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b^*_J Sets and b^*_J-Compact Ideal Spaces

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Abstract. We came up with the concept b^*-open set which has stricter condition with respect to the notion b-open sets, introduced by Andrijevic [2] as a generalization of Levine's [7] generalized closed sets. The condition imposes equality instead of inclusion.
In this study, we gave some important properties of b^*-open sets with respect to an ideal, and b^*-compact spaces.

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Key Words and Phrases: b^*-open sets, b^*_J-open sets, ideals, b^*-compact space, b^*_J-compact space

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1. Introduction

Topology is a relatively new branch of mathematics, being introduced in the 19th century. But topology is already seen in many areas of science [10]. It is applied in biochemistry [3] and information systems [15]. Topology as a mathematical system is fundamentally comprised of sets together with the operations union and intersection. Over time, open sets (elements of topology) were generalized in different directions. To name a few, Stone [16] presented regular open set. Levine [6] presented semi-open sets. Njasted [12] presented α-open sets. Mashhour et al. [8] presented pre-open sets. Abd El-Monsef et al. [1] presented β-open set.

It was in the year 1970, when Levine [7] presented the concept of generalized closed sets, and achoring on this notion, Andrijevic [2] presented yet another generalization of open sets called b-open sets. This study uses the notion of b-open sets to come up with a new concept called b^*-open sets.

The concept ideal topological spaces (or simply, ideal space) was first seen in [5]. Vaidyanathaswamy [19] investigated this concept in point set topology. Tripathy and Shravan [13, 14], Tripathy and Acharjee [17], Tripathy and Ray [18], Catalan et al. [4] among others, also made investigations in ideal topological spaces.

Several concepts in topology were generalized using this structure. One of which is the concept b^*-open sets. Consequently, using the notion of b^*-open sets, we introduced

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the concepts b^*-compact sets, compatible b^*_J-compact sets, countably b^*_J-compact sets, b^*_J-connected sets, in ideal generalized topological spaces.

Let W be a non-empty set. An ideal J on a set W is a non-empty collection of subsets of W which satisfies:

1. $B \in J$ and $D \subseteq B$ implies $D \in J$.
2. $B \in J$ and $D \in J$ implies $B \cup D \in J$.

Let W be a topological space and B be a subset of W. We say that B is *b^*-open set* if $B = \text{cl}(\text{int}(B)) \cup \text{int}(\text{cl}(B))$. For example, consider $W = \{a, b, c\}$ and the topology $\varsigma = \{\varnothing, \{a\}, \{b\}, \{a, b\}, W\}$ on W. Then the b^*-open subsets are $\varnothing$, $\{a, b\}$, $\{c\}$ and W.

Let W be a topological space and B be a subset of W. The set B is called *b^*-open relative to an ideal* J (or *b^*_J-open*), if there is an open set P with $P \subseteq \text{Int}(B)$, and a closed set S with $\text{Cl}(B) \subseteq S$ such that

1. $(\text{Int}(S) \cup \text{Cl}(\text{Int}(B)))\backslash B \in J$, and
2. $B\backslash(\text{Int}(\text{Cl}(B)) \cup \text{Cl}(P)) \in J$.

In addition, we say that a set B is a b^*_J-close set if B^C is b^*_J-open.

Consider the ideal space $(\{q, r, s\}, \{\varnothing, \{q\}, \{r\}, \{q, r\}, \{q, r, s\}\}, \{\varnothing, \{r\}\})$. Then $B = \{r, s\}$ is a b^*-open with respect to the ideal $J = \{\varnothing, \{r\}\}$. To see this, we let P be the open set $\{r\}$ and S be the closed set $\{r, s\}$. Then $\text{Int}(S) \cup \text{cl}(\text{int}(\{r, s\}))\backslash\{r, s\} = \text{int}(\{r, s\}) \cup \text{cl}(\{r\})\backslash\{r, s\} = \{r\} \cup \{r, s\}\backslash\{r, s\} = \{r, s\}\backslash\{r, s\} = \varnothing \in J$. Also, $\text{Int}(\text{cl}(\{r, s\}) \cup \text{cl}(P)\backslash\{r, s\} = \text{int}(\{r, s\}) \cup \text{cl}(\{r\})\backslash\{r, s\} = \{r\} \cup \{r, s\}\backslash\{r, s\} = \{r, s\}\backslash\{r, s\} = \varnothing \in J$. This shows that $B = \{r, s\}$ is a b^*_J-open.

Let W be a topological space and B be a subset of W. The set B is called *nearly b^*-open* relative to an ideal J (or nearly b^*_J-open) if there is an open set P with $P \subseteq \text{Int}(B)$, and a closed set S with $\text{Cl}(B) \subseteq S$ such that

1. $(\text{Int}(S) \cup \text{Cl}(\text{Int}(B)))\backslash\text{Cl}(B) \in J$, and
2. $B\backslash(\text{Int}(\text{Cl}(B)) \cup \text{Cl}(P)) \in J$.

Consider the ideal topological space $(\{1, 2, 3\}, \{\varnothing, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}, \{\varnothing, \{2\}\})$. Then $B = \{2, 3\}$ is a nearly b^*-open with respect to the ideal J (or nearly b^*_J-open). To see this, we let P be the open set $\{2\}$ and S be the closed set $\{2, 3\}$. Then $\text{Int}(S) \cup \text{cl}(\text{int}(\{2, 3\}))\backslash\text{cl}(\{2, 3\}) = \text{int}(\{2, 3\}) \cup \text{cl}(\{2\})\backslash\text{cl}(\{2, 3\}) = \{2\} \cup \{2, 3\}\backslash\text{cl}(\{2, 3\}) = \{2, 3\}\backslash\{2, 3\} = \varnothing \in J$. Also, $\text{Int}(\text{cl}(\{2, 3\}) \cup \text{cl}(P)\backslash\{2, 3\} = \text{int}(\{2, 3\}) \cup \text{cl}(\{2\})\backslash\{2, 3\} = \{2\} \cup \{2, 3\}\backslash\{2, 3\} = \{2, 3\}\backslash\{2, 3\} = \varnothing \in J$. This shows that $B = \{2, 3\}$ is a nearly b^*_J-open.

The set B is said to be b^*-compact if every cover of B by b^*-open sets, containing W, has a smaller finite sub-cover. The space W is said to be a b^*-compact space if W is b^*-compact set. Consider the topological space $(W = \{a, b, c\}, \{\varnothing, \{a\}, \{b, c\}, W\}, J = \{\varnothing, \{a\}\})$. Then $B = \{a\}$ is a b^*-compact set, while $D = \{a, b\}$ is not. To see this, we note that the

b^*-open sets of W are $\varnothing$, $\{a\}$, $\{b,c\}$ and W. Observe that the covering of B containing W is $\{\{a\},W\}$. Thus, $\{\{a\}\}$ is a smaller cover. Hence, $B=\{a\}$ is a b^*-compact set.

On the other hand, observe that the covering of D containing W are $\{\{a\},\{b,c\},W\}$ and $\{\{b,c\},W\}$. Since $\{\{b,c\},W\}$ has no smaller subcover, $D=\{a,b\}$ is not a b^*-compact set.

The set B is called b^*_J-compact if every cover of B by b^*_J-open sets which contains W, has a smaller finite sub-cover. The space W is called b^*_J-compact space if it is b^*_J-compact set. Consider the ideal topological space $(W=\{x,y,z\},\{\varnothing,\{x\},\{y\},\{x,y\},W\},\{\varnothing,\{y\}\})$. Then $B=\{y,z\}$ is a b^*_J-compact set where $J=\{\varnothing,\{y\}\}$. To see this, we note that the b^*_J-open sets of W are $\varnothing$, $\{y,z\}$ and W. Hence, every cover $\{P_\psi:\psi\in\Psi\}$ of B by b^*_J-open set must contain $\{y,z\}$ or W. Thus, each of the following is a covering of B: $\{\{y,z\}\}$; $\{\{y,z\},W\}$; and $\{W\}$. Note that $\{\{y,z\},W\}$ is a covering of B which has a smaller subcover $\{\{y,z\}\}$. This shows that $B=\{y,z\}$ is a b^*_J-compact set.

Now, consider the ideal topological space $(W=\{l,m,n\},\{\varnothing,\{l\},\{m\},\{l,m\},W\},\{\varnothing,\{m\}\})$. Then $B=\{l,m\}$ is a not b^*_J-compact set where $J=\{\varnothing,\{m\}\}$. To see this, we note again that the b^*_J-open sets of W are $\varnothing$, $\{m,n\}$ and W. Hence, every cover $\{P_\psi:\psi\in\Psi\}$ of B by b^*_J-open set must contain W. Thus, each of the following is a covering of B: $\{\{m,n\},W\}$; and $\{W\}$. Note that $\{\{m,n\},W\}$ has no smaller. This shows that $B=\{l,y\}$ is not a b^*_J-compact set.

The set B is said to be compatible b^*_J-compact (or simply cb^*_J-compact) if any cover $\{P_\psi:\psi\in\Psi\}$ of B by b^*_J-open sets containing W, Ψ has a smaller finite subset Ψ_0 such that $B\setminus\bigcup\{U_\psi:\psi\in\Psi_0\}\in J$. The topological space W is said to be a cb^*_J-compact space if it is cb^*_J-compact as a set. Consider the ideal topological space $(Z,\varsigma,J)=(\{h,i,j\},\{\varnothing,\{h\},\{i,j\},Z\},\{\varnothing,\{i\}\})$. Then $\{h,i\}$ is a compatible b^*_J-compact where $J=\{\varnothing,\{i\}\}$. To see this, we observe that the b^*_J-open sets of Z are $\varnothing$, $\{h\}$, $\{i,j\}$ and Z. Hence, every cover $\{P_\psi:\psi\in\Psi\}$ of Z by b^*_J-open set must contain $\{h\}$, $\{i,j\}$ or Z. Thus, $\{P_\psi:\psi\in\Psi\}$ is $\{\{h\},\{i,j\}\}$ or $\{\{h\},Z\}$ or $\{Z,\{i,j\},\{h\}\}$ or $\{Z,\{i,j\}\}$. In the first 3 cases, there is a smaller subset $\{\{h\}\}$ such that $\{h,i\}\setminus\{h\}=\{i\}\in J$, and for the last case, there exist a smaller subset $\{\{h,i\}\}$ such that $\{h,i\}\setminus\{h,i\}=\varnothing\in J$. This shows that $\{h,i\}$ is a compatible b^*_J-compact set. Next, consider the ideal topological space $(V=\{q,r,s\},\{\varnothing,\{q\},\{r,s\},V\},\{\varnothing,\{s\}\})$. Then $\{q,r\}$ is not compatible b^*_J-compact. To see this, we note that the b^*_J-open sets of V are $\varnothing$, $\{q\}$, $\{r,s\}$ and V. Hence, every cover $\{P_\psi:\psi\in\Psi\}$ of $\{q,r\}$ by b^*_J-open set must contain $\{q\}$, $\{r,s\}$ or V. Thus, $\{P_\psi:\psi\in\Psi\}$ is $\{\{q\},\{r,s\}\}$ or $\{\{q\},V\}$ or $\{V,\{r,s\},\{q\}\}$ or $\{V,\{r,s\}\}$. Consider the open cover $\{\{q\},\{r,s\}\}$. Note that its smaller covers are $\{\{q\}\}$ and $\{\{r,s\}\}$. Observe that $\{q,r\}\setminus\{q\}=\{r\}\notin J$ and $\{q,r\}\setminus\{r,s\}=\{q\}\notin J$. This shows that $\{q,r\}$ is not a compatible b^*_J-compact set.

2. Results

We present some of the important properties of b^*-open sets and b^*_J-open sets.

Lemma 1 is a characterization of b^*-open sets.

Lemma 1. *Let (Y, ς, J) be an ideal space and B be a subset of Y. Then B is an b^*-open set precisely when there is an open set P with $P \subseteq Int(B)$ and there is a close set S with $Cl(B) \subseteq S$ such that $Int(S) \cup Cl(Int(B)) \subseteq B \subseteq Int(Cl(B)) \cup Cl(P)$.*

Proof. Necessity. Let B is a b^*-open set. Then $B = \text{Int}(\text{Cl}(B)) \cup \text{Cl}(\text{Int}(B))$. Take the open set $P = \text{Int}(B)$ and the close set $S = \text{Cl}(B)$. Note that $\text{Int}(S) \cup \text{Cl}(\text{Int}(B)) \subseteq \text{Int}(\text{Cl}(B)) \cup \text{Cl}(\text{Int}(B)) = B$, and $\text{Int}(\text{Cl}(B)) \cup \text{Cl}(P) \supseteq \text{Int}(\text{Cl}(B)) \cup \text{Cl}(\text{Int}(B)) = B$. Hence, $\text{Int}(S) \cup \text{Cl}(\text{Int}(B)) \subseteq B \subseteq \text{Int}(\text{Cl}(B)) \cup \text{Cl}(P)$.

Sufficiency. Next, let P be an open set with $P \subseteq \text{Int}(B)$ and let S be a closed set with $\text{Cl}(B) \subseteq S$ such that $\text{Int}(S) \cup \text{Cl}(\text{Int}(B)) \subseteq B \subseteq \text{Int}(\text{Cl}(B)) \cup \text{Cl}(P)$. Then $B \supseteq \text{Int}(S) \cup \text{Cl}(\text{Int}(B)) \supseteq \text{Int}(\text{Cl}(B)) \cup \text{Cl}(\text{Int}(B))$, and $B \supseteq \text{Int}(\text{Cl}(B)) \cup \text{Cl}(P) \subseteq \text{Int}(\text{Cl}(B)) \cup \text{Cl}(\text{Int}(B))$.

Therefore, $B = \text{Int}(\text{Cl}(B)) \cup \text{Cl}(\text{Int}(B))$, that is B is a b^*-open set. □

An open set is *nearly* b^*_J-open. The next lemma, Lemma 2, shows this idea.

Lemma 2. *Let (Y, ς, J) be an ideal space. Then every open set is a b^*_J-open set.*

Proof. Let B be an open set, and consider $S = \varnothing = P$. Then S and P are both open and closed. Observed that $\text{int}(\text{cl}(B)) \cup \text{cl}(P) \supseteq \text{int}(B) \cup \text{cl}(\varnothing) = \text{int}(B) \cup \varnothing = \text{int}(B) = B$, and $\text{int}(S) \cup \text{cl}(\text{int}(B)) = \text{int}(\varnothing) \cup \text{cl}(\text{int}(B)) \subseteq \varnothing \cup \text{cl}(B) = \text{cl}(B)$.

Hence, we have $B \backslash \text{int}(\text{cl}(B)) \cup \text{cl}(P) = \varnothing \in J$, and $\text{int}(S) \cup \text{cl}(\text{int}(B)) \backslash \text{cl}(B) = \varnothing \in J$, that is, B is *nearly* b^*_J-open. □

An element of ideal J is *nearly* b^*_J-open set. The next lemma, Lemma 3, shows this idea. Please see [9] and [4] to have more insights.

Lemma 3. *Let (Y, ς, J) be an ideal space. Then each element of J is b^*_J-open.*

Proof. Let $B \in J$. Since $B - \text{int}(\text{cl}(B)) \cup \text{cl}(B) \subseteq B$, we have $\text{int}(\text{cl}(B)) \cup \text{cl}(B) \in J$. Next, consider $S = \varnothing$. Then $\text{int}(S) \cup \text{cl}(\text{int}(B)) \backslash \text{cl}(B) = \text{int}(\varnothing) \cup \text{cl}(\text{int}(B)) \backslash \text{cl}(B) = \varnothing \cup \text{cl}(\text{int}(B)) \backslash \text{cl}(B) = \text{cl}(\text{int}(B)) \backslash \text{cl}(B) = \varnothing \in J$. Therefore, B is *nearly* b^*_J-open. □

Lemma 4 says that each b^*-open set is b^*_J-open.

Lemma 4. *Let (Y, ς, J) be an ideal space. Then a b^*-open set is b^*_J-open.*

Proof. Let B be a b^*-open set. Then $\text{int}(\text{cl}(B)) \cup \text{cl}(\text{int}(B)) = B$. Consider $P = \text{int}(B)$ and $S = \text{cl}(B)$. Then P is open with $P \subseteq \text{int}(B)$, and S is closed with $S \subseteq \text{cl}(B)$. Observed that $\text{int}(\text{cl}(B)) \cup \text{cl}(P) = \text{int}(B) \cup \text{cl}(\text{int}(B)) = B$, and $\text{int}(S) \cup \text{cl}(\text{int}(B)) = \text{int}(\text{cl}(B)) \cup \text{cl}(\text{int}(B)) = B$.

Hence, we have $B \backslash \text{int}(\text{cl}(B)) \cup \text{cl}(P) = \varnothing \in J$, and $\text{int}(S) \cup \text{cl}(\text{int}(B)) \backslash B = \varnothing \in J$, that is, B is b^*_J-open. □

Lemma 5. *Let (Y, ς, J) be an ideal space with $J = \{\varnothing\}$. Then B is b^*-open precisely if B is b^*_J-open.*

Proof. Necessity. Let B be b^*_J-open. Then there is an open set P such that $P \subseteq \text{int}(B)$, and there is a close set S such that $S \subseteq \text{cl}(B)$. Hence, $B \subseteq \text{int}(\text{cl}(B)) \cup \text{cl}(P)$, and $\text{int}(S) \cup \text{cl}(\text{int}(B)) \subseteq B$. Thus, $\text{int}(\text{cl}(B)) \cup \text{cl}(\text{int}(B)) = \text{int}(S) \cup \text{cl}(\text{int}(B)) \subseteq B$, and $\text{int}(\text{cl}(B)) \cup \text{cl}(\text{int}(B)) = \text{int}(\text{cl}(B)) \cup \text{cl}(P) \supseteq B$. Therefore, $\text{int}(\text{cl}(B)) \cup \text{cl}(\text{int}(B)) = B$, that is, B is b^*-open.

Sufficiency. The converse follows from Lemma 4. □

If J is the minimal ideal, then the notions b^*-compact, b^*_J-compact and cb^*J-compact are the same. Theorem 1 shows this idea.

Theorem 1. *Let (Y, ς, J) be an ideal space with $J = \{\varnothing\}$. Then the following are equivalent.*

(i). (Y, ς, J) is a b^*-compact ideal space.

(ii). (Y, ς, J) is a b^*_J-compact ideal space.

(iii). (Y, ς, J) is a cb^*_J-compact ideal space.

Proof. (i) implies (ii): Let $\{U_\psi : \psi \in \Psi\}$ be a b^*_J-open covering Y. By Lemma 5, $\{U_\psi : \psi \in \Psi\}$ is also a b^*-open covering Y. Since Y is a b^*-compact ideal space, Ψ has a smaller finite subset, say Ψ_0, with $\{U_\psi : \psi \in \Psi_0\}$ still covering Y. Thus, by Lemma 5, $\{U_\psi : \psi \in \Psi_0\}$ is a smaller finite b^*_J-covering of Y. This shows that Y is a b^*_J compact set.

(ii) implies (iii): Let $\{U_\psi : \psi \in \Psi\}$ be a b^*_J-open covering Y. Since Y is a b^*_J-compact ideal space, Ψ has a smaller finite subset, say Ψ_0, with $\{U_\psi : \psi \in \Psi_0\}$ still covering Y. Thus, $Y - \bigcup\limits_{\psi \in \Psi_0} U_\psi = \varnothing \in J$. Therefore, Y is cb^*_J compact set.

(iii) implies (i): Let $\{U_\psi : \psi \in \Psi\}$ be a b^*-open covering Y. y Lemma 5, $\{U_\psi : \psi \in \Psi\}$ is also a b^*_J-open covering Y. Since Y is a cb^*_J-compact ideal space, Ψ has a smaller finite subset, say Ψ_0, with $Y - \bigcup\limits_{\psi \in \Psi_0} U_\psi = \varnothing \in J$, that is, $\{U_\psi : \psi \in \Psi_0\}$ is a smaller finite b^*-covering of Y. Therefore, Y is b^* compact set. □

Another characterization of b^*_J-compact topological spaces is presented in Theorem 2.

Theorem 2. *Let (Y, ς, J) be an ideal space. Then statement (i) is a necessary and sufficient condition for statement (ii).*

i. (Y, ς, J) is a b^*_J-compact space.

ii. If $\{S_\psi : \psi \in \Psi\}$ is a class of b^*_J-closed sets with $\bigcap\{S_\psi : \psi \in \Psi\} = \varnothing$, then Ψ has a smaller finite subset, say Ψ_0, with $\bigcap\{S_\psi : \psi \in \Psi_0\} = \varnothing$.

Proof. (i) implies (ii): Let $\{S_\psi : \psi \in \Psi\}$ be a class of b^*_J-closed sets with $\bigcap\{S_\psi : \psi \in \Psi\} = \varnothing$. Then $Y = \varnothing^C = (\bigcap\{S_\psi : \psi \in \Psi\})^C = \bigcup\{S^C_\psi : \psi \in \Psi\}$. Hence, $\{S^C_\psi : \psi \in \Psi\}$ is a class of b^*_J-open sets which covers of Y. By assumption, Ψ has a smaller finite subset, say Ψ_0, with the property $\bigcup\{S^C_\psi : \psi \in \Psi_0\} = X$. Hence, $(\bigcap\{S_\psi : \psi \in \Psi_0\} = \bigcup\{S^C_\psi : \psi \in \Psi_0\})^C = Y^C = \varnothing$.

(ii) implies (i): Let $\{P_\psi : \psi \in \Psi\}$ be a b^*_J-open covering of Y, i.e. $\bigcup\{P_\psi : \psi \in \Psi\} = Y$. Then $\bigcap\{P^C_\psi : \psi \in \Psi\} = (\bigcup\{P_\psi : \psi \in \Psi\})^C = \varnothing$. Note that P^C is b^*_J-close since P is b^*_J-open. By assumption, Ψ has a smaller finite subset, say Ψ_0, with the property that $\bigcap\{P^C_\psi : \psi \in \Psi_0\} = \varnothing$. Note that $\bigcup\{P_\psi : \psi \in \Psi_0\} = (\bigcap\{P^C_\psi : \psi \in \Psi_0\})^C = Y$. Hence, $\{P_\psi : \psi \in \Psi_0\}$ is a class of b^*_J-open sets that covers Y. □

Another characterization of cb^*_J-compact topological spaces is presented in Theorem 3.

Theorem 3. *Let (Y, ς, J) be an ideal topological space. Then (i) is a necessary and sufficient condition for statement (ii).*

i. (Y, ς, J) is cb^*_J-compact.

ii. If $\{S_\psi : \psi \in \Psi\}$ is a class of b^*_J-closed sets with $\bigcap\{S_\psi : \psi \in \Psi\} = \varnothing$, then Ψ has a smaller finite subset, say Λ_0, with the property that $\bigcap\{F_\lambda : \lambda \in \Lambda_0\} \in I$.

Proof. (i) implies (ii): Let $\{S_\psi : \psi \in \Psi\}$ be a class of b^*_J-closed sets such that $\bigcap\{S_\psi : \psi \in \Psi\} = \varnothing$. Note that $\bigcup\{S^C_\psi : \psi \in \Psi\} = (\bigcap\{S_\psi : \psi \in \Psi\})^C = Y$. Hence, $\{S^C_\psi : \psi \in \Psi\}$ is a class of b^*_J-open sets covering Y. By assumption, Ψ has a finite subset, say Ψ_0, with $Y - \bigcup\{S^C_\lambda : \psi \in \Psi_0\} \in J$, i.e. $\bigcap\{S_\psi : \psi \in \Psi_0\} \in J$.

(ii) implies (i): Let $\{P_\psi : \psi \in \Psi\}$ be a b^*_J-open covering of Y, i.e. $\bigcup\{P_\psi : \psi \in \Psi\} = Y$. Note that $\bigcap\{P^C_\psi : \psi \in \Psi\} = (\bigcup\{P_\psi : \psi \in \Psi\})^C = \varnothing$. By assumption, Ψ has a smaller finite subset, say Ψ_0, with $\bigcap\{P^C_\psi : \psi \in \Psi_0\} \in J$, i.e. $Y - \bigcup\{P_\psi : \psi \in \Psi_0\} \in J$. □

Remark 1. *[11] Let (Y, ς, J) and (W, ξ, K) be ideal topological spaces, and $\zeta : Y \to W$ be a mapping. Then:*

i. $\zeta(J) = \{\zeta(B) : B \in J\}$ is an ideal in W; And,

i. if ζ is a one to one correspondence, then $\zeta^{-1}(K) = \{\zeta^{-1}(D) : D \in K\}$ is an ideal in Y.

Definition 1. *Let (Y, ς, J) and (W, ξ, K) be ideal spaces. A mapping $\zeta : Y \to W$ is*

i. b^*_J-open if $\zeta(B)$ is b^*_K-open for every b^*_J-open set B in Y, and

ii. b^*_J-irresolute if $\zeta^{-1}(D)$ is b^*_J-open for each b^*_K-open set D in W.

If the domain of a b^*-irresolute map is cb^*_J-compact with respect to an ideal, then so is the image. We show this idea in Theorem 4.

Theorem 4. *Let* (Y, ς, J) *and* (W, ξ, K) *be ideal spaces, and* $\zeta : Y \to W$ *be a* b^*_J*-irresolute function with* $\zeta(J) = K$*. If* Y *is a* cb^*_J*-compact, then* $\zeta(Y)$ *is* cb^*_K*-compact.*

Proof. Let $\{P_\psi : \psi \in \Psi\}$ be a b^*_K-open covering of $\zeta(Y)$. Since ζ is b^*_J-irresolute, $\{\zeta^{-1}(P_\psi) : \psi \in \Psi\}$ is a b^*_J-open covering Y. By assumption, Ψ has a smaller finite subset, say Ψ_0, with $Y - \bigcup\{\zeta^{-1}(P_\psi) : \psi \in \Psi_0\} \in J$. And so by Remark 1 $\zeta(Y) \setminus \bigcup\{P_\psi : \psi \in \Psi_0\} = \zeta(Y - \bigcup\{\zeta^{-1}(P_\psi) : \psi \in \Psi_0\}) \in K$. □

If the co-domain of a b^*-open and onto map is cb^*_J-compact with respect to an ideal, then so is the domain. We show this idea in Theorem 5.

Theorem 5. *Let* (Y, ς, J) *and* (W, ξ, K) *be ideal spaces, and* $\zeta : Y \to W$ *be a* b^*_J*-open and onto map with* $\zeta(J) = K$*. If* W *is* cb^*_K*-compact, then* Y *is* cb^*_K*-compact.*

Proof. Let $\{P_\psi : \psi \in \Psi\}$ be a b^*_J-open covering of Y. Since ζ is a b^*_J-open and onto, $\{\zeta(P_\psi) : \psi \in \Psi\}$ is a b^*_K-open covering of W. By assumption, Ψ has a smaller finite subset, say Ψ_0, with $W \setminus \bigcup\{\zeta(P_\psi) : \psi \in \Psi_0\} \in K$. Thus, $Y \setminus \bigcup\{P_\psi : \psi \in \Psi_0\} = \zeta^{-1}(W \setminus \bigcup\{\zeta(P_\psi) : \psi \in \Psi_0\}) \in J$. □

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