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Analytical and approximate solutions for fractional systems of nonlinear differential equations

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Abstract. This paper studies the epidemic model via an efficient genius modern analytical approximate technique named Natural Transform Adomian Decomposition Method (NTADM). It is based on Caputo fractional derivative. To demonstrate the effectiveness of the present method, the results are displayed in graphs. Accordingly, the NTADM can be very easily applied to other nonlinear models.

2020 Mathematics Subject Classifications: 97M40

Key Words and Phrases: Natural transform, Adomian decomposition method, Mathematical epidemic model, Analytical solution

1. Introduction

Fractional calculus has become an important tool for modeling and solving problems in various areas of science, including biology, physics, chemistry, and engineering, among others [6–8, 10, 16, 28]. In biology, for example, fractional calculus has been used to model the behavior of cells, the spread of diseases, and the dynamics of biological systems. In physics, fractional calculus has been used to model the behavior of fluids, the diffusion of particles, and the properties of materials, among other things. One of the reasons why fractional calculus is so useful in modeling these types of problems is that it allows for the modeling of systems that exhibit memory or hereditary effects. Traditional calculus assumes that a system's behavior is determined solely by its current state, while fractional

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calculus allows for the influence of past states as well. This ability to model systems with memory effects has made fractional calculus an important tool in fields such as control theory, signal processing, and image analysis, where the behavior of systems depends on past inputs and states. Many challenges can disrupt daily life, and we need to control these challenges in order to improve our quality of life. One of these challenges is the spread of epidemics, the most recent being the COVID-19 pandemic. Many effective methods for obtaining numerical and analytical solutions for epidemic mathematical models with fractional-order derivatives have recently been used, for example, fractional natural decomposition method [11, 25], natural transform decomposition method [26], fractional residual power series method [14], fractional exponential function method [4], Laplace Adomian decomposition method [9], Natural transform homotopy analysis method [1], Sumudu transform [30], fractional novel analytical method [5], fractional Taylor series [29] and Caputo-Katugampola derivatives [2]. In this paper, we will study the epidemic mathematical model represented by system of nonlinear ordinary nonlinear differentials; studied recently through [13, 18], where Gao et al. [13] used the (q-HATM), while Liu et al. [18] presented some interesting results for the projected model, we present its treatment in fractional order via a novel smart technique named the Natural transform adomian decomposition method (NTADM). The Natural Transform (NT) is a mathematical tool that was originally defined by Khan and Khan [17] and has since been studied by other researchers [3, 15, 19–22, 24]. The inverse Natural Transform has also been defined by other researchers such as in [12, 23, 27].

2. Basic scheme of the suggested method

We will consider the non-linear equation in the form:

$$\ell\phi + r\phi + f\phi = \varpi(\chi,\tau), \tag{1}$$

where ℓ is the lowest order derivative, r is a linear operator of order lower than ℓ, $f\phi$ the nonlinear terms, and $\varpi(\chi,\tau)$ is the source term.

Applying the NT on both sides of Eq. (1), we get:

$$N[\ell\phi] + N[r\phi] + N[f\phi] = N[\varpi(\chi,\tau)]. \tag{2}$$

Using the property of the NT, as well as the initial conditions and arrangement, we have:

$$N[\phi(\chi,t)] = \frac{1}{s}\phi(\chi,0) + \frac{u}{s^2}\phi(\chi,0) + \ldots + \frac{u^{(n-1)}}{s^{(n-1)}}\phi^{(n-1)}(\chi,0) - \frac{u^n}{s^n}N[\phi] - \frac{u^n}{s^n}N[f\phi] + \frac{u^n}{s^n}N[\varpi(\chi,t)]. \tag{3}$$

Via Adomian producers, we have

$$\phi(\chi,\tau) = \sum_{n=0}^{\infty}\phi_n(\chi,\tau), \tag{4}$$

and the nonlinear term can be analyzed as:

$$f\phi(\chi,\tau)=\sum_{n=0}^{\infty}\Delta_n(\chi,\tau), \tag{5}$$

where Δ_n is an Adomian polynomial of $\phi_0, \phi_1, \dots, \phi_n$ that has the following relation:

$$\Delta_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[f\sum_{n=0}^{\infty}\lambda^n\phi_n\right]_{\lambda=0}, n=0,1,2,3,\dots. \tag{6}$$

Substituting Eqs. (5) and (4) into Eq. (3) yields:

$$N\left[\sum_{n=0}^{\infty}\phi_n(\chi,\tau)\right]=\frac{1}{s}\phi(\chi,0)+\frac{u}{s^2}\phi'(\chi,0)+\dots+\frac{u^{(n-1)}}{s^{(n-1)}}\phi^{(n-1)}(\chi,0)-\frac{u^n}{s^n}N[\phi]-\frac{u^n}{s^n}N[\sum_{n=0}^{\infty}\Delta_n]+\frac{u^n}{s^n}N[\varpi(\chi,\tau)], \tag{7}$$

i.e,

$$N\left[\sum_{n=0}^{\infty}\phi_0(\chi,\tau)\right]=\frac{1}{s}\phi(\chi,0)+\frac{u}{s^2}\phi'(\chi,0)+\dots+\frac{u^{(n-1)}}{s^{(n-1)}}\phi^{(n-1)}(\chi,0)+\frac{u^n}{s^n}N[\varpi(\chi,\tau)]=Y(\chi,u). \tag{8}$$

So,

$$N\left[\phi_1(\chi,\tau)\right]=-\frac{u^n}{s^n}N[r\phi_0(x,t)]-\frac{u^n}{s^n}N[\Delta_0], \tag{9a}$$

$$N\left[\phi_2(\chi,\tau)\right]=-\frac{u^n}{s^n}N[r\phi_1(x,t)]-\frac{u^n}{s^n}N[\Delta_1]. \tag{9b}$$

Finally, we obtain:

$$N\left[\phi_{n+1}(\chi,\tau)\right]=-\frac{u^n}{s^n}N[r\phi_n(x,t)]-\frac{u^n}{s^n}N[\Delta_n], \quad n\geq 0 \tag{10}$$

By using the inverse NT to (8) and (10), we obtain:

$$\phi_0(\chi,\tau)=\wp(\chi,\tau), \tag{11a}$$

$$\phi_{n+1}(\chi,\tau)=-N^{-1}[\frac{u^n}{s^n}N[r\phi_n(x,t)]+\frac{u^n}{s^n}N[\Delta_n]], n\geq 0. \tag{11b}$$

where $\wp(\chi,\tau)$ represents the term that is arising from the source term and prescribed initial conductions. By using the inverse NT to $\varpi(\chi,\tau)$ and the given conditions we get:

$$\delta=\eta+N^{-1}[\varpi(\chi,\tau)], \tag{12}$$

where

$$\delta=\delta_0+\delta_1+\dots+\delta_n, \tag{13}$$

$$\phi_0=\delta_k+\dots+\delta_{k+1}. \tag{14}$$

Finally, we proof that ϕ_0 satisfy the original equation.

3. The epidemic mathematical model Formulation

Now, we consider the epidemic mathematical model within fractional order represented by a system of nonlinear differential equations, and consists of four categories of individuals $\iota(\tau), A(\tau), \gamma(\tau)$, and $\alpha(\tau)$. The description of the interactions between the variables in this system is done by using the given nonlinear system [13, 18]:

$$D_\tau^\varepsilon \iota(\tau) = -\rho\iota(\tau)[A(\tau)+\alpha(\tau)], \tag{15a}$$

$$D_t^\varepsilon A(\tau) = \rho\iota(\tau)[A(\tau)+\alpha(\tau)] - \beta A(\tau), \tag{15b}$$

$$D_\tau^\varepsilon \gamma(\tau) = \beta_1 A(\tau) - \mu\gamma(\tau), \tag{15c}$$

$$D_\tau^\varepsilon \alpha(\tau) = \beta_2 A(\tau) - \mu\alpha(\tau), 0 < \varepsilon < 1. \tag{15d}$$

With the initial conditions

$$\iota(0) = t_0 = 11.081 \times 10^6, A(0) = A_0 = 3.62, \alpha(0) = \alpha_0 = 4.13, \text{and}\, \gamma(0) = \gamma_0 = 0. \tag{16}$$

The epidemic mathematical model system contains four components, representing individuals of different infection status: $\iota(\tau)$ individuals susceptible, $A(\tau)$ asymptomatic infectious, $\gamma(\tau)$ reported asymptomatic infectious, $\alpha(\tau)$ unreported asymptomatic infectious.

Applying NT of Eq. (15d), we obtain:

$$\frac{s^\varepsilon}{u^\varepsilon}N[\iota(\tau)] - \frac{s^{\varepsilon-1}}{u^\varepsilon}\iota(0) = -\rho N[\iota(\tau)A(\tau) + \iota(\tau)\alpha(\tau)], \tag{17a}$$

$$\frac{s^\varepsilon}{u^\varepsilon}N[A(\tau)] - \frac{s^{\varepsilon-1}}{u^\varepsilon}A(0) = \rho N[\iota(\tau)A(\tau) + \iota(\tau)\alpha(\tau)] - \beta N[A(\tau)], \quad 0 < \varepsilon < 1, \tag{17b}$$

$$\frac{s^\varepsilon}{u^\varepsilon}N[\gamma(\tau)] - \frac{s^{\varepsilon-1}}{u^\varepsilon}\gamma(0) = \beta_1 N[A(\tau)] - \mu N[\gamma(\tau)], \tag{17c}$$

$$\frac{s^\varepsilon}{u^\varepsilon}N[\alpha(\tau)] - \frac{s^{\varepsilon-1}}{u^\varepsilon}\alpha(0) = \beta_2 N[A(\tau)] - \mu N[\alpha(\tau)]. \tag{17d}$$

After rearrangement, we have

$$N[\iota(\tau)] = \frac{1}{s}\iota(0) - \rho\frac{u^\varepsilon}{s^\varepsilon}N[Q(t,A) + P(t,\alpha)],$$

$$N[A(\tau)] = \frac{1}{s}A(0) + \rho\frac{u^\varepsilon}{s^\varepsilon}N[Q(t,A) + P(t,\alpha)] - \beta\frac{u^\varepsilon}{s^\varepsilon}N[A(\tau)],$$

$$N[\gamma(\tau)] = \frac{1}{s}R(0) + \beta_1\frac{u^\varepsilon}{s^\varepsilon}N[\gamma(\tau)] - \mu\frac{u^\varepsilon}{s^\varepsilon}N[\gamma(\tau)],$$

$$N[\alpha(\tau)] = \frac{1}{s}\alpha(0) + \beta_2\frac{u^\varepsilon}{s^\varepsilon}N[A(\tau)] - \mu\frac{u^\varepsilon}{s^\varepsilon}N[\alpha(\tau)]. \tag{18a}$$

Using ADM, we have

$$\iota(\tau) = \sum_{n=0}^{\infty}\iota_n(\tau), A(\tau) = \sum_{n=0}^{\infty}A_n(\tau), \gamma(\tau) = \sum_{n=0}^{\infty}\gamma_n(\tau), \alpha(\tau) = \sum_{n=0}^{\infty}\alpha_n(\tau), \tag{19}$$

and the nonlinear terms are:

$$Q(t,A) = B_n, P(t,\alpha) = E_n, \tag{20}$$

where B_n, E_n are Adomian polynomials of $\iota, A, \alpha, \ldots$ which calculated by formulas:

$$B_n = \frac{1}{n!}\frac{d^n}{d\bar{\lambda}^n}\left[N(\sum_{n=0}^{\infty}\bar{\lambda}^n U_n)\right]_{\bar{\lambda}=0}, \tag{21a}$$

$$E_n = \frac{1}{n!}\frac{d^n}{d\bar{\lambda}^n}\left[N(\sum_{n=0}^{\infty}\bar{\lambda}^n U_n)\right]_{\bar{\lambda}=0}. \tag{21b}$$

Substituting Eqs. (19), (20) and (21) into Eq. (18) yields:

$$N[\sum_{n=0}^{\infty}\iota_n(\tau)] = \frac{1}{s}\iota_0 - \rho\frac{u^\varepsilon}{s^\varepsilon}N[Q+P], \tag{22a}$$

$$N[\sum_{n=0}^{\infty}A_n(\tau)] = \frac{1}{s}A_0 + \rho\frac{u^\varepsilon}{s^\varepsilon}N[Q+P] - \beta\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}A_n(\tau)], \tag{22b}$$

$$N[\sum_{n=0}^{\infty}\gamma_n(\tau)] = \frac{1}{s}\gamma_0 + \beta_1\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}A_n(\tau)] - \mu\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}\gamma_n(\tau)], \tag{22c}$$

$$N[\sum_{n=0}^{\infty}\alpha_n(\tau)] = \frac{1}{s}\alpha_0 + \beta_2\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}A_n(\tau)] - \mu\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}\alpha_n(\tau)]. \tag{22d}$$

By comparing both sides of Eq. (22) and throw ADM, we get:

$$\iota(0) = \iota_0, \qquad \iota_{n+1} = -\rho N^{-1}\left[\frac{u^\varepsilon}{s^\varepsilon}N[Q+P]\right], \tag{23a}$$

$$A(0) = A_0, \qquad A_{n+1} = \rho N^{-1}\left[\frac{u^\varepsilon}{s^\varepsilon}N[Q+P]\right] - \beta N^{-1}[\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}A_n(\tau)]], \tag{23b}$$

$$\gamma(0) = R_0, \qquad \gamma_{n+1} = \beta_1 N^{-1}\left[\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}A_n(\tau)]\right] - \mu N^{-1}[\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}\gamma_n(\tau)]], \tag{23c}$$

$$\alpha(0) = \alpha_0, \qquad \alpha_{n+1} = \beta_2 N^{-1}\left[\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}A_n(\tau)]\right] - \mu N^{-1}\left[\frac{u^\varepsilon}{s^\varepsilon}N[\sum_{n=0}^{\infty}\alpha_n(\tau)]\right]. \tag{23d}$$

i.e.

$$\iota_1 = -\rho N^{-1}[\frac{u^\varepsilon}{s^\varepsilon}N[\iota_0 A_0 + \iota_0\alpha_0]], \tag{24a}$$

$$A_1 = \rho N^{-1}[\frac{u^\varepsilon}{s^\varepsilon}N[\iota_0 A_0 + \iota_0\alpha_0]] - \beta N^{-1}[\frac{u^\varepsilon}{s^\varepsilon}N[A_0]], \tag{24b}$$

$$\gamma_1 = \beta_1 N^{-1}[\frac{u^\varepsilon}{s^\varepsilon} N[A_0]] - \mu N^{-1}[\frac{u^\varepsilon}{s^\varepsilon} N[\gamma_0]], \tag{24c}$$

$$\alpha_1 = \beta_2 N^{-1}[\frac{u^\varepsilon}{s^\varepsilon} N[A_0]] - \mu N^{-1}[\frac{u^\varepsilon}{s^\varepsilon} N[\alpha_0]]. \tag{24d}$$

Then, using parameters in the present model as in [18], we have $\rho = 4.48 \times 10^{-8}, \beta = \frac{1}{7}, \mu = \frac{1}{7}\beta_1 = 0.8\mu, \beta_2 = 0.8\mu$

Also, we obtain:

$$\iota_1 = -3.8432 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)}, \quad \iota_2 = -0.898197 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)}, \tag{25a}$$

$$A_1 = 2.29589 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)}, \quad A_2 = 0.570213 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)}, \tag{25b}$$

$$\gamma_1 = 0.413714 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)}, \quad \gamma_2 = 0.203285 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)}, \tag{25c}$$

$$\alpha_1 = -0.486571 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)}, \quad \alpha_2 = 1.35107 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)}. \tag{25d}$$

Then, we can write the solution of the epidemic model in series form as follows:

$$\iota(t) = 11.081 \times 10^6 - 3.8432 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)} - 0.898197 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)} + \ldots, \tag{26a}$$

$$A(t) = 3.62 + 2.29589 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)} + 0.570213 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)} + \ldots, \tag{26b}$$

$$\gamma(t) = 0 + 0.413714 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)} + 0.203285 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)} + \ldots, \tag{26c}$$

$$\alpha(t) = 4.13 - 0.486571 \frac{t^\varepsilon}{\Gamma(\varepsilon+1)} + 1.35107 \frac{t^{2\varepsilon}}{\Gamma(2\varepsilon+1)} + \ldots, \tag{26d}$$

which is the same exact solution obtained by using the q-homotopy analysis transform method (q-HAM).

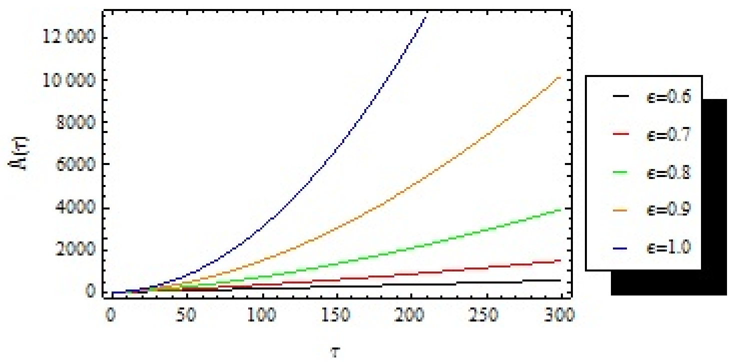

Figure 1: Natural solution of $A(\tau)$ for various values of ε that are similar results obtained via [13].

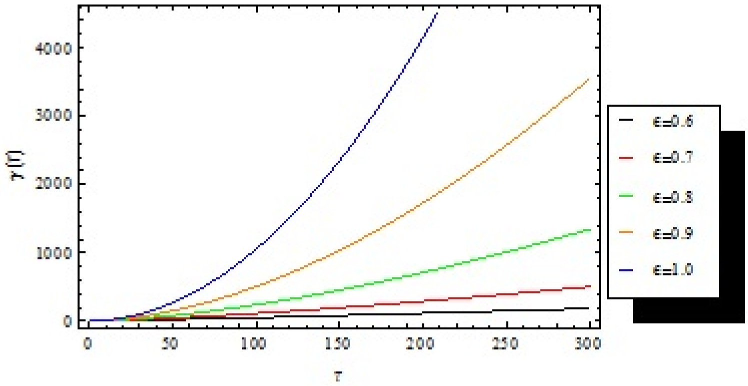

Figure 2: Natural solution of $\gamma(\tau)$ for various values of ε that are similar results obtained via [13].

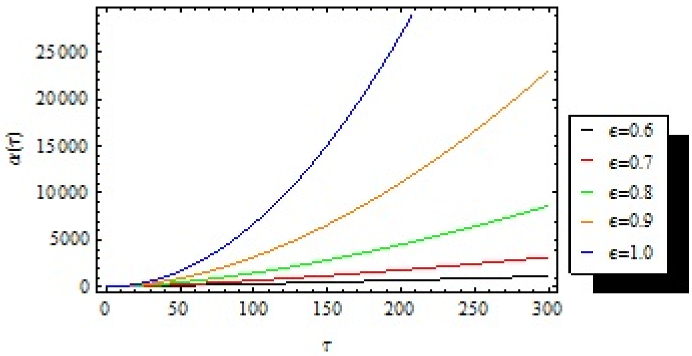

Figure 3: Natural solution of $\alpha(\tau)$ for various values of ε that are similar results obtained via [13].

4. Conclusion

The present paper deals with the epidemic mathematical model, which is solved via the Natural Transform Adomian Decomposition Method. It is found that the result of the proposed method is equivalent to those of q-HAM. The approximate analytical solutions of the fractional epidemic mathematical model gained via NTADM are equivalent to q-HAM, and suitable for various aspects of science. Finally, we can conclude that, the present technique is generalization of Laplace and Sumudu transforms. By setting $u = 1$ in Eq. (18) we have the solution via Laplace transform. Also by setting $s = 1$ in Eq. (18) we have the same solution via Sumudu transform. In the future, we will employ the NTADM to solve FDEs in various aspects of science.

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Declarations

Ethical Approval:
All the authors demonstrating that they have adhered to the accepted ethical standards of a genuine research study.

Consent to participate:
Being the corresponding author, I have consent to participate of all the authors in this research work.

Consent to publish:
All the authors are agreed to publish this research work.

Competing interests:
Authors declare no conflict of interest Authors' contributions All authors contributed equally.

Availability of data and materials:
Data will be provided on request to the corresponding author.

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