EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 16, No. 4, 2023, 2049-2065
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

Generalized Dense Sets in Bigeneralized Topological Spaces

Yasser Farhat[1], Vadakasi Subramanian[2,*]

[1] *Academic Support Department, Abu Dhabi Polytechnic, P. O. Box 111499, Abu Dhabi, United Arab Emirates*
[2] *Department of Mathematics, A.K.D.Dharma Raja Women's College, Rajapalayam, India*

Abstract. In this article, in a bigeneralized topological space, we introduce an interesting tool namely, (s, v)-dense set, and examine its significance of this set. Also, we give the relationships among nowhere-dense sets defined in both generalized and bigeneralized topological space and give some of their properties by using functions. Finally, we give some applications for (s, v)-dense and (s, v)-nowhere dense sets in a soft set theory.

2020 Mathematics Subject Classifications: 54A05, 54A10
Key Words and Phrases: Bigeneralized topological spaces, $\mu_{(s,v)}$-open, $\mu_{(s,v)}$-closed, $\mu_{(s,v)}$-dense, $g_{(s,v)}$-continuous function.

---

1. Introduction

In [2], Császár defined the notion of generalized topological space. Some researchers have found various new concepts in this space and examined their nature in a generalized topological space. Especially, nowhere dense and dense sets were introduced by Ekici in a generalized topological space [6]. He has given few results for nowhere-dense and dense sets in a generalized topological space.

Some researchers proved various properties for nowhere dense sets e.g. [9, 12, 14]. Inspired by this, Korczak-Kubiak, et al. introduced two new generalized topologies, namely, $\mu^\star$ and $\mu^{\star\star}$; then examined the nature of nowhere dense set using $\mu^\star$ and $\mu^{\star\star}$ [8].

In [7], J.C. Kelly introduced the notion of bitopological space. Motivated by this, C. Boonpok introduced the concept of bigeneralized topological space in 2010 [1]. He proved some results about (m, n)-closed sets in bigeneralized topological space.

In this paper, we define the generalization of dense sets, namely, (s, v)-dense in a bigeneralized topological space. In a bigeneralized topological space, various properties for (s, v)-dense and (s, v)-nowhere dense sets are launched.

---

*Corresponding author.
DOI: https://doi.org/10.29020/nybg.ejpam.v16i4.4911

Email addresses: farhat.yasser.1@gmail.com (Y. Farhat), vadakasivigneswaran@gmail.com (S.Vadakasi)

The basic definitions and results are presented in section 2 which is useful for the development of the following sections. In section 3, in a bigeneralized topological space, new results for (s, v)-dense sets are proven. The necessary conditions for a given set is (s, v)-dense are given. Section 4, some properties for (s, v)-nowhere dense sets are proven. In a bigeneralized topological space, the relationship between μ-nowhere dense and (s, v)-nowhere dense sets are examined. Finally, the set (s, v)-codense is defined and find few results for this set.

In section 5, the nature of (s, v)-dense and (s, v)-codense sets are examined by functions in a bigeneralized topological space. In the last section, we define a soft set using (s, v)-dense, (s, v)-nowhere dense, and (s, v)-codense sets are defined in a bigeneralized topological space.

2. Preliminaries

Let μ be the collection of subsets of a non-null set X. μ is called *generalized topology* [2] in X if it contains the empty set and is closed under arbitrary union. Then (X, μ) is called *generalized topological space* (GTS) [2]. If μ contains X, then (X, μ) is called as a *strong generalized topological space* (sGTS) [9].

In, [3], let Q be the subset of (X, μ),

- If $Q \in \mu$, then Q is called *μ-open.*
- If $X - Q \in \mu$, then Q is said to be *μ-closed.*
- The *interior of* Q denoted by $i_\mu Q$, is the union of all μ-open sets contained in Q.
- The *closure of* Q denoted by $c_\mu Q$, is the intersection of all μ-closed sets containing Q.

For ease of notation, we write $i(Q)$ and $c(Q)$ when no confusion can arise.

Korczak - Kubiak, et.al [8] defined the following notations;

$$\tilde{\mu} = \{L \in \mu \mid L \neq \emptyset\}.$$
$$\mu(x) = \{L \in \mu \mid x \in L\}.$$

Let Q be a subset of a generalized topological space (X, μ). Then Q is said to be ;

- *μ-nowhere dense* [6] if $ic(Q) = \emptyset$;
- *μ-dense* [6] if $cQ = X$;
- *μ-codense* [5] if $c(X - Q) = X$.

Let μ_1, μ_2 be two GT in a non-null set X. Then (X, μ_1, μ_2) is called as a *bigeneralized topological space* (BGTS) [1].

Let (X, μ_1, μ_2) be a BGTS, $D \subset X$. T The *closure of* D is notated by $c_s(D)$ and $i_s(D)$ denote the *interior of* D with respect to μ_s, respectively, for $s = 1, 2$ [1].

In a BGTS (X, μ_1, μ_2), let $Q, P \subset X$. Then

- Q is called *(s, v)-closed* [1] if $c_s(c_v(Q)) = Q$, where $s, v = 1$ or 2 ; $s \neq v$.

• If $X - Q$ is (s, v)-closed, then Q is called (s, v)-*open* [1] where $s, v = 1$ or 2 ; $s \neq v$.
• P is called $\mu_{(s,v)}$-*closed* [4] if $c_{\mu_v}(P) \subset K$ whenever $P \subset K$ and K is μ_s-open in X, for $s, v = 1, 2$; $s \neq v$.
• If $X - P$ is $\mu_{(s,v)}$-closed, then P is called $\mu_{(s,v)}$-*open* [4] where $s, v = 1$ or 2 ; $s \neq v$.

In [1], a subset Q of a BGTS (X, μ_1, μ_2) is called
• (s, v)-μ-*regular open* if $Q = i_s(c_v(Q))$ for $s, v = 1$ or 2 ; $s \neq v$.
• (s, v)-μ-*semi-open* if $Q \subseteq c_v(i_s(Q))$ for $s, v = 1$ or 2 ; $s \neq v$.
• (s, v)-μ-*preopen* if $Q \subseteq i_s(c_v(Q))$ for $s, v = 1$ or 2 ; $s \neq v$.
• (s, v)-μ-α-*open* if $Q \subseteq i_s(c_v(i_s(Q)))$ for $s, v = 1$ or 2 ; $s \neq v$.

Lemma 1. [Proposition 3.4, [1]] Let K be a subset of a BGTS (X, μ_1, μ_2). Then K is (s, v)-closed $\Leftrightarrow$ K is both μ-closed in (X, μ_s) and (X, μ_v) where $s, v = 1$ or 2 ; $s \neq v$.

Lemma 2. [Proposition 3.3, [4]] Let (X, μ_1, μ_2) be a BGTS, $K \subset X$. Then K is $\mu_{(s,v)}$-closed where $s, v = 1, 2$; $s \neq v$ whenever K is μ_v-closed.

Lemma 3. [Lemma 3.2, [9]] Let D, K be two subsets of a generalized topological space (X, μ). If $K \in \tilde{\mu}$ and $K \cap D = \emptyset$, then $K \cap cD = \emptyset$.

Lemma 4. [Proposition 3.3, [9]] In a GTS (X, μ), $Q \in \mathcal{D}(\mu) \Leftrightarrow H \cap Q \neq \emptyset$ for any $H \in \tilde{\mu}$ where $\mathcal{D}(\mu) = \{P \subset X \mid c_\mu(P) = X\}$.

Lemma 5. [Proposition 2.2, [10]] Let P, Q be two subsets of a GTS (X, μ). Then the followings are true:
(a) $c_\mu(X - P) = X - i_\mu(P)$; $i_\mu(X - P) = X - c_\mu(P)$.
(b) If $(X - P) \in \mu$, then $c_\mu(P) = P$ and if $P \in \mu$, then $i_\mu(P) = P$.
(c) If $P \subseteq Q$, then $c_\mu(P) \subseteq c_\mu(Q)$ and $i_\mu(P) \subseteq i_\mu(Q)$.
(d) $P \subseteq c_\mu(P)$ and $i_\mu(P) \subseteq P$.
(e) $c_\mu(c_\mu(P)) = c_\mu(P)$ and $i_\mu(i_\mu(P)) = i_\mu(P)$.

3. Nature of (s, v)-dense sets

Here, we define a generalized dense set using two generalized topologies namely, (s, v)-dense set, and analyze its nature in a BGTS (X, μ_1, μ_2).

Definition 1. Let D be a non-null subset of a bigeneralized topological space (X, μ_1, μ_2). Then D is called (s, v)-*dense* if $c_s(c_v(D)) = X$ where $s, v = 1, 2$ and $s \neq v$.

Moreover, $(s, v) - \mathcal{D}(X) = \{Q \subset X \mid Q$ is (s, v)-dense in $X\}$ for $s, v = 1, 2$; $s \neq v$.

Example 2. Consider the BGTS (X, μ_1, μ_2) where $X = \{e, f, k, l\}$;

$$\mu_1 = \{\emptyset, \{e\}, \{e, f\}, \{f, k\}, \{e, f, k\}\}$$

and

$$\mu_2 = \{\emptyset, \{e, f\}, \{f, l\}, \{e, f, l\}\}.$$

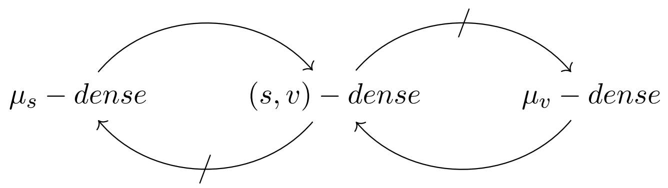
Then $(s, v) - \mathcal{D}(X) = \{Q \subset X \mid \text{either } e \in Q \text{ or } f \in Q\}$ where $s, v = 1, 2$; $s \neq v$.

In a GTS, every superset of a (s, v)-dense set is (s, v)-dense where $s, v = 1, 2$ and $s \neq v$.

Theorem 3. *Let (X, μ_1, μ_2) be a BGTS and Q be a non-null subset of X. Then Q is (s, v)-dense $\Leftrightarrow c_vQ \cap H \neq \emptyset$ for every H is a non-null μ_s-open set where $s, v = 1, 2$ and $s \neq v$.*

Proof. *Suppose $Q \in (s, v) - \mathcal{D}(X)$ for $s, v = 1, 2$; $s \neq v$, then $c_s(c_v(Q)) = X$ and so $X - (c_s(c_v(Q))) = \emptyset$ where $s, v = 1, 2$ and $s \neq v$. By Lemma 5, $X - (c_s(c_v(Q))) = i_s(X - (c_v(Q)))$, so that $i_s(X - (c_v(Q))) = \emptyset$ which implies that $c_v(Q) \cap H \neq \emptyset$ for every H is a non-null μ_s-open set where $s, v = 1, 2$ and $s \neq v$. Conversely, assume that, $c_v(Q) \cap H \neq \emptyset$ for every H is a non-null μ_s-open set where $s, v = 1, 2$ and $s \neq v$. Then $i_s(X - (c_v(Q))) = \emptyset$ and so $c_s(c_v(Q)) = X$, by Lemma 5 where $s, v = 1, 2$ and $s \neq v$. Hence Q is (s, v)-dense for $s, v = 1, 2$ and $s \neq v$.*

Theorem 4 and Example 5 are described in the below diagram.



Theorem 4. *In a BGTS (X, μ_1, μ_2), if K is either μ_s-dense or μ_v-dense, then K is (s, v)-dense where $s, v = 1, 2$; $s \neq v$.*

Proof. *Assume that, K is μ_s-dense where for $s = 1, 2$. Then $c_s(K) = X$ for $s = 1, 2$. Take $s = 2$ and $v = 1$. Then K is μ_2-dense. Since $K \subset c_1(K)$ we have $c_2(K) \subset c_2(c_1(K))$. Hence*

$$K \in (2, 1) - \mathcal{D}(X) \tag{1}$$

Take $s = 1$ and $v = 2$. Then K is μ_1-dense. Since $K \subset c_2(K)$ we have $c_1(K) \subset c_1(c_2(K))$. Thus,

$$K \in (1, 2) - \mathcal{D}(X) \tag{2}$$

From (1) & (2), K is (s, v)-dense where $s, v = 1, 2$ and $s \neq v$. Similarly, we can prove that K is (s, v)-dense if K is μ_v-dense where $s, v = 1, 2$ and $s \neq v$.

Example 5 describes that the Theorem 4 is not reversible. Generally, $(1, 2) - \mathcal{D}(X) \neq (2, 1) - \mathcal{D}(X)$ in a bigeneralized topological space as given in Example 6.

Example 5. Consider the bigeneralized topological space (X, μ_1, μ_2), $X = \{e, f, k, l\}$;

$$\mu_1 = \{\emptyset, \{e, l\}, \{f, l\}, \{e, f, l\}\}$$

and

$$\mu_2 = \{\emptyset, \{e,k\}, \{f,k\}, \{e,f,k\}\}.$$

Here $\{k\}$ is $(2,1)$-dense. But $\{k\}$ is not μ_1-dense. Also, $\{l\}$ is $(1,2)$-dense. But $\{l\}$ is not μ_2-dense.

Example 6. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{e, f, k, l\}$;

$$\mu_1 = \{\emptyset, \{e,f\}, \{f,k\}, \{e,f,k\}\}$$

and

$$\mu_2 = \{\emptyset, \{e\}, \{e,l\}, \{k,l\}, \{e,k,l\}\}.$$

Then

- $(1,2)-\mathcal{D}(X) = \{\{e\}, \{f\}, \{k\}, \{l\}, \{e,f\}, \{e,k\}, \{e,l\}, \{f,k\}, \{f,l\}, \{k,l\}, \{e,f,k\}, \{e,f,l\}, \{e,k,l\}, \{f,k,l\}, X\}$.
- $(2,1)-\mathcal{D}(X) = \{\{e\}, \{f\}, \{e,f\}, \{e,k\}, \{e,l\}, \{f,k\}, \{f,l\}, \{e,f,k\}, \{e,f,l\}, \{e,k,l\}, \{f,k,l\}, X\}$.

Thus, $(1,2) - \mathcal{D}(X) \neq (2,1) - \mathcal{D}(X)$.

Theorem 7. *Let μ_1 and μ_2 be two generalized topologies in X. If $\mu_s \subseteq \mu_v$, then $(v,s) - \mathcal{D}(X) \subseteq (s,v) - \mathcal{D}(X)$ where $s, v = 1, 2$ and $s \neq v$.*

Proof. *We give the detailed proof only for $s = 1$ and $v = 2$. Suppose that $\mu_1 \subseteq \mu_2$ and $Q \in (2,1) - \mathcal{D}(X)$, then $c_2(c_1(Q)) = X$. By Lemma 4, $c_1(Q) \cap H \neq \emptyset$ for every $H \in \tilde{\mu}_2$. Take $G \in \tilde{\mu}_1$ we get $G \in \tilde{\mu}_2$ for that $c_1(Q) \cap G \neq \emptyset$. Since $Q \subset c_2(Q)$ we have $c_1(Q) \subset c_1(c_2(Q))$. Thus, $c_1(c_2(Q)) \cap G \neq \emptyset$. Since G is an arbitrary non-null μ_1-open set we have $c_1(c_1(c_2(Q))) = X$, by Lemma 4. Hence $c_1(c_2(Q)) = X$, by Lemma 5(e). Therefore, $Q \in (1,2) - \mathcal{D}(X)$.*

Theorem 8. *Let (X, μ_1, μ_2) be a BGTS and D be a non-null subset of X. If $D \in (s,v) - \mathcal{D}(X)$, then $D \cap H \neq \emptyset$ for every H is a non-null (s,v)-open set in X for $s, v = 1, 2$; $s \neq v$.*

Proof. *Take $s = 1$ and $v = 2$. Assume that, D is $(1,2)$-dense. Then $c_1(c_2(D)) = X$. Let H be a non-null $(1,2)$-open set. By Lemma 1,*

$$H \in \tilde{\mu_1} \tag{3}$$

$$H \in \tilde{\mu_2} \tag{4}$$

Then $c_2(D) \cap H \neq \emptyset$, by Lemma 4 and (3). From (4) and $c_2(D) \cap H \neq \emptyset$ we have $D \cap H \neq \emptyset$, by Lemma 3. Thus, $D \cap H \neq \emptyset$ for every H is a non-null $(1,2)$-open set. Take $s = 2$ and $v = 1$. By similar considerations in the above case, we get the proof.

Theorem 9. *Let (X, μ_1, μ_2) be a BGTS, $D \subset X$. If $D \cap H \neq \emptyset$ for every $H \neq \emptyset$ is $\mu_{(s,v)}$-open, then $D \in (s,v) - \mathcal{D}(X)$; $s, v = 1, 2$ and $s \neq v$.*

Proof. *We give the detailed proof for $s = 1$ and $v = 2$ only. Suppose that $D \cap H \neq \emptyset$ for every H is non-null $\mu_{(1,2)}$-open. By Theorem 4, we have to prove D is μ_2-dense. Let $B \in \tilde{\mu_2}$. Then B is a non-null $\mu_{(1,2)}$-open set in X, by Lemma 2. By assumption, $D \cap B \neq \emptyset$. Therefore, D is a μ_2-dense set. Hence D is a $(1,2)$-dense set.*

The below Example 10 describes that the converse part of Theorem 9 is generally not true.

Example 10. Take $X = \{e, f, k, l\}$;

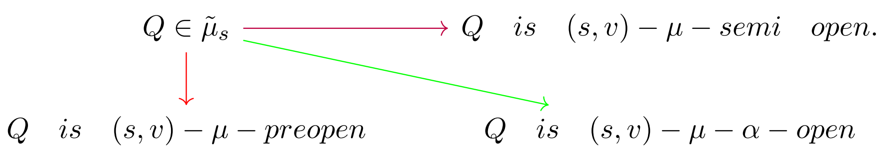
$$\mu_1 = \{\emptyset, \{e, f\}, \{f, l\}, \{e, f, l\}\}$$

and

$$\mu_2 = \{\emptyset, \{e, k\}, \{f, k\}, \{e, f, k\}\}.$$

Then $\mu_{(1,2)} = \{\emptyset, \{e\}, \{f\}, \{l\}, \{e, f\}, \{e, k\}, \{e, l\}, \{f, k\}, \{f, l\}, \{e, f, k\}, \{e, f, l\}\}$ and $\mu_{(2,1)} = \{\emptyset, \{e\}, \{f\}, \{k\}, \{e, f\}, \{e, k\}, \{f, k\}, \{f, l\}, \{e, f, k\}, \{e, f, l\}, \{f, k, l\}\}$.

Take $P = \{e\}$. Then $P \in (1, 2) - \mathcal{D}(X)$. But $P \cap Q = \emptyset$ where $Q = \{l\}$ is a non-null $\mu_{(1,2)}$-open set. Let $M = \{f\} \subset X$. Then $M \in (2, 1) - \mathcal{D}(X)$. But $M \cap L = \emptyset$ where $L = \{e\}$ is a non-null $\mu_{(2,1)}$-open set.



The following Lemma 6 describes the above diagram.

Lemma 6. *Let (X, μ_1, μ_2) be a BGTS. If $Q \in \tilde{\mu}_s$, then the below results are true.*
(a) Q is (s, v)-μ-semi open.
(b) Q is (s, v)-μ-preopen.
(c) Q is (s, v)-μ-α-open where $s, v = 1, 2$ and $s \neq v$.

Proof. We give the detailed proof for (b) only. Suppose that, $Q \in \tilde{\mu}_s$ for $s = 1, 2$. Then $i_s(Q) = Q$ for $s = 1, 2$. Since $Q \subset c_v(Q)$ for $v = 1, 2$ we have $i_s(Q) \subset i_s(c_v(Q))$ where $s, v = 1, 2$ and $s \neq v$. Thus, $Q \subset i_s(c_v(Q))$ where $s, v = 1, 2$ and $s \neq v$. Hence Q is a (s, v)-μ-preopen set in X for $s, v = 1, 2$; $s \neq v$.

Theorem 11. *Let (X, μ_1, μ_2) be a BGTS. Then $D \in (s, v) - \mathcal{D}(X)$ if any one of the following is true.*
(a) $D \cap M \neq \emptyset$ for every M is a non-null (s, v)-μ-semi open set in X
(b) $D \cap M \neq \emptyset$ for every M is a non-null (s, v)-μ-preopen set in X
(c) $D \cap M \neq \emptyset$ for every M is a non-null (s, v)-μ-α-open set in X where $s, v = 1, 2$; $s \neq v$.

Proof. We give the detailed proof for (b) only. Suppose that $D \cap M \neq \emptyset$ for every M is a non-null (s, v)-μ-preopen set in X where $s, v = 1, 2$ and $s \neq v$. It is enough to prove, D is μ_s-dense set in X for $s = 1, 2$, by Theorem 4. Let $B \in \tilde{\mu}_s$ for $s = 1, 2$. By Lemma 6, B is a non-null (s, v)-μ-preopen set in X where $s, v = 1, 2$ and $s \neq v$. By assumption, $D \cap B \neq \emptyset$. Therefore, D is a μ_s-dense set for $s = 1, 2$. Hence D is (s, v)-dense where $s, v = 1, 2$ and $s \neq v$.

Example 12 explains that the reverse part of Theorem 11 is generally not true.

Example 12. (a) Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{e, f, k, l, r\}$;

$$\mu_1 = \{\emptyset, \{e, f\}, \{e, l\}, \{f, l\}, \{e, f, l\}\}$$

and

$$\mu_2 = \{\emptyset, \{e, f, k\}, \{e, f, l\}, \{e, k, r\}, \{e, f, k, l\}, \{e, f, k, r\}, X\}.$$

Take $A = \{k, l, r\}$. Then A is $(1, 2)$-dense set. But $A \cap G = \emptyset$ where $G = \{e, f\}$ is a non-null $\mu_{(1,2)}$-μ-semi open set. Let $B = \{l, r\} \subset X$. Then B is $(2, 1)$-dense set. But $B \cap H = \emptyset$ where $H = \{e, f, k\}$ is a non-null $\mu_{(2,1)}$-μ-semi-open set.

(b) Consider the BGTS (X, μ_1, μ_2), $X = [0, 3]$;

$$\mu_1 = \{\emptyset, [0, 2), (1, 3], [0, 3]\}$$

and

$$\mu_2 = \{\emptyset, [0, \tfrac{3}{2}], (1, 2], [0, 2]\}.$$

Let $A = (0, 1) \cup (\frac{3}{2}, 3]$. Then $A \in (s, v) - \mathcal{D}(X)$ where $s, v = 1, 2$ and $s \neq v$. But $A \cap B = \emptyset$ where $B = \{\frac{3}{2}\}$ is a non-null (s, v)-μ-preopen set in X where $s, v = 1, 2$; $s \neq v$.

(c) Consider the BGTS (X, μ_1, μ_2), $X = [0, 4]$;

$$\mu_1 = \{\emptyset, [0, 2), (1, 2)\}$$

and

$$\mu_2 = \{\emptyset, [0, 2), (1, 2], (1, 3), [0, 2], [0, 3)\}.$$

Let $P = (0, 1) \cup [2, 4]$. Then $P \in (1, 2) - \mathcal{D}(X)$. But $P \cap Q = \emptyset$ where $Q = [1, 2)$ is a non-null (s, v)-μ-α-pen set in X where $s, v = 1, 2$ and $s \neq v$. Let $C = (0, 1) \cup [3, 4]$. Then C is $(2, 1)$-dense set in X. But $C \cap D = \emptyset$ where $D = [1, 3)$ is a non-null (s, v)-μ-α-pen set in X where $s, v = 1, 2$ and $s \neq v$.

4. Generalized nowhere dense sets

Here, we find the new results for (s, v)-nowhere dense set in a BGTS.

Definition 13. [13] Let (X, μ_1, μ_2) be a BGTS and $D \subset X$. Then D is called (s, v)-*nowhere dense* if $i_s(c_v(D)) = \emptyset$ where $s, v = 1, 2$ and $s \neq v$.

We notated, $(s, v) - \mathcal{N}(X) = \{Q \subset X \mid Q \text{ is } (s, v)\text{-nowhere dense in } X\}$ where $s, v = 1, 2$; $s \neq v$.

Example 14. Take $X = \{e, f, k, l\}$;

$$\mu_1 = \{\emptyset, \{e, f\}, \{e, k\}, \{e, f, k\}\}$$

and

$$\mu_2 = \{\emptyset, \{e, l\}, \{f, l\}, \{e, f, l\}\}.$$

Then $\{k\}$ is a non-null (s, v)-nowhere dense set in (X, μ_1, μ_2) where $s, v = 1, 2$; $s \neq v$.

In a bigeneralized topological space, if $Q \in (s, v) - \mathcal{N}(X)$ and $P \subset Q$, then $P \in (s, v) - \mathcal{N}(X)$ where $s, v = 1, 2$ and $s \neq v$.

Theorem 15. *In a BGTS* (X, μ_1, μ_2), $D \in (s, v) - \mathcal{N}(X)$ *if and only if* $c_v(D) \in (s, v) - \mathcal{N}(X)$ *where* $s, v = 1, 2$ *and* $s \neq v$.

In a BGTS (X, μ_1, μ_2), $(1, 2) - \mathcal{N}(X) \neq (2, 1) - \mathcal{N}(X)$ as shown by the below Example 16 . Also, this example shows that $(s, v) - \mathcal{N}(X)$ is not closed under finite union in general.

Example 16. Let (X, μ_1, μ_2) be a BGTS where $X = \{e, f, k, l\}$;

$$\mu_1 = \{\emptyset, \{e, l\}, \{f, l\}, \{e, f, l\}\}$$

and

$$\mu_2 = \{\emptyset, \{e, f\}, \{f, l\}, \{e, f, l\}\}.$$

Then
- $(1, 2) - \mathcal{N}(X) = \{\emptyset, \{e\}, \{k\}, \{l\}, \{e, k\}, \{k, l\}\}$
- $(2, 1) - \mathcal{N}(X) = \{\emptyset, \{e\}, \{f\}, \{k\}, \{e, k\}, \{f, k\}\}$.

Thus, $(2, 1) - \mathcal{N}(X) \neq (1, 2) - \mathcal{N}(X)$.

Here $\{e\}$ and $\{l\}$ are in $(1, 2) - \mathcal{N}(X)$. But $\{e, l\} \notin (1, 2) - \mathcal{N}(X)$. Also, $\{e\}$ and $\{f\}$ are in $(2, 1) - \mathcal{N}(X)$. But $\{e, f\} \notin (2, 1) - \mathcal{N}(X)$.

Theorem 17. *Let* μ_1 *and* μ_2 *be two generlized topologies on a non-null set X. If* $\mu_s \subseteq \mu_v$, *then* $(v, s) - \mathcal{N}(X) \subseteq (s, v) - \mathcal{N}(X)$ *where* $s, v = 1, 2$ *and* $s \neq v$.

Proof. We give the detailed proof only for $s = 1$ *and* $v = 2$. *Assume that,*

$$\mu_1 \subseteq \mu_2 \tag{5}$$

Let $D \in (2, 1) - \mathcal{N}(X)$. *Then* $i_2(c_1(D)) = \emptyset$. *Suppose* $i_1(c_2(D)) \neq \emptyset$. *There exists* $K \in \tilde{\mu}_1$ *such that* $K \subset c_2(D)$. *From (5),* $K \in \tilde{\mu}_2$. *Then* $i_2(c_2(D)) \neq \emptyset$. *By (5) we get* $c_2(D) \subset c_1(D)$. *Thus,* $i_2(c_1(D)) \neq \emptyset$ *which is not possible. Therefore,* $i_1(c_2(D)) = \emptyset$. *Hence* $D \in (1, 2) - \mathcal{N}(X)$.

The following Theorem 19 describes the below diagram.

$$\begin{array}{ccc} \mu_v - nowhere \quad dense & \longrightarrow & (s,v) - nowhere \quad dense \\ & & \uparrow \\ & & \mu_s - nowhere \quad dense \end{array}$$

The following Example 18 shows that the existence of the below Theorem 19.

Example 18. (a) Fix $s = 1, v = 2$. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;

$$\mu_1 = \{\emptyset, \{p, r\}, \{q, r\}, \{p, q, r\}\}$$

and

$$\mu_2 = \{\emptyset, \{p, r\}, \{q, r\}, \{p, s\}\{p, q, r\}, \{p, r, s\}, X\}.$$

Obviously, $\mu_1 \subset \mu_2$. Take $K = \{p, s\}$ and $L = \{q\}$. Then K is a μ_1-nowhere dense set and L is a μ_2-nowhere dense set. Here, both K and L are in $(1, 2) - \mathcal{N}(X)$.

(b) Fix $s = 2, v = 1$. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;

$$\mu_1 = \{\emptyset, \{p, s\}, \{r, s\}, \{q, s\}\{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$$

and

$$\mu_2 = \{\emptyset, \{q, s\}, \{r, s\}, \{q, r, s\}\}.$$

Clearly, $\mu_2 \subset \mu_1$. Take $H = \{r\}$ and $D = \{p, r\}$. Then H is a μ_1-nowhere dense set and D is a μ_2-nowhere dense set. Also, both H and D are in $(2, 1) - \mathcal{N}(X)$.

Theorem 19. *Let μ_1, μ_2 be two generlized topologies on X and $\mu_s \subseteq \mu_v$ where $s, v = 1, 2$ and $s \neq v$. If $P \subset X$ is μ_v-nowhere dense set or μ_s-nowhere dense set, then $P \in (s, v) - \mathcal{N}(X)$ where $s, v = 1, 2$ and $s \neq v$.*

Proof. We give the detailed proof only for $s = 2$ and $v = 1$. Assume that,

$$\mu_2 \subseteq \mu_1 \tag{6}$$

Let P be a μ_1-nowhere dense set. Then $i_1(c_1(P)) = \emptyset$. Suppose $i_2(c_1(P)) \neq \emptyset$. Then there is $Q \in \tilde{\mu}_2$ such that $Q \subset c_1(P)$. From (6), $Q \in \tilde{\mu}_1$. Then $i_1(c_1(P)) \neq \emptyset$ which is not possible. Therefore, $i_2(c_1(P)) = \emptyset$. Hence $P \in (2, 1) - \mathcal{N}(X)$.

Let P be a μ_2-nowhere dense set. Then $i_2(c_2(P)) = \emptyset$. Suppose $i_2(c_1(P)) \neq \emptyset$. Then there is a set $M \in \tilde{\mu}_2$ such that $M \subset c_1(P)$. By (6), $i_2(c_2(P)) \neq \emptyset$ which is not possible. Therefore, $i_2(c_1(P)) = \emptyset$. Hence $P \in (2, 1) - \mathcal{N}(X)$.

In Theorem 19, the condition "$\mu_s \subseteq \mu_v$" where $s, v = 1, 2$; $s \neq v$" is necessary as shown in Example 20.

Example 20. Take $X = \{e, f, k, l\}$;

$$\mu_1 = \{\emptyset, \{e, k\}, \{e, l\}, \{f, l\}, \{e, f, l\}, \{e, k, l\}, X\}$$

and

$$\mu_2 = \{\emptyset, \{e, f\}, \{f, k\}, \{e, l\}, \{f, l\}, \{e, f, k\}, \{e, f, l\}, \{e, k, l\}, \{f, k, l\}, X\}.$$

Let $P = \{f, k\}$. Then $i_1(c_1(P)) = i_1(\{f, k\}) = \emptyset$ and so P is μ_1-nowhere dense set. But $P \notin (2, 1) - \mathcal{N}(X)$. Let $M = \{e, k\}$. Then $i_2(c_2(M)) = i_2(\{e, k\}) = \emptyset$ and so M is a μ_2-nowhere dense set. But $M \notin (1, 2)-\mathcal{N}(X)$. Let $C = \{k, l\}$. Then $i_2(c_2(C)) = i_2(\{k, l\}) = \emptyset$ and so C is a μ_2-nowhere dense set. But $C \notin (2, 1) - \mathcal{N}(X)$.

Consider the BGTS (X, μ_1, μ_2), $X = [0, 3]$;

$$\mu_1 = \{\emptyset, [0, \tfrac{3}{2}), (1, 2], [0, 2]\}$$

and

$$\mu_2 = \{\emptyset, [0, 1), (1, 2), [0, 2)\}$$

Let $D = [\frac{3}{2}, 3]$. Then D is a μ_1-nowhere dense set in X. But $D \notin (1, 2) - \mathcal{N}(X)$.

$$\begin{array}{ccc} \mu_v - nowhere \quad dense & \longleftarrow & (s, v) - nowhere \quad dense \\ & & \downarrow \\ & & \mu_s - nowhere \quad dense \end{array}$$

The below Theorem 22 describes the above diagram. Example 21 proves the existence of the below Theorem 22.

Example 21. (a) Fix $s = 1, v = 2$. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;

$$\mu_1 = \{\emptyset, \{p, q\}, \{p, r\}, \{q, r\}, \{p, q, r\}\}$$

and

$$\mu_2 = \{\emptyset, \{p, r\}, \{q, r\}, \{p, q, r\}\}.$$

Obviously, $\mu_2 \subset \mu_1$. Consider, $L = \{q, s\}$. Then $i_1(c_2(L)) = \emptyset$ and so $L \in (1, 2) - \mathcal{N}(X)$. Here, $i_1(c_1(L)) = \emptyset$ and $i_2(c_2(L)) = \emptyset$. Thus, L is a μ_1-nowhere dense set and also a μ_2-nowhere dense set.

(b) Fix $s = 2, v = 1$. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;

$$\mu_1 = \{\emptyset, \{p, s\}, \{q, s\}, \{p, q, s\}\}$$

and

$$\mu_2 = \{\emptyset, \{p\}, \{p, s\}, \{q, s\}, \{p, q, s\}\}.$$

Clearly, $\mu_1 \subset \mu_2$. Take $K = \{q, r\}$ then we get $i_2(c_1(K)) = \emptyset$ and hence $K \in (2,1) - \mathcal{N}(X)$. Now, $i_1(c_1(K)) = \emptyset$ and $i_2(c_2(K)) = \emptyset$ which implies that K is a μ_1-nowhere dense set and also a μ_2-nowhere dense set.

Theorem 22. *Let μ_1, μ_2 be two generlized topologies on X and $\mu_v \subseteq \mu_s$ where $s, v = 1, 2$; $s \neq v$. If $Q \in (s, v) - \mathcal{N}(X)$, then Q is μ_v-nowhere dense and also μ_s-nowhere dense where $s, v = 1, 2$; $s \neq v$.*

Proof. We give the detailed proof for $s = 1$ and $v = 2$ only. Assume that, $\mu_2 \subseteq \mu_1$. Let Q be a $(1, 2)$-nowhere dense set. Then $i_1(c_2(Q)) = \emptyset$.

Suppose $i_1(c_1(Q)) \neq \emptyset$. By assumption, $i_1(c_2(Q)) \neq \emptyset$ which is a contradiction. Therefore, $i_1(c_1(Q)) = \emptyset$.

If $i_2(c_2(Q)) \neq \emptyset$, then there is a set $M \in \tilde{\mu}_2$ such that $M \subset c_2(Q)$. By assumption, $M \in \tilde{\mu}_1$. Thus, $i_1(c_2(Q)) \neq \emptyset$ which is a contradiction. Therefore, $i_2(c_2(Q)) = \emptyset$.

Theorem 23. *Let (X, μ_1, μ_2) be a BGTS and $K \subset X$. If $K \in (s, v) - \mathcal{N}(X)$ then $c_v(K) - K \in (s, v) - \mathcal{N}(X)$ where $s, v = 1, 2$ and $s \neq v$.*

Proof. Let $K \in (s, v) - \mathcal{N}(X)$ where $s, v = 1, 2$; $s \neq v$. Take $s = 1$ and $v = 2$. Then K is a $(1, 2)$-nowhere dense set in X. Since $c_2(K) - K \subset c_2(K)$ we have $c_2(c_2(K) - K) \subset c_2(c_2(K))$. By Lemma 5 (e), $c_2(c_2(K) - K) \subset c_2(K)$. Then $i_1(c_2(c_2(K) - K)) \subset i_1(c_2(K))$ and so $i_1(c_2(c_2(K) - K)) = \emptyset$, by assumption. Therefore, $c_2(K) - K \in (1, 2) - \mathcal{N}(X)$. By similar argument in the above case, we get $c_1(K) - K \in (2, 1) - \mathcal{N}(X)$.

Example 24. Consider the bigeneralized topological space (X, μ_1, μ_2), $X = \{e, f, k, l\}$;

$$\mu_1 = \{\emptyset, \{e, k\}, \{f, k\}, \{e, f, k\}\}$$

and

$$\mu_2 = \{\emptyset, \{k\}, \{e, k\}, \{f, k\}, \{e, f, k\}\}.$$

Take $Q = \{k\}$ we get $c_2(Q) - Q = \{e, f, l\}$ and so $i_1(c_2(c_2(Q) - Q)) = \emptyset$. Thus, $c_2(Q) - Q \in (1, 2) - \mathcal{N}(X)$. But $Q \notin (1, 2) - \mathcal{N}(X)$.

Choose $L = \{f, k\}$ so that $c_1(L) - L = \{e, l\}$ and so $i_2(c_1(c_1(L) - L)) = \emptyset$ implies that $c_1(L) - L \in (2, 1) - \mathcal{N}(X)$. But $L \notin (2, 1) - \mathcal{N}(X)$.

Theorem 25. *Let (X, μ_1, μ_2) be a BGTS. For $s, v = 1, 2$ and $s \neq v$, if $D \in (s, v) - \mathcal{N}(X)$, then the followings are true.*
(a) $K \nsubseteq D$ for all K is a non-null (s, v)-μ-preopen set in X.
(b) $K \nsubseteq D$ for all K is a non-null (s, v)-μ-regular open set in X.

(c) $K \nsubseteq D$ for all K is a non-null (s,v)-open set in X.
(d) $K \nsubseteq D$ for all K is a non-null (s,v)-μ-α-open set in X.

Proof. We give the detailed proof for (a) only. Assume that, $D \in (s,v)-\mathcal{N}(X)$ where $s,v=1,2$ and $s \neq v$. Then $i_s(c_v(D)) = \emptyset$ where $s,v=1,2$ and $s \neq v$. Suppose there is a non-null (s,v)-μ-preopen set M in X such that

$$M \subset D \tag{7}$$

where $s,v=1,2$ and $s \neq v$. Here,

$$M \subset i_s(c_v(M)) \tag{8}$$

where $s,v=1,2$ and $s \neq v$. From (7), we have $i_s(c_v(M)) \subset i_s(c_v(D))$ which implies that $M \subset i_s(c_v(D))$ where $s,v=1,2$ and $s \neq v$, by (8). Then $i_s(c_v(D)) \neq \emptyset$ which is not possible. Therefore, there is no non-null (s,v)-μ-preopen set M in X such that $M \subset D$ where $s,v=1,2$ and $s \neq v$. Hence D does not contain any non-null (s,v)-μ-preopen set in X where $s,v=1,2$ and $s \neq v$.

Theorem 26. *Let (X,μ_1,μ_2) be a BGTS. If $D \in (s,v)-\mathcal{N}(X)$, then $K \nsubseteq D$ for all $K \in \tilde{\mu_s}$ where $s,v=1,2$; $s \neq v$.*

Proof. Assume that, $D \in (s,v)-\mathcal{N}(X)$ where $s,v=1,2$; $s \neq v$. Take $s=1$ and $v=2$. Then $D \in (1,2)-\mathcal{N}(X)$. If there is $H \in \mu_1$ such that $H \subset D$, then $i_1(H) \subset D$ and so $i_1(H) \subset c_2(D)$. This implies $i_1(i_1(H)) \subset i_1(c_2(D))$. By Lemma 5 (e), $i_1(H) \subset i_1(c_2(D))$. By assumption, $H \subset i_1(c_2(D))$. Thus, $i_1(c_2(D)) \neq \emptyset$ which is not possible. Therefore, D does not contain any non-null μ_1-open set. Take $s=2$ and $v=1$. Then $D \in (2,1)-\mathcal{N}(X)$. By similar arguments in the above case, we get the proof.

In the rest of this section, we introduce a new tool namely, (s,v)-codense, and give some of its properties in a BGTS (X,μ_1,μ_2).

Definition 27. Let (X,μ_1,μ_2) be a BGTS and $E \subset X$. Then E is (s,v)-*codense* if $c_s(c_v(X-E)) = X$ where $s,v=1,2$ and $s \neq v$.

Example 28. Consider the bigeneralized topological space (X,μ_1,μ_2) where $X=\{e,f,k,l\}$;

$$\mu_1 = \{\emptyset, \{e,f\}, \{f,l\}, \{e,f,l\}\}$$

and

$$\mu_2 = \{\emptyset, \{e,k\}, \{f,k\}, \{e,f,k\}\}.$$

Take $A=\{k,l\}$ we get $X-A=\{e,f\}$ and so $c_1(c_2(\{e,f\})) = X$. Thus, A is a $(1,2)$-codense set in X. Also, $c_2(c_1(\{e,f\})) = X$. Therefore, A is a $(2,1)$-codense set. Hence A is (s,v)-codense where $s,v=1,2$ and $s \neq v$.

Theorem 29. *In a BGTS (X, μ_1, μ_2), if $E \in (s, v) - \mathcal{N}(X)$, then E is μ_s-codense where $s, v = 1, 2$ and $s \neq v$.*

Proof. *Given $E \in (s, v) - \mathcal{N}(X)$ for $s, v = 1, 2$; $s \neq v$. Then $i_s(c_v(E)) = \emptyset$ and so $X - (i_s(c_v(E))) = X$ where $s, v = 1, 2$ and $s \neq v$. This implies $c_s(X - (c_v(E))) = X$ where $s, v = 1, 2$ and $s \neq v$ which implies that $c_s(X - E) = X$ for $s = 1, 2$. Therefore, E is a μ_s-codense set in X for $s = 1, 2$.*

Example 30 explains that the reverse implication of Theorem 29 need not be true.

Example 30. Consider the BGTS (X, μ_1, μ_2) where $X = \{e, f, k, l, r\}$;

$$\mu_1 = \{\emptyset, \{e, f\}, \{e, k\}, \{e, f, k\}, \{e, f, l\}, \{e, f, k, l\}\}$$

and

$$\mu_2 = \{\emptyset, \{e, f\}, \{f, l\}, \{e, r\}, \{e, f, l\}, \{e, f, r\}, \{e, f, l, r\}\}.$$

Choose $P = \{f, k, r\}$, then $c_2(X - P) = X$. But $P \notin (2, 1) - \mathcal{N}(X)$. For, $i_2(c_1(P)) = i_2(X) = \{e, f, l, r\} \neq \emptyset$.

Consider, $Q = \{f, l, r\}$ we have $c_1(X - Q) = c_1(\{e, k\}) = X$. But $Q \notin (1, 2) - \mathcal{N}(X)$. For, $i_1(c_2(Q)) = i_1(X) = \{e, f, k, l\} \neq \emptyset$.

Proposition 31. *Let (X, μ_1, μ_2) be a BGTS. Then E is a (s, v)-codense set in X if and only if $i_s(i_v(E)) = \emptyset$ where $s, v = 1, 2$ and $s \neq v$.*

Proposition 32. *Let (X, μ_1, μ_2) be a BGTS. If $E \in (s, v) - \mathcal{N}(X)$, then E is a (s, v)-codense set in X.*

Proposition 33. *Let (X, μ_1, μ_2) be a BGTS. Then $E \in (s, v) - \mathcal{D}(X)$ if and only if $X - E$ is (s, v)-codense where $s, v = 1, 2$ and $s \neq v$.*

Proposition 34. *Let (X, μ_1, μ_2) be a BGTS. If E is a (s, v)-codense set in X, then there is no non-null (s, v)-open set H such that $H \subset E$ where $s, v = 1, 2$ and $s \neq v$.*

The reverse implication of Proposition 34 is generally not true as given by the below Example 35.

Example 35. (a) Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = [0, 4]$;

$$\mu_1 = \{\emptyset, [0, 2), (1, 3], [0, 3]\}$$

and

$$\mu_2 = \{\emptyset, [0, \tfrac{3}{2}), (1, 3], [0, 3]\}.$$

Let $A = [0, 2)$. Here $B = (1, 3]$ is $(2,1)$-open set. Also, $B \nsubseteq A$. But $i_2(i_1(A)) = [0, \frac{3}{2}) \neq \emptyset$.

(b) Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = [0, 3]$;

$$\mu_1 = \{\emptyset, [0, \tfrac{3}{2}), (1, 2), (1, 3), [0, 3)\}$$

and

$$\mu_2 = \{\emptyset, [0, 2), (1, 3), [0, 3)\}.$$

Take $A = [0, 2)$. Here $B = (1, 3)$ is $(1, 2)$-open set. Also, $B \not\subseteq A$. But $i_1(i_2(A)) = [0, \frac{3}{2}) \neq \emptyset$.

5. Sets via Functions

In this section, we give some properties for (s, v)-dense and (s, v)-nowhere dense sets under generalized continuous functions in a bigeneralized topological space.

Now, we recall some basic definitions defined in [4].

Let (X, μ^1_X, μ^2_X) and (Y, μ^1_Y, μ^2_Y) be two BGTS and $h : (X, \mu^1_X, \mu^2_X) \to (Y, \mu^1_Y, \mu^2_Y)$ be a map. Then

- h is called (s, v)-*generalized continuous* ($\mu_{(s,v)}$-*continuous*) if $h^{-1}(B)$ is $\mu_{(s,v)}$-closed in X for every μ_v-closed B of Y where $s, v = 1, 2$ and $s \neq v$.
- h is called as μ_s-*continuous* if $h^{-1}(C)$ is μ_s-closed in X for every μ_s-closed C of Y for $s = 1, 2$.
- h is said to be μ_s-*open* if $h(D)$ is μ_s-open of Y for every μ_s-open D of X for $s = 1, 2$.

Theorem 36. *Let (X, μ^1_X, μ^2_X) and (Y, μ^1_Y, μ^2_Y) be two bigeneralized topological spaces, $h : (X, \mu^1_X, \mu^2_X) \to (Y, \mu^1_Y, \mu^2_Y)$ be a $\mu_{(s,v)}$-continuous function where $s, v = 1, 2$ and $s \neq v$. If $Q \cap P \neq \emptyset$ for every P is non-null $\mu_{(s,v)}$-open, then $h(Q) \in (s, v) - \mathcal{D}(Y)$ where $Q \subset X$; $s, v = 1, 2$ and $s \neq v$.*

Proof. *It is enough to prove, $h(Q) \in \mathcal{D}(\mu^v_Y)$ where $v = 1, 2$, by Theorem 4. Take $v = 2$. Let $P \in \tilde{\mu}^2_Y$. Then $Y - P$ is μ^2_Y-closed. By hypothesis, $h^{-1}(Y - P)$ is $\mu_{(1,2)}$-closed in X. Then $h^{-1}(P)$ is non-null $\mu_{(1,2)}$-open. By hypothesis, $Q \cap h^{-1}(P) \neq \emptyset$. This implies $h^{-1}(h(Q)) \cap h^{-1}(P) \neq \emptyset$ which implies that $h^{-1}(h(Q) \cap P) \neq \emptyset$. Thus, $h(Q) \cap P \neq \emptyset$. Hence $h(Q) \in \mathcal{D}(\mu^2_Y)$. Take $v = 1$. Then by the same arguments in the above case, we get $h(Q) \in \mathcal{D}(\mu^1_Y)$. Hence $h(Q) \in \mathcal{D}(\mu^v_Y)$ where $v = 1, 2$.*

Theorem 37. *Let (X, μ^1_X, μ^2_X) and (Y, μ^1_Y, μ^2_Y) be two bigeneralized topological spaces, $P, Q \subset X$, $h : (X, \mu^1_X, \mu^2_X) \to (Y, \mu^1_Y, \mu^2_Y)$ be a μ_s-continuous function for $s = 1, 2$. Then the followings are true.*

(a) If $P \in \mathcal{D}(\mu_v)$, then $h(P) \in (s, v) - \mathcal{D}(Y)$ where $s, v = 1, 2$ and $s \neq v$.

(b) If Q is μ_v-codense and h is one-one, then $h(Q)$ is (s, v)-codense in Y where $s, v = 1, 2$ and $s \neq v$.

Proof. (a). It is enough to prove, $h(P) \in \mathcal{D}(\mu_v)$ in Y where $v = 1, 2$, by Theorem 4. Assume that, $P \in \mathcal{D}(\mu_v)$ in X for $v = 1, 2$. Take $v = 1$. Then $P \in \mathcal{D}(\mu_1)$ in X. Let $M \in \tilde{\mu}_1$. Then $Y - M$ is μ_1-closed in Y. By hypothesis, $h^{-1}(Y - M)$ is μ_1-closed set in X. Then $h^{-1}(M)$ is a non-null μ_1-open set in X. By hypothesis, $P \cap h^{-1}(M) \neq \emptyset$. This implies

$h^{-1}(h(P)) \cap h^{-1}(M) \neq \emptyset$, since $P \subset h^{-1}(h(P))$ which implies that $h^{-1}(h(P) \cap M) \neq \emptyset$. Thus, $h(P) \cap M \neq \emptyset$. Hence $h(P) \in \mathcal{D}(\mu_1)$ in Y. Take $v = 2$. Then by similar arguments in the above case, we get $h(P) \in \mathcal{D}(\mu_2)$ in Y. Hence $h(P) \in \mathcal{D}(\mu_v)$ in Y where $v = 1, 2$.

(b) Let Q be a μ_v-codense set in X for $v = 1, 2$. Then $X - Q \in \mathcal{D}(\mu_v)$ in X for $v = 1, 2$. By (a), $h(X - Q) \in (s, v) - \mathcal{D}(Y)$ where $s, v = 1, 2$ and $s \neq v$. Since h is one-one, $h(X) - h(Q) \in (s, v) - \mathcal{D}(Y)$ where $s, v = 1, 2$ and $s \neq v$. Therefore, $Y - h(Q) \in (s, v) - \mathcal{D}(Y)$ where $s, v = 1, 2$ and $s \neq v$. Hence $h(Q)$ is (s, v)-codense in Y where $s, v = 1, 2$; $s \neq v$.

Theorem 38. *Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be two bigeneralized topological spaces, $K, L \subset Y$, $h : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a μ_s-open, one-one function for $s = 1, 2$. Then the followings are true.*
(a) If $K \in \mathcal{D}(\mu_v)$ in Y, then $h^{-1}(K) \in (s, v) - \mathcal{D}(X)$ where $s, v = 1, 2$ and $s \neq v$.
(b) If L is μ_v-codense in Y, then $h^{-1}(L)$ is (s, v)-codense where $s, v = 1, 2$ and $s \neq v$.

Proof. The trivial proof is omitted.

6. (s, v)-dense sets applications

In 1999, Molodstov introduced a new mathematical tool namely, soft set theory [11]. It has been used for dealing with uncertainty. Most of the researchers presented an application of soft sets in decision-making problems.

Motivated, by this we try to give an example of the soft set using (s, v)-dense and (s, v)-nowhere dense sets in a bigeneralized topological space.

Example 39. Consider the BGTS (X, μ_1, μ_2) where $X = \{a, b, c, d\}$;

$$\mu_1 = \{\emptyset, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\};$$

and

$$\mu_2 = \{\emptyset, \{b, c\}, \{b, d\}, \{b, c, d\}\}.$$

Here,
• $(1, 2) - \mathcal{D}(X) = exp(X)$ where $exp(X)$ is the power set of X.
• $(2, 1) - \mathcal{D}(X) = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$.

Let $U = \{a, c\}$ be a subset of X and $E = \{(1, 2)$-dense set, $(2, 1)$-dense set, both$\} = \{e_1, e_2, e_3\}$ is the set of parameters. Define a map F from E to $exp(U)$ by, $F(e_1) = \{c\}; F(e_2) = \{a\}; F(e_3) = \{a, c\}$. Then the pair (F, E) is a soft set over U.

Example 40. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{a, b, c, d\}$;

$$\mu_1 = \{\emptyset, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$$

and

$$\mu_2 = \{\emptyset, \{a\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}.$$

Here,
• $(1,2)-\mathcal{N}(X) = \{\emptyset, \{a\}, \{d\}, \{a,d\}\}$;
• $(2,1)-\mathcal{N}(X) = \{\emptyset, \{b\}, \{c\}, \{d\}, \{b,d\}, \{c,d\}\}$.

Let $U = \{a,c,d\}$ be a subset of X and $E = \{(1,2)$-nowhere dense set, $(2,1)$-nowhere dense set, both$\} = \{e_1, e_2, e_3\}$ is the set of parameters. Consider the map F from E into the power set of U. Defined by $F(e_1) = \{a\}; F(e_2) = \{c\}; F(e_3) = \{d\}$. Then (F,E) is a soft set over U.

Example 41. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{a,b,c,d\}$;

$$\mu_1 = \{\emptyset, \{a\}, \{a,d\}, \{c,d\}, \{a,c,d\}\}$$

and

$$\mu_2 = \{\emptyset, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}.$$

Here,
• $(1,2)$-codense sets $= \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}$.
• $(2,1)$-codense sets $= \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a, b,d\}\}$.

Let $U = \{a,c,d\}$ be a subset of X and $E = \{(1,2)$-codense set, $(2,1)$-codense set, $(1,2)$-codense but not $(2,1)$-codense, $(2,1)$-codense but not $(1,2)$-codense, $(1,2)$-codense and $(2,1)$-codense $\} = \{e_1, e_2, e_3, e_4, e_5\}$ is the set of parameters. Consider the map F from E into the power set of U. Defined by $F(e_1) = \{a\}; F(e_2) = \{c\}; F(e_3) = \{c,d\}; F(e_4) = \{a,c\}; F(e_5) = \{d\}$. Then we get the pair (F,E) is a soft set over U.

Example 42. Consider the generalized topological space (X, η_1, η_2) where $X = \{a,b,c,d\}$; η_1 and η_2 are defined in above Example 40, that is; we take $\eta_1 = \mu_2$ and $\eta_2 = \mu_1$. Then we get;

• η_1-nowhere dense sets $= \{\emptyset, \{b\}, \{d\}, \{b,d\}\}$;
• η_2-nowhere dense sets $= \{\emptyset, \{c\}, \{d\}, \{c,d\}\}$;
• η_1-dense sets $= \{\{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, X\}$;
• η_2-dense sets $= \{\{a,b\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, X\}$;
• $(1,2)-\mathcal{N}(X) = \{\emptyset, \{b\}, \{c\}, \{d\}, \{b,d\}, \{c,d\}\}$;
• $(2,1)-\mathcal{N}(X) = \{\emptyset, \{a\}, \{d\}, \{a,d\}\}$.

Let $U = \{a,b,c\}$ be a non-null subset of X and $E = \{\eta_1$-nowhere dense set, η_2-nowhere dense set, η_1-dense set, η_2-dense set, $(1,2)$-nowhere dense set, $(2,1)$-nowhere dense set$\} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ is the set of parameters. Take F be a function defined from E to the subsets of U by; $F(e_1) = \{b\}; F(e_2) = \{c\}; F(e_3) = \{a,c\}; F(e_4) = \{b,c\}; F(e_5) = \{d\}; F(e_6) = \{a\}$. Thus, (F,E) is a soft set over U.

7. Conclusion

In this article, various properties for (s, v)-dense and (s, v)-nowhere dense sets are proved, which are useful to easily check the characterization of a given set in a bigeneralized topological space.

References

[1] Chawalit Boonpok. Weakly open functions on bigeneralized topological spaces. *Int. Journal of Math. Analysis*, 4(18):891–897, 2010.

[2] Akos Császár. Generalized open sets. *Acta mathematica hungarica*, 75, 1997.

[3] Akos Császár. Generalized open sets in generalized topologies. *Acta mathematica hungarica*, 106, 2005.

[4] Wichai Dungthaisong, Chawalit Boonpok, and Chokchai Viriyapong. Generalized closed sets in bigeneralized topological spaces. *International Journal of Mathematical Analysis*, 5(24):1175–1184, 2011.

[5] E. Ekici. Generalized submaximal spaces. *Acta Math. Hungar.*, 134:132 – 138, 2012.

[6] Erdal Ekici. Generalized hyperconnectedness. *Acta Mathematica Hungarica*, 133, 2011.

[7] J.C. Kelly. Bitopological spaces. *Pro. London Math. Soc.*, 3(13):71 – 79, 1969.

[8] Ewa Korczak-Kubiak, Anna Loranty, and Ryszard J Pawlak. Baire generalized topological spaces, generalized metric spaces and infinite games. *Acta Mathematica Hungarica*, 140(3):203–231, 2013.

[9] Zhaowen Li and Funing Lin. Baireness on generalized topological spaces. *Acta Mathematica Hungarica*, 139(4), 2013.

[10] W. K. Min. Almost continuity on generalized topological spaces. *Acta Math. Hungar.*, 125:121 – 125, 2009.

[11] D. Molodtsov. Soft set theory-first results. *Comput. Math. Appl.*, 37:19 – 31, 1999.

[12] V Renukadevi and S Vadakasi. Modifications of strongly nodec spaces. *Communications in Advanced Mathematical Sciences*, 2:99–112, 2018.

[13] Binod Chandra Tripathy S. Acharjee and Kyriakos Papadopoulos. Two forms of pairwise lindelöfness and some results related to hereditary class in a bigeneralized topological space. *New Mathematics and Natural Computation*, 13(2):181–193, 2017.

[14] Preecha Yupapin Vadakasi Subramanian, Yasser Farhat. On nowhere dense sets. *European Journal of Pure and Applied Mathematics*, 15(2):403–414, 2022.