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Analysis of Novel 4D Rabinovich-Fabrikant Continuous Dynamical System with Coexistence Attractors

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Abstract. In this paper a new Rabinovitch-Fabrikant (R-F) four dimensional (4D) continuous time dynamical system was generated from three dimensional (3D) Rabinovitch-Fabrikant dynamical system using the state augmentation technique by adding new state variables u. The system employs thirteen terms includes five cross-product terms and one irreversible function. The dynamical behaviors of the system were investigated which include equilibrium points, stability analysis, wave form analysis, phase space analysis, multistability, Hopf-bifurcation, the Lyapunov exponent and Lyapunov dimension. The values of Lyapunov exponents are:$L_1 = 14.025946, L_2 = 0.295151, L_3 = -2.854401, L_4 = -13.736833.$ and Lyapunov dimension is (3.83474), so the system is unstable and hyperchaotic with coexistence attractors. Chaos was handled in two ways: adaptive control and adaptive synchronization, it was found that the new system is stable and achieved good results.

2020 Mathematics Subject Classifications: 65K10, 49M37, 90C06

Key Words and Phrases: Stability, Multistability, Lyapunov exponent, Adaptive Control, Synchronization

1. Introduction

The origin of nonlinear dynamics goes back to French scientists (Henri poincar) [14], [2]. Dynamic systems are evolutionary process, which are generally characterized by differential equation including mechanics and fluid flow [17], [16], [18], [1], [5]. Chaos theory is now attractive to various fields such as physics, chemistry, medicine, biology, mathematics, and engineering [13], [3], [6], [7]. Chaos control has received widespread attention of research because contrability of chaotic attractors are index of utility in very totally different designs like in communication and arificial intelligence [9], [12], [8], [4]. The synchronization of

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chaotic system is a very energetic subject in non-linear science, this phenomenon happens where two non-linear system are combined, there are different types of synchronization phenomena including complete synchronization, linear feedback synchronization, Hybrid synchronization, Anti- synchronization, etc [10]. Due to the increasing importance in many fields at the present time to generate a high dimensional system, a new 4D (R-F) dynamical system was generated and it seen from the comparison table 1 that the new 4D (R-F) is chaotic system with unstable real equilibrium points, several tools were used to analyze the new system. It was found that the new 4D (R-F) coexistence of self-existing chaotic attractors Table 1, comparison of the new 4D (R-F) dynamical system with results of published papers.

Table 1: Comparision on new R-F system with published papers.

Dimension of the system	**System nature**	**References of papers and year**
3D (R-F)	1- System with real equilibrium points 2- Composition of a controlled (R-F) system with a fractional time derivative 3- Synchronization between fractional order and chaos controlled (R-F) SYSTEMS	[20] In 2014
3D (R-F)	1- System with real equilibrium points 2- Hidden transient chaotic attractors	[11] In 2016
3D (R-F)	1- System with real equilibrium points 2- The system is unstable by Routh-Hurwitz criterion 3- Configure an electrical circuit	[19] In 2021
4D (R-F)	1- Perform hybrid projective synchronization for 3D (R-F) system and 4D (R-F) system	[15] In 2017
4D New (R-F)	1- System with real equilibrium points. 2- The system is unstable by: the roots of the characteristic equation, Routh stability criterion, Hurwitz stability criterion, and Lyapunov function 3- coexistence of self-existing chaotic attractors 4- the system is highly chaotic because of the two positives of Lyapunov exponent. 5- The system has hopf-bifurcation. 6- By adaptive control and adaptive synchronizal the new system is stable and achieved good results	New 4D (R-F) dynamical system

2. New 4D Rabinovitch-Fabrikant(R-F) System

To construct a 4D system from 3D continuous time dynamical system, state augmentation technique [15], was used by adding new state variable(u), with irreversible function (trigonometric function) term, to the 3D dynamical system, that exhibits similar dynamics.

The new thirteen terms system (1) with six non-linearities terms, given by four differential equations with three parameters a, b, k and x, y, z, u are state variables.

Expressed as:

$$\begin{aligned}\dot{x} &= y(z-1+x^2)+ax\\ \dot{y} &= x(3z+1-x^2)+ay\\ \dot{z} &= -2z(b+xy)\\ \dot{u} &= a(x-y)+k\sin(u)\end{aligned} \tag{1}$$

where $a = 0.077, b = 0.1, k = 4$.

3. System Analysis

To analyze the system we follow the steps:

3.1. Equilibria

We set the system equal to zero and find the equilibrium points.

So, the equilibrium points of the system are : $E_0 = (0,0,0,0)$,
$E_{1,2} = (\pm1.5378, \mp0.065, -0.26909, \mp1.768)$, $E_{3,4} = (\pm0.0438, \mp2.2831, 1.7635, \mp2.5673)$.

3.2. Analysis of stability

3.2.1. Characteristic equation

The Jacobian matrix of new system (1), at equilibrium point $E_0(0,0,0,0)$ is:

$$\begin{bmatrix} 0.077 & -1 & 0 & 0\\ 1 & 0.077 & 0 & 0\\ 0 & 0 & -0.2 & 4\\ 0 & 0 & -0.2 & 4 \end{bmatrix}$$

It's characteristic equation
$\lambda^4 - 3.954\lambda^3 + 0.791129\lambda^2 - 3.6993302\lambda - 0.8047432 = 0$. The roots of characteristic equation are:

$\lambda_1 = 4, \lambda_2 = -0.2, \lambda_3 = 0.077 + i, \lambda_4 = 0.077 - i$ since λ_1 is positive and λ_3, λ_4 are complex numbers with positive real part, Therefor the new (R-F) system (1) is not stable.

3.2.2. Criterion of Routh stability

Routh criteria states that new R-F system (1) is steady that each term in Routh Array table's first column are positive values, is necessary and sufficient condition.

$$a_4 = 1, a_3 = -3.95, a_2 = 0.791129, a_1 = -3.6993302, a_0 = -0.8047432$$

$$b_1 = \frac{a_3a_2 - a_4a_1}{a_3} = 1.72672, c_1 = \frac{b_1a_1 - a_3b_2}{b_1} = -5.5421$$

Since the characteristic equation contains different signs and the first column contains negative values. So, the new system (1) is unstable.

Table 2: Routh Arrays for new R-F system.

λ^4	1	0.791129	-0.8047432
λ^3	-3.954	-3.6993302	0
λ^2	1.72672	-0.8047432	
λ^1	-5.5421	0	
λ^0	-0.8047432		

3.2.3. Hurwitz stability criteria

Using the coefficients of the characteristic equation, we form a square matrix in which the number of rows and columns equals the degree of the equation:

$\Delta_1 = a_3 = -3.954 < 0, \Delta_2 = \begin{vmatrix} a_3 & a_1 \\ a_4 & a_2 \end{vmatrix} = -6.827454266 < 0, \Delta_3 = \begin{vmatrix} a_3 & a_1 & 0 \\ a_4 & a_2 & a_0 \\ 0 & a_3 & a_1 \end{vmatrix} =$ $10.4683797, \Delta_4 = \begin{vmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{vmatrix} = 30.4502404$. Since $(\Delta_1, \Delta_2) < 0$, the new system (1) is unstable.

3.2.4. Lyapunov function

Let the Lyapunov function of new R-F system (1) is

$$V(x,y,z,u) = \frac{1}{2}(x^2 + y^2 + z^2 + u^2) \tag{2}$$

$$\dot{V}(x,y,z,u) = x\dot{x} + y\dot{y} + z\dot{z} + u\dot{u} \tag{3}$$

Substitute the new system (1) in equation (3) we get:

$$\dot{V}(x,y,z.u) = ax^2 + ay^2 - 2bz^2 + 4xyz - 2xyz^2 + axu - ayu + ku\ sin(u) \tag{4}$$

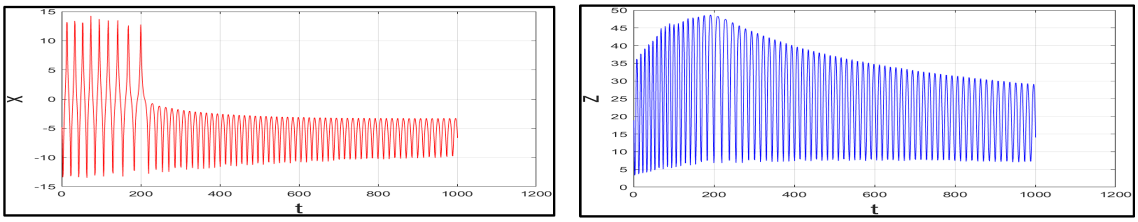
substitute the initial conditions in (4) we get: $\dot{V}(x,y,z,u) > 0$. Hence the system is unstable.

3.3. Graphical numerical analysis

New system (1) has been solved by Runge-Kutta technique of fourth order with initial values $[x_0, y_0, z_0, u_0] = [4, -11, 18, 3]$.

3.3.1. Wave-form

The wave form $x(t), z(t)$ for system (1) is characterized with a non-aperiodic shape, shown in figure (1), which is one of the basic characteristic of chaotic dynamical system :

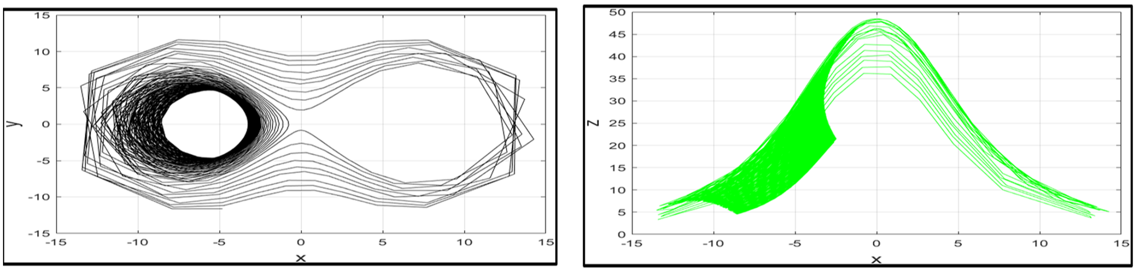


(a) (b)

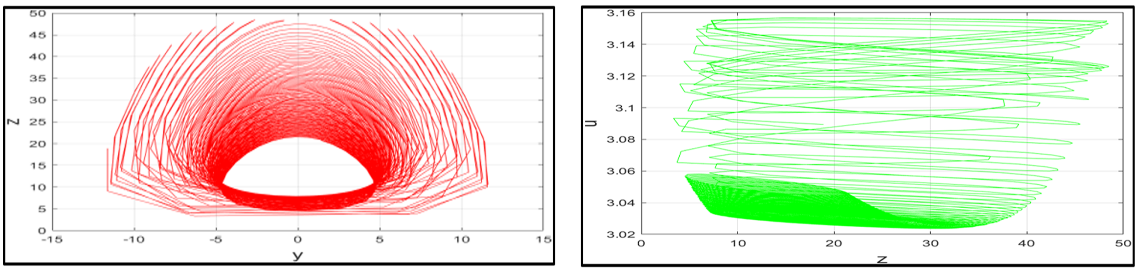
Figure 1: The wave-form of new system (1): $(1a)x - t, (1b)z - t$.

3.3.2. Phase space of new system (1)

I this paragraph figure (2)(a,b) shows the chaotic attractors in $(x-y), (x-z), (y-z), (z-u)$ plane, and figure (3) in $(x, y, z), (y, z, u)$ space:



(a) (b)

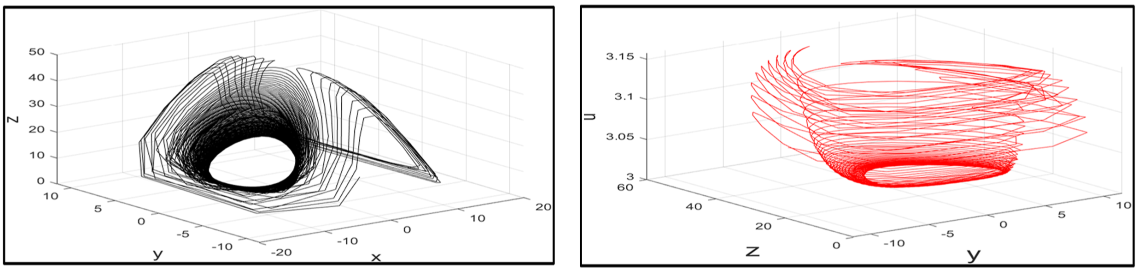


(c) (d)

Figure 2: 2-D for the chaotic attractor for new system (1) $(2a)x - y(2b)x - z.(2c)y - z.(2d)z - u.$

3.4. Multistability

Multistability (coexisting attractors) of nonlinear dynamical system mean , by changing the initial condition for same parameters of the system , and changing the initial condition with changing parameters is achieve the coexistence of many aspects according to the table (2), as in figure (4).



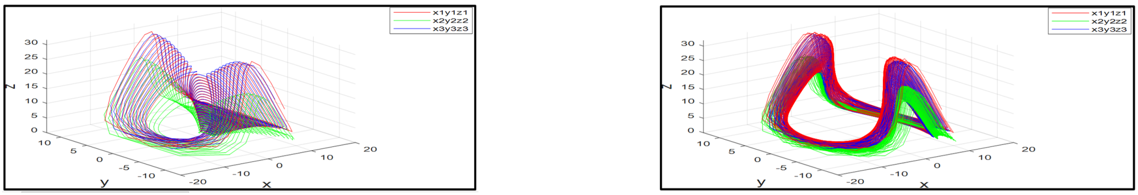
(a) (b)

Figure 3: 3-D chaotic attractor for new system (1).

The presence of multistability does not necessarily indicate chaos, but it can provide a favorable environment for the emergence of chaos in dynamical system. Multistability can increase the complexity and richness of the system's behavior and provide a basis for emergence of more complex dynamics, such as chaos.

Table 3: Coexistence with different parameters and initial conditions.

Initial conditions	Parameters	Color	Figures
$x_0 = 5, y_0 = -12, z_0 = 15, u_0 = 9$ $x_0 = 6, y_0 = -13, z_0 = 5, u_0 = 2$ $x_0 = 4, y_0 = -11, z_0 = 18, u_0 = 3$	a=0.077 b=0.1 k=4	Red Green Blue	Figure (4) (a)
$x_0 = 5, y_0 = -12, z_0 = 15, u_0 = 9$ $x_0 = 6, y_0 = -13, z_0 = 5, u_0 = 2$ $x_0 = 4, y_0 = -11, z_0 = 18, u_0 = 3$	a=-0.4 b=-0.0001 k=0.2	Red Green Blue	Figure (4) (b)



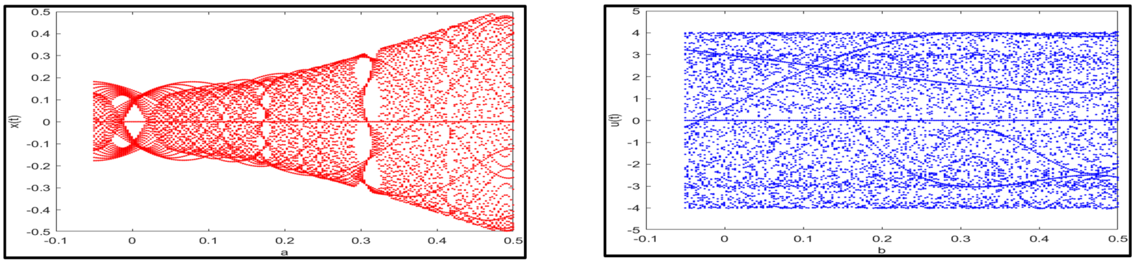
(a) (b)

Figure 4: Multistability of three attractors with three initial conditions corresponding to table (3).

3.5. Hopf-bifurcation

The Jacobian matrix(J) have pair of complex eigenvalues with modulus approximately equal one, so the new system (1) has Hopf-Bifurcation. Analyzing new system (1) using bifurcation diagrams. The bifurcation parameter (a) is changed interval [-0.5-0.05] with the rest of the parameters fixed, shown in figure (5a), also the bifurcation parameter (b)

is changed with the rest of parameters fixed show in figure(5a). Which shows the chaotic behavior of the system (1).



(a) (b)

Figure 5: Bifurcation diagram for the system (1).

3.6. Lyapunov Exponent

The Lyapunov exponent is simple way of describing the dynamics of a chaotic system. The system is chaotic if at least one Lyapunov exponent greater than zero. The values of Lyapunoc exponents are $L_1 = 14.025946, L_2 = 0.295151, L_3 = -2.854401, L_4 = -13.736833$. and $\sum_{i=1}^{4} L_i = -2.270137$therefore the Lyapunov dimension is $D_L = 3 + \frac{L_1+L_2+L_3}{|L_4|} = 3.83474$. So, the system (1) is Hyper chaotic system since the necessary condition are satisfied: Two Lyapunov exponent are positive from the four Lyapunov exponents, as shown in figure (6).

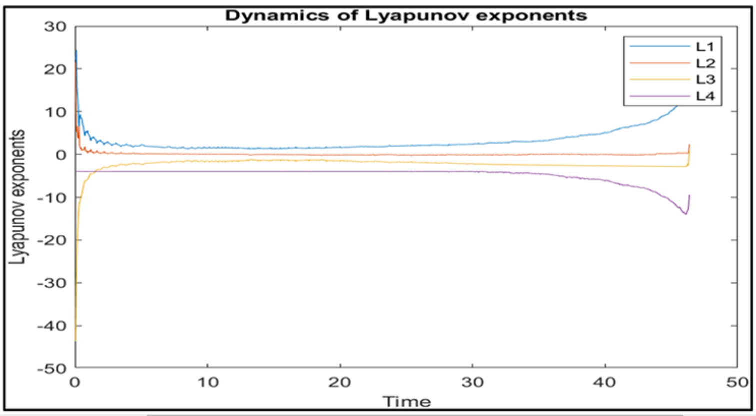


Figure 6: Lyapunov Exponent of new system(1).

4. ADAPTIVE CONTROL TECHNIQUE

4.1. Theoretical results

To achieve the stability of the new hyperchaotic R-F system (1) by adaptive control technology, with the undefined parameter (b).

$$\begin{aligned} \dot{x} &= y(z-1+x^2)+0.077x+v_1(t) \\ \dot{y} &= x(3z+1-x^2)+0.077y+v_2(t) \\ \dot{z} &= -2z(b+xy)+v_3(t) \\ \dot{u} &= ax-ay+k\ sin(u)+v_4(t) \end{aligned} \tag{5}$$

where $(v_1(t), v_2(t), v_3(t), v_4(t)$ are feedback controllers and (x, y, z, u) are state variable.

The adaptive control functions are:

$$\begin{aligned} v_1(t) &= -yz+y-x^2y-0.077x-\beta_1 x \\ v_2(t) &= -3xz-x+x^3-0.077y-\beta_2 y \\ v_3(t) &= 2\hat{b}z+2xyz-\beta_3 z \\ v_4(t) &= -ax+ay-k\ sin(u)-\beta_4 u \end{aligned}$$

where the constants $(\beta_1, \beta_2, \beta_3, \beta_4) > 0$, and $\hat{b}$ is the estimate parameter of (b): Substituting the adaptive control functions into (5), we get:

$$\begin{aligned} \dot{x} &= -\beta_1 x \\ \dot{y} &= -\beta_2 y \\ \dot{z} &= 2(\hat{b}-b)z-\beta_3 z \\ \dot{u} &= -\beta_4 u \end{aligned} \tag{6}$$

Let the parameter estimation error:

$$e_b = b - \hat{b} \tag{7}$$

Substituting (7) in (6) we obtain:

$$\begin{aligned} \dot{x} &= -\beta_1 x \\ \dot{y} &= -\beta_2 y \\ \dot{z} &= -2e_b z-\beta_3 z \\ \dot{u} &= -\beta_4 u \end{aligned} \tag{8}$$

Consider the Lyapunov function: $V(x, y, z, u, e) = \frac{1}{2}(x^2 + y^2 + z^2 + u^2, e_b^2)$
V is positive-definite on R^5.
Differentiating V we get:

$$\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + u\dot{u} + e_b\dot{e}_b \tag{9}$$

Also,

$$\dot{e}_{\boldsymbol{b}} = -\dot{\hat{b}} \tag{10}$$

We substituting the equation (10) and the system (8) in equation (9) we get:

$$\dot{V} = -\beta_1 x - \beta_2 y - \beta_3 z - \beta_4 u - e_b(2z^2 + \dot{\hat{b}}) \tag{11}$$

Not that:

$$\dot{\hat{b}} = -2z^2 + \beta_5 e_b \tag{12}$$

Substitute (12) in (11) we obtain:

$$\dot{V} = -\beta_1 x^2 - \beta_2 y^2 - \beta_3 z^2 - \beta_4 u^2 - \beta_5 e_b^2 \tag{13}$$

where (β_5) is positive.

We substitute ($\beta_1, \beta_2, \beta_3, \beta_4.\beta_5$) and the initial value in equation (13) we get $\dot{V}(x, y, z, u, e_b) < 0$. So, the following proposition 1 is proved.

Proposition 1: By adaptive control design, the chaotic system (5) with unknown parameter is stabilized for every initial value, where the estimated parameter is obtaind by (12) and ($\beta_1, \beta_2, \beta_3, \beta_4.\beta_5$) are greater than zero.

4.2. Numerical results

Simulation for controlled hyperchaotic system (8) with initial values $x_0 = 3, y_0 = -2, z_0 = -5, u_0 = 7, \beta_1 = 10, \beta_2 = 15, \beta_3 = 20, \beta_4 = 5, \beta_5 = 10$ and unknown parameter ($\hat{b} = 0.9$). Figure (7) shown the controlled state path of new system:

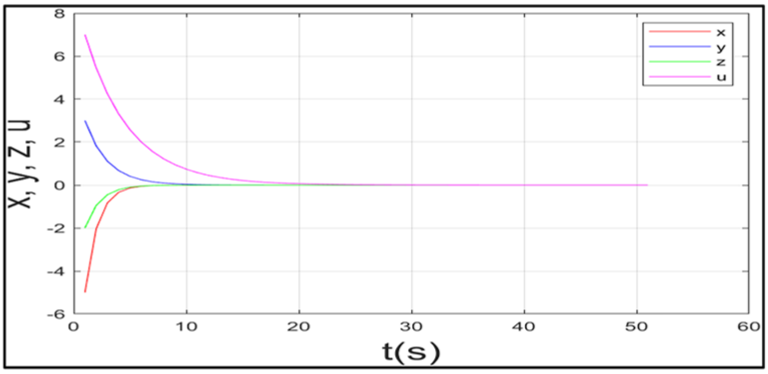


Figure 7: Paths of state variable x, y, z, u for controlled system.

5. Strategy for adaptive synchronization

5.1. Theoretical results

This section adaptive synchronization of hyperchaotic system, when the parameter (b) is unknown parameter.

The drive system, we consider the chaotic new (R-F) dynamical system described by:

$$\begin{aligned} \dot{x}_1 &= y_1z_1 - y_1 + x_1^2y_1 + 0.077x_1 \\ \dot{y}_1 &= 3x_1z_1 + x_1 - x_1^3 + 0.077y_1 \\ \dot{z}_1 &= -0.2z_1 - 2x_1y_1z_1 \\ \dot{u}_1 &= 0.077x_1 - 0.077y_1 + 4sinu_1 \end{aligned} \tag{14}$$

where (x_1, y_1, z_1, u_1) are state variables. The response system, is given by:

$$\begin{aligned} \dot{x}_2 &= y_2z_2 - y_2 + x_2^2y_2 + 0.077x_2 + v_1(t) \\ \dot{y}_2 &= 3x_2z_2 + x_2 - x_2^3 + 0.077y_2 + v_2(t) \\ \dot{z}_2 &= -2bz_2 - 2x_2y_2z_2 + v_3(t) \\ \dot{u}_2 &= 0.077x_2 - 0.077y_2 + 4sinu_2 + v_4(t) \end{aligned} \tag{15}$$

where $(v_1(t), v_2(t), v_3(t), v_4(t))$ are the feedback control and (x_2, y_2, z_2, u_2).

Are state variables.

adaptive synchronization error given by $e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1, e_4 = u_2 - u_1$.

Hence, the dynamical system of synchronization error:

$$\begin{aligned} \dot{e}_1 &= e_2e_3 + z_1e_2 + y_1e_3 - e_2 + e_1^2e_2 + y_1e_1 + x_1^2e_2 + 0.077e_1 + v_1(t) \\ \dot{e}_2 &= 3(e_1e_3 + z_1e_1 + x_1e_3) + e_1 - e_1^3 + 0.077e_2 + v_2(t) \\ \dot{e}_3 &= -2be_3 - 2(e_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3) + v_3(t) \\ \dot{e}_4 &= 0.077e_1 - 0.077e_2 + 4sine_4 + v_4(t) \end{aligned} \tag{16}$$

Defined adaptive control function:

$$\begin{aligned} v_1(t) &= -e_2e_3 - z_1e_2 - y_1e_3 + e_2 - e_1^2e_2 - y_1e_1 - x_1^2e_2 - 0.077e_1 - \beta_1e_1 \\ v_2(t) &= -3(e_1e_3 + z_1e_1 + x_1e_3) - e_1 - e_1^3 - 0.077e_2 - \beta_2e_2 \\ v_3(t) &= 2\hat{b}e_3 + 2(e_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3) - \beta_3e_3 \\ v_4(t) &= -0.077e_1 + 0.077e_2 - 4sine_4 - \beta_4e_4 \end{aligned} \tag{17}$$

where $(\beta_1, \beta_2, \beta_3, \beta_4)$ are positive real values, and $(\hat{b})$ is the estimated value of the parameter (b).

Substituting (17) into (16) we get the dynamical system of synchronization error:

$$\begin{aligned} \dot{e}_1 &= -\beta_1e_1 \\ \dot{e}_2 &= -\beta_2e_2 \\ \dot{e}_3 &= -2(b - \hat{b})e_3 - \beta_3e_3 \\ \dot{e}_4 &= -\beta_4e_4 \end{aligned} \tag{18}$$

The parameter estimation error is:

$$e_b = b - \hat{b} \tag{19}$$

Substitute (19) into (18) we obtain synchronization error system is:

$$\begin{aligned}
\dot{e}_1 &= -\beta_1 e_1 \\
\dot{e}_2 &= -\beta_2 e_2 \\
\dot{e}_3 &= -2e_b e_3 - \beta_3 e_3 \\
\dot{e}_4 &= -\beta_4 e_4
\end{aligned} \tag{20}$$

To prove the stability of the system (20), Consider the quadratic Lyapunov function is considered as:

$$V(e_1, e_2, e_3, e_4, e_k) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_b^2) \tag{21}$$

which is a positive definite on R^5. Not that:

$$\dot{e}_b = -\dot{\hat{b}} \tag{22}$$

Differentiating equation (21) and substituting the system (20) and (22) we get:

$$\dot{V} = -\beta_1 e_1^2 - \beta_2 e_2^2 - \beta_3 e_3^2 - \beta_4 e_4^2 - e_b(\dot{\hat{b}} + 2e_3) \tag{23}$$

Hence, the estimated parameter is:

$$\dot{\hat{b}} = -2e_3 + \beta_5 e_b \tag{24}$$

Where (β_5) is a positive.

Substitute (24) into (23) we obtion:

$$\dot{V} = -\beta_1 e_1^2 - \beta_2 e_2^2 - \beta_3 e_3^2 - \beta_4 e_4^2 - \beta_5 e_b^2 \tag{25}$$

Which is a negative on R^5. Hence, by Lyapunov stability its immediately that the parameter error and error synchronization decay exponentially to zero. Thus the proposition 2 is proved.

Proposition 2: The identical chaotic of the drive and the response systems, with the unknown parameter (b) are synchronized for all initial value by adaptive control (17), where the estimated given by (24) and constant $\beta_i, (i = 1, 2, 3, 4, 5)$ are positive, since $lim_{i\to\infty} e_i \to 0$, Therefor the system (20) is stable.

5.2. Numerical results

By the 4th-order Runge-kutta method to solve the master system (14), slave system (15), and synchronization error system. We take initial value of the system (14) $(x_{1(0)}, y_{1(0)}, z_{1(0)}, u_{1(0)}) = [3, 6, 5, 12]$ and initial value of the system (15). $(x_{2(0)}, y_{2(0)}, z_{2(0)}, u_{2(0)}) = [19, 10, -4, 5]$,the parameter unknown $\hat{b} = 2.4$ and $\beta_i = 6$ for $i = 1, 2, 3, 4, 5$. Dynamic synchronization error system shown in figure (8).

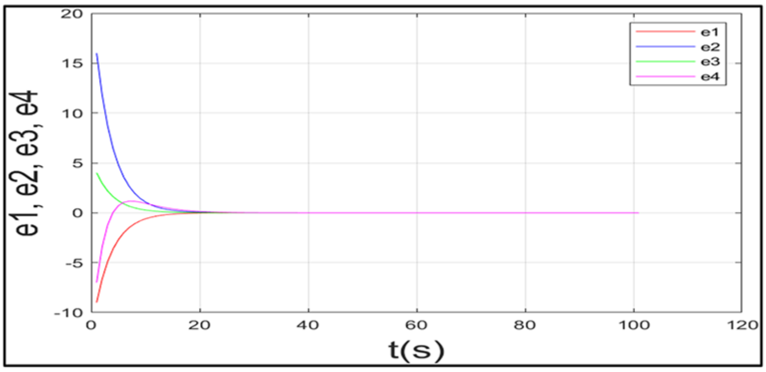


Figure 8: The convergent for system (16) with adaptive control (17).

6. Conclusion

This paper present the successful creation of a new 4D Rabinovich-Fabrikant dynamical system from a 3D version using the state augmentation technique. A comprehensive investigation of the system's dynamical behaviors was conducted, revealing an unstable hyperchaotic with coexistence self-excited attractors, the system multistability, Hopf-bifurcation, wave form analysis and phase space analysis. Were meticulously analyzed. Importantly, the Lyapunov exponents are: $L_1 = 14.025946, L_2 = 0.295151, L_3 = -2.854401, L_4 = -13.736833$. confirmed the system's unstability. While the Lyapunov dimension $D_L = 3.83474$. underscord. It's hyperchaotic state. Despite the inherent chaos, the system's stability was effectively attained through the implementation of adaptive control and adaptive synchronization techniques, illustration the potential of these methods for controlling and managing complex dynamical systems.

References

[1] Mohammad Alkasassbeh, Zurni Omar, Fateh Mebarek-Oudina, Jawad Raza, and Ali Chamkha. Heat transfer study of convective fin with temperature-dependent internal heat generation by hybrid block method. *Heat Transfer—Asian Research*, 48(4):1225–1244, 2019.

[2] Kathleen T Alligood, Tim D Sauer, James A Yorke, and David Chillingworth. Chaos: an introduction to dynamical systems. *SIAM Review*, 40(3):732–732, 1998.

[3] Shelly Arora, Fateh M Oudina, Saroj Sahani, et al. Super convergence analysis of fully discrete hermite splines to simulate wave behavior of kuramoto-shivashinsky equation. *Wave Motion*, page 103187, 2023.

[4] Maysoon Aziz and Muhammad Hamid. The possibility of increasing the predictability indices after control of 3d-continuous-time system. In *2019 International Conference on Computing and Information Science and Technology and Their Applications (ICCISTA)*, pages 1–5. IEEE, 2019.

[5] Maysoon M Aziz. Engineering mathematics for radiative heat transfer in the magnetohydrodynamic. *Computer Science*, 18(3):563–570, 2023.

[6] Maysoon M Aziz and Mansour N Faraj. Difficulty of predicting earthquakes in mosul dam. *International Journal of Technical Research and Applications*, 3(6):29–36, 2015.

[7] Maysoon M Aziz and Arqam Salim Mahmood. Mathematical model of epidemic disease covid-19. In *AIP Conference Proceedings*, volume 2414. AIP Publishing, 2023.

[8] Maysoon M Aziz and Dalya M Merie. Stability and chaos with mathematical control of 4-d dynamical system. *Indonesian Journal of Electrical Engineering and Computer Science*, 20(3):1242–1251, 2020.

[9] Jianghong Bao and Yongjian Liu. Multistability and bifurcations in a 5d segmented disc dynamo with a curve of equilibria. *Advances in Difference Equations*, 2019:1–15, 2019.

[10] Hsien-Keng Chen. Synchronization of two different chaotic systems: a new system and each of the dynamical systems lorenz, chen and lü. *Chaos, Solitons & Fractals*, 25(5):1049–1056, 2005.

[11] Marius-F Danca. Hidden transient chaotic attractors of rabinovich–fabrikant system. *Nonlinear Dynamics*, 86(2):1263–1270, 2016.

[12] Selcuk Emiroglu and Yılmaz Uyaroğlu. Analysis and control of a 4d hyperchaotic system with passive control. *International Journal of Nonlinear Analysis and Applications*, 12(1):1087–1095, 2021.

[13] M Farhan, Zurni Omar, F Mebarek-Oudina, J Raza, Z Shah, RV Choudhari, and OD Makinde. Implementation of the one-step one-hybrid block method on the nonlinear equation of a circular sector oscillator. *Computational Mathematics and Modeling*, 31:116–132, 2020.

[14] Jack K Hale. Dynamical systems and stability. Technical report, National Science Foundation, 1968.

[15] A Khan, Neetu Aneja, P Tripathi, and J Biswas. A new hyper chaotic system and study of hybrid projective synchronization behavior. *Nonlinear Dynamics and Systems Theory*, page 266, 2017.

[16] Fateh Mebarek-Oudina. Numerical modeling of the hydrodynamic stability in vertical annulus with heat source of different lengths. *Engineering science and technology, an international journal*, 20(4):1324–1333, 2017.

[17] Fateh Mebarek-oudina and Rachid Bessaïh. Numerical modeling of mhd stability in a cylindrical configuration. *Journal of the Franklin Institute*, 351(2):667–681, 2014.

[18] Fateh Mebarek-Oudina and Oluwole Daniel Makinde. Numerical simulation of oscillatory mhd natural convection in cylindrical annulus: Prandtl number effect. In *Defect and Diffusion Forum*, volume 387, pages 417–427. Trans Tech Publ, 2018.

[19] Kiran Ravichandran. Rabinovich-fabrikant chaotic system and its application to secure communication. *Research Square*, 2021.

[20] M Srivastava, SK Agrawal, K Vishal, and S Das. Chaos control of fractional order rabinovich–fabrikant system and synchronization between chaotic and chaos controlled fractional order rabinovich–fabrikant system. *Applied Mathematical Modelling*, 38(13):3361–3372, 2014.