EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 16, No. 4, 2023, 2286-2305
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

On Dense Sets

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Abstract. In this paper, we introduce one interesting mathematical tool namely, $(s, v)^{\star}$-dense, and analyze its nature in a bigeneralized topological space. Further, we prove some properties of this set and give the relationship between (s, v)-dense and $(s, v)^{\star}$-dense sets. Finally, we give applications for various sets defined in a bigeneralized topological space.

2020 Mathematics Subject Classifications: AMS Subject Classification(2020):Primary: 54A05, 54A10.

Key Words and Phrases: Bigeneralized topological space, $\mu_{(s,v)}$-open, (s, v)-open, (s, v)-nowhere dense.

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1. Introduction

The concept of a generalized topological space was introduced by Császár in [3]. Some researchers have defined various concepts in this space and examined their significance in a generalized topological space. Especially, in a generalized topological space, dense sets were introduced by Ekici [8]. He has proven few results for dense sets in a generalized topological space. Based on this, some mathematicians have proved various properties for dense sets e.g. [11, 12, 15, 17, 19].

In [10], J.C. Kelly introduced the concept namely, a bitopological space. Using these aspects, Boonpok founded the notion of a bigeneralized topological space in 2010 [2]. He examines the significance of (m, n)-closed sets in a bigeneralized topological space.

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DOI: https://doi.org/10.29020/nybg.ejpam.v16i4.4944

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Inspired by all this, we define a new dense set, namely, $(s,v)^\star$-dense set using semi-open sets in a bigeneralized topological space. Find various interesting results for $(s,v)^\star$-dense sets.

Next section, the preliminary definitions, and lemmas are remembered.

In sections 3 & 4, in a bigenerlized topological space, examined the significance of $(s,v)^\star$-dense set. The relationship between μ-dense and $(s,v)^\star$-dense sets are proven. Further, few results for $(s,v)^\star$-dense sets using functions are launched. In the last section, we defined a soft set using $(s,v)^\star$-dense sets and various types of open sets defined in a bigeneralized topological space.

2. Preliminaries

In [3], let X be any non-null set. A family μ of subsets of X is a *generalized topology* in X if it contains the empty set and is closed under arbitrary union. The pair (X,μ) is called a *generalized topological space* (GTS). If $X \in \mu$, then (X,μ) is called a *strong generalized topological space* (sGTS).

In [6], if $Q \in \mu$, then Q is called a *μ-open set* and if $X - Q \in \mu$, then Q is said to be a *μ-closed set.* The *interior of* $Q \subset X$ denoted by $i_\mu(D)$, is the union of all μ-open sets contained in D and the *closure of* D denoted by $c_\mu(D)$, is the intersection of all μ-closed sets containing D [12]. Here, the interior and closure of the set Q are notated by iQ and cQ, respectively, when no confusion can arise.

In [11], notated by;

$$\tilde{\mu} = \{D \in \mu \mid D \neq \emptyset\};$$

$$\mu(x) = \{D \in \mu \mid x \in D\}.$$

Definition 1. [8] A subset Q of a GTS (X,μ) is said to be;
- *μ-nowhere dense* if $icQ = \emptyset$.
- *μ-dense* if $cQ = X$.
- *μ-codense* [7] if $c(X - Q) = X$.

Definition 2. [11] A subset Q of X is called as;
- *μ-meager* if $Q = \bigcup_{m\in\mathbb{N}} Q_m$ where each Q_m is a μ-nowhere dense set.
- *μ-second category* if Q is not μ-meager.

In [11], defined two new generalized topologies;

$$\mu^\star = \{\bigcup_t (L_1^t \cap L_2^t \cap L_3^t \cap ... \cap L_{n_t}^t) \mid L_1^t, L_2^t, ..., L_{n_t}^t \in \mu\};$$

$$\mu^{\star\star} = \{D \subset X \mid D \text{ is of } \mu\text{-II category}\}.$$

Obviously, $\mu \subset \mu^\star$ and $\mu^\star$ is closed under finite intersection [11].

Definition 3. [6] Let (X, μ) be a GTS and $Q \subset X$ is called;
• μ-*semi-open* if $Q \subset c_\mu(i_\mu(Q))$.
• μ-*pre-open* if $Q \subset i_\mu(c_\mu(Q))$.
• μ-α-*open* if $Q \subset i_\mu(c_\mu(i_\mu(Q)))$.
• μ-β-*open* if $Q \subset c_\mu(i_\mu(c_\mu(Q)))$.
• μ-*b-open* [1] if $Q \subset c_\mu(i_\mu(Q)) \cup i_\mu(c_\mu(Q))$.

Moreover, $\sigma(\mu)$ or $\sigma(\mu(X)) = \{Q \subset X \mid Q$ is μ-semi-open set in $X\}$ [12]. The μ-semi-interior of a subset Q of (X, μ), denoted by $i_\sigma(Q)$, is defined by the union of all μ-semi-open subsets of X contained in Q [12].

Definition 4. [2] Let μ_1 and μ_2 be two generalized topologies defined a non-null set X. A triple (X, μ_1, μ_2) is called a *bigeneralized topological* space (briefly, BGTS).

• The *closure* and *interior* of $Q \subset X$ with respect to μ_s are denoted by $c_s(Q)$ and $i_s(Q)$, respectively, for $s = 1, 2$.
• Q is called (s, v)-*closed* if $c_s(c_v(Q)) = D$, where $s, v = 1$ or 2 ; $s \neq v$.
• Q is called (s, v)-*open* if $X - Q$ is (s, v)-closed where $s, v = 1$ or 2 ; $s \neq v$.

A subset Q of a BGTS (X, μ_1, μ_2) is said to be
(1) (s, v)-μ-*regular open* if $Q = i_s(c_v(Q))$ where $s, v = 1$ or 2 ; $s \neq v$.
(2) (s, v)-μ-*semi-open* if $Q \subseteq c_v(i_s(Q))$ where $s, v = 1$ or 2 ; $s \neq v$.
(3) (s, v)-μ-*preopen* if $Q \subseteq i_s(c_v(Q))$ where $s, v = 1$ or 2 ; $s \neq v$.
(4) (s, v)-μ-α-*open* if $Q \subseteq i_s(c_v(i_s(Q)))$ where $s, v = 1$ or 2 ; $s \neq v$ [2].

Lemma 1. [2, Proposition 3.4] Let (X, μ_1, μ_2) be a BGTS and $Q \subset X$. Then Q is (s, v)-closed if and only if Q is both μ-closed in (X, μ_s) and (X, μ_v) where $s, v = 1, 2$; $s \neq v$.

Lemma 2. [5] In a GTS (X, μ), $r \in cP$ if and only if $L \cap P \neq \emptyset$ for all $L \in \tilde{\mu}(r)$.

Lemma 3. [12, Lemma 3.2] Let (X, μ) be a GTS and $K, P \subset X$. If $K \in \tilde{\mu}$ and $K \cap P = \emptyset$, then $K \cap cP = \emptyset$.

Lemma 4. [13, Proposition 2.2] Let (X, μ) be a GTS. For subsets $Q, P \subset X$, then the following properties holds:
(a) $c_\mu(X - Q) = X - i_\mu(Q)$ and $i_\mu(X - Q) = X - c_\mu(Q)$.
(b) If $X - Q \in \mu$, then $c_\mu(Q) = Q$ and if $Q \in \mu$, then $i_\mu(Q) = Q$.
(c) If $Q \subseteq P$, then $c_\mu(Q) \subseteq c_\mu(P)$ and $i_\mu(Q) \subseteq i_\mu(P)$.
(d) $Q \subseteq c_\mu(Q)$ and $i_\mu(Q) \subseteq Q$.
(e) $c_\mu(c_\mu(Q)) = c_\mu(Q)$ and $i_\mu(i_\mu(Q)) = i_\mu(Q)$.

3. Nature of (s, v)*-dense sets

Here, we define another branch of dense set namely, $(s, v)^\star$-dense set and study its significance in a BGTS.

In a bigeneralized topological space, various interesting results for $(s,v)^\star$-dense sets are derived which is helpful for examining the given set is $(s,v)^\star$-dense or not.

Definition 5. A GTS (X,μ) is called as;
• *hyperconnected* [8] if $c_\mu(Q)=X$ whenever $Q\in\tilde{\mu}$.
• *generalized submaximal* [7] if $Q\in\tilde{\mu}$ whenever $c_\mu(Q)=X$.

Definition 6. [16] A GT μ on X is said to satisfy the $\mathcal{I}$-*property* whenever $W_1,W_2,..,W_m\in\mu$ with $W_1\cap W_2\cap\cdots\cap W_m\neq\emptyset, i_\mu(W_1\cap W_2\cap\cdots\cap W_m)\neq\emptyset$.

Definition 7. [9] A non-null subset Q of a BGTS (X,μ_1,μ_2) is called (s,v)-*dense* if $c_s(c_v(Q))=X$ where $s,v=1,2$ and $s\neq v$.

Moreover, $(s,v)-\mathcal{D}(X)=\{Q\subset X \mid Q \text{ is a } (s,v)\text{-dense set in } X\}$ where $s,v=1,2$; $s\neq v$.

Definition 8. Let Q be a non-null subset of a bigeneralized topological space (X,μ_1,μ_2). Then Q is called $(\mu_s,\mu_v)^\star$-*dense* (briefly, $(s,v)^\star$-*dense*) if $c_v(Q)\cap M\neq\emptyset$ for every $M\in\tilde{\sigma}_s$ where $s,v=1,2$; $s\neq v;\sigma_s=\sigma(\mu_s)$.

For simplification we noted;

$$(s,v)^\star-\mathcal{D}(X)=\{Q\subset X\mid Q \text{ is a } (s,v)^\star\text{-dense set in } X\}$$

where $s,v=1,2$; $s\neq v$.

Remark 9. *In a BGTS, if $P\in(s,v)^\star-\mathcal{D}(X)$ and $P\subset Q$, then $Q(s,v)^\star-\mathcal{D}(X)$.*

Example 10. Consider the bigeneralized topological space (X,μ_1,μ_2) where $X=\{p,q,r,s\}$;

$$\mu_1=\{\emptyset,\{p,q\},\{q,r\},\{p,q,r\}\}$$

and

$$\mu_2=\{\emptyset,\{p,s\},\{q,s\},\{p,q,s\}\}.$$

Then

$$\sigma_1=\{\emptyset,\{p,q\},\{q,r\},\{p,q,r\},\{p,q,s\},\{q,r,s\},X\}.$$

Take $K=\{q,r\}$. Then $c_2(K)=K$. Also, $K\cap M\neq\emptyset$ for all $M\in\tilde{\sigma}_1$. Thus, $c_2(K)\cap M\neq\emptyset$ for all $M\in\tilde{\sigma}_1$. Therefore, $K\in(1,2)^\star-\mathcal{D}(X)$.

(b) Consider the bigeneralized topological space (X,μ_1,μ_2) where $X=\{p,q,r,s\}$;

$$\mu_1=\{\emptyset,\{q,r\},\{q,s\},\{q,r,s\}\}$$

and

$$\mu_2=\{\emptyset,\{p,q\},\{p,r\},\{p,q,r\}\}.$$

Then

$$\sigma_2 = \{\emptyset, \{p,q\}, \{p,r\}, \{p,q,r\}, \{p,q,s\}, \{p,r,s\}, X\}.$$

Take $J = \{p,r\}$. Here $c_1(J) \cap H \neq \emptyset$ for all $H \in \tilde{\sigma}_2$. Hence $J \in (2,1)^\star - \mathcal{D}(X)$.

Theorem 11. *Let (X, μ_1, μ_2) be a BGTS and $c_{\mu_s}(Q) = X$. If μ_s is a sGT, then $Q \in (s,v)^\star - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$.*

Proof. *Take $s = 1$ and $v = 2$. Assume that, $c_{\mu_1}(Q) = X$ and μ_1 is a sGT. Let $P \in \tilde{\sigma}_1$. Then $P \subset c_{\mu_1}(i_{\mu_1}(P))$ and so $i_{\mu_1}(P) \neq \emptyset$, since μ_1 is a sGT. This implies $i_{\mu_1}(P) \in \tilde{\mu}_1$ which implies that $i_{\mu_1}(P) \cap Q \neq \emptyset$. Thus, $Q \cap P \neq \emptyset$. Therefore, $Q \in (1,2)^\star - \mathcal{D}(X)$.*

Take $s = 2$ and $v = 1$. Suppose $c_{\mu_2}(Q) = X$ and μ_2 is a sGT. Let $M \in \tilde{\sigma}_2$. Then $M \subset c_{\mu_2}(i_{\mu_2}(M))$ and so $i_{\mu_2}(M) \neq \emptyset$, since μ_2 is a sGT. Thus, $i_{\mu_2}(M) \in \tilde{\mu}_2$ so that $i_{\mu_2}(M) \cap Q \neq \emptyset$. This implies $Q \cap M \neq \emptyset$ which implies that $Q \in (2,1)^\star - \mathcal{D}(X)$.

The below Example 12 shows that the hypothesis in Theorem 11 can not be dropped.

Example 12. (a). Consider the BGTS (X, μ_1, μ_2) where $X = \{p,q,r,s\}$;

$$\mu_1 = \{\emptyset, \{q,s\}, \{r,s\}, \{q,r,s\}\}$$

and

$$\mu_2 = \{\emptyset, \{p,r\}, \{q,r\}, \{p,q,r\}, \{p,q,s\}, X\}.$$

Fix $s = 1; v = 2$. Obviously,

$$\sigma_1 = \{\emptyset, \{p\}, \{q,s\}, \{r,s\}, \{p,q,s\}, \{p,r,s\}, \{q,r,s\}, X\}.$$

Choose $L = \{q,s\}$ so that $c_{\mu_1}L = X$ and $c_{\mu_2}L = L$. Thus, $L = \{q,s\}$ is μ_1-dense. But $c_{\mu_2}L \cap \{p\} = \emptyset$ where $\{p\} \in \tilde{\sigma}_1$ for that $L \notin (1,2)^\star - \mathcal{D}(X)$.

(b). Consider the BGTS (X, μ_1, μ_2) where $X = \{p,q,r,s\}$;

$$\mu_1 = \{\emptyset, \{p,r\}, \{q,r\}, \{r,s\}, \{p,q,r\}, \{p,r,s\}, \{q,r,s\}, X\}$$

and

$$\mu_2 = \{\emptyset, \{p,s\}, \{q,s\}, \{p,q,s\}\}.$$

Fix $s = 2; v = 1$. Obviously,

$$\sigma_2 = \{\emptyset, \{r\}, \{p,s\}, \{q,s\}, \{p,q,s\}, \{p,r,s\}, \{q,r,s\}, X\}.$$

Choose $K = \{s\}$ so that $c_{\mu_1}K = K$ and $c_{\mu_2}K = X$. Here, $K = \{s\}$ is μ_2-dense. But $c_{\mu_1}K \cap \{r\} = \emptyset$ where $\{r\} \in \tilde{\sigma}_2$ for that $K \notin (2,1)^\star - \mathcal{D}(X)$.

Theorem 13. *Let (X, μ_1, μ_2) be a BGTS. Then the following are true.*
(a) If $c_{\mu_v}(Q) = X$, then $Q \in (s, v)^{\star} - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$.
(b) If $\mu_s \subset \mu_v$, then every $(s, v)^{\star}$-dense is μ_s-dense where $s, v = 1, 2$; $s \neq v$.

Proof. (a). Assume that, $c_{\mu_v}(Q) = X$ for $v = 1, 2$.

Fix $s = 1$ and $v = 2$. We get $c_{\mu_2}(Q) = X$ so that $c_{\mu_2}(Q) \cap H \neq \emptyset$ for all $H \in \tilde{\sigma}_1$. Therefore, Q is $(1, 2)^{\star} - \mathcal{D}(X)$.

Take $s = 2$ and $v = 1$. Then $c_{\mu_1}(Q) = X$ and so $c_{\mu_1}(Q) \cap K \neq \emptyset$ for all $K \in \tilde{\sigma}_2$. Therefore, Q is $(2, 1)^{\star} - \mathcal{D}(X)$.

(b). Suppose that $\mu_s \subset \mu_v$ for $s, v = 1, 2$; $s \neq v$. Let $K \in (s, v)^{\star} - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$.

Consider $s = 1$ and $v = 2$. Then $\mu_1 \subset \mu_2$ and $K \in (1, 2)^{\star} - \mathcal{D}(X)$. Let $G \in \tilde{\mu}_1$. Then $G \in \tilde{\sigma}_1$ so that $G \cap c_{\mu_2} K \neq \emptyset$. By hypothesis and Lemma 3, $G \cap K \neq \emptyset$. Hence K is μ_1-dense.

Take $s = 2$ and $v = 1$. Then $\mu_2 \subset \mu_1$ and $K \in (2, 1)^{\star} - \mathcal{D}(X)$. Let $H \in \tilde{\mu}_2$. Then $H \in \tilde{\sigma}_2$ so that $H \cap c_{\mu_1} K \neq \emptyset$. By hypothesis and Lemma 3, $H \cap K \neq \emptyset$. Hence K is μ_2-dense.

The below Example 14 (b) shows that the converse part of Theorem 13 (a) need not be true and the hypothesis of Theorem 13 (b) can not be neglected as shown by Example 14 (a).

Example 14. Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;

$$\mu_1 = \{\emptyset, \{p, r\}, \{p, s\}, \{p, r, s\}\}$$

and

$$\mu_2 = \{\emptyset, \{q, r\}, \{q, s\}, \{r, s\}, \{q, r, s\}\}.$$

We get

$$\sigma_1 = \{\emptyset, \{q\}, \{p, r\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, X\}$$

and

$$\sigma_2 = \{\emptyset, \{p\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}.$$

(a). Fix $s = 1; v = 2$. Here, $\mu_1 \not\subseteq \mu_2$. Choose $Q = \{q, r\}$ we get $Q \in (1, 2)^{\star} - \mathcal{D}(X)$. Because, $c_2 Q \cap L \neq \emptyset$ for all $L \in \tilde{\sigma}_1$. But $c_1 Q = Q \neq X$ so that Q is not μ_1-dense.

Take $s = 2, v = 1$ and $L = \{p, q\}$. Here, $c_1 L \cap D \neq \emptyset$ for each $D \in \tilde{\sigma}_2$ so that $L \in (2, 1)^{\star} - \mathcal{D}(X)$. Since $c_2 L = L \neq \emptyset$ we have L is not μ_2-dense.

(b). Fix $s = 1; v = 2$. Choose $W = \{p, q\}$ we get $W \in (1, 2)^{\star} - \mathcal{D}(X)$, since $c_2 W \cap L \neq \emptyset$ for all $L \in \tilde{\sigma}_1$. Here, $c_2 W = W \neq X$ so that W is not a μ_2-dense set.

Take $s = 2; v = 1$. Consider the BGTS (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;

$$\mu_1 = \{\emptyset, \{p, r\}, \{p, s\}, \{r, s\}, \{p, r, s\}\}$$

and

$$\mu_2 = \{\emptyset, \{q, r\}, \{q, s\}, \{q, r, s\}\}.$$

Clearly, we have

$$\sigma_1 = \{\emptyset, \{q\}, \{p, r\}, \{p, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$$

and

$$\sigma_2 = \{\emptyset, \{p\}, \{q, r\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}, X\}.$$

Consider $K = \{p, q\}$. Since $c_1 K \cap M \neq \emptyset$ for all $M \in \tilde{\sigma}_2$ we get $K \in (2, 1)^{\star} - \mathcal{D}(X)$. But $c_1 K = K \neq X$. Thus, K is not μ_1-dense.

Theorem 15. *Let (X, μ_1, μ_2) be a BGTS. Then $(s, v)^{\star} - \mathcal{D}(X) \subset (s, v) - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$.*

Proof. *Let $Q \in (s, v)^{\star} - \mathcal{D}(X)$.*

Take $s = 1; v = 2$. Then $Q \in (1, 2)^{\star} - \mathcal{D}(X)$ so that $c_2(Q) \cap M \neq \emptyset$ for every $M \in \tilde{\sigma}_1$. Since $\mu_1 \subset \sigma_1$, $c_2(Q) \cap K \neq \emptyset$ for every $K \in \tilde{\mu}_1$. Therefore, $Q \in (1, 2) - \mathcal{D}(X)$.

Fix $s = 2; v = 1$. We get $Q \in (2, 1)^{\star} - \mathcal{D}(X)$ for that $c_1(Q) \cap L \neq \emptyset$ for every $L \in \tilde{\sigma}_2$. Since $\mu_2 \subset \sigma_2$, $c_1(Q) \cap G \neq \emptyset$ for every $G \in \tilde{\mu}_2$. Therefore, $Q \in (2, 1) - \mathcal{D}(X)$.

The below Example 16 shows that in a bigeneralized topological space, the reverse implication of the above Theorem 15 need not be true in general.

Example 16. *Consider the bigeneralized topological space (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;*

$$\mu_1 = \{\emptyset, \{p, r\}, \{r, s\}, \{p, r, s\}\}$$

and

$$\mu_2 = \{\emptyset, \{p, q\}, \{p, r\}, \{q, r\}, \{p, q, r\}\}.$$

Then

$$\sigma_1 = \{\emptyset, \{q\}, \{p, r\}, \{r, s\}, \{p, q, r\}, \{p, r, s\}, \{q, r, s\}, X\}$$

and

$$\sigma_2 = \{\emptyset, \{s\}, \{p,q\}, \{p,r\}, \{q,r\}, \{p,q,r\}, \{p,q,s\}, \{p,r,s\}, \{q,r,s\}, X\}.$$

• *Fix $s = 1$ and $v = 2$. Choose $K = \{p, s\}$ we get K is $(1,2)$-dense. But $K \notin (1,2)^\star - \mathcal{D}(X)$. For, if we choose $D = \{q\}$, then $D \in \tilde{\sigma}_1$. But $D \cap c_2(K) = \emptyset$. Thus, there is $D \in \tilde{\sigma}_1$ such that $D \cap c_2(K) = \emptyset$.*

• *Fix $s = 2$ and $v = 1$. Take $L = \{p, q\}$, then we get $L \in (1,2) - \mathcal{D}(X)$. Here, we take $H = \{s\}$ so that $H \in \tilde{\sigma}_2$ but $H \cap c_1(L) = \emptyset$. Thus, $L \notin (1,2)^\star - \mathcal{D}(X)$.*

Theorem 17. *Let (X, μ_1, μ_2) be a BGTS. If μ_s is a sGT, then $(s,v) - \mathcal{D}(X) \subset (s,v)^\star - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$.*

Proof. *Assume that, μ_s is sGT and $Q \in (s,v) - \mathcal{D}(X)$.*

Take $s = 1; v = 2$. Then $Q \in (1,2) - \mathcal{D}(X)$ so that $c_1(c_2(Q)) = X$. Thus, $c_2(Q) \cap M \neq \emptyset$ for all $M \in \tilde{\mu}_1$. Let $H \in \tilde{\sigma}_1$. Suppose $H \in \tilde{\mu}_1$. Then there is nothing to prove. Suppose $H \notin \tilde{\mu}_1$. Here $H \subset c_1(i_1(H))$. This implies $c_1(i_1(H)) \neq \emptyset$ which implies that $i_1(H) \neq \emptyset$, by hypothesis. Thus, $i_1(H) \in \tilde{\mu}_1$ so that $c_2(Q) \cap H \neq \emptyset$. Thus, $c_2(Q) \cap H \neq \emptyset$ for all $H \in \tilde{\sigma}_1$. Hence $Q \in (1,2)^\star - \mathcal{D}(X)$.

Fix $s = 2; v = 1$. We get $Q \in (2,1) - \mathcal{D}(X)$ such that $c_2(c_1(Q)) = X$ which implies $c_1(Q) \cap L \neq \emptyset$ for all $L \in \tilde{\mu}_2$. Let $K \in \tilde{\sigma}_2$. Suppose $K \in \tilde{\mu}_2$. Then there is nothing to prove. If $K \notin \tilde{\mu}_2$, then from the definition of K such that $K \subset c_2(i_2(K))$. This implies $c_2(i_2(K)) \neq \emptyset$ which implies that $i_2(K) \neq \emptyset$ since μ_2 is a sGT. Thus, $i_2(K) \in \tilde{\mu}_2$ so that $c_1(Q) \cap K \neq \emptyset$. Thus, $c_1(Q) \cap K \neq \emptyset$ for all $K \in \tilde{\sigma}_2$. Hence $Q \in (2,1)^\star - \mathcal{D}(X)$.

The above Example 16 also proves that the hypothesis of Theorem 17 can not be dropped.

Theorem 18. *Let (X, μ_1, μ_2) be a BGTS and $\mu_1 \subset \mu_2$. If $\mu_1 \subset (s,v)^\star - \mathcal{D}(X)$, then (X, μ_1) is hyperconnected for $s, v = 1, 2$; $s \neq v$.*

Proof. *Let $Q \in \tilde{\mu}_1$.*

Choose $s = 1$ and $v = 2$. Then $Q \in (1,2)^\star - \mathcal{D}(X)$. By hypothesis and Theorem 13, Q is μ_1-dense so that (X, μ_1) is a hyperconnected space.

Fix $s = 2$; $v = 1$. Then $Q \in (2,1)^\star - \mathcal{D}(X)$ so that $c_1(Q) \cap M \neq \emptyset$ for every $M \in \tilde{\sigma}_2$. Let $K \in \tilde{\mu}_1$. By hypothesis, $K \in \tilde{\mu}_2$ which implies $K \in \tilde{\sigma}_2$, since $\mu_2 \subset \sigma_2$ which turn implies that $c_1(Q) \cap K \neq \emptyset$. Thus, $Q \cap K \neq \emptyset$. Since K is an arbitrary non-null μ_1-open set, Q is μ_1-dense. Therefore, (X, μ_1) is a hyperconnected space.

Theorem 19. *Let (X, μ_1, μ_2) be a BGTS. If $\mu_2 \subset \mu_1$ and if $\mu_2 \subset (s,v)^\star - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$, then (X, μ_2) is hyperconnected.*

Proof. *Let* $P \in \tilde{\mu}_2$.

Take $s = 1$ *and* $v = 2$. *Then* $P \in (1,2)^\star - \mathcal{D}(X)$ *so that* $c_2(P) \cap M \neq \emptyset$ *for every* $M \in \tilde{\sigma}_1$. *Let* $K \in \tilde{\mu}_2$. *By hypothesis,* $K \in \tilde{\mu}_1$ *which implies* $K \in \tilde{\sigma}_1$, *since* $\mu_1 \subset \sigma_1$ *which turn implies that* $c_2(P) \cap K \neq \emptyset$. *Thus,* $P \cap K \neq \emptyset$. *Since* K *is an arbitrary non-null* μ_2*-open set,* P *is* μ_2*-dense. Hence* (X, μ_2) *is hyperconnected.*

Now we choose $s = 2$; $v = 1$. *We get* $P \in (2,1)^\star - \mathcal{D}(X)$. *By Theorem 13 and hypothesis, we get* Q *is* μ_2*-dense. Therefore,* (X, μ_2) *is a hyperconnected space.*

Theorem 20. *Let* (X, μ_1, μ_2) *be a BGTS and* $Q \in (s,v) - \mathcal{D}(X)$; $Q \in \mu_v$; $J \in (v,s)^\star - \mathcal{D}(X)$. *If* $\mu_v \subset \mu_s$ *and if* μ_v *has the* $\mathcal{I}$*-property, then* $Q \cap J \in (v,s) - \mathcal{D}(X)$ *where* $s, v = 1, 2$; $s \neq v$.

Proof. *Fix* $s = 1, v = 2$. *Assume that,* $Q \in (1,2) - \mathcal{D}(X)$; $Q \in \mu_2$ *and* $J \in (2,1)^\star - \mathcal{D}(X)$. *Then*

$$(a) \quad c_1(c_2(Q)) = X.$$

$$(b) \quad c_1 J \cap M \neq \emptyset \quad for \quad all \quad M \in \tilde{\sigma}_2.$$

Suppose μ_2 *has the* $\mathcal{I}$*-property and* $\mu_2 \subset \mu_1$. *Let* $K \in \tilde{\mu}_2$. *By hypothesis,* $K \in \tilde{\mu}_1$ *so that* $K \cap c_2(Q) \neq \emptyset$, *by (a) which implies that* $K \cap Q \neq \emptyset$, *by Lemma 3. By our assumption,* $i_2(K \cap Q) \neq \emptyset$. *By (b),* $c_1 J \cap i_2(K \cap Q) \neq \emptyset$ *which implies* $c_2 J \cap i_2(K \cap Q) \neq \emptyset$ *by hypothesis which turn implies that* $J \cap i_2(K \cap Q) \neq \emptyset$, *by Lemma 3. Thus,* $J \cap (K \cap Q) \neq \emptyset$ *so that* $(J \cap Q) \cap K \neq \emptyset$. *Therefore,* $c_1(J \cap Q) \cap K \neq \emptyset$. *Hence* $Q \cap J \in (2,1) - \mathcal{D}(X)$.

Take $s = 2, v = 1$. *Assume that,* $Q \in (2,1) - \mathcal{D}(X)$; $Q \in \mu_1$ *and* $J \in (1,2)^\star - \mathcal{D}(X)$. *We get*

$$(c) \quad c_2(c_1(Q)) = X.$$

$$(d) \quad c_2 J \cap H \neq \emptyset \quad for \quad all \quad H \in \tilde{\sigma}_1.$$

Suppose μ_1 *has the* $\mathcal{I}$*-property and* $\mu_1 \subset \mu_2$. *Let* $L \in \tilde{\mu}_1$. *By hypothesis,* $L \in \tilde{\mu}_2$ *so that* $L \cap c_1(Q) \neq \emptyset$, *by (c) which implies that* $L \cap Q \neq \emptyset$, *by Lemma 3. By our assumption,* $i_1(L \cap Q) \neq \emptyset$. *By (d),* $c_2 J \cap i_1(L \cap Q) \neq \emptyset$. *This implies* $c_1 J \cap i_1(L \cap Q) \neq \emptyset$ *by hypothesis which implies that* $J \cap i_1(L \cap Q) \neq \emptyset$, *by Lemma 3. Thus,* $J \cap (L \cap Q) \neq \emptyset$ *so that* $(J \cap Q) \cap L \neq \emptyset$. *Therefore,* $c_2(J \cap Q) \cap L \neq \emptyset$. *Hence* $Q \cap J \in (1,2) - \mathcal{D}(X)$.

Moreover, in a BGTS every μ_v-dense set is (s,v)-preopen where $s, v = 1, 2$; $s \neq v$.

Theorem 21. *Let* (X, μ_1, μ_2) *be a BGTS and* $\eta_1 = \{Q \subset X \mid Q \in (1,2)^\star - \mathcal{D}(X); \eta_2 = \{P \subset X \mid P \in (2,1)^\star - \mathcal{D}(X)\}$. *Then*
(a) If $\zeta = \eta_1 \cup \{\emptyset\}$ and if $\emptyset \neq \zeta \subset \mu_1 \cap \mu_2$, then (X, ζ) is a hyperconnected space.
(b) If $\zeta = \eta_2 \cup \{\emptyset\}$ and if $\emptyset \neq \zeta \subset \mu_1 \cap \mu_2$, then (X, ζ) is a hyperconnected space.

Proof. (a) Assume that, $\zeta = \eta_1 \cup \{\emptyset\}$ and $\emptyset \neq \zeta \subset \mu_1 \cap \mu_2$. Let $K \in \tilde{\zeta}$. Then $K \in (1,2)^{\star} - \mathcal{D}(X)$ and so $c_2 K \cap J \neq \emptyset$ for all $J \in \tilde{\sigma}_1$. Let $D \in \tilde{\zeta}$. By hypothesis, $D \in \mu_1$ so that $D \in \tilde{\sigma}_1$. Thus, $c_2 K \cap D \neq \emptyset$. Since $D \in \mu_2$ we have $K \cap D \neq \emptyset$, by Lemma 3. Hence K is ζ-dense. Therefore, (X, ζ) is a hyperconnected space.

(b) Suppose that, $\zeta = \eta_2 \cup \{\emptyset\}$ and $\emptyset \neq \zeta \subset \mu_1 \cap \mu_2$. Let $P \in \tilde{\zeta}$. Then $P \in (2,1)^{\star} - \mathcal{D}(X)$ and so $c_1 P \cap J \neq \emptyset$ for all $J \in \tilde{\sigma}_2$. Let $M \in \tilde{\zeta}$. By hypothesis, $M \in \mu_2$ so that $M \in \tilde{\sigma}_2$. Thus, $c_1 P \cap M \neq \emptyset$. Since $M \in \mu_1$ we have $P \cap M \neq \emptyset$, by Lemma 3. Hence P is ζ-dense. Therefore, (X, ζ) is a hyperconnected space.

Definition 22. *Let (X, μ) be a GTS. A GT μ is said to satisfy the $\mathcal{I}_{\mathcal{D}}$-property if $P \in \tilde{\mu}$ and $c_\mu Q = X$, then $i_\mu(P \cap Q) \neq \emptyset$.*

Theorem 23. *Let (X, μ_1, μ_2) be a bigeneralized topological space and $\eta_1 = \{P \subset X \mid c_{\mu_1} P = X\}; \eta_2 = \{Q \subset X \mid c_{\mu_2} Q = X\}$. Then*
(a) If $\emptyset \neq \zeta = \eta_1 \cup \{\emptyset\}$ and if μ_1 has $\mathcal{I}_{\mathcal{D}}$-property, then (X, ζ) is a hyperconnected space.
(b) If $\emptyset \neq \zeta = \eta_2 \cup \{\emptyset\}$ and if μ_2 has $\mathcal{I}_{\mathcal{D}}$-property, then (X, ζ) is a hyperconnected space.

Proof. (a) Suppose $\emptyset \neq \zeta = \eta_1 \cup \{\emptyset\}$ and if μ_1 has $\mathcal{I}_{\mathcal{D}}$-property. Let $K \in \tilde{\zeta}$. Then $c_{\mu_1} K = X$ and so $K \cap J \neq \emptyset$ for every $J \in \tilde{\mu}_1$. Take $H \in \tilde{\zeta}$ which implies that $H \cap M \neq \emptyset$ for all $M \in \tilde{\mu}_1$. Thus, there is $D \in \tilde{\mu}_1$ such that $K \cap D \neq \emptyset$ and $H \cap D \neq \emptyset$. Since $c_{\mu_1} H = X$ and $D \in \tilde{\mu}_1$ we have $i_{\mu_1}(H \cap D) \neq \emptyset$, by hypothesis. Thus, $i_{\mu_1}(H \cap D) \in \tilde{\mu}_1$ which implies that $K \cap i_{\mu_1}(H \cap D) \neq \emptyset$ which turn implies that $K \cap H \neq \emptyset$. Therefore, K is ζ-dense. Hence (X, ζ) is a hyperconnected space.

(b) Assume that, $\emptyset \neq \zeta = \eta_2 \cup \{\emptyset\}$ and if μ_2 has $\mathcal{I}_{\mathcal{D}}$-property. Let $L \in \tilde{\zeta}$. Then $c_{\mu_2} L = X$ and so $L \cap J \neq \emptyset$ for every $J \in \tilde{\mu}_2$. Take $H \in \tilde{\zeta}$ which implies that $H \cap K \neq \emptyset$ for all $K \in \tilde{\mu}_2$. Thus, there is $D \in \tilde{\mu}_2$ such that $L \cap D \neq \emptyset$ and $H \cap D \neq \emptyset$. Since $c_{\mu_2} H = X$ and $D \in \tilde{\mu}_2$ we have $i_{\mu_2}(H \cap D) \neq \emptyset$, by hypothesis. Thus, $i_{\mu_2}(H \cap D) \in \tilde{\mu}_2$ which implies that $L \cap i_{\mu_2}(H \cap D) \neq \emptyset$ which turn implies that $L \cap H \neq \emptyset$. Therefore, L is ζ-dense. Hence (X, ζ) is a hyperconnected space.

Theorem 24. *Let (X, μ_1, μ_2) be a bigeneralized topological space where $\mu_1 = \mu$ and $\mu_2 = \mu^{\star\star} \neq \emptyset$, μ is a generalized topology on X. Then every $\mu^{\star\star}$-dense set is $(2,1)^{\star}$-dense set in X.*

Proof. Let K be a $\mu^{\star\star}$-dense set. Then $c_2(K) = X$. By hypothesis, μ_2 is a sGT. By Theorem 11, K is a $(2,1)^{\star}$-dense set in X.

Theorem 25. *Let (X, μ_1, μ_2) satisfy the condition; if $P \in \tilde{\mu}_1; Q \in \tilde{\mu}_2$ and $P \cap Q \neq \emptyset$, then $i_{\mu_1}(P \cap Q) \neq \emptyset$ here $\mu_1 = \mu$ and $\mu_2 = \mu^{\star\star} \neq \emptyset$ where μ is a GT on X. Then every $(1,2)^{\star}$-dense set is μ_2-dense set in X.*

Proof. Let $P \in (1,2)^\star - \mathcal{D}(X)$. Then $c_2 P \cap K \neq \emptyset$ for all $K \in \tilde{\sigma}_1$. Let $L \in \tilde{\mu}_2$. Then L is of μ-second category and so L is not a μ-meager set which implies $i_1(c_1(L)) \neq \emptyset$. Take $D = i_1(c_1(L))$. Then $D \in \tilde{\mu}_1$ so that $D \cap c_2 P \neq \emptyset$. Thus, $c_1 L \cap c_2 P \neq \emptyset$. Choose $t \in (c_1 L \cap c_2 P)$. Then $t \in c_1 L$ which implies $H \cap L \neq \emptyset$ for every $H \in \mu_1(t)$, by Lemma 2. By hypothesis, $i_{\mu_1}(H \cap L) \neq \emptyset$. This implies $c_2 P \cap i_{\mu_1}(H \cap L) \neq \emptyset$ which implies $c_2 P \cap (H \cap L) \neq \emptyset$ which turn implies that $c_2 P \cap L \neq \emptyset$. Since $L \in \tilde{\mu}_2$ we have $P \cap L \neq \emptyset$, by Lemma 3. Hence P is μ_2-dense.

Theorem 26. *Let (X, μ_1, μ_2) be a BGTS here $\mu_1 = \mu$ and $\mu_2 = \mu^\star$ where μ is a GT on X. If μ_1 has the $\mathcal{I}$-property, then every $(1,2)^\star$-dense set is μ_2-dense in X.*

Proof. Let $Q \in (1,2)^\star - \mathcal{D}(X)$. Then $c_2 Q \cap K \neq \emptyset$ for every $K \in \tilde{\sigma}_1$. Let $L \in \tilde{\mu}_2$. Then $L = \bigcup_t (L_1^t \cap L_2^t \cap \cdots \cap L_{n_t}^t)$ where each $L_i^t \in \tilde{\mu}_1$ for $i = 1$ to n_t. Choose $D = L_1^k \cap L_2^k \cap \cdots \cap L_{n_k}^k$ for some k ; each $L_m^k \in \tilde{\mu}_1$ for $m = 1$ to n_k with $D \neq \emptyset$. By hypothesis, $i_{\mu_1} D \neq \emptyset$ which implies that $i_{\mu_1} D \in \tilde{\mu}_1$ which turn implies that $i_{\mu_1} D \cap c_2 Q \neq \emptyset$. Thus, $c_2 Q \cap D \neq \emptyset$ so that $c_2 Q \cap L \neq \emptyset$. Since $L \in \tilde{\mu}_2$ we have $Q \cap L \neq \emptyset$, By Lemma 3. Therefore, Q is μ_2-dense.

Theorem 27. *Let (X, μ_1, μ_2) be a BGTS here $\mu_1 = \mu$ and $\mu_2 = \mu^{\star\star}$ where μ is a generalized topology on X. If (X, μ_1) is a hyperconnected space and if μ_1 is a sGT, then every non-null μ_2-open set is $(1,2)^\star$-dense in X.*

Proof. Let $P \in \tilde{\mu}_2$. Then P is of μ_1-second category so that P is not a μ_1-meager set which implies $i_{\mu_1}(c_{\mu_1}(P)) \neq \emptyset$. Thus, $i_{\mu_1}(c_{\mu_1}(P)) \in \tilde{\mu}_1$. Let $K \in \tilde{\sigma}_1$. By hypothesis, $i_{\mu_1} K \in \tilde{\mu}_1$. Since (X, μ_1) is a hyperconnected space we have $i_{\mu_1} K$ is μ_1-dense. Therefore, $i_{\mu_1}(c_{\mu_1}(P)) \cap i_{\mu_1} K \neq \emptyset$. This implies $c_{\mu_1} P \cap i_{\mu_1} K \neq \emptyset$ which implies that $i_{\mu_1} K \cap P \neq \emptyset$, by Lemma 3. Thus, $P \cap K \neq \emptyset$. Therefore, $P \in (1,2)^\star - \mathcal{D}(X)$.

Theorem 28. *Let (X, μ_1, μ_2) be a BGTS here $\mu_1 = \mu$ and $\mu_2 = \mu^\star$ where μ is a sGT on X. If (X, μ_1) is a hyperconnected space and if μ_1 has $\mathcal{I}$-property, then every non-null μ_2-open set is a $(1,2)^\star$-dense set in X.*

Proof. Let $P \in \tilde{\mu}^\star$. Then $P = \bigcup_t (P_1^t \cap P_2^t \cap \cdots \cap P_{n_t}^t)$ where each $P_i^t \in \tilde{\mu}_1$ for $i = 1$ to n_t. Choose $D = P_1^k \cap P_2^k \cap \cdots \cap P_{n_k}^k$ for some k ; each $P_m^k \in \tilde{\mu}_1$ for $m = 1$ to n_k with $D \neq \emptyset$. By hypothesis, $i_{\mu_1} D \neq \emptyset$ which implies that $i_{\mu_1} D \in \tilde{\mu}_1$. Since (X, μ_1) is a hyperconnected space, $i_{\mu_1} D$ is μ_1-dense which implies P is μ_1-dense. By hypothesis and Theorem 11, P is $(1,2)^\star$-dense.

Theorem 29. *Let (X, μ_1, μ_2) be a bigeneralized topological space. If (X, μ_s) is a hyperconnected space and if μ_s is a sGT for $s = 1, 2$, then*

(a) Every non-null μ_s-semi-open set is $(s,v)^\star$-dense.
(b) Every non-null μ_s-pre-open set is $(s,v)^\star$-dense.
(c) Every non-null μ_s-α-open set is $(s,v)^\star$-dense.
(d) Every non-null μ_s-β-open set is $(s,v)^\star$-dense.

(e) Every non-null μ_s-b-open set is $(s,v)^\star$-dense where $s,v=1,2$; $s\neq v$.

Proof. Assume that, (X,μ_s) is a hyperconnected space and μ_s is a sGT for $s=1,2$.. Choose $s=2$ and $v=1$. Then (X,μ_2) is a hyperconnected space, μ_2 is a strong generalized topology.

(a). Let Q be a non-null μ_s-semi-open set. Then Q is μ_2-semi-open set in X. Let $H\in\tilde{\sigma}_2$. Suppose $H\in\tilde{\mu}_2$. Then there is nothing to prove. Suppose that, $H\notin\tilde{\mu}_2$. Here $H\subset c_2(i_2(H))$ which implies $i_2(H)\in\tilde{\mu}_2$, by our assumption. Since (X,μ_2) is a hyperconnected space we have i_2H is a μ_2-dense set in X. Also, $Q\subset c_2(i_2(Q))$ which implies $i_2(Q)\in\tilde{\mu}_2$ which turn implies that $i_2(Q)\cap i_2H\neq\emptyset$. Thus, $Q\cap i_2H\neq\emptyset$. Therefore, $c_1Q\cap H\neq\emptyset$. Hence $Q\in(2,1)^\star-\mathcal{D}(X)$.

(b). Let P be a non-null μ_s-preopen set. Then P is μ_2-preopen set in X. Let $G\in\tilde{\sigma}_2$. If $G\in\tilde{\mu}_2$, then the proof is trivial. Assume that, $G\notin\tilde{\mu}_2$. Here $G\subset c_2(i_2(G))$ which implies $i_2(G)\in\tilde{\mu}_2$, by our assumption which turn implies that i_2G is a μ_2-dense set in X. Also, $P\subset i_2(c_2(P))$ so that $i_2(c_2(P))\in\tilde{\mu}_2$ for that $i_2(c_2(P))\cap i_2G\neq\emptyset$. Thus, $c_2P\cap i_2G\neq\emptyset$ so that $P\cap i_2G\neq\emptyset$, by Lemma 3. Therefore, $c_1P\cap G\neq\emptyset$. Hence $P\in(2,1)^\star-\mathcal{D}(X)$.

(c). Let K be a non-null μ_s-α-open set. Then K is μ_2-α-open set in X which implies K is μ_2-semi-open set. Hence $K\in(2,1)^\star-\mathcal{D}(X)$, by (a).

(d). Choose L be a non-null μ_s-β-open set. Then L is μ_2-β-open set in X. Let $M\in\tilde{\sigma}_2$. If $M\in\tilde{\mu}_2$, then there is nothing to prove. Suppose $M\notin\tilde{\mu}_2$. Since $M\subset c_2(i_2(M))$ we have $i_2(M)\in\tilde{\mu}_2$, by our assumption. By our assumption, i_2M is a μ_2-dense set in X. Also, $L\subset c_2(i_2(c_2(L)))$ for that $i_2(c_2(L))\in\tilde{\mu}_2$ which turn implies that $i_2(c_2(L))\cap i_2M\neq\emptyset$. Thus, $c_2L\cap i_2M\neq\emptyset$ so that $L\cap i_2M\neq\emptyset$, by Lemma 3. Therefore, $c_1L\cap M\neq\emptyset$. Hence $L\in(2,1)^\star-\mathcal{D}(X)$.

(e). Take F be a non-null μ_s-b-open set. We get F is μ_2-b-open set in X. Let $V\in\tilde{\sigma}_2$. If $V\in\tilde{\mu}_2$, then the proof is obvious. Assume that, $V\notin\tilde{\mu}_2$ then $V\subset c_2(i_2(V))$ so that $i_2(V)\in\tilde{\mu}_2$, by our assumption. Thus, i_2V is a μ_2-dense set in X. Here, $F\subset c_2(i_2(F))\cup i_2(c_2(F))$ which implies

$$(1)\quad i_2(c_2(F))\in\tilde{\mu}_2$$

or

$$(2)\quad i_2(F)\in\tilde{\mu}_2$$

or

$$(3)\quad i_2(c_2(F))\in\tilde{\mu}_2\quad and\quad i_2(F)\in\tilde{\mu}_2$$

From the above three cases, we get $F\cap i_2V\neq\emptyset$. Therefore, $c_1F\cap V\neq\emptyset$. Hence $F\in(2,1)^\star-\mathcal{D}(X)$.

By similar considerations, we can prove this theorem for the case $s=1$ and $v=2$.

Theorem 30. *Let (X, μ_1, μ_2) be a BGTS. If (X, μ_s) is a hyperconnected space and if μ_s is a sGT for $s = 1, 2$, then*
(a) Every non-null (s, v)-μ-pre-open set is $(s, v)^\star$-dense.
(b) Every non-null (s, v)-μ-α-open set is $(s, v)^\star$-dense where $s, v = 1, 2$; $s \neq v$.

Proof. Assume that, (X, μ_s) is a hyperconnected space and μ_s is a strong generalized topological space for $s = 1, 2$. Take $s = 2$ and $v = 1$. Then (X, μ_2) is a hyperconnected space, μ_2 is a sGT.

(a). Let Q be a non-null (s, v)-μ-pre-open set where $s, v = 1, 2$; $s \neq v$. Then Q is $(2, 1)$-μ-pre-open. Let $K \in \tilde{\sigma}_2$. If $K \in \tilde{\mu}_2$, then there is nothing to prove. Assume that, $K \notin \tilde{\mu}_2$. Here $K \subset c_2(i_2(K))$. By our assumption, i_2K is μ_2-dense. Since $Q \subset i_2(c_1(Q))$ we have $i_2(c_1(Q)) \in \tilde{\mu}_2$. This implies $i_2(c_1(Q)) \cap i_2K \neq \emptyset$ which implies $c_1(Q) \cap i_2K \neq \emptyset$ which turn implies that $c_1Q \cap K \neq \emptyset$. Hence $Q \in (2, 1)^\star - \mathcal{D}(X)$.

(b). Take P be a non-null (s, v)-μ-α-open set where $s, v = 1, 2$; $s \neq v$. We get P is $(2, 1)$-μ-α-open. Let $G \in \tilde{\sigma}_2$. If $G \in \tilde{\mu}_2$, then the proof is trivial. Suppose $G \notin \tilde{\mu}_2$. By our assumption, i_2G is μ_2-dense. Since $P \subset i_2(c_1(i_2(P)))$ we have $i_2(c_1(i_2(P))) \in \tilde{\mu}_2$. This implies $i_2(c_1(i_2(P))) \cap i_2G \neq \emptyset$ which implies $c_1(i_2(P)) \cap i_2G \neq \emptyset$ which turn implies that $c_1P \cap i_2G \neq \emptyset$. Thus, $c_1P \cap G \neq \emptyset$. Hence $P \in (2, 1)^\star - \mathcal{D}(X)$.

Similarly we can prove this theorem for the case $s = 1$ and $v = 2$.

Theorem 31. *Let (X, μ_1, μ_2) be a BGTS. If (X, μ_s) is a hyperconnected space, $\mu_s \subset \mu_v$ and if μ_s is a strong generalized topology, then every non-null (s, v)-μ-semi-open set is $(s, v)^\star$-dense where $s, v = 1, 2$; $s \neq v$.*

Proof. Assume that, (X, μ_s) is a hyperconnected space; $\mu_s \subset \mu_v$ and μ_s is a strong generalized topological space for $s = 1, 2$.

Take $s = 1$ and $v = 2$. Then (X, μ_1) is a hyperconnected space; $\mu_1 \subset \mu_2$ and μ_1 is a sGT.

Let Q be a non-null (s, v)-μ-semi-open set where $s, v = 1, 2$; $s \neq v$. Then Q is $(1, 2)$-μ-semi-open. Let $H \in \tilde{\sigma}_1$. Suppose $H \in \tilde{\mu}_1$, then there is nothing to prove. Assume that, $H \notin \tilde{\mu}_1$. Here $H \subset c_1(i_1(H))$. By our assumption, i_1H is μ_1-dense. Since $Q \subset c_2(i_1(Q)))$ we have $i_1(Q) \in \tilde{\mu}_1$, $\mu_1 \subset \mu_2$ and μ_1 is a sGT. This implies $i_1(Q) \cap i_1H \neq \emptyset$ which implies $c_2Q \cap H \neq \emptyset$. Hence $Q \in (1, 2)^\star - \mathcal{D}(X)$.

Choose $s = 2$ and $v = 1$. We get (X, μ_2) is a hyperconnected space; $\mu_2 \subset \mu_1$ and μ_2 is a sGT.

Consider P is a non-null (s, v)-μ-semi-open set where $s, v = 1, 2$; $s \neq v$. Then P is $(2, 1)$-μ-semi-open. Let $G \in \tilde{\sigma}_2$. If $G \in \tilde{\mu}_2$, then the proof is obvious. Suppose $G \notin \tilde{\mu}_2$.

Here $G \subset c_2(i_2(G))$. By hypothesis, i_2G is μ_2-dense. Since $P \subset c_1(i_2(P)))$ we have $i_2(P) \in \tilde{\mu}_2$, by hypothesis so that $i_2(P) \cap i_2G \neq \emptyset$ which implies that $c_1P \cap G \neq \emptyset$. Hence $P \in (2,1)^\star - \mathcal{D}(X)$.

In the rest of this section, we analyze the nature of $(s,v)^\star$-dense sets in a subspace.

Let (X,μ) be a GTS, $Q \subset X$ and $\mu_Q = \{P \cap Q \mid P \in \mu\}$. Then μ_Q is called relative generalized topology on Q [7].

Theorem 32. *Let (X,μ_1,μ_2) be a bigeneralized topological space, Q be a μ_s-dense subspace of X for $s = 1,2$. If P is a μ_{s_Q}-dense and μ_s is a sGT, then $P \in (s,v)^\star - \mathcal{D}(X)$ where $s,v = 1,2$; $s \neq v$.*

Proof. *Assume that, Q is μ_s-dense in X and μ_s is a strong generalized topology for $s = 1,2$. Let P be a μ_{s_Q}-dense set in Q where $s = 1,2$.*

Take $s = 1$ and $v = 2$. Then Q is μ_1-dense, μ_1 is a sGT and P is μ_{1_Q}-dense in Q. Let $K \in \tilde{\sigma}_1$. If $K \in \tilde{\mu}_1$, then further proof investigation no longer required. Suppose $K \notin \tilde{\mu}_1$. By hypothesis, $i_{\mu_1}K \in \tilde{\mu}_1$ which implies $i_{\mu_1}K \cap Q \in \tilde{\mu_{1_Q}}$. Take $L = i_{\mu_1}K \cap Q$. Then $L \cap P \neq \emptyset$ so that $i_{\mu_1}K \cap P \neq \emptyset$. This implies $K \cap P \neq \emptyset$ which implies $K \cap c_2P \neq \emptyset$. Therefore, $P \in (1,2)^\star - \mathcal{D}(X)$.

Fix $s = 2, v = 1$. Then Q is μ_2-dense, μ_2 is a sGT and P is μ_{2_Q}-dense in Q. Let $M \in \tilde{\sigma}_2$. If $M \in \tilde{\mu}_2$, then the proof is directly follows. Assume that, $M \notin \tilde{\mu}_2$. By hypothesis, $i_{\mu_2}M \in \tilde{\mu}_2$ which implies $i_{\mu_2}M \cap Q \in \tilde{\mu_{2_Q}}$. Take $V = i_{\mu_2}M \cap Q$. Then $V \cap P \neq \emptyset$ so that $i_{\mu_2}M \cap P \neq \emptyset$ which implies $M \cap P \neq \emptyset$ which turn implies that $M \cap c_2P \neq \emptyset$. Hence, $P \in (2,1)^\star - \mathcal{D}(X)$.

Theorem 33. *Let (X,μ_1,μ_2) be a BGTS, μ_s satisfy the $\mathcal{I}$-property and Q be a μ_s-open subset of X for $s = 1,2$. If $\mu_s \subset \mu_v$ and if $P \in (s,v)^\star - \mathcal{D}(X)$, then P is μ_{s_Q}-dense set in Q where $P \subset Q$; $s,v = 1,2$; $s \neq v$.*

Proof. *Assume that, Q is μ_s-open subset of $X; \mu_s \subset \mu_v$ and $P \in (s,v)^\star - \mathcal{D}(X)$ for $s,v = 1,2$; $s \neq v$.*

Choose $s = 1$ and $v = 2$. Then $Q \in \tilde{\mu}_1$; $\mu_1 \subset \mu_2$ and $P \in (1,2)^\star - \mathcal{D}(X)$. Let $L \in \tilde{\mu_{1_Q}}$. Then $L = K \cap Q$ where $K \in \tilde{\mu}_1$. By hypothesis, $i_{\mu_1}L \in \tilde{\mu}_1$. This implies $L \cap c_2P \neq \emptyset$ which implies that $L \cap P \neq \emptyset$, by Lemma 3. Hence P is a μ_{1_Q}-dense set in Q.

Fix $s = 2$ and $v = 1$. We get $Q \in \tilde{\mu}_2$; $\mu_2 \subset \mu_1$ and $P \in (2,1)^\star - \mathcal{D}(X)$. Let $V \in \tilde{\mu_{2_Q}}$. Then $V = M \cap Q$ where $M \in \tilde{\mu}_2$. By assumption, $i_{\mu_2}V \in \tilde{\mu}_2$ so that $V \cap c_1P \neq \emptyset$ which implies that $V \cap P \neq \emptyset$, by Lemma 3. Therefore, P is a μ_{2_Q}-dense set in Q.

Theorem 34. *Let (X, μ_1, μ_2) be a bigeneralized topological space, Q be a μ_s-open subset of X and μ_s satisfy the $\mathcal{I}$-property for $s = 1, 2$. If μ_{s_Q} is a sGT and $P \in (s, v)^\star - \mathcal{D}(X)$, then $P \in (\mu_{s_Q}, \mu_v) - \mathcal{D}(Q)$ where $P \subset Q$; $s, v = 1, 2$; $s \neq v$.*

Proof. *Suppose that, $Q \in \tilde{\mu}_s$, μ_s satisfy the $\mathcal{I}$-property and μ_{s_Q} is a strong generalized topology for $s = 1, 2$. Let $P \in (s, v)^\star - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$.*

Choose $s = 1$ and $v = 2$. Then $Q \in \tilde{\mu}_1$, μ_1 satisfy the $\mathcal{I}$-property, μ_{1_Q} is a sGT and $P \in (1, 2)^\star - \mathcal{D}(X)$. Let $J \in \tilde{\sigma}_{1_Q}$. If $J \in \tilde{\mu}_{1_Q}$, then there is nothing to prove. Suppose $J \notin \tilde{\mu}_{1_Q}$. Since $J \in \tilde{\sigma}_{1_Q}$ and μ_{1_Q} is a strong subspace generalized topology we have $i_{1_Q} J \in \tilde{\mu}_{1_Q}$. Take $K = i_{1_Q} J$. Then $K \neq \emptyset$ and $K = L \cap Q$ where $L \in \tilde{\mu}_1$. Since $L, Q \in \tilde{\mu}_1$ and μ_1 satisfy the $\mathcal{I}$-property, $i_{\mu_1}(K) \in \tilde{\mu}_1$. This implies $i_{\mu_1} K \cap c_2 P \neq \emptyset$ which implies $K \cap c_2 P \neq \emptyset$ which turn implies that $J \cap c_2 P \neq \emptyset$. Hence $P \in (\mu_{1_Q}, \mu_2)^\star - \mathcal{D}(Q)$.

Take $s = 2$ and $v = 1$. We get $Q \in \tilde{\mu}_2$, μ_2 satisfy the $\mathcal{I}$-property, μ_{2_Q} is a sGT and $P \in (2, 1)^\star - \mathcal{D}(X)$. Let $V \in \tilde{\sigma}_{2_Q}$. If $V \in \tilde{\mu}_{2_Q}$, then the proof is obvious. Assume $V \notin \tilde{\mu}_{2_Q}$. by the definition of V and μ_{2_Q} is a strong subspace generalized topology we have $i_{2_Q} V \in \tilde{\mu}_{2_Q}$. Take $L = i_{2_Q} V$. Then $L \neq \emptyset$ and $L = M \cap Q$ where $M \in \tilde{\mu}_2$. Here, $M, Q \in \tilde{\mu}_2$ and μ_2 satisfy the $\mathcal{I}$-property, $i_{\mu_2}(L) \in \tilde{\mu}_2$ so that $i_{\mu_2} L \cap c_1 P \neq \emptyset$ which implies $L \cap c_1 P \neq \emptyset$ which turn implies that $V \cap c_1 P \neq \emptyset$. Therefore, $P \in (\mu_{2_Q}, \mu_1)^\star - \mathcal{D}(Q)$.

Theorem 35. *Let (X, μ_1, μ_2) be a BGTS and Q be a μ_s-dense subset of X for $s = 1, 2$. If μ_s is a strong generalized topology and if $P \in (\mu_{s_Q}, \mu_{v_Q})^\star - \mathcal{D}(Q)$, then $P \in (s, v)^\star - \mathcal{D}(X)$ for $s, v = 1, 2$; $s \neq v$.*

Proof. *Assume that, $P \in (\mu_{s_Q}, \mu_{v_Q})^\star - \mathcal{D}(Q)$ where $s, v = 1, 2$; $s \neq v$.*

Choose $s = 1$ and $v = 2$. Then $P \in (\mu_{1_Q}, \mu_{2_Q})^\star - \mathcal{D}(Q)$. Let $H \in \tilde{\sigma}_1$. Suppose $H \in \tilde{\mu}_1$. Then $H \cap Q \in \tilde{\mu}_{1_Q}$. Take $K = H \cap Q$. Then $K \cap c_{2_Q} P \neq \emptyset$ so that $K \cap c_2 P \neq \emptyset$. This implies $H \cap c_2(P) \neq \emptyset$ which implies that $P \in (1, 2)^\star - \mathcal{D}(X)$. If $H \notin \tilde{\mu}_1$, then $i_1 H \in \tilde{\mu}_1$. Take $L = i_1 H$. Then by similar arguments in the above case, we get $P \in (1, 2)^\star - \mathcal{D}(X)$.

Fix $s = 2$ and $v = 1$. We get $P \in (\mu_{2_Q}, \mu_{1_Q})^\star - \mathcal{D}(Q)$. Let $G \in \tilde{\sigma}_2$. Suppose $G \in \tilde{\mu}_2$ we get $G \cap Q \in \tilde{\mu}_{2_Q}$. Choose $K = G \cap Q$ so that $K \cap c_{1_Q} P \neq \emptyset$ which implies that $K \cap c_1 P \neq \emptyset$. Thus, $G \cap c_1(P) \neq \emptyset$ so that $P \in (2, 1)^\star - \mathcal{D}(X)$. Assume that, $G \notin \tilde{\mu}_2$, then $i_2 G \in \tilde{\mu}_2$. Take $L = i_2 G$. By similar considerations, we get $P \in (2, 1)^\star - \mathcal{D}(X)$.

4. Images of $(s, v)^\star$-dense sets

A function $f : (X, \mu) \to (Y, \eta)$ is said to be (μ, η)-*continuous* [4] (resp. (μ, η)-*open*) [18] if $f^{-1}(Q) \in \mu$ whenever $Q \in \eta$ (resp. $f(P) \in \eta$ whenever $P \in \mu$).

Lemma 5. [12, Lemma 7.3] A map $f : (X, \mu) \to (Y, \eta)$ is (μ, η)-open if and only if $f^{-1}(cP) \subset c(f^{-1}(P))$ for any $P \subset Y$.

Theorem 36. *Let (X, μ_1, μ_2) and (Y, η_1, η_2) be two BGTSs. If $f : X \to Y$ is (μ_t, η_t)-continuous for $t = 1, 2$ and η_s is sGT for $s = 1, 2$, then image of a $(s, v)^\star$-dense set is $(s, v)^\star$-dense where $s, v = 1, 2$; $s \neq v$.*

Proof. *Assume that, f is (μ_t, η_t)-continuous for $t = 1, 2$. Let $Q \in (s, v)^\star - \mathcal{D}(X)$ where $s, v = 1, 2$; $s \neq v$.*

Fix $s = 1$ and $v = 2$. We get $Q \in (1, 2)^\star - \mathcal{D}(X)$ so that $c_{\mu_2}Q \cap H \neq \emptyset$ for $H \in \tilde{\sigma}_{\mu_1}$. Let $K \in \tilde{\sigma}_{\eta_1}$. By assumption, η_1 is a sGT so that $i_{\eta_1}K \in \tilde{\eta_1}$. This implies $f^{-1}(i_{\eta_1}K) \in \tilde{\mu}_1$, by hypothesis which implies that $c_{\mu_2}Q \cap f^{-1}(i_{\eta_1}K) \neq \emptyset$. Thus, $f(c_{\mu_2}Q \cap f^{-1}(i_{\eta_1}K)) \neq \emptyset$ so that $f(c_{\mu_2}Q) \cap i_{\eta_1}K \neq \emptyset$. Since f is (μ_1, η_1)-continuous we have $c_{\eta_2}(f(Q)) \cap i_{\eta_1}K \neq \emptyset$. Therefore, $f(Q) \in (1, 2)^\star - \mathcal{D}(Y)$.

Take $s = 2$ and $v = 1$. Then $Q \in (2, 1)^\star - \mathcal{D}(X)$ and so $c_{\mu_1}Q \cap M \neq \emptyset$ for $M \in \tilde{\sigma}_{\mu_2}$. Choose $L \in \tilde{\sigma}_{\eta_2}$. By hypothesis, η_2 is a sGT so that $i_{\eta_2}L \in \tilde{\eta_2}$ which implies $f^{-1}(i_{\eta_2}L) \in \tilde{\mu}_2$, by assumption which turn implies that $c_{\mu_1}Q \cap f^{-1}(i_{\eta_2}L) \neq \emptyset$. Thus, $f(c_{\mu_1}Q \cap f^{-1}(i_{\eta_2}L)) \neq \emptyset$ for that $f(c_{\mu_1}Q) \cap i_{\eta_2}L \neq \emptyset$. By hypothesis, $c_{\eta_1}(f(Q)) \cap i_{\eta_2}L \neq \emptyset$. Hence $f(Q) \in (2, 1)^\star - \mathcal{D}(Y)$.

Theorem 37. *Let (X, μ_1, μ_2) and (Y, η_1, η_2) be two bigeneralized topological spaces. If $f : X \to Y$ is (μ_t, η_t)-open for $t = 1, 2$; one-one map and μ_s is sGT for $s = 1, 2$, then inverse image of a $(s, v)^\star$-dense set is $(s, v)^\star$-dense.*

Proof. *Let $P \in (s, v)^\star - \mathcal{D}(Y)$ for $s, v = 1, 2$; $s \neq v$.*

Fix $s = 1$ and $v = 2$. Then $P \in (1, 2)^\star - \mathcal{D}(Y)$ so that $c_{\eta_2}P \cap L \neq \emptyset$ for all $L \in \tilde{\sigma}_{\eta_1}$. Let $D \in \tilde{\sigma}_{\mu_1}$ so that $i_{\mu_1}D \in \tilde{\mu}_1$, by assumption. Since f is (μ_1, η_1)-open we have $f(i_{\mu_1}D) \in \tilde{\eta_1}$. This implies $c_{\eta_2}P \cap f(i_{\mu_1}D) \neq \emptyset$ which implies that $f^{-1}(c_{\eta_2}P) \cap f^{-1}(f(i_{\mu_1}D)) \neq \emptyset$. Here f is an injective map, $f^{-1}(c_{\eta_2}P) \cap i_{\mu_1}D \neq \emptyset$. By Lemma 5, $c_{\mu_2}(f^{-1}(P)) \cap \imath_{\mu_1}D \neq \emptyset$. Hence $f^{-1}(P) \in (1, 2)^\star\mathcal{D}(X)$.

Choose $s = 2$ and $v = 1$. We get $P \in (2, 1)^\star - \mathcal{D}(Y)$ implies that $c_{\eta_1}P \cap M \neq \emptyset$ for all $M \in \tilde{\sigma}_{\eta_2}$. Choose $V \in \tilde{\sigma}_{\mu_2}$ so that $i_{\mu_2}V \in \tilde{\mu}_2$, by hypothesis which implies $f(i_{\mu_2}V) \in \tilde{\eta_2}$. Thus, $c_{\eta_1}P \cap f(i_{\mu_2}V) \neq \emptyset$ so that $f^{-1}(c_{\eta_1}P) \cap f^{-1}(f(i_{\mu_2}V)) \neq \emptyset$. Since f is an injective map, $f^{-1}(c_{\eta_1}P) \cap i_{\mu_2}V \neq \emptyset$. By Lemma 5, $c_{\mu_1}(f^{-1}(P)) \cap \imath_{\mu_2}V \neq \emptyset$. Therefore, $f^{-1}(P) \in (2, 1)^\star - \mathcal{D}(X)$.

5. Applications for $(s, v)^\star$-dense sets

In 1999, Molodstov introduced a new mathematical tool namely, soft set theory [14]. It has been used for dealing with uncertainty. Most of the researchers presented an application of soft sets in decision-making problems.

Motivated, by this we try to give an example of the soft set using $(s, v)^{\star}$-dense and some subsets defined in a bigeneralized topological space and also in generalized topological space.

Example 38. Consider the BGTS (X, μ_1, μ_2) where $X = \{a, b, c, d\}$;

$$\mu_1 = \{\emptyset, \{b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\};$$

and

$$\mu_2 = \{\emptyset, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}.$$

Here,
- $\sigma_1 = \{\emptyset, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$.
- $\sigma_2 = \{\emptyset, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$.

Then we get,
- $(1, 2)^{\star} - \mathcal{D}(X) = \{\{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, X\}$.
- $(2, 1)^{\star} - \mathcal{D}(X) = \{\{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$.

Let $U = \{a, c, d\}$ be a subset of X and $E = \{(1, 2)^{\star}$-dense set, $(2, 1)^{\star}$-dense set, $(1, 2)^{\star}$-dense but not $(2, 1)^{\star}$-dense, $(2, 1)^{\star}$-dense but not $(1, 2)^{\star}$-dense, $(1, 2)^{\star}$-dense and $(2, 1)^{\star}$-dense $\} = \{e_1, e_2, e_3, e_4, e_5\}$ is the set of parameters. Define a map F from E to $exp(U)$ by, $F(e_1) = \{a, c\}; F(e_2) = \{d\}; F(e_3) = \{a, c\}; F(e_4) = \{c, d\}, F(e_5) = \{a, c, d\}$. Then the pair (F, E) is a soft set over U.

Example 39. Consider the BGTS (X, μ_1, μ_2) where $X = \{p, q, r, s\}$;

$$\mu_1 = \{\emptyset, \{p\}, \{p, s\}, \{q, s\}, \{p, q, s\}\}$$

and

$$\mu_2 = \{\emptyset, \{q\}, \{p, r\}, \{q, r\}, \{p, q, r\}\}.$$

Here,
- μ_1-semi-open sets $= \{\emptyset, \{p\}, \{r\}, \{p, r\}, \{p, s\}, \{q, s\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}, X\}$.
- μ_1-pre-open sets $= \{\emptyset, \{p\}, \{s\}, \{p, q\}, \{p, s\}, \{q, s\}, \{p, q, s\}\}$.
- $\mu_1 - \alpha$-open sets $= \{\emptyset, \{p\}, \{p, s\}, \{q, s\}, \{p, q, s\}\}$.
- $\mu_1 - \beta$-open sets $= exp(X) - \{\{q\}, \{q, r\}\}$.
- $\mu_1 - b$-open sets $= exp(X) - \{q\}$.

Let $U = \{q, r, s\}$ be a subset of X and $E = \{\mu_1$-semi-open set, μ_1-pre-open set, $\mu_1 - \alpha$-open set, $\mu_1 - \beta$-open set, $\mu_1 - b$-open set $\} = \{e_1, e_2, e_3, e_4, e_5\}$ is the set of parameters. Define a function F from a set E to $exp(U)$ by, $F(e_1) = \{r\}; F(e_2) = \{s\}; F(e_3) = \{q, s\}; F(e_4) =$

$\{r,s\}; F(e_5) = \{q,r\}$. Then the pair (F,E) is a soft set over U.

Here,
- μ_2-semi-open sets $= \{\emptyset, \{q\}, \{s\}, \{p,r\}, \{q,r\}, \{q,s\}, \{p,q,r\}, \{p,r,s\}, \{q,r,s\}, X\}$.
- μ_2-pre-open sets $= \{\emptyset, \{q\}, \{r\}, \{p,q\}, \{p,r\}, \{q,r\}, \{p,q,r\}\}$.
- $\mu_2 - \alpha$-open sets $= \{\emptyset, \{q\}, \{p,r\}, \{q,r\}, \{p,q,r\}\}$.
- $\mu_2 - \beta$-open sets $= exp(X) - \{\{p\}, \{p,s\}\}$.
- $\mu_2 - b$-open sets $= exp(X) - \{\{p\}, \{p,s\}\}$.

Let $U = \{p,r,s\}$ be a subset of X and $E = \{\mu_2$-semi-open set, μ_2-pre-open set, $\mu_2 - \alpha$-open set, $\mu_2 - \beta$-open set, $\mu_2 - b$-open set $\} = \{e_1, e_2, e_3, e_4, e_5\}$ is the set of parameters. Define a function F from a set E to $exp(U)$ by, $F(e_1) = \{s\}; F(e_2) = \{r\}; F(e_3) = \{q\}; F(e_4) = \{r,s\}; F(e_5) = \{p,r\}$. Then the pair (F,E) is a soft set over U.

Example 40. Consider the BGTS (X, μ_1, μ_2) where $X = \{p,q,r,s\}$;

$$\mu_1 = \{\emptyset, \{r\}, \{p,s\}, \{r,s\}, \{p,r,s\}\}$$

and

$$\mu_2 = \{\emptyset, \{q\}, \{q,s\}, \{r,s\}, \{q,r,s\}\}.$$

Here,
- (s,v)-μ_1-regular open sets $= \{\emptyset, \{r\}, \{p,r,s\}\}$.
- (s,v)-μ_1-semi-open sets $= \{\emptyset, \{p\}, \{r\}, \{p,r\}, \{p,s\}, \{r,s\}, \{p,r,s\}\}$.
- (s,v)-μ_1-pre-open sets $= \{\emptyset, \{r\}, \{s\}, \{p,s\}, \{r,s\}, \{p,r,s\},\}$.
- (s,v)-μ_1-α-open sets $= \{\emptyset, \{r\}, \{p,s\}, \{r,s\}, \{p,r,s\}\}$.

Let $U = \{p,r,s\}$ be a subset of X and $E = \{(s,v)$-μ_1-regular open, (s,v)-μ_1-semi-open set, (s,v)-μ_1-pre-open set, (s,v)-$\mu_1 - \alpha$-open set $\} = \{e_1, e_2, e_3, e_4,\}$ is the set of parameters. Define a map F from a non-null set E to $exp(U)$ by, $F(e_1) = \{r\}; F(e_2) = \{p\}; F(e_3) = \{s\}; F(e_4) = \{r,s\}$. Then the pair (F,E) is a soft set over U.

Now,
- (s,v)-μ_2-regular open sets $= \{\{q\}, \{q,s\}, \{q,r,s\}\}$.
- (s,v)-μ_2-semi-open sets $= \{\emptyset, \{q\}, \{q,s\}, \{r,s\}, \{p,q,s\}, \{p,r,s\}, \{q,r,s\}, X\}$.
- (s,v)-μ_2-pre-open sets $= \{\emptyset, \{q\}, \{s\}, \{q,s\}, \{r,s\}, \{q,r,s\},\}$.
- (s,v)-μ_2-α-open sets $= \{\emptyset, \{q\}, \{q,s\}, \{r,s\}, \{q,r,s\}\}$.

Let $U = \{q,r,s\}$ be a subset of X and $E = \{(s,v)$-μ_2-regular open, (s,v)-μ_2-semi-open set, (s,v)-μ_2-pre-open set, (s,v)-$\mu_2 - \alpha$-open set $\} = \{e_1, e_2, e_3, e_4,\}$ is the set of parameters. Define a map F from a set E to $exp(U)$ by, $F(e_1) = \{q\}; F(e_2) = \{q,s\}; F(e_3) = \{s\}; F(e_4) = \{r,s\}$. Then the pair (F,E) is a soft set over U.

6. Conclusion

In this article, we are given additional tricks for finding the significance of a given set in a bigeneralized topological space. Also, we have proven some results for checking whether the given set is $(s,v)^{\star}$-dense or not. Finally, we defined soft sets using various open sets and $(s,v)^{\star}$-dense sets.

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