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Relations Between Derivations and Homomorphisms of Ordered Hyperrings

Ruiqi Cai[1], Mashaer Alsaeedi[2], Maryam Akhoundi[3,*]

[1] *Institute of Computing Science and Technology, Guangzhou University, Guangzhou 510006, China*
[2] *Department of Mathematics, College of Sciences and Humanities, Prince Sattam bin Abdulaziz University, Al-Kharj, Saudi Arabia*
[3] *Clinical Research Development Unit of Rouhani Hospital, Babol University of Medical Sciences, Babol, Iran*

Abstract. The present study investigates the relation between derivations and hyperideals on ordered hyperrings with no zero divisors. Also, we identify some results for the ordered hyperrings induced by the homomorphism of the ordered hyperrings by derivations. The present work explores some aspects of derivations in ordered hyperrings. Also, we establish some results in connection with homomorphisms and hyperideals. Furthermore, we describe prime hyperideals associated to a derivation d on an ordered hyperring T and derive several results about homomorphisms and derivations on ordered hyperrings.

2020 Mathematics Subject Classifications: 13N15, 16Y99

Key Words and Phrases: Krasner hyperring, ordered hyperring, derivation, homomorphism, hyperideal

1. Introduction

Marty presented the hypergroup ideas in 1934 [1]. Krasner originally considered hyperring, which is a development of ring, in [2].

The study of hyperideals have been made by Heidari and Davvaz in the context of ordered semihypergroups in [3]. The study also demonstrated that the direct product of ordered hyperstructures are ordered hyperstructures. Later on, Davvaz et al. [4] utilized pseudoorders to construct strongly regular relations in ordered semihypergroups and examined the relationships between ordered hyperstructures and ordered structures. Also, see [5, 6]. Al-Tahan and Davvaz [7] use the ordered hyperstructure to communicate with biological inheritance and genetics to do research, and to access applications.

*Corresponding author.
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Email addresses: cairic@e.gzhu.edu.cn (R. Cai), m.alsaedi@psau.edu.sa (M. Alsaeedi), Maryam.akhoundi@mubabol.ac.ir (M. Akhoundi)

Derivation in rings was first explored by Posner [8] and later on hyperrings by Asokkumar [9] and Kamali and Davvaz [10]. Derivation on ordered semihyperring was presented by Rao et al. in [11]. Omidi and Davvaz considered ordered hyperring, which is a development of ordered ring, in [12]. Also, see [13–16].

The present work explores some aspects of derivations in ordered hyperrings. Also, we establish some results in connection with homomorphisms and hyperideals.

2. Preliminaries

We set that

OKH: the set of all ordered Krasner hyperrings $(E, \oplus, \odot, \leq)$,

Der(E): the set of all derivations of E.

Definition 1. *[2] $(E, \oplus, \odot)$ is a Krasner hyperring if:*

(1) $(E, \oplus)$ is a canonical hypergroup;

(2) $(E, \odot)$ is a semigroup and $z \odot 0 = 0 \odot z = \{0\}, \forall z \in E$;

(3) $\odot$ is distributive with respect to the hyperaddition $\oplus$.

Definition 2. *[12] Let $(E, \oplus, \odot)$ be a Krasner hyperring. $(E, \oplus, \odot, \leq) \in$ **OKH** if*

(1) $(E, \leq)$ is a partially ordered set;

(2) $(l, l') \in \leq \Rightarrow l \oplus t \preceq l' \oplus t, \forall l, l', t \in E$;

(3) $(l, l') \in \leq$ and $(0, t) \in \leq \Rightarrow (l \odot t, l' \odot t) \in \leq$ and $(t \odot l, t \odot l') \in \leq$.

Note that for every $\emptyset \neq L, L' \subseteq E$,

$$L \preceq L' \Leftrightarrow \forall l \in L, \exists l' \in L' \text{ such that } (l, l') \in \leq.$$

Definition 3. *[12] Let $(E, \oplus, \odot, \leq)$ and $(E', \oplus', \odot', \leq') \in$ **OKH**. A function $\Lambda : E \to E'$ is a homomorphism if $\forall l, l' \in E$,*

(1) $\Lambda(l \oplus l') \subseteq \Lambda(l) \oplus' \Lambda(l')$;

(2) $\Lambda(l \odot l') = \Lambda(l) \odot' \Lambda(l')$;

(3) $(l, l') \in \leq \Rightarrow (\Lambda(l), \Lambda(l')) \in \leq'$.

Definition 4. *[3] Let $(E, \oplus, \odot, \leq) \in$ **OKH**. $\emptyset \neq X \subseteq E$ is a hyperideal of E if*

(1) $(X, \oplus)$ is a canonical subhypergroup of $(E, \oplus)$;

(2) $l \odot x, x \odot l \in X, \forall l \in E, \forall x \in X$;

(3) $(X] := \{l \in E \mid l \leq x, \text{ for some } x \in X\} \subseteq X$.

Definition 5. *[11] Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$. $d \in \mathbf{Der(E)}$ *if for all* $l, l' \in E$,

(1) $d(l \oplus l') \subseteq d(l) \oplus d(l')$;

(2) $d(l \odot l') \in d(l) \odot l' \oplus l \odot d(l')$;

(3) $(l, l') \in \leq \Rightarrow (d(l), d(l')) \in \leq$.

3. Main Results

Let $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$. Then, $0 \neq z \in E$ is a zero divisor if

$$\exists\ 0 \neq v \in E \text{ such that } z \odot v = 0 = v \odot z.$$

Theorem 1. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *with no zero divisors and* $0 \neq d \in \mathbf{Der(E)}$. *If* Y *is a proper hyperideal of* E*, then* d *is nonzero on* Y.

Proof. Let

$$d(m) = 0, \forall\ 0 \neq m \in Y.$$

As Y is a hyperideal of E, $m \odot g \in Y$, $\forall\ g \in E$. Thus,

$$d(m \odot g) = 0.$$

So,

$$\begin{aligned} d(m \odot g) &\in d(m) \odot g \oplus m \odot d(g) \\ &= 0 \odot g \oplus m \odot d(g) \\ &= 0 \oplus m \odot d(g) \\ &= m \odot d(g). \end{aligned}$$

Hence,

$$m \odot d(g) = d(m \odot g) = 0.$$

By hypothesis, E has no zero divisors. Thus,

$$d(g) = 0,\ \forall\ g \in E$$

a contradiction. Therefore, d is nonzero on Y.

Theorem 2. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *and*

$$g \in g \oplus g, \forall g \in E.$$

$Id_E(g) = g$ for any $g \in E$, is a homomorphism iff $Id_E \in \mathbf{Der}(\mathbf{E})$.

Proof. Let Id_E be a homomorphism and $g, g' \in E$. Then,

$$\begin{aligned} Id_E(g \odot g') &= Id_E(g) \odot Id_E(g') \\ &= g \odot g' \\ &\in (g \odot g') \oplus (g \odot g') \\ &= Id_E(g) \odot g' \oplus g \odot Id_E(g'). \end{aligned}$$

Hence, $Id_E \in \mathbf{Der}(\mathbf{E})$.

Conversely, let $g, g' \in E$. Then

$$Id_E(g \odot g') = g \odot g' = Id_E(g) \odot Id_E(g').$$

So, Id_E is a homomorphism.

Theorem 3. *Let $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ be commutative and and*

$$g \in g \oplus g, \forall g \in E.$$

For a given $t \in E$, we set

$$d_t(g) = t \odot g, \ \forall g \in E.$$

Then $d_t \in \mathbf{Der}(\mathbf{E})$.

Proof. Let $g, g' \in E$. For a given $t \in E$, we have

$$\begin{aligned} d_t(g \oplus g') &= t \odot (g \oplus g') \\ &= t \odot g \oplus t \odot g' \\ &= d_t(g) \oplus d_t(g'), \end{aligned}$$

and

$$\begin{aligned} d_t(g \odot g') &= t \odot (g \odot g') \\ &\in t \odot (g \odot g') \oplus t \odot (g \odot g') \\ &= (t \odot g) \odot g' \oplus (t \odot g) \odot g' \\ &= (t \odot g) \odot g' \oplus (g \odot t) \odot g' \\ &= (t \odot g) \odot g' \oplus g \odot (t \odot g') \\ &= d_t(g) \odot g' \oplus g \odot d_t(g'). \end{aligned}$$

Let $g, g' \in E$ and $(g, g') \in \leq$. Then

$$d_t(g) = t \odot g \leq t \odot g' = d_t(g')$$

by Definition 2, and hence $d_t \in \mathbf{Der(E)}$.

Corollary 1. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *be commutative and*

$$g \in g \oplus g, \forall g \in E.$$

Then, the identity function Id_E *defined by* $Id_E(g) = g$ *for any* $g \in E$*, is a homomorphism.*

Proof. We have

$$d_1(g) = 1 \odot g = g = Id_E(g).$$

By Theorem 3, $Id_E = d_1 \in \mathbf{Der(E)}$. Now, by Theorem 2, Id_E is a homomorphism.

Corollary 2. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *be commutative and*

$$g \in g \oplus g, \forall g \in E.$$

If $d = d_t$*, where* $t \in E$*, satisfies the following condition*

$$(g', g) \in \leq \text{ and } d(g) = g \Rightarrow d(g') = g',$$

then

$$Fix_d(E) = \{x \in E \mid d(x) = x\}$$

is a hyperideal of E*.*

Proof. By Theorem 3, $d(g) = d_t(g) = t \odot g$, for any $g \in E$. Let $g, g' \in Fix_d(E)$. Then $d(g) = g$ and $d(g') = g'$. We have

$$\begin{aligned} d(g \ominus g') &= d_t(g \ominus g') \\ &= t \odot (g \ominus g') \\ &= t \odot g \ominus t \odot g' \\ &= d_t(g) \ominus d_t(g') \\ &= d(g) \ominus d(g') \\ &= g \ominus g'. \end{aligned}$$

So, $g \ominus g' \subseteq Fix_d(E)$.

Now, let $g \in Fix_d(E)$ and $q \in E$. Then,

$$\begin{aligned} d(g \odot q) &= d_t(g \odot q) \\ &= t \odot (g \odot q) \\ &= (t \odot g) \odot q \\ &= d_t(g) \odot q \\ &= d(g) \odot q \\ &= g \odot q. \end{aligned}$$

So, $g \odot q \in Fix_d(E)$.

Let $g \in Fix_d(E)$, $q \in E$ and $q \leq g$. Then,

$$d(q) = d_t(q) \leq d_t(g) = d(g) = g.$$

By hypothesis, $d(q) = q$. So, $q \in Fix_d(E)$. Hence, $Fix_d(E)$ is a hyperideal of E.

Example 1. *Let* $E = \{0, 1, f, f'\}$ *and*

$\oplus$	0	1	f	f'
0	0	1	f	f'
1	1	$\{0, f\}$	$\{1, f'\}$	f
f	f	$\{1, f'\}$	$\{0, f\}$	1
f'	f'	f	1	0

$\odot$	0	1	f	f'
0	0	0	0	0
1	0	1	f	f'
f	0	f	f	0
f'	0	f'	0	f'

$$\leq := \{(z, z) \mid z \in E\} \cup \{(0, f), (f', 1)\}.$$

Then $(E, \oplus, \odot, \leq) \in$ **OKH**. *We set*

$$d(z) = d_f(z) = \begin{cases} 0, & z = 0, f' \\ f, & z = 1, f. \end{cases}$$

Then, $d = d_f \in$ **Der(E)**. *Indeed:*

$$d_f(0) = f \odot 0 = 0,$$

$$d_f(1) = f \odot 1 = f,$$

$$d_f(f) = f \odot f = f,$$

$$d_f(f') = f \odot f' = 0.$$

Clearly, $Fix_d(E) = \{0, f\}$ *is a hyperideal of* E*.*

Theorem 4. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *and* $d \in \mathbf{Der(E)}$ *with*

$$d(g) = d(1) \odot g;\ \forall g \in E.$$

If d *is a homomorphism, then* d *is idempotent.*

Proof. Let $g \in E$. We have

$$\begin{aligned} d^2(g) &= d(d(g)) \\ &= d(1 \odot d(g)) \\ &= d(1) \odot (1 \odot d(g)) \\ &= (d(1) \odot 1) \odot d(g) \\ &= d(1) \odot d(g) \\ &= d(1 \odot g) \\ &= d(g). \end{aligned}$$

Hence, $d^2 = d$.

Example 2. *In Example 1,*

$$d(1 \odot 1) = d(1) = f = f \odot f = d(1) \odot d(1),$$

$$d(1 \odot f) = d(f) = f = f \odot f = d(1) \odot d(f),$$

$$d(1 \odot f') = d(f') = 0 = f \odot 0 = d(1) \odot d(f'),$$

$$d(f \odot f) = d(f) = f = f \odot f = d(f) \odot d(f),$$

$$d(f \odot f') = d(0) = 0 = f \odot 0 = d(f) \odot d(f').$$

Hence, d *is a homomorphism of* E*. Also,*

$$d(g) = d(1) \odot g;\ \forall g \in E.$$

Now, by Theorem 4, d is idempotent.

Definition 6. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *and* $d \in \mathbf{Der(E)}$ *be a homomorphism. A proper hyperideal* W *of* E *is said to be a prime hyperideal associated to* d *if*

$$g \odot g' \in W \Rightarrow g \in W \text{ or } d(g') \in W, \ \forall g, g' \in E.$$

Theorem 5. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *and* $d \in \mathbf{Der(E)}$ *be a homomorphism. Then* Y *is a prime hyperideal of* E *associated to* d *iff for any hyperideals* G *and* G' *of* E*, we have*

$$G \odot G' \subseteq Y \Rightarrow G \subseteq Y \text{ or } d(G') \subseteq Y.$$

Proof. ($\Rightarrow$): Let Y be a prime hyperideal of E associated to d, $G \odot G' \subseteq Y$ and $G \nsubseteq Y$, where G, G' are hyperideals of E. As $G \nsubseteq Y$,

$$\exists g \in G \text{ such that } g \notin Y.$$

Take any $g' \in G'$. Then,

$$g \odot g' \in G \odot G' \subseteq Y.$$

Since Y is a prime hyperideal associated to d and $g \notin Y$, we get

$$d(g') \in Y.$$

Hence, $d(G') \subseteq Y$.

($\Leftarrow$): Suppose that $g \odot g' \in Y$ for some $g, g' \in E$. Then $< g \odot g' > \subseteq Y$. So,

$$< g > \odot < g' > \subseteq < g \odot g' > \subseteq Y.$$

Hence, $< g > \subseteq Y$ or $d(< g' >) \subseteq Y$. Thus, $g \in Y$ or $d(g') \in Y$. Therefore, Y is a prime hyperideal associated to d.

Example 3. *In Example 1,* $Y = \{0, f\}$ *is a prime hyperideal associated to* d*.*

Theorem 6. *Let* $(E, \oplus, \odot, \leq) \in \mathbf{OKH}$ *and* $d \in \mathbf{Der(E)}$ *be a homomorphism. If* W *is a prime hyperideal associated to* d*, then*

$$\sqrt{W} := \{t \in E \mid \exists n \in \mathbb{N} \text{ such that } t^n \in W\}$$

is a prime hyperideal of E *associated to* d*.*

Proof. Let $z, z' \in \sqrt{W}$. By the proof of Lemma 3.19 in [17],

$$z \oplus z' \subseteq \sqrt{W} \text{ and } \ominus z \in \sqrt{W}.$$

Also, for any $t \in E$, $t \odot z, z \odot t \in \sqrt{W}$.

Now, let $q \in (\sqrt{W}]$. Then $q \leq z$ for some $z \in \sqrt{W}$. As $z \in \sqrt{W}$,

$$\exists n \in \mathbb{N} \text{ such that } z^n \in W.$$

Since $q \leq z$, we get

$$q^n \leq z^n \in W.$$

Thus, $q^n \in W$. So, $q \in \sqrt{W}$ and hence $(\sqrt{W}] \subseteq \sqrt{W}$.

Let $g \odot g' \in \sqrt{W}$ and $g \notin \sqrt{W}$ for $g, g' \in E$.

Claim: $d(g') \in \sqrt{W}$.

As $g \odot g' \in \sqrt{W}$,

$$\exists n \in \mathbb{N} \text{ such that } (g \odot g')^n \in W.$$

So, $g^n \odot g'^n \in W$. As W is a prime hyperideal associated to d and $g^n \notin W$, $d(g'^n) \in W$. Since d is a homomorphism of T, we obtain

$$(d(g'))^n = d(g'^n) \in W.$$

Thus, $d(g') \in \sqrt{W}$. So, $\sqrt{W}$ is a prime hyperideal associated to d.

Let Ω be an index set and $(T_i, \oplus_i, \odot_i, \leq_i) \in \mathbf{OKH}$, for all $i \in \Omega$. Then,

$$\prod_{i \in \Omega} T_i = \{(t_i)_{i \in \Omega} \mid t_i \in T_i\} \in \mathbf{OKH}.$$

Indeed: for any $(w_i)_{i\in\Omega}, (w'_i)_{i\in\Omega} \in \prod\limits_{i\in\Omega} T_i$,

(i) $(w_i)_{i\in\Omega} \oplus (w'_i)_{i\in\Omega} = \{(t_i)_{i\in\Omega} \mid t_i \in w_i \oplus_i w'_i\}$;

(ii) $(w_i)_{i\in\Omega} \odot (w'_i)_{i\in\Omega} = (w_i \odot_i w'_i)_{i\in\Omega}$;

(iii) $(w_i)_{i\in\Omega} \leq (w'_i)_{i\in\Omega} \Leftrightarrow w_i \leq_i w'_i, \forall i \in \Omega$.

Define the map $\pi_i : \prod\limits_{i\in\Omega} T_i \to T_i$ by $\pi_i((w_i)_{i\in\Omega}) = w_i$, for each $(w_i)_{i\in\Omega} \in \prod\limits_{i\in\Omega} T_i$ and $i \in \Omega$ and define the map $\rho_i : T_i \to \prod\limits_{i\in\Omega} T_i$ by

$$(\rho_i(t))(j) = \begin{cases} t, & \text{if } i = j \\ \\ 0_j, & \text{otherwise} \end{cases}$$

for each $t \in T_i$.

Theorem 7. *Let Ω be an index set and $(T_i, \oplus_i, \odot_i, \leq_i) \in \mathbf{OKH}$, for all $i \in \Omega$. If $d \in \mathbf{Der}(\prod\limits_{i\in\Omega} \mathbf{T_i})$, then $d_i = \pi_i d \rho_i \in \mathbf{Der}(\mathbf{T_i})$, for all $i \in \Omega$.*

Proof. Let $d \in \mathbf{Der}(\prod_{i\in\Omega} \mathbf{T_i})$ and $z, z' \in T_i$, for all $i \in \Omega$. Then,

$$\begin{aligned} d_i(z \oplus_i z') &= \pi_i d\rho_i(z \oplus_i z') \\ &= \pi_i d(\rho_i(z \oplus_i z')) \\ &= \pi_i(d(\rho_i(z) \oplus_i \rho_i(z'))) \\ &\subseteq \pi_i(d(\rho_i(z)) \oplus_i d(\rho_i(z'))) \\ &= \pi_i d\rho_i(z) \oplus_i \pi_i d\rho_i(z') \\ &= d_i(z) \oplus_i d_i(z'), \end{aligned}$$

and

$$\begin{aligned} d_i(z \odot_i z') &= \pi_i d\rho_i(z \odot_i z') \\ &= \pi_i d(\rho_i(z \odot_i z')) \\ &= \pi_i(d(\rho_i(z) \odot_i \rho_i(z'))) \\ &\in \pi_i((d(\rho_i(z)) \odot_i \rho_i(z')) \oplus_i (\rho_i(z) \odot_i d(\rho_i(z')))) \\ &= (\pi_i d\rho_i(z) \odot_i \pi_i\rho_i(z')) \oplus_i (\pi_i\rho_i(z) \odot_i \pi_i d\rho_i(z')) \\ &= (\pi_i d\rho_i(z) \odot_i z') \oplus_i (z \odot_i \pi_i d\rho_i(z')) \\ &= (d_i(z) \odot_i z') \oplus_i (z \odot_i d_i(z')). \end{aligned}$$

Since $d \in \mathbf{Der}(\prod_{i\in\Omega} \mathbf{T_i})$, it follows that d is isotone. Also, since π_i and ρ_i are isotone, we get $\pi_i d\rho_i$ is isotone. Therefore, $d_i \in \mathbf{Der}(\mathbf{T_i})$, for all $i \in \Omega$.

Let Ω be an index set, $(T_i, \oplus_i, \odot_i, \leq_i) \in \mathbf{OKH}$ and $d_i \in \mathbf{Der}(\mathbf{T_i})$, for all $i \in \Omega$. Define $\prod_{i\in\Omega} d_i : \prod_{i\in\Omega} T_i \to \prod_{i\in\Omega} T_i$ by $(\prod_{i\in\Omega} d_i)((w_i)_{i\in\Omega}) = (d_i(w_i))_{i\in\Omega}$, for each $(w_i)_{i\in\Omega} \in \prod_{i\in\Omega} T_i$.

Corollary 3. *Let* Ω *be an index set and* $(T_i, \oplus_i, \odot_i, \leq_i) \in \mathbf{OKH}$*, for all* $i \in \Omega$*. If* $d \in \mathbf{Der}(\prod_{i\in\Omega} \mathbf{T_i})$*, then* $d = \prod_{i\in\Omega} \pi_i d\rho_i$ *iff* $d \in \prod_{i\in\Omega} \mathbf{Der}(\mathbf{T_i})$.

Proof. ($\Rightarrow$): Let $d \in \mathbf{Der}(\prod_{i\in\Omega} \mathbf{T_i})$ and $d = \prod_{i\in\Omega} \pi_i d\rho_i$. By Theorem 7, we have

$$\pi_i d\rho_i \in \mathbf{Der}(\mathbf{T_i}), \ \forall i \in \Omega.$$

Thus, $d \in \prod_{i\in\Omega} \mathbf{Der}(\mathbf{T_i})$.

($\Leftarrow$): Let $d \in \prod_{i\in\Omega} \mathbf{Der(T_i)}$ and $w \in T_i$. Then $(\pi_i(\prod_{i\in\Omega} d_i)\rho_i)(w) = d_i(w)$, where $d_i \in \mathbf{Der(T_i)}$. So, $(\pi_i(\prod_{i\in\Omega} d_i)\rho_i) = d_i$. Thus, $d = \prod_{i\in\Omega} d_i$ for some $d_i \in \mathbf{Der(T_i)}$. Hence, $d = \prod_{i\in\Omega} \pi_i d\rho_i$.

4. Conclusions

This study was conducted to investigate the significant relationship between homomorphisms and derivations in ordered hyperrings. Moreover, we investigated the relation between derivations and hyperideals on ordered hyperrings with no zero divisors. Furthermore, we described prime hyperideals associated to derivations on ordered hyperrings and derive several results about homomorphisms and derivations on ordered hyperrings. One can further apply these notions on fuzzy prime hyperideals associated to derivations in ordered hyperrings.

References

[1] Marty, F. *Sur une Generalization de la Notion de Groupe*; 8iem congres Math. Scandinaves: Stockholm, Sweden, 1934; pp. 45–49.

[2] Krasner, M. A class of hyperrings and hyperfields. *Intern. J. Math. Math. Sci.* **1983**, *6*, 307–312.

[3] Heidari, D.; Davvaz, B. On ordered hyperstructures. *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **2011**, *73*, 85–96.

[4] Davvaz, B.; Corsini, P.; Changphas, T. Relationship between ordered semihypergroups and ordered semigroups by using pseudoorder. *Eur. J. Combin.* **2015**, *44*, 208–217.

[5] Gu, Z.; Tang, X. Ordered regular equivalence relations on ordered semihypergroups. *J. Algebra* **2016**, *450*, 384–397.

[6] Tang, J.; Feng, X.; Davvaz, B.; Xie, X.Y. A further study on ordered regular equivalence relations in ordered semihypergroups. *Open Math.* **2018**, *16*, 168–184.

[7] Al-Tahan, M.; Davvaz, B. On quasi-ordering hypergroups, ordered hyperstructures and their applications in genetics. *Mathematics Interdisciplinary Research.* **2022**, *7*, 1–19.

[8] Posner, E.C. Derivations in prime rings, *Proc. Am. Math. Soc.* **1957**, 8, 1093–1100.

[9] Asokkumar, A. Derivations in hyperrings and prime hyperrings. *Iran. J. Math. Sci. Inform.* **2013**, *8*, 1–13.

[10] Kamali Ardekani, L.; Davvaz, B. Some notes on differential hyperrings, *Iran. J. Sci. Technol. Trans. A Sci.* **2015**, 39(1), 101–111.

[11] Rao, Y.; Kosari, S.; Shao, Z.; Omidi, S. Some properties of derivations and m-k-hyperideals in ordered semihyperrings. *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **2021**, *83*, 87–96.

[12] Omidi, S.; Davvaz, B. Ordered Krasner hyperrings, *Iran. J. Math. Sci. Inform.* **2017**, *12*, 35–49.

[13] Kosari, S.; Gheisari, M.; Maedeh Mirmohseni, S.; Zavieh, H.; Riskhan, B.; Faizan Khan, M.; Liu, Y. A survey on weak pseudoorders in ordered hyperstructures, *Artificial Intelligence and Applications.* **2023**, 1–5. https://doi.org/10.47852/bonviewAIA3202535.

[14] Z. Shao, X. Chen, S. Kosari and S. Omidi, *On some properties of right pure (bi-quasi-)hyperideals in ordered semihyperrings*, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys. **83(4)** (2021), 95–104.

[15] Chen, C.; Kosari, S.; Omidi, S.; Davvaz, B.; Akhoundi, M. A study on interior hyperfilters in ordered Γ-semihypergroups, *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **2022**, *84(1)*, 71–80.

[16] Rao, Y.; Kosari, S.; Shao, Z.; Akhoundi, M.; Omidi, S. A study on A-**I**-Γ-hyperideals and (m,n)-Γ-hyperfilters in ordered Γ-Semihypergroups. *Discrete Dyn. Nat. Soc.* **2021**, 10.

[17] Jun, J. Algebraic geometry over hyperrings, *Adv. Math.* **2018**, 323, 142–192.