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# Finite Minimal Simple Groups non Satisfying the Basis Property

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**Abstract.** Let $G$ be a finite group. We say that $G$ has the Basis Property if every subgroup $H$ of $G$ has a minimal generating set (basis), and any two bases of $H$ have the same cardinality. A group $G$ is called minimal not satisfying the Basis Property if it does not satisfy the Basis Property, but all its proper subgroups satisfy the Basis Property. We prove that the following groups $PSL(2,5) \cong A_5$, $PSL(2,8)$ , are minimal groups non satisfying the Basis Property, but the groups $PSL(2,9)$, $PSL(2,17)$ and $PSL(3,4)$ are not minimal and not satisfying the Basis Property.

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**Key Words and Phrases**: Simple group, Minimal group, Group with the Basis Property

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## 1. Introduction

The Burnside Basis Theorem tells us that the generating sets for $p$-groups shares many property with the bases of vector spaces. In particular, if $G$ is a finite $p$-group, then the minimal generating sets (sets that no smaller proper subset can generate $G$ as well) have the same cardinality. We will say that an arbitrary finite group has the Generation Property if its minimal generating sets have the same cardinality. A finite group $G$ has the Basis Property if $G$ and all its subgroups have the Generation Property. In [12], Jones has introduced the Basis Property and considered it in the context of inverse semigroups. Also, Jones in [13] proved that if $G$ is a group with the Basis Property, then every element of $G$ must has a prime power order, after that, he established that the Basis Property is inherited by quotients and a group with the Basis Property is soluble as well. The Basis Property for groups has been developed by many authors as in the articles [1, 2, 4, 11, 14, 15] and we shall mention some of this work below. A variant of these properties is the concept of a Matroid group, which is a group that satisfies the Generation Property and the additional condition that every independent subset of $G$

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is contained in a minimal generating set. Scapellato and Verardi through the articles [17, 18] have studied Matroid groups. In more details, they provide a full characterization of Matroid groups that a Matroid group has the Basis Property. Alkhalaf in [3] has provided a pleasant characterization of groups with the Basis Property based on results of Higman [7], who has classified the soluble groups with all elements of prime-power order. Also A. Alkhalaf generalized many of the results related to groups that satisfies the Basis Property, and we can find them in [5, 6]. The purpose of this research is to initiate a study of groups with minimal group that not satisfying the Basis Property. Since every image of a homomorphism group with the Basis Property is a group with the Basis Property, then the group $G$ can be a minimal group not satisfying the Basis Property if its image under a homomorphism of every proper subgroup $H$ from $G$ must be satisfied the Basis Property. Likewise, all subgroups of a group $G$ must satisfy that. It follows, from all of these previous results, a minimal simple group does not satisfy the Basis Property if and only if every maximal subgroups is a group with the Basis Property. Be noted that, along this paper, the finite groups will be considered only, hence any proper subgroup will be contained in a maximal subgroup and we can see that in [9] and since the Basis Property is an inherited property as in [13], then it is enough to verify that the maximal subgroups satisfies the Basis Property. It should be mentioned that simple groups are not soluble groups, unless the simple groups with prime orders. Therefore, they can not be simple and non-prime and satisfying the Basis Property at the same time, but here we are trying to obtain a description of simple groups close to groups that are satisfying the Basis Property, in other words, we will study the simple groups that are not satisfying the Basis Property. A finite group $G$ is called a semi prime if the order of every element is a power of a prime number, this means, every element will be either $p$-element or $q$-element. Therefore, every cyclic subgroup of a group $G$ is a primary. A finite group $G$ is called a semi simple if it does not have any soluble normal non-trivial subgroups. A finite group $G$ is called a completely decomposable if it is decomposed into a direct product of a finite number of simple groups. [16]

## 2. Preliminaries

The previous concepts have studied by many authors, which they considered the Basis Properties of groups in their works, as written in [2–5]. Now, we are willing to prove our theorems, for that, we need to state some lemmas as the following.

**Lemma 1.** *[3, Theorem(2.5)] Let a finite group $G$ be a semi direct product of a $p$-group $P = Fit(G)$ (Fitting subgroup of $G$) by a cyclic $q$-group $< y >$ of order $q^b$, where $p \neq q$ ($p$ and $q$ are prime numbers), $b \in N$. Then the group $G$ has the Basis Property if and only if for any element $u \in < y >, u \neq e$ and for any invariant subgroup $H$ of $P$, the automorphism $\varphi_u$ must define an isotopic representation on every quotient Frattini subgroup $H$.*

In [5], the author used known results for the nilpotency class of the kernel of a Frobenius group to describe the nilpotency class of the Fitting subgroup of the group with the Basis Property.

**Lemma 2.** *[19, Theorem 16] Let $G$ be a simple group, assume that every nonidentity element composite order of $G$ is a prime power order. Then $G$ is isomorphic with one of the following groups*

$$PSL(2,5), PSL(2,7), PSL(2,2^3), PSL(2,3^2),$$

$$PSL(2,17), PSL(3,2^2), Sz(2^3), Sz(2^6).$$

## 3. The properties of simple groups

**Lemma 3.** *Let $G$ be a minimal simple group that does not satisfy the Basis Property. Then $G$ is a semi-prime.*

**p**roof. Suppose the group $G$ is a simple and non semi-prime. Then, for the two prime numbers $p$ and $q$ ($p \neq q$), there is a cyclic subgroup $< y >$ of $G$ with order $pq$, which does not satisfy the Basis Property, also, since $G$ is the minimal group, which does not satisfy the Basis Property, hence the group $G$ is a biprimary cyclic group whose order is $pq$, and this contradicts the hypothesis. Therefore, $G$ is a semi-prime group.

**Lemma 4.** *Let $G$ be a minimal group neither satisfying the Basis Property, nor soluble. Then $G$ is not a commutative simple group.*

**proof**. Suppose $G$ is not a soluble group and let $H$ be a maximal normal subgroup, which is a soluble subgroup of $G$. If $|H| \neq e$ and since $G$ is a minimal that is not satisfying the Basis Property, then $G/H$ is a group the Basis Property, therefore $G/H$ is a soluble, see the book [16] and $G$ must be a soluble group as an extension of a soluble subgroup by the soluble group, but this contradicts the hypothesis that $G$ is not soluble. So $G$ does not contain any soluble normal subgroup, hence it is a semi- simple, by using [9]. Thus $G$ contains a maximal normal subgroup $A$ that is completely decompose without center, so the group $G$ is embedding in the group automorphisms $AutA$. If $A$ coincides the group of inner automorphisms of $A$, then

$$A = A_1 \cdot A_2 \cdot \ldots \cdot A_m,$$

where $A_i$ are simple groups. Since a group $A$ without center, then $A$ has no a subgroup of a prime order. Thus $A_i$ are simple and non commutative subgroups. In the case of $m > 1$, then $A_1$ is a proper subgroup of $G$, therefore, according to the definition of a minimal simple group that does not satisfy the Basis Property, hence $A_1$ is a group with the Basis Property, but this contradicts the concept that the group that achieves the Basis Property is a soluble as in [13]. So the group $A$ is simple. Suppose that $G \neq A$. Then $A$ is a proper subgroup of the group $G$, therefore, it is a group with the Basis Property. This is a contradiction of the solubility of the group with the Basis Property.

**Corollary 1.** *Let $G$ be a minimal group that does not satisfy the Basis Property, then $G$ is semi-prime, and if $G$ is a not soluble, then it must be simple and noncommutative.*

**Remark 1.** *The general linear projective groups $PGL(2, q)$, where $q \geq 5$, which $q$ is an odd number, are not simple, see that in [19, 20], but, $PSL(2, q)$ is simple for $q \geq 4$.*

We can also getting on more details about this kind of groups in [19]. Since the groups $PGL(2, q)$ for each $q \geq 5$ and $q$ is odd are not simple. So, we can remove it from the class of the minimal simple group that does not satisfy the Basis Property.

**Example 1.** *If $G = S_4 =< \alpha, \beta >$ is a symmetric group $S_4$ does not satisfy the Basis Property, such that $\alpha = (1234), \beta = (142)$, then $\alpha^2 = (13)(24)$ and $\alpha^2\beta = (34)$, $\beta\alpha^3\beta = (13)$. for more, we can find that*

$$G = S_4 =< \alpha^2, \alpha^2\beta, \beta\alpha^3\beta >$$

*That means, there are two bases for the symmetric group $S_4$, first of them consists of three elements and the other one has two elements, but, this contradicts the concept of the Basis Property. So the group does not satisfy the Basis Property.*

**Lemma 5.** *The groups*

$$PSL(2, 7), PSL(2, 3^2), PSL(2, 17),$$

*are semi-prime that not satisfy the Basis Property, but, they are not minimal.*

**proof**. According to the article [19], we saw that the three previous groups are semi-prime, also, since they are simple, then, they must not to be a soluble. According to [13], the groups do not satisfy the Basis Property. Now by [19], the order of the field $GF(q)$ has the form $q = 8h \pm 1$, where $q = 7$, $q = 9$ or $q = 17$ , then each of the previous groups must contains a proper subgroup that is isomorphic to the symmetric group $S_4$, and according to Example 1, the group $S_4$ does not satisfy the Basis Property, therefore the groups $q = 7$, $q = 9$ or $q = 17$ where $q = 7$, $q = 9$ or $q = 17$ are not minimal.

## 4. The Minimal simple groups

**Theorem 1.** *A group $G = PSL(2, 5)$ is a minimal does not satisfy the Basis Property.*

**p**roof. According to [19], the groups, which of the form $PSL(2, q^n)$, where $q$ is a prime have a subgroup of the form $A_5$, and since $A_5$ is a simple, then, it is non a soluble and since every a non soluble group does not satisfy the Basis Property, depending on [13]. On the other hand, the order of the group $PSL(2, q)$ is given by the following relationship.

$$|PSL(2, q)| = \frac{q(q^2 - 1)}{2}.$$

Hence,

$$|PSL(2, 5)| = \frac{5(5^2 - 1)}{2} = 60.$$

Since $|A_5| = 60$, so $PSL(2, 5) \approx A_5$ (is equal to its subgroup of the same order).

**Theorem 2.** *Let $A_5$ be the alternating group. Then $A_5$ is minimal not satisfy the Basis Property.*

**proof**. We will find all subgroups of the alternating group $A_5$. According to [19], the subgroups of the group $PSL(2,5) \approx A_5$ can be one of the following groups:

- The order of the even groups $D_2$ or $D_3$ (dihedral groups), where $q = 5 \neq 1$ are 4 or 6 respectively and the 2-group $D_2$ satisfies the Basis Property, because it is a primary group, see [3]. Likewise, the 2-group $D_3$ satisfies the Basis Property, because it is a metacyclic group, depending on [3].

- The subgroup of a non-commutative group $H$ with the order $\frac{5(5-1)}{2} = 10$, is a 5-Sylow subgroup $Q$ , which is a primary commutative group isomorphic to $C_5$, and since$|H : Q| = 2$. Then $Q \trianglelefteq H$ that means $H$ is the even group of the order 10, and also $H$ is metacyclic group that satisfies the Basis Property [3].

- The groups $PSL(2,5) \cong A_5$, $PGL(2,5) \geq 60$ and $PGL(2,5)$ are not a subgroup of $A_5$. Therefore, we find that all the proper subgroups of the group $A_5$ has the Basis Property, but $A_5$ does not satisfy the Basis Property, because it is simple, as it is not soluble.

- The alternating group $A_4$ , the symmetric group $S_4$, or the alternating group $A_5$ itself. Since $|S_4| = 24$, then$|S_4| \nmid |A_5|$ the symmetric group $S_4$ is not a subgroup of the group $A_5$ . As the group $A_5$ is not proper subgroup of the group$A_5$.

Now we study the alternating group $A_4$ where $|A_4| = 12 = 2^2 \cdot 3$, which is a semidirect product of the Klein group $K$ by the cyclic group $< y >=< (123) >$ whose order is equal to 3. On the other hand, we have Klein's group

$$K =< (12)(34), (13)(24) >= \{(1), (12)(34), (13)(24)(14)(23)\}$$

is a primary 2-group of order 4 as that $K \trianglelefteq A_4$. Then the group $K$ can be considered as a vector space of dimension 2 over the field $GF(2)$. Since $K$ is a $\trianglelefteq A_4$, then $y$ acts as a linear operator on a vector space of the form $\phi_y : \alpha \longrightarrow y^{-}1\alpha y$, for all $\alpha \in K$ such that $\varphi_y^3 = idK$ and $\varphi(\alpha) \neq \alpha$ for all $\alpha \in K - \{(1)\}$, $\varphi_y^3 = idK$. In fact

$$(123)(12)(34)(123) = (14)(32)$$

$$(132)(14)(23)(123) = (13)(24)$$

$$(132)(13)(24)(123) = (12)(34).$$

Thus, from the equality

$$(\varphi_y^3 - idK)(\alpha) = (\varphi_y - idK)(\varphi^2 + \varphi + idK)(\alpha) = 0,$$

for all $\alpha \in K$.

From the fact that the operator $(\varphi_y - idK)$ is not singular, then $\varphi^2 + \varphi + idK = 0$, therefore the polynomial $z^2 + z + 1$ on the field $GF(2)$ is a minimal polynomial of the operator $\varphi_y$ since it is not decomposing over the field $GF(2)$ [10] on non zero vector of $K$. Thus, the condition I1) for automorphism $\varphi_y$ for the group, $K$ is fulfilled.

If $\alpha \in K$ and $u \in A_4 \setminus K$, then the vectors $\alpha$ and $u^-1\alpha u$ are linearly independent (otherwise, since $|u| = 3$ and $|\alpha| = 2, u\alpha = \alpha u$, and the order of $u\alpha$ is equal 6, thus this is not possible in the group$A_4$ ). So, there are no proper normal subgroups in the group K with respect to automorphism.

Therefore, the condition I1) for all elements and for any normal subgroups with respect to automorphism $\phi_y$, because we can in the previous study with respect to automorphism $\varphi_y$ substitute $y$ by $u$.

Now according to [3, Theorem 2.5], the group has the Basis Property.

So $A_5$ is a minimal that not satisfy the Basis Property.

**Theorem 3.** *Let $G = PSL(3, 4)$ be a group. Then, a group $PSL(3, 4)$ is not minimal not satisfy the Basis Property.*

**proof**. We study the group $SL(2, 4)$. Since the identity $x^2 - 1 = 0$ has unique solution on the field $GF(2)$, then the center of the group $SL(2, 4)$ coincides with the identity, so $PSL(2, 4) \approx SL(2, 4)$ [19, Theorem6.14] and $PSL(2, 4) \approx A_5$, since $|PSL(2, 4)| = 4(4^2 - 1) = 60$. Define the mapping $\xi : SL(2, 4) \to SL(3, 4)$ by:

$$\xi(\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \gamma & \delta \end{bmatrix}$$

Let's define the natural homomorphism $\eta : SL(3, 4) \to \frac{SL(3,4)}{Z(SL(3,4)} \approx PSL(3, 4)$. Thus $\phi = \eta \circ \xi \mid_{SL(2,4)}$ is a subgroup of a group $T = PSL(3, 4)$, let $\phi = \eta \circ \xi \mid_{SL(2,4)}$ and since the group $SL(2, 4)$ is a simple, then either $|T| = 1$ or $\phi$ is onto, but

$$\phi(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \not\subseteq Z(SL(3, 4)$$

Therefore, the situation $|T| = 1$ is not possible, and $\phi$ is onto and within the subgroups of the group $T = PSL(3, 4)$, there is a group isomorphic to $A_5$ , and so

$$|PSL(3, 4)| = \frac{|SL(3, 4)|}{3} = \frac{(2^6 - 1)(2^6 - 2^2)2^2}{3} > 60.$$

Hence $PSL(3, 4)$ has a proper subgroup not satisfying the Basis Property and $PSL(3, 4)$ is not minimal and does not satisfy the Basis Property.

**Theorem 4.** *Let $G = PSL(2, 8)$ be a group. Then, a group $PSL(2, 8)$ is minimal not satisfying the Basis Property.*

**p**roof. Consider the group $PSL(2,8)$. From [8]

$$|PSL(2,8)| = (8+1)8(8-1) = 9 \cdot 8 \cdot 7 = 504.$$

Now, by using [19, Theorem 6.25] the subgroups of the group $PSL(2,8)$ can be one of the following groups.

- The even groups (dihedral groups) with orders $2(q \pm 1)$ i.e. the groups with orders $2 \cdot 7$ or $2 \cdot 9$ which form groups the Basis Property, because they are metacyclic groups.

- The group $H$ of order $\frac{q(q-1)}{d}$ , where $d = (2, q-1) = (2,7) = 1$. Thus the group $H$ of order $(8)(7) = 56$ and it is an extension of the primary abelian 2-group $Q$ such that $|Q| = 8$, where $Q \triangleright H$ by a group with order 7. Since $|PSL(2,8)| = (7)(8)(9)$, then the group $Q$ is a 2-Sylow subgroup of $PSL(2,8)$. According to [8, Theorem7.1] 2- Sylow subgroup of $PSL(2,8)$ is composed of matrices of the form

$$B_\beta = \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} \beta \in GF(8).$$

(Note that the center of the group $SL(2,8)$ is equal to the identity, therefore, we can be considered $SL(2,8) = PSL(2,8)$).Thus we can write that

$$Q = \{B_\beta : \beta \in GF(8)\}.$$

Thus, according to the same theory the normalizer $N(Q)$ is composed of matrices of the form

$$B_\beta = \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} \beta \in GF(8).$$

$$\begin{bmatrix} \alpha & \beta \\ 0 & \delta \end{bmatrix}, \alpha \neq 0, \delta \neq 0.$$

Since

$$det \begin{bmatrix} \alpha & \beta \\ 0 & \delta \end{bmatrix} = \alpha\delta = 1$$

Then $\delta = \alpha^{-1}$ and

$$N(Q) = \{ \begin{bmatrix} \alpha & \beta \\ 0 & \delta \end{bmatrix} : \alpha \in F^*, \beta \in GF(8)\}.$$

We study the matrix $A_\alpha, \alpha \neq 0, \alpha \in GF(8)$

$$A_\alpha = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{bmatrix}.$$

It is clear that $|\alpha| = 8 - 1 = 7$ and $|A_\alpha| = 7, A_\alpha^7 = 1$ Now, the matrix of the form $A_\alpha$ generate a cyclic group of order 7, and so on...

$$A_\alpha B_\beta = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha^-1 \end{bmatrix} \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & \alpha\beta \\ 0 & \alpha^{-1} \end{bmatrix}.$$

Then $N(Q) = <A_\alpha>Q$ , since

$$A_\alpha B_\beta A_{\alpha^{-1}} = \begin{bmatrix} \alpha & \alpha\beta \\ 0 & \alpha^{-1} \end{bmatrix} \begin{bmatrix} \alpha^{-1} & 0 \\ 0 & \alpha \end{bmatrix} = \begin{bmatrix} 1 & \alpha^2\beta \\ 0 & 1 \end{bmatrix},$$

and

$$<A_\alpha> \cap Q = \{1\}, Q \trianglelefteq N(Q), N(Q) = Q\lambda<A_\alpha>.$$

Thus, we can consider that $H = N(Q)$, and $K = <A_\alpha>$. Since

$$B_\beta B_\gamma = B_{\beta+\gamma}. \tag{1}$$

Then it can be considered that $Q$ is a vector space over the field $GF(8)$ and the element $A_\alpha$ of order 7 acts on $Q$ by the rule according to (1)

$$\phi_\alpha : B_\beta \to B_{\alpha^{-2}\beta}, f(\alpha^2) = 0.$$

Thus if $g(z)$ is a minimum polynomial on the field $GF(2)$, then if $\beta \neq 0$, then

$$g(\phi_\alpha)B_\beta = 0 \Leftrightarrow g(\alpha^{-2})\beta = 0 \Leftrightarrow g(\alpha^{-2}) = 0.$$

Since $\alpha^{-2} \neq 0, \alpha^{-2}$ is the element of the field $GF(8)$ has a minimal polynomial over the field $GF(2)$, which is irreducible over the field $GF(2)$ and according to [10], we take the minimum polynomial of the form

$$f_0(z) = z^3 + z + 1$$

or

$$f_0(z) = z^3 + z^2 + 1.$$

Thus

$$deg f_0(z) = min\{l \in \mathbb{N} : z^l \equiv 1(mod 7).$$

Hence, the vector space $Q$ is not reducible, and it follows that all the conditions of the [3, Theorem 2.5] satisfy for the group $H$ and is a group with the Basis Property.

- The group $A_4$, was indicated earlier in theorem 1 that it is a group with the Basis Property. The group $S_4$ does not contain in a group $PSL(2, 8)$, then by Theorem 6-26 Suzuki [20], since in this case $q^2 = 64 \neq 1(mod 16)$ also the group $PSL(2, 8)$ does not contain a group $A_5$ by the same theorem, since $q(q^2 - 1) = (8)(63) \neq 0(mod 5)$.

- If $r < 8$ ,and $q = r^m$, then $r = 2$ so we consider the groups $PSL(2,2), PGL(2,2)$. Since the center of the group $GL(2,2)$ is the identity and $GF(2) = \{0,1\}$, then

$$PSL(2,2) \cong PGL(2,2) \cong GL(2,2) = 2(2^2 - 1) = 6.$$

Since the group $GL(2,2)$ is not abelian of order 6, and has the form

$$\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\}.$$

Hence, it is metacyclic so it is a group with the Basis Property.

Therefore, all cases have been studied, and so the group $PSL(2,8)$ is minimal not satisfying the Basis Property.

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