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Image Impulse Noise Reduction Using a Conjugate Gradient of Alternative Parameter

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Abstract. Conjugate gradient approaches emphasise the conjugate formula. This study creates a new conjugate coefficient for the conjugate gradient approach to restore pictures using Perry's conjugacy condition and a quadratic model. Algorithms have global convergence and descent. The new technique performed better in numerical testing. The new conjugate gradient technique outperforms the FR method. The new technique performed better in numerical testing. The new conjugate gradient technique outperforms the FR method.

2020 Mathematics Subject Classifications: 65K10, 49M37, 90C06

Key Words and Phrases: Alternative Parameter gradient, Global convergence, Image restoration problems

1. Introduction

Gradient methods are a type of first-order approach that have been proven to be effective in solving nonlinear optimization problems, which are often very large in size. These methods are commonly used in image processing applications.

Adaptive median filter and variational technique benefits are combined in a two-phase strategy in [1]. The first step for salt-and-pepper noise uses an adaptive median filter [15]. Let X represent the genuine picture and $A = \{1, 2, 3, \cdots M\} \times \{1, 2, 3, \cdots N\}$ represent X's index set. The collection of noise pixel indices discovered in the first phase is denoted by the symbol $N \subset A$. The current challenge is to reduce the functional as much as feasible:

$$f_\alpha(u) = \sum_{(i,j)\in N} \left[|u_{i,j} - y_{i,j}| + \frac{\beta}{2}(2 \times S^1_{i,j} + S^2_{i,j}) \right] \tag{1}$$

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a regularization parameter β, $S^1_{i,j} = 2\sum_{(m,n)\in P_{i,j}\cap \mathrm{N}^c \varphi_\alpha(u_{i,j}-y_{m,n}}$, $S^2_{i,j} = \sum_{(m,n)\in P_{i,j}\cap N} \phi_\alpha(u_{i,j} - y_{m,n})$ and an edge-preserving potential function $\varphi_\alpha = \sqrt{\alpha + x^2}, \alpha > 0$ to improve the accuracy of a system. Let $y_{i,j}$ represent the observed pixel value of the image at position (i, j), and let $P_{i,j}$ represent the set of the pixel's four closest neighbours at location $(i, j) \in A$ and $u_{i,j} = \lfloor u_{i,j} \rfloor_{(i,j)\in N}$ represent a lexicographically arranged column vector of length c. The number of items in N is given by c.

The term $|u_{i,j} - y_{i,j}|$ in equation (1) enables noisy pixels to be recognised but provides a modest bias on damaged pixel restoration [1],[15]. It advises removing the term from the equation and considering the functional of the following form:

$$f_\alpha(u) = \sum_{(i,j)\in N} \lfloor (2 \times S^1_{i,j} + S^2_{i,j}) \rfloor \tag{2}$$

One of the important iterative approaches for conjugate gradient (CG) removes impulsive noise:

$$f(u^*) = \min_{x\in R^N} f(u) \tag{3}$$

The conjugate gradient approach is a kind of iterative algorithm that generates a sequence using the following format:

$$u_{k+1} = u_k + \alpha_k d_k \tag{4}$$

where d_k denotes the direction of the search and α_k denotes the average step size uncovered by a reliable exact line search, as in:

$$\alpha_k = -\frac{g_k^T d_k}{d_k^T Q d_k} \tag{5}$$

Formula [13] adds details. Under the Wolfe scenario, step length is defined as α_k:

$$\begin{aligned} f(u_k + \alpha_k d_k) &\le f(u_k) + \delta \alpha_k g_k^T d_k \\ d_k^T g(u_k + \alpha_k d_k) &\ge \sigma\ d_k^T g_k \end{aligned} \tag{6}$$

where $0 < \delta < \sigma < 1$. You may find further information in [11]. The conjugate gradient method's formula for choosing the search direction is as follows:

$$d_{k+1} = -g_{k+1} + \beta_k d_k \tag{7}$$

where β_k is a scalar.

Two examples of formulas for are the Dai-Yuan (DY) technique [2] and the Fletcher-Reeves (FR) method [4]. They take the shape of the following:

$$\beta_k^{FR} = \frac{\|g_{k+1}\|^2}{\|g_k\|^2}, \beta_{k+1}^{DY} = \frac{\|g_{k+1}\|^2}{d_k^T y_k} \tag{8}$$

Many studies have been done on the characteristics of convergence shown by conjugate gradient methods. These investigations got underway with Zoutendijk [16], who showed how the FR approach converges globally when precise line searching is carried out. Up to now, a number of researchers have created brand-new conjugate gradient coefficient equations that perform superbly numerically and lead to the global solution. While this is only a prototype, the conjugate gradient technique has known more advanced adaptations [3], [5] and [12]. Accurate descent conjugate gradient methods have been developed developed in a number of different ways. Examples of CG approaches given by Wu and Chen [14] include the following:

$$\beta_k^{WC} = \frac{y_{k+1}^T g_{k+1}}{d_k^T y_k} + \frac{2(f_k - f_{k+1}) + g_k^T s_k}{d_k^T y_k} \tag{9}$$

Both in terms of their theoretical use and their practical usefulness, these strategies excel. The calculation of the search direction is the primary point of differentiation between the conjugate gradient technique and the Wu and Chen algorithm. Please see [6], [9] and [7] for further information on the optimization techniques and references.

We create a new class of formulas by basing them on the quadratic model, and then we examine and report on the theoretical properties as well as the numerical performance of these formulas.

2. A New Parameter For β_k

The new parameter will be derived by using Taylor series as defined by:

$$f(u) = f(u_{k+1}) + g_{k+1}^T(u - u_{k+1}) + \frac{1}{2}(u - u_{k+1})^T Q(u_k)(u - u_{k+1}) \tag{10}$$

Now, choosing $u = u_k$, and we can calculate the derivative as:

$$g_{k+1} = g_k + Q(u_k)s_k \tag{11}$$

From (10) and (11), yield:

$$s_k^T Q(u_k)s_k = 2(f_k - f_{k+1}) + 2y_k^T s_k + 2g_k^T s_k \tag{12}$$

Based on (11) and (12), yield: d_{k+1}

$$-s_k^T g_{k+1} = 2(f_{k+1} - f_k) - 2y_k^T s_k - 3g_k^T s_k. \tag{13}$$

Perry's conjugecy condition is defined by:

$$d_{k+1}^T y_k = -s_k^T g_{k+1} \tag{14}$$

Based on (12) and (14), yield:

$$\beta_k d_k^T y_k = g_{k+1}^T y_k + 2(f_{k+1} - f_k) - 2y_k^T s_k - 3g_k^T s_k. \tag{15}$$

As resulted:

$$\beta_k = \frac{g_{k+1}^T y_k + 2(f_{k+1} - f_k) - 2y_k^T s_k - 3g_k^T s_k}{d_k^T y_k}. \tag{16}$$

The ways that the aforementioned parameter produced are known as the New.

3. Global convergence

The aim of this part is to study the global convergence aspects of the method. We start by doing the following:

Hypotheses

(i) In the level set $\Omega = \{u : u \in R^n, f(u) \leq f(u_1)\}$ is bounded.

(ii) In some neighborhood Λ in Ω, the gradient $g(u)$ of the function $f(u)$ is satisfying Lipchitz condition as:

$$\|g(t_1) - g(t_2)\| \leq L\|t_1 - t_2\|, \forall t_1, t_2 \in \Lambda \tag{17}$$

where L is Lipchitz constant. The Assumption 1 above implies that there exists $\mu > 0$ such that:

$$(\nabla f(r_1) - \nabla f(r_2))^T \geq \mu\|r_1 - r_2\|^2, \forall r_1, r_2 \in R^n \tag{18}$$

See [8] and [10].

Theorem 1. *Let our assumption hold. Then:*

$$d_{k+1}^T g_{k+1} \leq -c\|g_{k+1}\|^2. \tag{19}$$

Proof. If $k = 0$ after that $g_0^T d_0 = -\|g_0\|^2$. Let $d_k^T g_k < 0$ for all k. Multiply (7) by g_{k+1}, we obtain:

$$d_{k+1}^T g_{k+1} = -g_{k+1}^T g_{k+1} + \beta_k s_k^T g_{k+1} \tag{20}$$

By substituting (16) into (20) and using (11), we obtain:

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + (\frac{g_{k+1}^T y_k}{s_k^T y_k} - \frac{s_k^T g_{k+1}}{s_k^T y_k}) s_k^T g_{k+1} \tag{21}$$

It's implies:

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \frac{g_{k+1}^T y_k s_k^T g_{k+1}}{s_k^T y_k} - \frac{(s_k^T g_{k+1})^2}{s_k^T y_k} \tag{22}$$

Apply Cauchy-Schwartz inequality $w^T v \leq \frac{1}{2}(\|w\|^2 + \|v\|^2$, where $w = (y_k^T s_k) g_{k+1}$ and $v = (s_k^T g_{k+1}) y_k$ we get:

$$\frac{g_{k+1}^T y_k s_k^T g_{k+1}}{s_k^T y_k} \leq \frac{\frac{1}{2}[\|g_{k+1}\|^2 (y_k^T s_k)^2 + (s_k^T g_{k+1})^2 \|y_k\|^2]}{(s_k^T y_k)^2} \tag{23}$$

Putting (23) in (22) we have:

$$d_{k+1}^T g_{k+1} \leq -\|g_{k+1}\|^2 + \frac{1/2[\|g_{k+1}\|^2 (y_k^T s_k)^2 + (s_k^T g_{k+1})^2 \|y_k\|^2]}{(s_k^T y_k)^2} - \frac{(s_k^T g_{k+1})^2}{s_k^T y_k} \tag{24}$$

Using (17) in (24), it's ensures:

$$d_{k+1}^T g_{k+1} \leq -\frac{1}{2}\|g_{k+1}\|^2 + [\frac{1}{2}L - 1]\frac{(s_k^T g_{k+1})^2}{s_k^T y_k} \tag{25}$$

As results:

$$d_{k+1}^T g_{k+1} \leq -c\|g_{k+1}\|^2 \tag{26}$$

so it has been proved.

Any conjugate gradient technique with Wolfe line search converges. A weak Zoutendijk condition [16] is sufficient.

Lemma 1. *In the event that the hypotheses are true, let* $\{u_k\}$ *produced by (3),* d_k *be the direction of descent,* α_k *meet Wolfe conditions, and if:*

$$\sum_{k\geq 0} \frac{1}{\|d_{k+1}\|^2} = \infty \tag{27}$$

Then

$$\lim_{k\to\infty} inf\|g_k\| = 0. \tag{28}$$

Theorem 2. *New Algorithm converges globally whenever our assumption hold, i.e.:*

$$\lim_{k\to\infty} inf\|g_k\| = 0 \tag{29}$$

Proof. But it holds from (9), that:

$$\|d_{k+1}\| = \| - g_{k+1} + \beta_k^{New} s_k\| \tag{30}$$

Putting (16) in (30) by using (11), implies:

$$\|d_{k+1}\| = \| - g_{k+1} + \frac{g_{k+1}^T y_k}{d_k^T y_k} s_k - \frac{s_k^T g_{k+1}}{d_k^T y_k} s_k\| \tag{31}$$

Finally, using (17) and (18), it gets as:

$$c\|d_{k+1}\| \leq \|g_{k+1}\| + \frac{\|g_{k+1}\|L\|s_k\|^2}{\mu\|s_k\|^2} + \frac{\|g_{k+1}\|\|s_k\|^2}{\mu\|s_k\|^2} \quad \leq (1 + \frac{L}{\mu} + \frac{1}{\mu})\|g_{k+1}\| \leq [\frac{\mu + L + 1}{\mu}]\|g_{k+1}\| \tag{32}$$

Therefore,

$$\sum_{k\geq 1} \frac{1}{\|d_k\|^2} \geq (\frac{\mu}{\mu + L + 1})\frac{1}{\Gamma}\sum_{k\geq 1} 1 = \infty \tag{33}$$

Applying Lemma, this research concludes that $\lim_{k\to\infty} inf\|g_k\| = 0$.

4. Numerical Results

In order to demonstrate how well New reduces salt-and-pepper impulse noise, we provide some numerical data. We evaluate the FR approach as well as the New method. We function in this manner. With MATLAB r2017a, each code is present. Then they are executed by a computer. These are the circumstances that will cause both techniques to stop:

$$\|f(u_k)\| \le 10^{-4}(1+|f(u_k)|) \ and \ \frac{|f(u_k)-f(u_{k-1})|}{|f(u_k)|} \le 10^{-4} \tag{34}$$

In addition to the test text, the test images include Lena, House, the Cameraman, and Elaine. Also featured is the test photograph. In order to provide a qualitative assessment of the performance of the restoration, we use the PSNR (peak signal to noise ratio) in a manner that is analogous to [1], [15]. The following definition applies to the restoration performance:

$$PSNR = 10log_{10}\frac{255^2}{\frac{1}{MN}\sum_{i,j}(u^r_{i,j}-u^*_{i,j})^2} \tag{35}$$

Although the pixel values of the original image are represented by $u^r_{i,j}$, those of the image that was restored to its original condition are represented by $u^*_{i,j}$. In this research, we show how many function evaluations and iterations are needed to finish the whole denoising procedure as well as the image's PSNR results from this method. The new approach outperforms the FR methodology, which takes a very long time to finish, in terms of speed. Table (1) may be accessed here, and it contains the supporting data. The PSNR values produced with the new technique and those acquired with the FR method are quite comparable to one another.

Table 1: Numerical results of FR, New algorithms.

Image	Noise level r (%)	FR-Method			New-Method		
		NI	NF	PSNR (dB)	NI	NF	PSNR (dB)
Le	50	82	153	30.5529	56	113	30.3944
	70	81	155	27.4824	61	109	27.4183
	90	108	211	22.8583	59	97	22.7628
Ho	50	52	53	30.6845	39	53	34.9847
	70	63	116	31.2564	51	53	31.0019
	90	111	214	25.287	62	64	24.7366
El	50	35	36	33.9129	34	35	33.8806
	70	38	39	31.864	36	37	31.826
	90	65	114	28.2019	50	51	28.1194
c512	50	59	87	35.5359	37	47	35.2094
	70	78	142	30.6259	50	52	30.6319
	90	121	236	24.3962	57	114	25.026

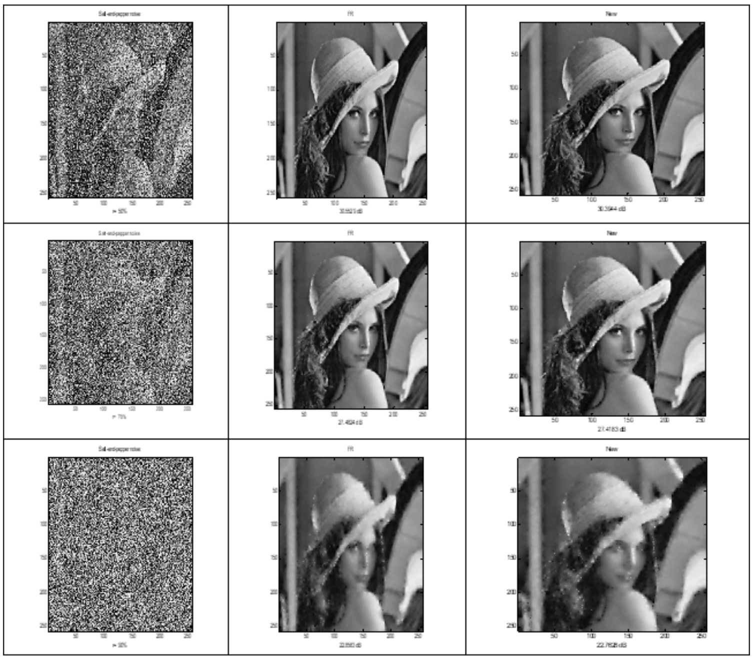

Figure 1: Lena image.

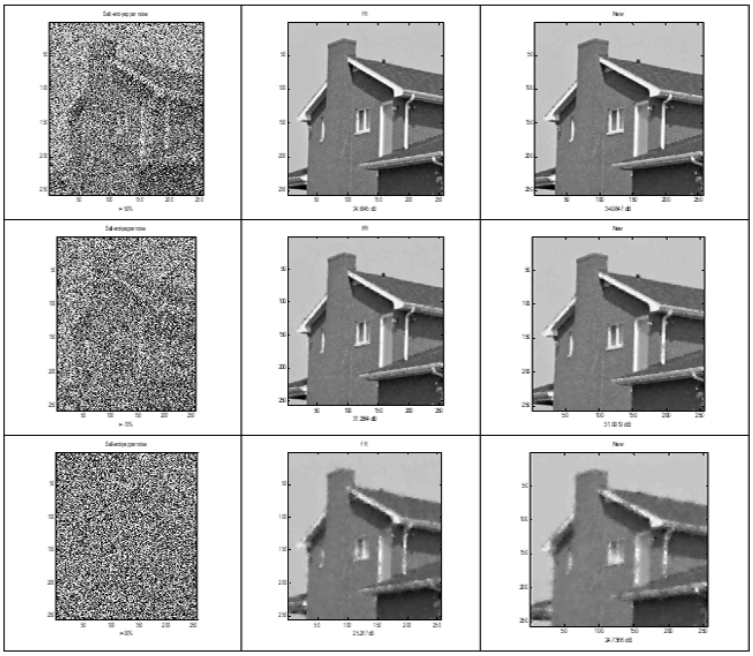

Figure 2: House image.

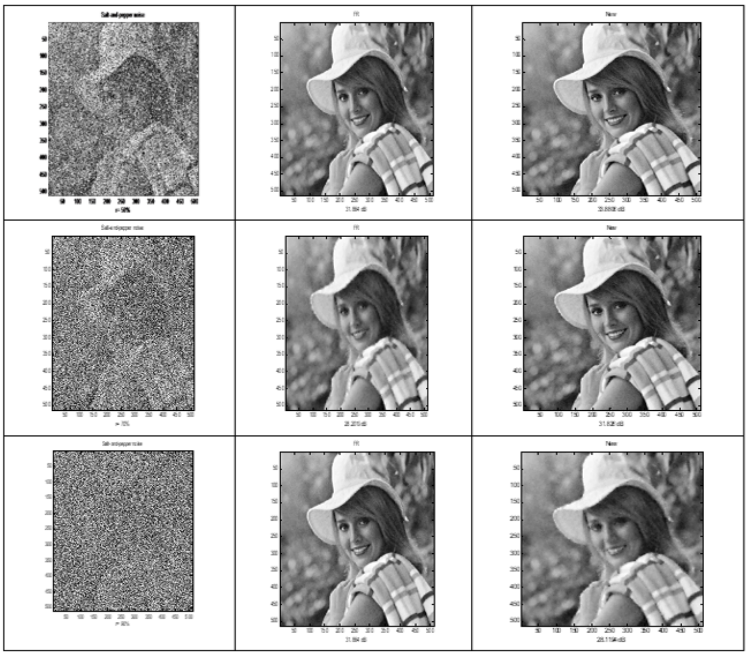

Figure 3: Elaine image.

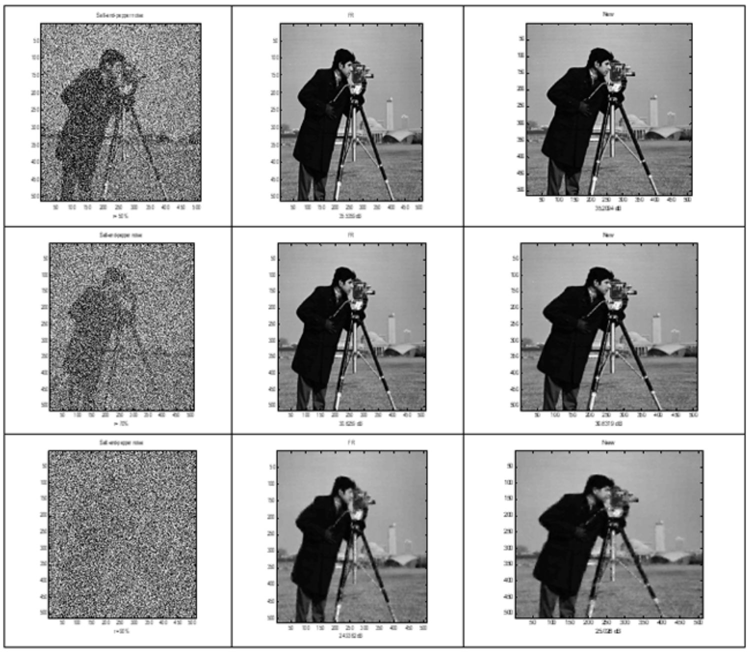

Figure 4: Cameraman image.

5. Conclusions

We also discussed the conjugate gradient approach new, in addition to a newly discovered formula for a conjugate gradient. We were able to find the Wolfe line's global convergence using search criteria. It has been shown that new may significantly reduce the quantity of simulation evaluations of functions and iterations while keeping the visual quality constant.

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