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On The Representations and Characters of Quaternions Group $\mathbf{Q_8}$

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Abstract. We are interested in studying the linear representations of the Quaternions group $\mathbf{Q_8}$ by determining it's charater table and irreducible representations which allows us to construct some representations of degre 6 and 8 of $\mathbf{Q_8}$.

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1. Introduction

Group representation theory allows the study of abstract groups by representing their elements by invertible matrices. We then have the methods of linear algebra which often make the study of these groups easier and make it possible to obtain new properties. The idea is to make a group G act on a vector space V such that the action of each element is compatible with the structure of the vector space, that is to say it is an element of $\mathbf{GL(V)}$ the group of linear automorphisms of V and more only a bijection of V on V.

This concept emerges at the end of the 19th century and the general study of the representations of a group is largely developed by William Burnside and Ferdinand Georg Frobenius at the beginning of the 20th century[3].

Representation theory has enabled remarkable advances, first of all in group theory. In particular, it plays a fundamental role in the classification theorem of finite simple groups[5].

The theory of representations of general groups (not necessarily finite) and algebras also has numerous applications in crystallographic chemistry, in engineering and especially in

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quantum physics: the theory of representation makes it possible to analyze the symmetries of a related physical problem with a group by classifying the solutions of this problem according to the irreducible representations[2].

In this work, we recall first some results on the theory of reprentations and character of finite groups, and by the end we arrive to determine the irreducible representations and to draw up the character table of the quaternions group $\mathbf{Q_8}$ which allows us to construct some representations of degre 6 and 8 of this group.

2. Preleminaries

Let E be a vector space on $\mathbb{K}$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, and let $\mathbf{GL}(E)$ the group of isomorphisms of E on itself.

Definition 1 (See [3])**.** *A representation of dimension n of a group G is the data of a complex vector space E of dimension n, and of a morphism of groups*

$$\tau : G \to \boldsymbol{GL}(E)$$

For example, the standard representation of S_n is given as follows [4]:

$$\tau : S_n \to \mathbf{GL}_n(\mathbb{C}) \qquad \tau_\sigma(e_i) = e_{\sigma(i)}$$

We obtain the matrix of τ_σ by permuting the columns of the identity matrix in accordance to σ. For example, for $n = 3$, we have:

$$\tau_{(12)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \tau_{(123)} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Furthermore, if $t \in \mathbf{S_3}$ is the transposition $123 \to 132$ and c be the circular permutation $123 \to 231$ which generate $\mathbf{S_3}$. We set $j = e^{\frac{2i\pi}{3}}$. We can represent $\mathbf{S_3}$ in $\mathbb{C}^2$ by setting:

$$\tau(e) = I, \tau(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tau(c) = \begin{bmatrix} j & 0 \\ 0 & j^2 \end{bmatrix}$$

In particular, if the vector space E is of dimension $n = |G|$ with basis indexed by the elements of G, the representation is called the regular representation of G.

Definition 2 (See [3])**.** *Let $\tau : G \to \boldsymbol{GL}(E)$ be a linear representation, and F a vector subspace of E stable for the operations of G, then $\tau^F : G \to \boldsymbol{GL}(F)$ is a linear representation of G in F; called a* ***sub-representation*** *of E.*

In addition, (τ, E) is called irreducible if E is not reduced to 0, and if no vector subspace of E is stable by G, except 0 and E. And we have the following theorem:

Theorem 1 (See [2], Maschke theorem)**.** *every representation is a direct sum of irreducible representations.*

Recall here the notion of the character which is a function on G with complex values characterizing the representation.

Definition 3 (See [1]). *Let $\tau : G \to \boldsymbol{GL}(E)$ be a linear representation of a finite group G in the vector space E. The character of G is defined as follow:*

$$For\ all\ g \in G, \qquad \chi_\tau(g) = Tr(\tau_g).$$

Furthermore, two representations of the same character are isomorphic, and we have:

Proposition 1 (See [2]). *The character R_G of the regular representation is given by:*

$$R_G(1) = |G|,\ and\ R_G(g) = 0\ if\ g \neq 1.$$

The importance of the regular representation lies in the fact that an irreducible representation F_i is contained in it a number of times equal to its degree n_i.

Proposition 2 (See [2]). *The degrees n_i verify the relation $\sum n_i^2 = |G|$.*

Thus, we define the character table of a finite group G as follows:

Let $c = |Conj(G)|$ the number of conjugation classes of G. The character map of G is an array $c \times c$ of which the entries are the values of the irreducible characters on the conjugation classes of G, the coefficient at the intersection of the column corresponding to the character χ of the line corresponding to the conjugation class C, being $\chi(C)$. It's sort of the card of the group G[3].

For example, the group $\{\pm 1\}$ has two conjugation classes 1 and -1, and two irreducible characters 1 and χ (of dimension 1 since $\{\pm 1\}$ is commutative); its character table is very easy to establish:

	1	-1
χ_1	1	1
χ_2	1	-1

The example of the group $\{\pm 1\}$ is a little too trivial to give an idea of how we can construct the character table of a group. There are other groups when we use other techniques (Burnside formula, character orthogonality relationships, etc.) to establish character tables. For example the character table of the symmetric group σ_3 is given as follows :

First recall that:

$$\sigma_3 = \{1, (12), (23), (13), (123), (132)\}$$

σ_3 has 3 characters because that is its number of conjugation classes. First there is the trivial character χ_1. Then the morphism χ_2 given by the signature of the elements of σ_3. Then by Burnside's formula we obtain that the third character χ_3 is such that $n_3 = 2$. We therefore obtain for the moment the following character table[1].

	1	(12)	(123)
χ_1	1	1	1
χ_2	1	-1	1
χ_3	2	a	b

To have a and b just use the orthogonality of the character table columns. So we get $a = 0$ and $b = -1$.

Hence finally:

	1	(12)	(123)
χ_1	1	1	1
χ_2	1	-1	1
χ_3	2	0	-1

3. Representations and the characters of the quaternion group $\mathbf{Q_8}$

Recall that the non-commutative field $\mathbb{H}$ of **quaternions** can be obtained from the field $\mathbb{C}$ by the construction of **Cayley-Dickson**: we provide the set $\mathbb{C}^2$ of pairs (z, w) of complex numbers of the following addition and multiplication:

$$(z, w) + (z', w') = (z + z', w + w')$$

$$(z, w).(z', w') = (zz' - w'w, w'z + wz')$$

Note that this construction carried out from the field $\mathbb{R}$ of the real numbers produces the field $\mathbb{C}$.

The application $z \to (z, 0)$ allows us to canonically identify $\mathbb{C}$ to a subfield of $\mathbb{H}$.

We set $j = (0, 1)$ we have $j^2 = -1$ and $\overline{j}z = zj$ for all $z \in \mathbb{C}$ (in particular $k = ij = -ji$).

Any element $h = (z, w)$ of $\mathbb{H}$ is then uniquely written $h = z + jw$ with $z, w \in \mathbb{C}$ so that $(1, j)$ is a basis of the $\mathbb{C}$-vector space $\mathbb{H}$ while $(1, i, j, k)$ is a basis of the $\mathbb{R}$-vector space $\mathbb{H}$.

We have:

$$\begin{cases} i^2 = j^2 = k^2 = -1 \\ ij = -ji = k \\ jk = -kj = i \\ ki = -ik = j \end{cases}$$

We again set $\overline{h} = z - jw$ so that $\overline{\overline{h}} = h$ and $\overline{hh'} = \overline{h'h}$ for all $h, h' \in \mathbb{H}$. We then have $N(h) = h\bar{h} \in \mathbb{R}_+$ and $N(hh') = N(h)N(h')$. Then all $h \in \mathbb{H}^*$ is invertible and we have $h^{-1} = \dfrac{1}{N(h)}\overline{h}$.

Finally, we have the canonical representation of the $\mathbb{R}$-algebra $\mathbb{H}$:

$$\pi : \mathbb{H} \to \mathbf{M_2}(\mathbb{C})$$

$$h = z + jw \to \begin{pmatrix} z & -\overline{w} \\ w & \overline{z} \end{pmatrix}$$

Note that we have $N(h) = det(\pi(h))$ so that the matrix $\pi(h)$ is invertible if and only if $h \neq 0$. In particular, we have:

$$\pi(\pm 1) = \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \pi(\pm i) = \pm \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$\pi(\pm j) = \pm \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \pi(\pm k) = \pm \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$

The quaternion group :

$$\mathbf{Q_8} =< i, j >= \{1, -1, i, -i, j, -j, k, -k\}$$

The group $\mathbf{Q_8}$ has 5 conjugation classes:

$$K_1 = \{1\} \qquad K_2 = \{-1\} \qquad K_3 = \{i, -i\} \qquad K_4 = \{j, -j\} \qquad K_5 = \{k, -k\}$$

3.1. Character table of group $\mathbf{Q_8}$

We have the group $\{\pm 1\}$ is distinguished in $\mathbf{Q_8}$ and $\mathbf{Q_8}/\{\pm 1\} \approx V$ (V : Klein group) and according to the Burnside formula ($n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2 = 8$), we will have $n_5 = 2$ So the character table of $\mathbf{Q_8}$ is of the form:

	K_1	K_2	K_3	K_4	K_5
χ_1	1	1	1	1	1
χ_2	1	1	1	-1	-1
χ_3	1	1	-1	1	-1
χ_4	1	1	-1	-1	1
χ_5	2	a	b	c	d

Then by orthogonality of the columns we obtain $a = -2$ et $b = c = d = 0$. Finally the character table of $\mathbf{Q_8}$ is as follows:

	K_1	K_2	K_3	K_4	K_5
χ_1	1	1	1	1	1
χ_2	1	1	1	-1	-1
χ_3	1	1	-1	1	-1
χ_4	1	1	-1	-1	1
χ_5	2	-2	0	0	0

3.2. Irreducible representations of the group $\mathbf{Q_8}$

3.2.1. Realization of irreducible representations of degree 1

Let $\tau : \mathbb{H} \to \mathbb{C}^*$ be a 1-dimensional representation of $\mathbb{H}$. Then τ cannot be faithful, otherwise $\mathbb{H}$ would be abelian.

Therefore, $Ker\tau$ is a distinguished subgroup of $\mathbb{H}$, $Ker\tau \neq \{1\}$. If $Ker\tau = \mathbb{H}$ we obtain the trivial representation τ_1. Others possibilities for $Ker\tau$ are:

$$Z(\mathbb{H}) = \{\pm 1\}, \quad I = \{\pm 1, \pm i\}$$

$$J = \{\pm 1, \pm j\}, \quad K = \{\pm 1, \pm k\}$$

But, $\mathbb{H}/I \approx \mathbb{H}/J \approx \mathbb{H}/K \approx \mathbf{Z_2}$ and $\mathbb{H}/Z(\mathbb{H}) \approx \mathbf{Z_2} \times \mathbf{Z_2}$. Obviously, τ is factorized as the composite of the projection on the quotient $\mathbb{H}/Ker\tau \to \mathbb{C}^*$. Furthermore, there is no only one way to represent $\mathbf{Z_2}$ in a non-trivial way in $\mathbb{C}$ and therefore we obtain the following representations:

When $ker\tau = I$

$$\tau_2(\{\pm 1, \pm i\}) = 1, \quad \tau_2(\{\pm j, \pm k\}) = -1$$

When $ker\tau = J$

$$\tau_3(\{\pm 1, \pm j\}) = 1, \quad \tau_3(\{\pm i, \pm k\}) = -1$$

When $ker\tau = K$

$$\tau_4(\{\pm 1, \pm k\}) = 1, \quad \tau_4(\{\pm i, \pm j\}) = -1$$

When $ker\tau = Z(\mathbb{H})$, the representations obtained by composing the projection on the quotient by a representation of E in $\mathbb{C}^*$ still give the three representations above. The four representations thus found are not isomorphic with each other.

3.2.2. Realization of the irreducible representation of degree 2

$\mathbf{Q_8}$ operates by left multiplication on the space $\mathbb{H}$ of quaternions, which is a vector space on the right on $\mathbb{C} = \mathbb{R} \oplus \mathbb{R}i$, of dimension 2. In the basis $\{1, j\}$, we obtain the following matrix representation:

$$\tau_5(\pm 1) = \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \tau_5(\pm i) = \pm \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$\tau_5(\pm j) = \pm \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \tau_5(\pm k) = \pm \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$

3.3. Some representations of the group $\mathbf{Q_8}$

We know that any representation is the sum of irreducible representations, therefore, from these we can construct the desired representations.

3.3.1. The regular representation of $\mathbf{Q_8}$

We have $|\mathbf{Q_8}| = 8$, therefore the degree of the regular representation equal to 8, and we know that every irreducible representation is contained in the regular representation a number of times equal to its degree, therefore:

$$\tau_{reg} = \tau_1 \oplus \tau_2 \oplus \tau_3 \oplus \tau_4 \oplus 2\tau_5$$

so:

$$\tau_{reg} : \mathbf{Q_8} \to \mathbf{GL_8}(\mathbb{C})$$

defined by :

$$\tau_{reg}(1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\tau_{reg}(-1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\tau_{reg}(i) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \end{pmatrix}$$

$$\tau_{reg}(-i) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \end{pmatrix}$$

$$\tau_{reg}(j) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\tau_{reg}(-j) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\tau_{reg}(k) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \end{pmatrix}$$

$$\tau_{reg}(-k) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \end{pmatrix}$$

3.3.2. The representation of degree 6 of $\mathbf{Q_8}$

Let θ be the representation of degree 6 of $\mathbf{Q_8}$, then:

$$\theta = \tau_1 \oplus \tau_2 \oplus \tau_3 \oplus \tau_4 \oplus \tau_5$$

so:

$$\theta : \mathbf{Q_8} \to \mathbf{GL_6}(\mathbb{C})$$

defined by :

$$\theta(1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} ; \quad \theta(-1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\theta(i) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \end{pmatrix} ; \quad \theta(-i) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & i \end{pmatrix}$$

$$\theta(j) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} ; \quad \theta(-j) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\theta(k) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & -i & 0 \end{pmatrix} ; \quad \theta(-k) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & i & 0 \end{pmatrix}$$

References

[1] Yvette Kosmann-Schwarzbach. *Groups and symmetries.* Springer Science and Business Media, 2009.

[2] David Renard. *Groups and representation.* Ecole Polytechnique, 2010.

[3] Jean-Pierre Serre. *Linear representations of finite groups*. Herman collections methods, Paris, 1998.

[4] Benjamin Steinberg. *Representation Theory of Finite Groups An Introductory Approach*. 2011.

[5] W.Fulton and J.Harris. *Representation Theory, graduate text in mathematics*. springer, Paris, 1991.