



Properties of weakly $\beta(\Lambda, p)$ -open functions and weakly $\beta(\Lambda, p)$ -closed functions

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Abstract. This paper deals with the concepts of weakly $\beta(\Lambda, p)$ -open functions and weakly $\beta(\Lambda, p)$ -closed functions. Moreover, some properties of weakly $\beta(\Lambda, p)$ -open functions and weakly $\beta(\Lambda, p)$ -closed functions are investigated.

2020 Mathematics Subject Classifications: 54A05; 54C10

Key Words and Phrases: Weakly $\beta(\Lambda, p)$ -open function, weakly $\beta(\Lambda, p)$ -closed function

1. Introduction

It is well-known that the branch of mathematics called topology is related to all questions directly or indirectly concerned with openness and closedness. Semi-open sets, pre-open sets, α -open sets, β -open sets, b -open sets, δ -open sets and θ -open sets play an important role in the researches of generalizations of open functions and closed functions. By using these sets, many authors introduced and studied various types of open functions and closed functions. The concept of weakly open functions was first introduced by Rose [15]. Rose and Janković [16] investigated some of the fundamental properties of weakly closed functions. Caldas and Navalagi [7] introduced two new classes of functions called weakly preopen functions and weakly preclosed functions as a generalization of weak openness and weak closedness due to [15] and [16], respectively. Moreover, Caldas and Navalagi [8] introduced and investigated the concepts of weakly semi-open functions and weakly semi-closed functions as a new generalization of weakly open functions and weakly closed functions, respectively. Noiri et al. [14] introduced and studied two new classes of functions called weakly b - θ -open functions and weakly b - θ -open functions by utilizing the notions of b - θ -open sets and the b - θ -closure operator. Weak b - θ -openness (resp. b - θ -closedness) is a generalization of both θ -preopenness and weak semi- θ -openness (resp. θ -preclosedness and weak semi- θ -closedness). Caldas and Navalagi [6] introduced and investigated the notions of weakly β -open functions and weakly β -closed functions.

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DOI: <https://doi.org/10.29020/nybg.ejpam.v17i1.4974>

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The concepts of (Λ, s) -open sets, $s(\Lambda, s)$ -open sets, $p(\Lambda, s)$ -open sets, $\alpha(\Lambda, s)$ -open sets, $\beta(\Lambda, s)$ -open sets and $b(\Lambda, s)$ -open sets were studied in [5]. In [1], the present authors investigated some properties of (Λ, sp) -open sets, $s(\Lambda, sp)$ -open sets, $p(\Lambda, sp)$ -open sets, $\alpha(\Lambda, sp)$ -open sets, $\beta(\Lambda, sp)$ -open sets and $b(\Lambda, sp)$ -open sets. Srisarakham and Boonpok [17] studied several properties of $\delta p(\Lambda, s)$ -closed sets and the $\delta p(\Lambda, s)$ -closure operator. Khampakdee and Boonpok [11] introduced and investigated the concept of (Λ, p) -closed functions. The notion of weakly $b(\Lambda, p)$ -open functions was studied by Chutiman and Boonpok [9]. Some characterizations of weakly $\delta(\Lambda, p)$ -open functions and weakly $\delta(\Lambda, p)$ -closed functions were presented in [18] and [12], respectively. Moreover, several characterizations $\theta p(\Lambda, p)$ -open functions and $\theta p(\Lambda, p)$ -closed functions were established in [2]. In [3], the authors introduced and investigated the concepts of weakly $p(\Lambda, p)$ -open functions and weakly $p(\Lambda, p)$ -closed functions. In this paper, we introduce the notions of weakly $\beta(\Lambda, p)$ -open functions and weakly $\beta(\Lambda, p)$ -closed functions. Furthermore, some characterizations of weakly $\beta(\Lambda, p)$ -open functions and weakly $\beta(\Lambda, p)$ -closed functions are discussed.

2. Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ) , $\text{Cl}(A)$ and $\text{Int}(A)$, represent the closure and the interior of A , respectively. A subset A of a topological space (X, τ) is said to be *preopen* [13] if $A \subseteq \text{Int}(\text{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [10] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p -set [4] (*pre- Λ -set* [10]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p) -closed [4] if $A = T \cap C$, where T is a Λ_p -set and C is a preclosed set. The complement of a (Λ, p) -closed set is called (Λ, p) -open. The family of all (Λ, p) -open (resp. (Λ, p) -closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ) . A point $x \in X$ is called a (Λ, p) -cluster point [4] of A if $A \cap U \neq \emptyset$ for every (Λ, p) -open set U of X containing x . The set of all (Λ, p) -cluster points of A is called the (Λ, p) -closure [4] of A and is denoted by $A^{(\Lambda, p)}$. The union of all (Λ, p) -open sets of X contained in A is called the (Λ, p) -interior [4] of A and is denoted by $A_{(\Lambda, p)}$. A subset A of a topological space (X, τ) is said to be $s(\Lambda, p)$ -open [4] (resp. $p(\Lambda, p)$ -open [4], $\beta(\Lambda, p)$ -open [4], $\alpha(\Lambda, p)$ -open [20], $r(\Lambda, p)$ -open [4]) if $A \subseteq [A_{(\Lambda, p)}]^{(\Lambda, p)}$ (resp. $A \subseteq [A^{(\Lambda, p)}]_{(\Lambda, p)}$, $A \subseteq [[A^{(\Lambda, p)}]_{(\Lambda, p)}]^{(\Lambda, p)}$, $A \subseteq [[A_{(\Lambda, p)}]^{(\Lambda, p)}]_{(\Lambda, p)}$, $A = [A^{(\Lambda, p)}]_{(\Lambda, p)}$). The union of all $\beta(\Lambda, p)$ -open sets of X contained in A is called the $\beta(\Lambda, p)$ -interior of A and is denoted by $A_{\beta(\Lambda, p)}$. The complement of a $s(\Lambda, p)$ -open (resp. $p(\Lambda, p)$ -open, $\beta(\Lambda, p)$ -open, $\alpha(\Lambda, p)$ -open, $r(\Lambda, p)$ -open) set is called $s(\Lambda, p)$ -closed (resp. $p(\Lambda, p)$ -closed, $\beta(\Lambda, p)$ -closed, $\alpha(\Lambda, p)$ -closed, $r(\Lambda, p)$ -closed). The intersection of all $\beta(\Lambda, p)$ -closed sets of X containing A is called the $\beta(\Lambda, p)$ -closure of A and is denoted by $A^{\beta(\Lambda, p)}$. Let A be a subset of a

topological space (X, τ) . The $\theta(\Lambda, p)$ -closure [4] of A , $A^{\theta(\Lambda, p)}$, is defined as follows:

$$A^{\theta(\Lambda, p)} = \{x \in X \mid A \cap U^{(\Lambda, p)} \neq \emptyset \text{ for each } (\Lambda, p)\text{-open set } U \text{ containing } x\}.$$

A subset A of a topological space (X, τ) is called $\theta(\Lambda, p)$ -closed [4] if $A = A^{\theta(\Lambda, p)}$. The complement of a $\theta(\Lambda, p)$ -closed set is said to be $\theta(\Lambda, p)$ -open. Let A be a subset of a topological space (X, τ) . A point $x \in X$ is called a $\theta(\Lambda, p)$ -interior point [19] of A if $x \in U \subseteq U^{(\Lambda, p)} \subseteq A$ for some $U \in \Lambda_p O(X, \tau)$. The set of all $\theta(\Lambda, p)$ -interior points of A is called the $\theta(\Lambda, p)$ -interior [19] of A and is denoted by $A_{\theta(\Lambda, p)}$.

Lemma 1. [19] *For subsets A and B of a topological space (X, τ) , the following properties hold:*

- (1) $X - A^{\theta(\Lambda, p)} = [X - A]_{\theta(\Lambda, p)}$ and $X - A_{\theta(\Lambda, p)} = [X - A]^{\theta(\Lambda, p)}$.
- (2) A is $\theta(\Lambda, p)$ -open if and only if $A = A_{\theta(\Lambda, p)}$.
- (3) $A \subseteq A^{(\Lambda, p)} \subseteq A^{\theta(\Lambda, p)}$ and $A_{\theta(\Lambda, p)} \subseteq A_{(\Lambda, p)} \subseteq A$.
- (4) If $A \subseteq B$, then $A^{\theta(\Lambda, p)} \subseteq B^{\theta(\Lambda, p)}$ and $A_{\theta(\Lambda, p)} \subseteq B_{\theta(\Lambda, p)}$.
- (5) If A is (Λ, p) -open, then $A^{(\Lambda, p)} = A^{\theta(\Lambda, p)}$.

3. Properties of weakly $\beta(\Lambda, p)$ -open functions

In this section, we introduce the notion of weakly $\beta(\Lambda, p)$ -open functions. Moreover, some properties of weakly $\beta(\Lambda, p)$ -open functions are discussed.

Definition 1. A functions $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be weakly $\beta(\Lambda, p)$ -open if

$$f(U) \subseteq [f(U^{(\Lambda, p)})]_{\beta(\Lambda, p)}$$

for each (Λ, p) -open set U of X .

Theorem 1. For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties are equivalent:

- (1) f is weakly $\beta(\Lambda, p)$ -open;
- (2) $f(A_{\theta(\Lambda, p)}) \subseteq [f(A)]_{\beta(\Lambda, p)}$ for every subset A of X ;
- (3) $[f^{-1}(B)]_{\theta(\Lambda, p)} \subseteq f^{-1}(B_{\beta(\Lambda, p)})$ for every subset B of Y ;
- (4) $f^{-1}(B^{\beta(\Lambda, p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda, p)}$ for every subset B of Y ;
- (5) for each $x \in X$ and each (Λ, p) -open set U of X containing x , there exists a $\beta(\Lambda, p)$ -open set V of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda, p)})$;
- (6) $f(K_{(\Lambda, p)}) \subseteq [f(K)]_{\beta(\Lambda, p)}$ for every (Λ, p) -closed set K of X ;

(7) $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}$ for every (Λ,p) -open set U of X ;

(8) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}$ for every $p(\Lambda,p)$ -open set U of X ;

(9) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}$ for every $\alpha(\Lambda,p)$ -open set U of X .

Proof. (1) \Rightarrow (2): Let A be any subset of X and $x \in A_{\theta(\Lambda,p)}$. Then, there exists a (Λ,p) -open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$. Then, $f(x) \in f(U) \subseteq f(U^{(\Lambda,p)}) \subseteq f(A)$. Since f is weakly $\beta(\Lambda,p)$ -open, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)} \subseteq [f(A)]_{\beta(\Lambda,p)}$. It implies that $f(x) \in [f(A)]_{\beta(\Lambda,p)}$. Thus, $x \in f^{-1}([f(A)]_{\beta(\Lambda,p)})$ and hence $A_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{\beta(\Lambda,p)})$. This shows that $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\beta(\Lambda,p)}$.

(2) \Rightarrow (1): Let U be any (Λ,p) -open set of X . As $U \subseteq [U^{(\Lambda,p)}]_{\theta(\Lambda,p)}$ implies

$$\begin{aligned} f(U) &\subseteq f([U^{(\Lambda,p)}]_{\theta(\Lambda,p)}) \\ &\subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}. \end{aligned}$$

Thus, f is weakly $\beta(\Lambda,p)$ -open.

(2) \Rightarrow (3): Let B be any subset of Y . Then by (2), $f([f^{-1}(B)]_{\theta(\Lambda,p)}) \subseteq B_{\beta(\Lambda,p)}$. Thus, $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\beta(\Lambda,p)})$.

(3) \Rightarrow (2): This is obvious.

(3) \Rightarrow (4): Let B be any subset of Y . Using (3), we have

$$\begin{aligned} X - [f^{-1}(B)]^{\theta(\Lambda,p)} &= [X - f^{-1}(B)]_{\theta(\Lambda,p)} \\ &= [f^{-1}(Y - B)]_{\theta(\Lambda,p)} \\ &\subseteq f^{-1}([Y - B]_{\beta(\Lambda,p)}) \\ &= f^{-1}(Y - B^{\beta(\Lambda,p)}) \\ &= X - f^{-1}(B^{\beta(\Lambda,p)}) \end{aligned}$$

and hence $f^{-1}(B^{\beta(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$.

(4) \Rightarrow (3): Let B be any subset of Y . By (4), $X - f^{-1}(B_{\beta(\Lambda,p)}) \subseteq X - [f^{-1}(B)]_{\theta(\Lambda,p)}$. Thus, $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\beta(\Lambda,p)})$.

(1) \Rightarrow (5): Let $x \in X$ and U be any (Λ,p) -open set of X containing x . Since f is weakly $\beta(\Lambda,p)$ -open, $f(x) \in f(U) \subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}$. Put $V = [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}$. Then, V is a $\beta(\Lambda,p)$ -open set of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$.

(5) \Rightarrow (1): Let U be any (Λ,p) -open set of X and $y \in f(U)$. It following from (5) $V \subseteq f(U^{(\Lambda,p)})$ for some $\beta(\Lambda,p)$ -open set V of Y containing y . Thus,

$$y \in V \subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}$$

and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\beta(\Lambda,p)}$. This shows that f is weakly $\beta(\Lambda,p)$ -open.

(1) \Rightarrow (6) \Rightarrow (7) \Rightarrow (8) \Rightarrow (9) \Rightarrow (1): This is obvious.

Theorem 2. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective function. Then, the following properties are equivalent:*

- (1) f is weakly $\beta(\Lambda, p)$ -open;
- (2) $[f(U)]^{\beta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every (Λ, p) -open set U of X ;
- (3) $[f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(K)$ for every (Λ, p) -closed set K of X .

Proof. (1) \Rightarrow (3): Let K be any (Λ, p) -closed set of X . Then, we have

$$\begin{aligned} f(X - K) &= Y - f(K) \\ &\subseteq [f([X - K]^{(\Lambda, p)})]_{\beta(\Lambda, p)} \end{aligned}$$

and hence $Y - f(K) \subseteq Y - [f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)}$. Thus, $[f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(K)$.

(3) \Rightarrow (2): Let U be any (Λ, p) -open set of X . Since $U^{(\Lambda, p)}$ is (Λ, p) -closed and $U \subseteq [U^{(\Lambda, p)}]_{(\Lambda, p)}$, by (3) we have $[f(U)]^{\beta(\Lambda, p)} \subseteq [f([U^{(\Lambda, p)}]_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$.

(2) \Rightarrow (1): Let U be any (Λ, p) -open set of X . By (2), we have

$$[f(X - U^{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f([X - U^{(\Lambda, p)}]^{(\Lambda, p)}).$$

Since f is bijective, $[f(X - U^{(\Lambda, p)})]^{\beta(\Lambda, p)} = Y - [f(U^{(\Lambda, p)})]_{\beta(\Lambda, p)}$ and

$$\begin{aligned} f([X - U^{(\Lambda, p)}]^{(\Lambda, p)}) &= f(X - [U^{(\Lambda, p)}]_{(\Lambda, p)}) \\ &\subseteq f(X - U) \\ &= Y - f(U). \end{aligned}$$

Thus, $f(U) \subseteq [f(U^{(\Lambda, p)})]_{\beta(\Lambda, p)}$ and hence f is weakly $\beta(\Lambda, p)$ -open.

4. Properties of weakly $\beta(\Lambda, p)$ -closed functions

We begin this section by introducing the concept of weakly $\beta(\Lambda, p)$ -closed functions.

Definition 2. *A functions $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be weakly $\beta(\Lambda, p)$ -closed if $[f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(K)$ for each (Λ, p) -closed set K of X .*

Theorem 3. *For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties are equivalent:*

- (1) f is weakly $\beta(\Lambda, p)$ -closed;
- (2) $[f(U)]^{\beta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every (Λ, p) -open set U of X ;
- (3) $[f(U)]^{\beta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every $r(\Lambda, p)$ -open set U of X ;
- (4) for each subset B of Y and each (Λ, p) -open set U of X with $f^{-1}(B) \subseteq U$, there exists a $\beta(\Lambda, p)$ -open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U^{(\Lambda, p)}$;

- (5) for each point $y \in Y$ and each (Λ, p) -open set U of X with $f^{-1}(y) \subseteq U$, there exists a $\beta(\Lambda, p)$ -open set V of Y containing y such that $f^{-1}(V) \subseteq U^{(\Lambda, p)}$;
- (6) $[f([U^{(\Lambda, p)}]_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every (Λ, p) -open set U of X ;
- (7) $[f([U^{\theta(\Lambda, p)}]_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(U^{\theta(\Lambda, p)})$ for every (Λ, p) -open set U of X ;
- (8) $[f(U)]^{\beta(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$ for every $\beta(\Lambda, p)$ -open set U of X .

Proof. (1) \Rightarrow (2): Let U be any (Λ, p) -open set of X . Then by (1),

$$\begin{aligned} [f(U)]^{\beta(\Lambda, p)} &= [f(U_{(\Lambda, p)})]^{\beta(\Lambda, p)} \\ &\subseteq [f([U^{(\Lambda, p)}]_{(\Lambda, p)})]^{\beta(\Lambda, p)} \\ &\subseteq f(U^{(\Lambda, p)}). \end{aligned}$$

(2) \Rightarrow (1): Let K be any (Λ, p) -closed set of X . Using (2), we have

$$\begin{aligned} [f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)} &\subseteq f([K_{(\Lambda, p)}]^{(\Lambda, p)}) \\ &\subseteq f(K^{(\Lambda, p)}) \\ &= f(K). \end{aligned}$$

This shows that f is weakly $\beta(\Lambda, p)$ -closed.

It is clear that (1) \Rightarrow (7), (4) \Rightarrow (5) and (1) \Rightarrow (6) \Rightarrow (8) \Rightarrow (3) \Rightarrow (1). To show that (3) \Rightarrow (4): Let B be any subset of Y and U be any (Λ, p) -open set of X with $f^{-1}(B) \subseteq U$. Then, $f^{-1}(B) \cap [X - U^{(\Lambda, p)}]^{(\Lambda, p)} = \emptyset$ and $B \cap f([X - U^{(\Lambda, p)}]^{(\Lambda, p)}) = \emptyset$. Since $X - U^{(\Lambda, p)}$ is $r(\Lambda, p)$ -open, $B \cap [f(X - U^{(\Lambda, p)})]^{\beta(\Lambda, p)} = \emptyset$ by (3). Put $V = Y - [f(X - U^{(\Lambda, p)})]^{\beta(\Lambda, p)}$. Then, V is $\beta(\Lambda, p)$ -open set of Y such that $B \subseteq V$ and

$$\begin{aligned} f^{-1}(V) &\subseteq X - f^{-1}([f(X - U^{(\Lambda, p)})]^{\beta(\Lambda, p)}) \\ &\subseteq X - f^{-1}(f(X - U^{(\Lambda, p)})) \\ &\subseteq U^{(\Lambda, p)}. \end{aligned}$$

(7) \Rightarrow (1): It suffices see that $U^{\theta(\Lambda, p)} = U^{(\Lambda, p)}$ for every (Λ, p) -open set U of X .

(5) \Rightarrow (1): Let K be any (Λ, p) -closed set U of X and $y \in Y - f(K)$. Since

$$f^{-1}(y) \subseteq X - K,$$

there exists a $\beta(\Lambda, p)$ -open set V of Y such that $y \in V$ and

$$f^{-1}(V) \subseteq [X - K]^{(\Lambda, p)} = X - K_{(\Lambda, p)}$$

by (5). Thus, $V \cap f(K_{(\Lambda, p)}) = \emptyset$ and hence $y \in Y - [f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)}$. Therefore, $[f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(K)$. This shows that f is weakly $\beta(\Lambda, p)$ -closed.

(7) \Rightarrow (8): This is obvious since $U^{\theta(\Lambda, p)} = U^{(\Lambda, p)}$ for every $\beta(\Lambda, p)$ -open set U of X .

The following theorem the proof is mostly straightforward and is omitted.

Theorem 4. For a function $f : (X, \tau) \rightarrow (Y, \sigma)$, the following properties are equivalent:

- (1) f is weakly $\beta(\Lambda, p)$ -closed;
- (2) $[f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(K)$ every $\beta(\Lambda, p)$ -closed set K of X ;
- (3) $[f(K_{(\Lambda, p)})]^{\beta(\Lambda, p)} \subseteq f(K)$ every $\alpha(\Lambda, p)$ -closed set K of X .

Acknowledgements

This research project was financially supported by Mahasarakham University.

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