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Weakly $\theta b(\Lambda, p)$-open functions and weakly $\theta b(\Lambda, p)$-closed functions

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Abstract. This article is concerned with the concepts of weakly $\theta b(\Lambda, p)$-open functions and weakly $\theta b(\Lambda, p)$-closed functions. Moreover, some characterizations of weakly $\theta b(\Lambda, p)$-open functions and weakly $\theta b(\Lambda, p)$-closed functions are established.

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1. Introduction

Topology is concerned with all questions directly or indirectly related to openness and closedness. Semi-open sets, preopen sets, α-open sets, β-open sets, b-open sets, δ-open sets and θ-open sets play an important role in the researches of generalizations of open functions and closed functions. By using these sets, many authors introduced and studied various types of open functions and closed functions. In 1996, Andrijević [1] introduced a new class of generalized open sets called b-open sets in a topological space. Park [16] introduced the notion of b-θ-open sets and showed that b-θ-cluster points can be characterized by b-regular sets. In 1983, Rose [17] introduced and studied the notions of weakly open functions and almost open functions. In 1987, Rose and Janković [18] investigated some of the fundamental properties of weakly closed functions. In 2006, Caldas et al. [8] introduced and studied the concepts of θ-preopen functions and θ-preclosed functions by using the notions of pre-θ-interior and pre-θ-closure. Moreover, Caldas et al. [7] introduced and investigated the concepts of weakly semi-θ-open functions and weakly semi-θ-closed functions. In 2009, Noiri et al. [15] introduced and studied two new classes of functions called weakly b-θ-open functions and weakly b-θ-closed functions by utilizing the notions of b-θ-open sets and the b-θ-closure operator. Weak b-θ-openness (resp. b-θ-closedness) is a generalization of both θ-preopenness and weak semi-θ-openness (resp. θ-preclosedness and weak semi-θ-closedness). In [3], the present authors introduced and studied the notions of

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$\theta p(\Lambda,p)$-open functions and $\theta p(\Lambda,p)$-closed functions. Khampakdee and Boonpok [11] investigated some properties of (Λ,p)-closed functions. Chutiman and Boonpok [9] obtained several properties of weakly $b(\Lambda,p)$-open functions. Furthermore, some characterizations of weakly $\delta(\Lambda,p)$-open functions and weakly $\delta(\Lambda,p)$-closed functions were presented in [19] and [12], respectively. Klanarong and Boonpok [13] studied the notions of weakly $s(\Lambda,p)$-open functions and weakly $s(\Lambda,p)$-closed functions by utilizing $s(\Lambda,p)$-open sets and the $s(\Lambda,p)$-closure operator. In [4], the authors introduced and investigated the concepts of weakly $\beta(\Lambda,p)$-open functions and weakly $\beta(\Lambda,p)$-closed functions. Moreover, several characterizations of weakly $p(\Lambda,p)$-open (resp. weakly $p(\Lambda,p)$-closed) functions and weakly $\theta s(\Lambda,p)$-open (resp. weakly $\theta s(\Lambda,p)$-closed) functions were established in [5] and [2], respectively. In this article, we introduce the notions of weakly $\theta b(\Lambda,p)$-open functions and $\theta b(\Lambda,p)$-closed functions. In particular, several characterizations of weakly $\theta b(\Lambda,p)$-open functions and $\theta b(\Lambda,p)$-closed functions are investigated.

2. Preliminaries

Throughout the present paper, spaces (X,τ) and (Y,σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X,τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X,τ) is said to be *preopen* [14] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X,τ) is denoted by $PO(X,\tau)$. A subset $\Lambda_p(A)$ [10] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X,\tau)\}$. A subset A of a topological space (X,τ) is called a Λ_p-*set* [6] (*pre-*Λ*-set* [10]) if $A = \Lambda_p(A)$. A subset A of a topological space (X,τ) is called (Λ,p)-*closed* [6] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ,p)-closed set is called (Λ,p)-*open*. The family of all (Λ,p)-open (resp. (Λ,p)-closed) sets in a topological space (X,τ) is denoted by $\Lambda_pO(X,\tau)$ (resp. $\Lambda_pC(X,\tau)$). Let A be a subset of a topological space (X,τ). A point $x \in X$ is called a (Λ,p)-*cluster point* [6] of A if $A \cap U \neq \emptyset$ for every (Λ,p)-open set U of X containing x. The set of all (Λ,p)-cluster points of A is called the (Λ,p)-*closure* [6] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ,p)-open sets of X contained in A is called the (Λ,p)-*interior* [6] of A and is denoted by $A_{(\Lambda,p)}$. Let A be a subset of a topological space (X,τ). The $\theta(\Lambda,p)$-*closure* [6] of A, $A^{\theta(\Lambda,p)}$, is defined as follows: $A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap U^{(\Lambda,p)} \neq \emptyset$ for each (Λ,p)-open set U containing $x\}$. A subset A of a topological space (X,τ) is called $\theta(\Lambda,p)$-*closed* [6] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda,p)$-closed set is said to be $\theta(\Lambda,p)$-*open*. A point $x \in X$ is called a $\theta(\Lambda,p)$-*interior point* [20] of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_pO(X,\tau)$. The set of all $\theta(\Lambda,p)$-interior points of A is called the $\theta(\Lambda,p)$-*interior* [20] of A and is denoted by $A_{\theta(\Lambda,p)}$.

Lemma 1. [20] *For subsets A and B of a topological space (X,τ), the following properties hold:*

(1) $X - A^{\theta(\Lambda,p)} = [X - A]_{\theta(\Lambda,p)}$ *and* $X - A_{\theta(\Lambda,p)} = [X - A]^{\theta(\Lambda,p)}$.

(2) A *is* $\theta(\Lambda,p)$*-open if and only if* $A = A_{\theta(\Lambda,p)}$.

(3) $A \subseteq A^{(\Lambda,p)} \subseteq A^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq A_{(\Lambda,p)} \subseteq A$.

(4) If $A \subseteq B$*, then* $A^{\theta(\Lambda,p)} \subseteq B^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq B_{\theta(\Lambda,p)}$.

(5) If A *is* (Λ,p)*-open, then* $A^{(\Lambda,p)} = A^{\theta(\Lambda,p)}$.

A subset A of a topological space (X,τ) is said to be $s(\Lambda,p)$*-open* [6] (resp. $r(\Lambda,p)$*-open* [6], $p(\Lambda,p)$*-open* [6], $\alpha(\Lambda,p)$*-open* [21]) if $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp. $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$). The family of all $s(\Lambda,p)$-open (resp. $r(\Lambda,p)$-open, $p(\Lambda,p)$-open, $\alpha(\Lambda,p)$-open) sets in a topological space (X,τ) is denoted by $s(\Lambda,p)O(X,\tau)$ (resp. $r(\Lambda,p)O(X,\tau)$, $p(\Lambda,p)O(X,\tau)$, $\alpha(\Lambda,p)O(X,\tau)$). The union of all $s(\Lambda,p)$-open (resp. $p(\Lambda,p)$-open, $\alpha(\Lambda,p)$-open) sets of X contained in A is called the $s(\Lambda,p)$*-interior* (resp. $p(\Lambda,p)$*-interior*, $\alpha(\Lambda,p)$*-interior*) of A and is denoted by $A_{s(\Lambda,p)}$ (resp. $A_{p(\Lambda,p)}$, $A_{\alpha(\Lambda,p)}$). The complement of a $s(\Lambda,p)$-open (resp. $r(\Lambda,p)$-open, $p(\Lambda,p)$-open, $\alpha(\Lambda,p)$-open) set is called $s(\Lambda,p)$*-closed* (resp. $r(\Lambda,p)$*-closed*, $p(\Lambda,p)$*-closed*, $\alpha(\Lambda,p)$*-closed*). The family of all $s(\Lambda,p)$-closed (resp. $r(\Lambda,p)$-closed, $p(\Lambda,p)$-closed, $\alpha(\Lambda,p)$-closed) sets in a topological space (X,τ) is denoted by $s(\Lambda,p)C(X,\tau)$ (resp. $r(\Lambda,p)C(X,\tau)$, $p(\Lambda,p)C(X,\tau)$, $\alpha(\Lambda,p)C(X,\tau)$). The intersection of all $s(\Lambda,p)$-closed (resp. $p(\Lambda,p)$-closed, $\alpha(\Lambda,p)$-closed) sets of X containing A is called the $s(\Lambda,p)$*-closure* (resp. $p(\Lambda,p)$*-closure*, $\alpha(\Lambda,p)$*-closure*) of A and is denoted by $A^{s(\Lambda,p)}$ (resp. $A^{p(\Lambda,p)}$, $A^{\alpha(\Lambda,p)}$).

Lemma 2. *For subsets* A *and* B *of a topological space* (X,τ)*, the following properties hold:*

(1) $A_{\alpha(\Lambda,p)} = A \cap [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$;

(2) $A_{s(\Lambda,p)} = A \cap [A_{(\Lambda,p)}]^{(\Lambda,p)}$;

(3) $A_{p(\Lambda,p)} = A \cap [A^{(\Lambda,p)}]_{(\Lambda,p)}$.

A subset A of a topological space (X,τ) is said to be $b(\Lambda,p)$*-open* if

$$A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)} \cup [A_{(\Lambda,p)}]^{(\Lambda,p)}.$$

The family of all $b(\Lambda,p)$-open sets in a topological space (X,τ) is denoted by $b(\Lambda,p)O(X,\tau)$. The union of all $b(\Lambda,p)$-open sets of X contained in A is called the $b(\Lambda,p)$*-interior* of A and is denoted by $A_{b(\Lambda,p)}$. The complement of a $b(\Lambda,p)$-open set is called $b(\Lambda,p)$*-closed*. The family of all $b(\Lambda,p)$-closed sets in a topological space (X,τ) is denoted by $b(\Lambda,p)C(X,\tau)$. The intersection of all $b(\Lambda,p)$-closed sets of X containing A is called the $b(\Lambda,p)$*-closure* of A and is denoted by $A^{b(\Lambda,p)}$.

Lemma 3. *For subsets* A *and* B *of a topological space* (X,τ)*, the following properties hold:*

(1) $A_{b(\Lambda,p)} = A_{s(\Lambda,p)} \cup A_{p(\Lambda,p)}$;

(2) $A^{b(\Lambda,p)} = A^{s(\Lambda,p)} \cap A^{p(\Lambda,p)}$*;*

(3) $[X - A]^{b(\Lambda,p)} = X - A_{b(\Lambda,p)}$*;*

(4) $x \in A^{b(\Lambda,p)}$ *if and only if* $A \cap U \neq \emptyset$ *for every* $U \in b(\Lambda,p)O(X,\tau)$ *containing* x*;*

(5) $A \in b(\Lambda,p)C(X,\tau)$ *if and only if* $A = A^{b(\Lambda,p)}$*;*

(6) $[A^{b(\Lambda,p)}]_{p(\Lambda,p)} = [A_{p(\Lambda,p)}]^{b(\Lambda,p)}$.

Let A be a subset of a topological space (X,τ). A point $x \in X$ is called a $\theta b(\Lambda,p)$-*cluster point* of A if $A \cap U^{b(\Lambda,p)} \neq \emptyset$ for every $b(\Lambda,p)$-open set U of X containing x. The set of all $\theta b(\Lambda,p)$-cluster points of A is called the $\theta b(\Lambda,p)$-*closure* of A and is denoted by $A^{\theta b(\Lambda,p)}$. If $A = A^{\theta b(\Lambda,p)}$, then A is called $\theta b(\Lambda,p)$-*closed*. The complement of a $\theta b(\Lambda,p)$-closed set is called $\theta b(\Lambda,p)$-*open*. The $\theta b(\Lambda,p)$-*interior* of A is defined by the union of all $\theta b(\Lambda,p)$-open sets of X contained in A and is denoted by $A_{\theta b(\Lambda,p)}$. The family of all $\theta b(\Lambda,p)$-open (resp. $\theta b(\Lambda,p)$-closed) sets in a topological space (X,τ) is denoted by $\theta b(\Lambda,p)O(X,\tau)$ (resp. $\theta b(\Lambda,p)C(X,\tau)$).

Lemma 4. *For subsets* A *and* $C_\gamma(\gamma \in \Gamma)$ *of a topological space* (X,τ)*, the following properties hold:*

(1) If $C_\gamma \in \theta b(\Lambda,p)O(X,\tau)$ *for each* $\gamma \in \Gamma$*, then* $\cup_{\gamma\in\Gamma} C_\gamma \in \theta b(\Lambda,p)O(X,\tau)$.

(2) If $A \in b(\Lambda,p)C(X,\tau)$*, then* $A_{b(\Lambda,p)} = A_{\theta b(\Lambda,p)}$.

(3) $A^{\theta b(\Lambda,p)} \in \theta b(\Lambda,p)C(X,\tau)$.

3. Weakly $\theta b(\Lambda,p)$-open functions

In this section, we introduce the concept of weakly $\theta b(\Lambda,p)$-open functions. Moreover, some characterizations of weakly $\theta b(\Lambda,p)$-open functions are discussed.

Definition 1. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is said to be weakly* $\theta b(\Lambda,p)$*-open if* $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$ *for each* (Λ,p)*-open set* U *of* X.

Theorem 1. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is weakly* $\theta b(\Lambda,p)$*-open;*

(2) $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\theta b(\Lambda,p)}$ *for every subset* A *of* X*;*

(3) $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\theta b(\Lambda,p)})$ *for every subset* B *of* Y*;*

(4) $f^{-1}(B^{\theta b(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$ *for every subset* B *of* Y.

Proof. (1) $\Rightarrow$ (2): Let A be any subset of X and $x \in A_{\theta(\Lambda,p)}$. Then, there exists a (Λ, p)-open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$. Then, $f(x) \in f(U) \subseteq f(U^{(\Lambda,p)}) \subseteq f(A)$. Since f is weakly $\theta b(\Lambda, p)$-open, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)} \subseteq [f(A)]_{\theta b(\Lambda,p)}$. This implies that $f(x) \in [f(A)]_{\theta b(\Lambda,p)}$. Thus, $x \in f^{-1}([f(A)]_{\theta b(\Lambda,p)})$ and hence $A_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{\theta b(\Lambda,p)})$. This shows that $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\theta b(\Lambda,p)}$.

(2) $\Rightarrow$ (3): Let B be any subset of Y. Then by (2), $f([f^{-1}(B)]_{\theta(\Lambda,p)}) \subseteq B_{\theta b(\Lambda,p)}$. Thus, $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\theta b(\Lambda,p)})$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. Using (3), we have

$$\begin{aligned} X - [f^{-1}(B)]^{\theta(\Lambda,p)} &= [X - f^{-1}(B)]_{\theta(\Lambda,p)} \\ &= [f^{-1}(Y - B)]_{\theta(\Lambda,p)} \\ &\subseteq f^{-1}([Y - B]_{\theta b(\Lambda,p)}) \\ &= f^{-}(Y - B^{\theta b(\Lambda,p)}) \\ &= X - f^{-1}(B^{\theta b(\Lambda,p)}). \end{aligned}$$

Thus, $f^{-1}(B^{\theta b(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$.

(4) $\Rightarrow$ (1): Let U be any (Λ, p)-open set of X. By (4),

$$f^{-1}([Y - f(U^{(\Lambda,p)})]^{\theta b(\Lambda,p)}) \subseteq [f^{-1}(Y - f(U^{(\Lambda,p)}))]^{\theta(\Lambda,p)}.$$

Thus, $f^{-1}(Y - [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}) \subseteq [X - f^{-1}(f(U^{(\Lambda,p)}))]^{\theta(\Lambda,p)} \subseteq [X - U^{(\Lambda,p)}]^{\theta(\Lambda,p)}$ and hence $U \subseteq [U^{(\Lambda,p)}]_{\theta(\Lambda,p)} \subseteq f^{-1}([f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)})$. Therefore, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$. This shows that f is weakly $\theta b(\Lambda, p)$-open.

Theorem 2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $\theta b(\Lambda, p)$-open;

(2) for each $x \in X$ and each (Λ, p)-open set U of X containing x, there exists a $\theta b(\Lambda, p)$-open set V of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$.

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and U be any (Λ, p)-open set of X containing x. Since f is weakly $\theta b(\Lambda, p)$-open, $f(x) \in f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$. Let $V = [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$. Then, V is $\theta b(\Lambda, p)$-open and $f(x) \in V \subseteq f(U^{(\Lambda,p)})$.

(2) $\Rightarrow$ (1): Let U be any (Λ, p)-open set of X and $y \in f(U)$. It follows from (2) that $V \subseteq f(U^{(\Lambda,p)})$ for some $\theta b(\Lambda, p)$-open set V of Y containing y. Thus, $y \in V \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$ and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$. This shows that f is weakly $\theta b(\Lambda, p)$-open.

Theorem 3. *For a bijective function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is weakly $\theta b(\Lambda, p)$-open;

(2) $[f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(K)$ for each (Λ, p)-closed set K of X;

(3) $[f(U)]^{\theta b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ *for each* (Λ,p)*-open set* U *of* X*.*

Proof. (1) $\Rightarrow$ (2): Let K be any (Λ,p)-closed set of X. Then, we have

$$f(X-K) = Y - f(K) \subseteq [f([X-K]^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$$

and hence $Y - f(K) \subseteq Y - [f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)}$. Thus, $[f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(K)$.

(2) $\Rightarrow$ (3): Let U be any (Λ,p)-open set of X. Since $U^{(\Lambda,p)}$ is a (Λ,p)-closed set and $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$, by (2) we have $[f(U)]^{\theta b(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$.

(3) $\Rightarrow$ (1): Let U be any (Λ,p)-open set of X. Using (3), we have

$$\begin{aligned} Y - [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)} &= [Y - f(U^{(\Lambda,p)})]^{\theta b(\Lambda,p)} \\ &= [f(X - U^{(\Lambda,p)})]^{\theta b(\Lambda,p)} \\ &\subseteq f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) \\ &= f(X - [U^{(\Lambda,p)}]_{(\Lambda,p)}) \\ &\subseteq f(X-U) \\ &= Y - f(U) \end{aligned}$$

and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$. This shows that f is weakly $\theta b(\Lambda,p)$-open.

The proof of the following theorem is straightforward and thus is omitted.

Theorem 4. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is weakly* $\theta b(\Lambda,p)$*-open;*

(2) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$ *for each* $p(\Lambda,p)$*-open set* U *of* X*;*

(3) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$ *for each* $\alpha(\Lambda,p)$*-open set* U *of* X*;*

(4) $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{\theta b(\Lambda,p)}$ *for each* (Λ,p)*-open set* U *of* X*;*

(5) $f(K_{(\Lambda,p)}) \subseteq [f(K)]_{\theta b(\Lambda,p)}$ *for each* (Λ,p)*-closed set* K *of* X*.*

4. Weakly $\theta b(\Lambda,p)$-closed functions

In this section, we introduce the concept of weakly $\theta b(\Lambda,p)$-closed functions. Furthermore, some characterizations of weakly $\theta b(\Lambda,p)$-closed functions are investigated.

Definition 2. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is said to be weakly* $\theta b(\Lambda,p)$*-closed if* $[f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(K)$ *for each* (Λ,p)*-closed set* K *of* X*.*

Theorem 5. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is weakly* $\theta b(\Lambda,p)$*-closed;*

(2) $[f(U)]^{\theta b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ *for every* (Λ,p)*-open set* U *of* X.

Proof. (1) ⇒ (2): Let U be any (Λ,p)-open set of X. Since $U^{(\Lambda,p)}$ is a (Λ,p)-closed set and $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$, we have $[f(U)]^{\theta b(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$.

(2) ⇒ (1): Let K be any (Λ,p)-closed set of X. Then, we have

$$[f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)}) \subseteq f(K^{(\Lambda,p)}) = f(K)$$

and hence f is weakly $\theta b(\Lambda,p)$-closed.

Corollary 1. *A bijective function* $f : (X,\tau) \to (Y,\sigma)$ *is weakly* $\theta b(\Lambda,p)$*-open if and only if* f *is weakly* $\theta b(\Lambda,p)$*-closed.*

Proof. This is an immediate consequence of Theorem 3 and 5.

The proof of the following theorem is straightforward and thus is omitted.

Theorem 6. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is weakly* $\theta b(\Lambda,p)$*-closed;*

(2) $[f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(K)$ *for every* $p(\Lambda,p)$*-closed set* K *of* X*;*

(3) $[f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(K)$ *for every* $\alpha(\Lambda,p)$*-closed set* K *of* X*;*

(4) $[f([A^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(A^{(\Lambda,p)})$ *for every subset* A *of* X*;*

(5) $[f(U)]^{\theta b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ *for every* $p(\Lambda,p)$*-open set* U *of* X.

Theorem 7. *For a function* $f : (X,\tau) \to (Y,\sigma)$*, the following properties are equivalent:*

(1) f *is weakly* $\theta b(\Lambda,p)$*-closed;*

(2) $[f(U)]^{\theta b(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ *for every* $r(\Lambda,p)$*-open set* U *of* X*;*

(3) for each subset B *of* Y *and each* (Λ,p)*-open set* U *of* X *with* $f^{-1}(B) \subseteq U$*, there exists a* $\theta b(\Lambda,p)$*-open set* V *of* Y *such that* $B \subseteq V$ *and* $f^{-1}(V) \subseteq U^{(\Lambda,p)}$*;*

(4) for each point $y \in Y$ *and each* (Λ,p)*-open set* U *of* X *with* $f^{-1}(y) \subseteq U$*, there exists a* $\theta b(\Lambda,p)$*-open set* V *of* Y *containing* y *and* $f^{-1}(V) \subseteq U^{(\Lambda,p)}$.

Proof. (1) ⇒ (2): By Theorem 5.

(2) ⇒ (3): Let B be any subset of Y and U be any (Λ,p)-open set of X with $f^{-1}(B) \subseteq U$. Then, we have $f^{-1}(B) \cap [X - U^{(\Lambda,p)}]^{(\Lambda,p)} = \emptyset$ and hence $B \cap f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) = \emptyset$. Since $X - U^{(\Lambda,p)}$ is $r(\Lambda,p)$-open, $B \cap [f(X - U^{(\Lambda,p)})]^{\theta b(\Lambda,p)} = \emptyset$. Let

$$V = Y - [f(X - U^{(\Lambda,p)})]^{\theta b(\Lambda,p)}.$$

Then, V is a $\theta b(\Lambda,p)$-open set with $B \subseteq V$ and

$$\begin{aligned} f^{-1}(V) &\subseteq X - f^{-1}([f(X - U^{(\Lambda,p)})]^{\theta b(\Lambda,p)}) \\ &\subseteq X - f^{-1}(f(X - U^{(\Lambda,p)})) \\ &\subseteq U^{(\Lambda,p)}. \end{aligned}$$

(3) $\Rightarrow$ (4): This is obvious.

(4) $\Rightarrow$ (1): Let K be any (Λ,p)-closed set of Y and $y \in Y - f(K)$. Since $f^{-1}(y) \subseteq X - K$, by (4) there exists a $\theta b(\Lambda,p)$-open set V of Y such that $y \in V$ and

$$f^{-1}(V) \subseteq [X - K]^{(\Lambda,p)} = X - K_{(\Lambda,p)}.$$

Thus, $V \cap f(K_{(\Lambda,p)}) = \emptyset$ and hence $y \notin [f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)}$. Therefore, $[f(K_{(\Lambda,p)})]^{\theta b(\Lambda,p)} \subseteq f(K)$. This shows that f is weakly $\theta b(\Lambda,p)$-closed.

Theorem 8. *If $f : (X,\tau) \to (Y,\sigma)$ is a bijective weakly $\theta b(\Lambda,p)$-closed function, then for every subset B of Y and every (Λ,p)-open set U of X with $f^{-1}(B) \subseteq U$, there exists a $\theta b(\Lambda,p)$-closed set K of Y such that $B \subseteq K$ and $f^{-1}(K) \subseteq U^{(\Lambda,p)}$.*

Proof. Let B be any subset of Y and U be any (Λ,p)-open set of X with $f^{-1}(B) \subseteq U$. Put $K = [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta b(\Lambda,p)}$. Then, K is a $\theta b(\Lambda,p)$-closed set of Y such that $B \subseteq K$, since $B \subseteq f(U) \subseteq f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta b(\Lambda,p)} = K$. Since f is weakly $\theta b(\Lambda,p)$-closed, by Theorem 6 we have $f^{-1}(K) \subseteq U^{(\Lambda,p)}$.

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