroup S. Then $\xi = (S;\xi^p,\xi^n)$ is a BF almost (α,β)-bi-ideal of S if and only if $\text{supp}(\xi)$ is an almost (α,β)-bi-ideal of S.*

Proof. Let $\xi = (S;\xi^p,\xi^n)$ be a BF almost (α,β)-bi-ideal of S, and $u \in S$. Then $(\xi^p \circ_\alpha x_t^p \circ_\beta \xi^p) \wedge \xi^p \neq 0$ and $(\xi^n \circ_\alpha x_s^n \circ_\beta \xi^n) \vee \xi^n \neq 0$. Thus, there exists $r \in S$ such that $((\xi^p \circ_\alpha x_t^p \circ_\beta \xi^p) \wedge \xi^p)(r) \neq 0$, and $((\xi^n \circ_\alpha x_s^n \circ_\beta \xi^n) \vee \xi^n)(r) \neq 0$. So, there exists $k_1, k_2 \in S$ such that $r = k_1\alpha\beta k_2$, $\xi^p(r) \neq 0$, $\xi^n(r) \neq 0$, and $\xi^p(k) \neq 0$, $\xi^n(k) \neq 0$. It implies that $r, k_1, k_2 \in \text{supp}(\xi)$. Thus, $(\lambda^p_{\text{supp}(\xi)} \circ_\alpha x_t^p \circ_\beta \lambda^p_{\text{supp}(\xi)})(r) \neq 0$ and $\lambda^p_{\text{supp}(\xi)} \neq 0$. Similarly, $(\lambda^n_{\text{supp}(\xi)} \circ_\alpha x_s^n \circ_\beta \lambda^n_{supp(\xi)})(r) \neq 0$, and $\lambda^n_{\text{supp}(\xi)} \neq 0$. Hence, $[(\lambda^p_{\text{supp}(\xi)} \circ_\alpha x_t^p \circ_\beta \lambda^p_{\text{supp}(\xi)})] \wedge \lambda^p_{\text{supp}(\xi)} \neq 0$, and $[(\lambda^n_{\text{supp}(\xi)} \circ_\alpha x_s^n \circ_\beta \lambda^n_{\text{supp}(\xi)})] \vee \lambda^n_{supp(\xi)} \neq 0$. Therefore, $\lambda_{\text{supp}(\xi)}$ is a BF almost (α,β)-bi-ideal of S. This shows that $\text{supp}(\xi)$ is an almost (α,β)-bi-ideal of S.

Conversely, let $\text{supp}(\xi)$ be an almost (α,β)-bi-ideal of S. Then, by Theorem 20, $\lambda_{\text{supp}(\xi)}$ is a BF almost (α,β)-bi-ideal of S. Thus, $[(\lambda^p_{\text{supp}(\xi)} \circ_\alpha x_t^p \circ_\beta \lambda^p_{\text{supp}(\xi)})] \wedge \lambda^p_{\text{supp}(\xi)} \neq 0$, and $[(\lambda^n_{\text{supp}(\xi)} \circ_\alpha x_s^n \circ_\beta \lambda^n_{\text{supp}(\xi)})] \vee \lambda^n_{\text{supp}(\xi)} \neq 0$. So, there exists $r \in S$ such that $((\lambda^p_{\text{supp}(\xi)} \circ_\alpha x_t^p \circ_\beta \lambda^p_{\text{supp}(\xi)}) \wedge \lambda^p_{\text{supp}(\xi)})(r) \neq 0$, and $((\lambda^n_{\text{supp}(\xi)} \circ_\alpha x_s^n \circ_\beta \lambda^n_{\text{supp}(\xi)}) \vee \lambda^n_{supp(\xi)})(r) \neq 0$. It implies that $(\lambda^p_{\text{supp}(\xi)} \circ_\alpha x_t^p \circ_\beta \lambda^p_{\text{supp}(\xi)})(r) \neq 0$, and $\lambda^p_{supp(\xi)}(r) \neq 0$. Similarly, $(\lambda^n_{\text{supp}(\xi)} \circ_\alpha x_s^n \circ_\beta \lambda^n_{\text{supp}(\xi)})(r) \neq 0$, and $\lambda^n_{\text{supp}(\xi)}(r) \neq 0$. Thus, there exist $k_1, k_2 \in S$ such that $r = k_1\alpha u\beta k_2$, $\xi^p(r) \neq 0$, $\xi^n(r) \neq 0$, and $\xi^p(k) \neq 0$, $\xi^n(k) \neq 0$.
Hence, $(\xi^p \circ_\alpha x_t^p \circ_\beta \xi^p) \wedge \xi^p \neq 0$, and $(\xi^n \circ_\alpha x_s^n \circ_\beta \xi^n) \vee \xi^n \neq 0$. Therefore, ξ is a BF almost (α,β)-bi-ideal of S.

Definition 19. *A BF almost (α,β)-bi-ideal $\xi = (S;\xi^p,\xi^n)$ of a Γ-semigroup S is* minimal *if for all BF almost (α,β)-bi-ideal $\varsigma = (S;\varsigma^p,\varsigma^n)$ of S such that $\varsigma \subseteq \xi$, then $supp(\varsigma) = supp(\xi)$.*

Theorem 22. *Let K be a non-empty subset of a Γ-semigroup S. Then K is a minimal almost (α, β)-bi-ideal if and only if $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost (α, β)-bi-ideal of S.*

Proof. Suppose that K is a minimal almost (α, β)-bi-ideal of S. Then K is an almost (α, β)-quasi-ideal of S. Thus, by Theorem 20, $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a BF (α, β)-quasi-ideal of S. Let $\varsigma = (S; \varsigma^p, \varsigma^n)$ be a BF (α, β)-bi-ideal of S such that $\varsigma \subseteq \lambda_K$. Then, by Theorem 21, $\text{supp}(\varsigma)$ is an almost (α, β)-bi-ideal of S. Thus, $\text{supp}(\varsigma) \subseteq \text{supp}(\lambda_K) = K$. By assumption, $\text{supp}(\varsigma) = K = \text{supp}(\lambda_K)$. Thus, $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost (α, β)-bi-ideal of S.

Conversely, let $\lambda_K = (S; \lambda_K^p, \lambda_K^n)$ be a minimal BF almost (α, β)-bi-ideal of S. Then, by Theorem 20, K is an almost (α, β)-bi-ideal of S. Let J be an almost (α, β)-bi-ideal of S such that $J \subseteq K$. Then by Theorem 20, λ_J is a BF (α, β)-bi-ideal of S such that $\lambda_J \subseteq \lambda_K$. Thus, $J = \text{supp}(\lambda_J) = \text{supp}(\lambda_K) = K$. Hence, K is a minimal almost (α, β)-bi-ideal of S.

Corollary 3. *Let S be a Γ-semigroup S. Then S has no proper almost (α, β)-bi-ideal of S if and only if for any BF almost (α, β)-bi-ideal $\xi = (S; \xi^p, \xi^n)$ of S, $\text{supp}(\xi) = S$.*

5. Conclusion

In this article, we introduce the concept of a new bipolar fuzzy ideal and bipolar fuzzy almost ideals in Γ-semigroups. We also study properties of new bipolar fuzzy ideals and bipolar fuzzy almost ideals. We hope that the present study will be useful mathematical tools. In further, we extend to hesitant fuzzy almost ideals and algebraic systems.

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References

[1] R. Chinram. On quasi-gamma-ideals in γ-semigroups. *Science Asia*, 32:351–353, 2016.

[2] T. Gaketem and P. Khamrot. On some semigroups characterized in terms of biplar fuzzy weakly interior ideals. *IAENG International Journal of Computer Science*, 48(2):250–256, 2021.

[3] T. Gaketem and P. Khamrot. New types of intuitionistic fuzzy ideals in γ-semigroups. *International Journal of Fuzzy Logic and Intelligent Systems*, 22(2):135–143, 2022.

[4] A. Iampan. Note on bi-ideal in γ-semigroups. *International journal of algebra*, 3(4):181–188, 2009.

[5] P. Khamrot and T. Gaketem. A novel of cubic ideals in γ-semigroups. *International Journal of analysis and applications*, 20:1–15, 2012.

[6] P. Khamrot and T. Gaketem. A new types interval valued fuzzy ideals in γ-semigroups. *IAENG International Journal of Computer Sciences*, 50(2), 2023.

[7] P. Kummoon and T. Changphas. Bi-bases of γ-semigroups. *Thai journal of mathematics*, pages 75–86, 2017.

[8] K. Lee. Bipolar-valued fuzzy sets and their operations. *In proceeding International Conference on Intelligent Technologies Bangkok, Thailand*, pages 307–312, 2000.

[9] SK. Majumder. Bipolar valued fuzzy sets in γ-semigroups. *Mathematica Aeterna*, 2(3):203–213, 2013.

[10] J.N. Mordeson, D. S. Malik, and N. Kuroki. Fuzzy semigroup. *Springer Science and Business Media*, 2003.

[11] A. Simuen, A. Iampan, and R. Chinram. A novel of ideals and fuzzy ideals of γ-semigroups. *Journal of mathematics*, pages 1–14, 2021.

[12] L.A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

[13] W. Zhang. Bipolar fuzzy sets and relations: A computational framework forcognitive modeling and multiagent decision analysis. *In proceedings of IEEE conference*, pages 305–309, 1994.