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Weakly quasi (τ_1, τ_2)-continuous multifunctions

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Abstract. Our main purpose is to introduce the notion of weakly quasi (τ_1, τ_2)-continuous multifunctions. Furthermore, several characterizations of weakly quasi (τ_1, τ_2)-continuous multifunctions are established.

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1. Introduction

The concept of quasi continuous functions was introduced by Marcus [28]. Popa [33] introduced and investigated the notion of almost quasi continuous functions. Neubrunnovaá [29] showed that quasi continuity is equivalent to semi-continuity due to Levine [27]. Popa and Stan [36] introduced and studied the notion of weakly quasi continuous functions. Weak quasi continuity is implied by quasi continuity and weak continuity [26] which are independent of each other. It is shown in [30] that weak quasi continuity is equivalent to weak semi-continuity due to Arya and Bhamini [1] and Kar and Bhattacharyya [24]. Duangphui et al. [23] introduced and investigated the notion of weakly $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, some characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, (Λ, sp)-continuous functions, $\delta p(\Lambda, s)$-continuous functions, $(\Lambda, p(\star))$-continuous functions, pairwise weakly M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions and weakly (τ_1, τ_2)-continuous functions were presented in [38], [40], [12], [37], [18], [11], [10], [6], [3], [43], [39], [9], [4], [19], [17] and [13], respectively.

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The concept of almost quasi continuous multifunctions was introduced by Popa and Noiri [35]. Noiri and Popa [31] introduced and studied the notion of weakly quasi continuous multifunctions. Several characterizations of weakly quasi continuous multifunctions have been obtained in [35]. Popa and Noiri [34] introduced and investigated the concepts of upper and lower θ-quasi continuous multifunctions. In particular, some characterizations of upper and lower θ-quasi continuous multifunctions were established in [32]. In [8], the present author introduced and studied the concepts of almost quasi $\star$-continuous multifunctions and weakly quasi $\star$-continuous multifunctions. Laprom et al. [25] introduced and investigated the notion of almost $\beta(\tau_1, \tau_2)$-continuous multifunctions. Viriyapong and Boonpok [42] introduced and studied the concept of weakly $(\tau_1, \tau_2)\alpha$-continuous multifunctions. Furthermore, several characterizations of weakly $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, almost weakly $\star$-continuous multifunctions, weakly $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, weakly $\imath^\star$-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions and weakly (τ_1, τ_2)-continuous multifunctions were investigated in [7], [21], [20], [5], [15], [14], [44], [16] and [41], respectively. In this paper, we introduce the concept of weakly quasi (τ_1, τ_2)-continuous multifunctions. Moreover, some characterizations of weakly quasi (τ_1, τ_2)-continuous multifunctions are discussed.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [22] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open.* Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [22] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [22] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [22] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-Cl(A) and $\tau_1\tau_2$-Cl$(\tau_1\tau_2$- Cl$(A)) = \tau_1\tau_2$-Cl(A).

(2) If $A \subseteq B$, then $\tau_1\tau_2$- Cl$(A) \subseteq \tau_1\tau_2$-Cl$(B)$.

(3) $\tau_1\tau_2$-Cl(A) is $\tau_1\tau_2$-closed.

(4) A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2$-Cl(A).

(5) $\tau_1\tau_2$-Cl$(X - A) = X - \tau_1\tau_2$-Int$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)r$-*open* [42] (resp. $(\tau_1, \tau_2)s$-*open* [7], $(\tau_1, \tau_2)p$-*open* [7], $(\tau_1, \tau_2)\beta$-*open* [7], $\alpha(\tau_1, \tau_2)$-*open* [45]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open, $\alpha(\tau_1, \tau_2)$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*, $\alpha(\tau_1, \tau_2)$-*closed*). Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)s$-closed sets of X containing A is called the $(\tau_1, \tau_2)s$-*closure* [7] of A and is denoted by $(\tau_1, \tau_2)\text{-sCl}(A)$. The union of all $(\tau_1, \tau_2)s$-open sets of X contained in A is called the $(\tau_1, \tau_2)s$-*interior* [7] of A and is denoted by $(\tau_1, \tau_2)\text{-sInt}(A)$.

Lemma 2. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $(\tau_1, \tau_2)\text{-}sCl(A) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)) \cup A$ [21];

(2) $(\tau_1, \tau_2)\text{-}sInt(A) = \tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(A)) \cap A$.

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $(\tau_1, \tau_2)\theta$-*cluster point* [42] of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$-*closure* [42] of A and is denoted by $(\tau_1, \tau_2)\theta\text{-Cl}(A)$. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$-*closed* [42] if $(\tau_1, \tau_2)\theta\text{-Cl}(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$-*open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $(\tau_1, \tau_2)\theta$-*interior* [42] of A and is denoted by $(\tau_1, \tau_2)\theta\text{-Int}(A)$.

Lemma 3. [42] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2\text{-}Cl(A) = (\tau_1, \tau_2)\theta\text{-}Cl(A)$.

(2) $(\tau_1, \tau_2)\theta\text{-}Cl(A)$ is $\tau_1\tau_2$-closed in X.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [2] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and

$$F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}.$$

In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x\in A}F(x)$. Let $\mathscr{P}(X)$ be the collection of all nonempty subsets of X. For any $\tau_1\tau_2$-open set V of a bitopological space (X, τ_1, τ_2), we denote $V^+ = \{B \in \mathscr{P}(X) \mid B \subseteq V\}$ and $V^- = \{B \in \mathscr{P}(X) \mid B \cap V \neq \emptyset\}$.

3. Weakly quasi (τ_1, τ_2)-continuous multifunctions

In this section, we introduce the concept of weakly quasi (τ_1, τ_2)-continuous multifunctions. Moreover, some characterizations of weakly quasi (τ_1, τ_2)-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly quasi (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$, $F(G) \subseteq \sigma_1\sigma_2\text{-}Cl(V_1)$ and $\sigma_1\sigma_2\text{-}Cl(V_2) \cap F(z) \neq \emptyset$ for every $z \in G$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly quasi (τ_1, τ_2)-continuous if F is weakly quasi (τ_1, τ_2)-continuous at each point of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) F is weakly quasi (τ_1, τ_2)-continuous;

(2) for each $x \in X$ and every $\sigma_1\sigma_2$-open sets V_1, V_2 of Y such that $F(x) \in V_1^+ \cap V_2^-$, there exists a $(\tau_1, \tau_2)s$-open set of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V_1)$ and $\sigma_1\sigma_2\text{-}Cl(V_2) \cap F(z) \neq \emptyset$ for every $z \in U$;

(3) $\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(F^-(\sigma_1\sigma_2\text{-}Int(K_1)) \cup F^+(\sigma_1\sigma_2\text{-}Int(K_2)))) \subseteq F^-(K_1) \cup F^+(K_2)$ for every $\sigma_1\sigma_2$-closed sets K_1, K_2 of Y;

(4) $F^+(V_1) \cap F^-(V_2) \subseteq (\tau_1, \tau_2)\text{-}sInt(F^+(\sigma_1\sigma_2\text{-}Cl(V_1)) \cap F^-(\sigma_1\sigma_2\text{-}Cl(V_2)))$ for every $\sigma_1\sigma_2$-open sets V_1, V_2 of Y;

(5) $(\tau_1, \tau_2)\text{-}sCl(F^-(V_1) \cup F^+(V_2)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1)) \cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))$ for every $\sigma_1\sigma_2$-open sets V_1, V_2 of Y.

Proof. (1) $\Rightarrow$ (2): Let $\mathscr{U}(x)$ the family of all $\tau_1\tau_2$-open sets of X containing x. Let V_1, V_2 be any $\sigma_1\sigma_2$-open sets of Y such that $F(x) \in V_1^+ \cap V_2^-$. For each $H \in \mathscr{U}(x)$, there exists a nonempty $\tau_1\tau_2$-open set G_H such that $G_H \subseteq H$, $F(G_H) \subseteq \sigma_1\sigma_2\text{-Cl}(V_1)$ and $\sigma_1\sigma_2\text{-Cl}(V_2) \cap F(y) \neq \emptyset$ for each $y \in G_H$. Let $W = \cup\{G_H \mid H \in \mathscr{U}(x)\}$. Then, W is $\tau_1\tau_2$-open in X, $x \in \tau_1\tau_2\text{-Cl}(W)$, $F(W) \subseteq \sigma_1\sigma_2\text{-Cl}(V_1)$ and $\sigma_1\sigma_2\text{-Cl}(V_2) \cap F(w) \neq \emptyset$ for every $w \in W$. Put $U = W \cup \{x\}$, then $W \subseteq U \subseteq \tau_1\tau_2\text{-Cl}(W)$. Thus, U is a $(\tau_1, \tau_2)s$-open set of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V_1)$ and $\sigma_1\sigma_2\text{-Cl}(V_2) \cap F(z) \neq \emptyset$ for every $z \in U$.

(2) $\Rightarrow$ (4): Let V_1, V_2 be any $\sigma_1\sigma_2$-open sets of Y and $x \in F^+(V_1) \cap F^-(V_2)$. Then, $F(x) \in V_1^+ \cap V_2^-$ and there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V_1)$ and $\sigma_1\sigma_2\text{-Cl}(V_2) \cap F(z) \neq \emptyset$ for each $z \in U$. Thus, $x \in U \subseteq (\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2)))$ and so

$$F^+(V_1) \cap F^-(V_2) \subseteq (\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2))).$$

(4) ⇒ (5): Let V_1, V_2 be any $\sigma_1\sigma_2$-open sets of Y. Then by (4), we have

$$\begin{aligned}
&X - (F^-(\sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2)))\\
&= (X - F^-(\sigma_1\sigma_2\text{-Cl}(V_1))) \cap (X - F^+(\sigma_1\sigma_2\text{-Cl}(V_2)))\\
&= F^+(Y - \sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(Y - \sigma_1\sigma_2\text{-Cl}(V_2))\\
&\subseteq (\tau_1,\tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V_1))) \cap F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V_2))))\\
&= (\tau_1,\tau_2)\text{-sInt}(F^+(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1))) \cap F^-(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2))))\\
&\subseteq (\tau_1,\tau_2)\text{-sInt}(F^+(Y - V_1) \cap F^-(Y - V_2))\\
&= (\tau_1,\tau_2)\text{-sInt}((X - F^-(V_1)) \cap (X - F^+(V_2)))\\
&= (\tau_1,\tau_2)\text{-sInt}(X - (F^-(V_1) \cup F^+(V_2)))\\
&= X - (\tau_1,\tau_2)\text{-sCl}(F^-(V_1) \cup F^+(V_2))
\end{aligned}$$

and hence $(\tau_1,\tau_2)\text{-sCl}(F^-(V_1) \cup F^+(V_2)) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2))$.

(5) ⇒ (3): Let K_1, K_2 be any $\sigma_1\sigma_2$-closed sets of Y. By (5) and Lemma 2,

$$\begin{aligned}
&\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}(K_1)) \cup F^+(\sigma_1\sigma_2\text{-Int}(K_2))))\\
&\subseteq (\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(K_1)) \cup F^+(\sigma_1\sigma_2\text{-Int}(K_2)))\\
&\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K_1))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K_2)))\\
&\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(K_1)) \cup F^+(\sigma_1\sigma_2\text{-Cl}(K_2))\\
&= F^-(K_1) \cup F^+(K_2).
\end{aligned}$$

(3) ⇒ (4): Let V_1, V_2 be any $\sigma_1\sigma_2$-open sets of Y. By (3) and Lemma 2,

$$\begin{aligned}
&X - (\tau_1,\tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2)))\\
&= (\tau_1,\tau_2)\text{-sCl}(F^-(Y - \sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(Y - \sigma_1\sigma_2\text{-Cl}(V_2)))\\
&\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V_1))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V_2)))\\
&= F^-(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1))) \cup F^+(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2)))\\
&\subseteq F^-(Y - V_1) \cup F^+(Y - V_2)\\
&= (X - F^+(V_1)) \cup (X - F^-(V_2))\\
&= X - (F^+(V_1) \cap F^-(V_2))
\end{aligned}$$

and hence $F^+(V_1) \cap F^-(V_2) \subseteq (\tau_1,\tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2)))$.

(4) ⇒ (1): Let $x \in X$ and V_1, V_2 be any $\sigma_1\sigma_2$-open sets of Y such that $F(x) \in V_1^+ \cap V_2^-$. By (4), we have $F^+(V_1) \cap F^-(V_2) \subseteq (\tau_1,\tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2)))$. Put $U = (\tau_1,\tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2)))$. Then, U is $(\tau_1,\tau_2)s$-open set of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Cl}(V_1)$ and $\sigma_1\sigma_2\text{-Cl}(V_2) \cap F(z) \neq \emptyset$ for every $z \in U$. This shows that F is weakly quasi (τ_1,τ_2)-continuous.

Theorem 2. *For a multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is weakly quasi (τ_1,τ_2)-continuous;

(2)

$$(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int((\sigma_1,\sigma_2)\theta\text{-}Cl(B_1)))\cup F^+(\sigma_1\sigma_2\text{-}Int((\sigma_1,\sigma_2)\theta\text{-}Cl(B_2))))$$
$$\subseteq F^-((\sigma_1,\sigma_2)\theta\text{-}Cl(B_1))\cup F^+((\sigma_1,\sigma_2)\theta\text{-}Cl(B_2))$$

for every subsets B_1, B_2 of Y;

(3)

$$(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B_1)))\cup F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B_2))))$$
$$\subseteq F^-((\sigma_1,\sigma_2)\theta\text{-}Cl(B_1))\cup F^+((\sigma_1,\sigma_2)\theta\text{-}Cl(B_2))$$

for every subsets B_1, B_2 of Y;

(4)

$$(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_1)))\cup F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_2))))$$
$$\subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1))\cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))$$

for every $\sigma_1\sigma_2$-open sets V_1, V_2 of Y;

(5)

$$(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_1)))\cup F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_2))))$$
$$\subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1))\cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))$$

for every $(\sigma_1,\sigma_2)p$-open sets V_1, V_2 of Y;

(6)

$$(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(K_1))\cup F^+(\sigma_1\sigma_2\text{-}Int(K_2)))\subseteq F^-(K_1)\cup F^+(K_2)$$

for every $(\sigma_1,\sigma_2)r$-closed sets K_1, K_2 of Y.

Proof. (1) $\Rightarrow$ (2): Let B_1, B_2 be any subsets of Y. Since $(\sigma_1,\sigma_2)\theta$-Cl$(B_1)$ and $(\sigma_1,\sigma_2)\theta$-Cl$(B_2)$ are $\sigma_1\sigma_2$-closed in Y, by Theorem 1

$$\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1,\sigma_2)\theta\text{-Cl}(B_1)))\cup F^+(\sigma_1\sigma_2\text{-Int}((\sigma_1,\sigma_2)\theta\text{-Cl}(B_2)))))$$
$$\subseteq F^-((\sigma_1,\sigma_2)\theta\text{-Cl}(B_1))\cup F^+((\sigma_1,\sigma_2)\theta\text{-Cl}(B_2))$$

and by Lemma 2, we have

$$(\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1,\sigma_2)\theta\text{-Cl}(B_1)))\cup F^+(\sigma_1\sigma_2\text{-Int}((\sigma_1,\sigma_2)\theta\text{-Cl}(B_2))))$$
$$\subseteq F^-((\sigma_1,\sigma_2)\theta\text{-Cl}(B_1))\cup F^+((\sigma_1,\sigma_2)\theta\text{-Cl}(B_2)).$$

(2) $\Rightarrow$ (3): This is obvious since $\sigma_1\sigma_2\text{-Cl}(B) \subseteq (\sigma_1,\sigma_2)\theta\text{-Cl}(B)$ for every subset B of Y.

(3) $\Rightarrow$ (4): This is obvious since $\sigma_1\sigma_2\text{-Cl}(V) = (\sigma_1,\sigma_2)\theta\text{-Cl}(V)$ for every $\sigma_1\sigma_2$-open set V of Y.

(4) $\Rightarrow$ (5): Let V_1, V_2 be any $(\sigma_1,\sigma_2)p$-open sets of Y. Then, we have

$$V_i \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_i))$$

and $\sigma_1\sigma_2\text{-Cl}(V_i) = \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_i)))$ for $i = 1, 2$. Now, put

$$G_i = \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_i)),$$

then G_i is $\sigma_1\sigma_2$-open in Y and $\sigma_1\sigma_2\text{-Cl}(G_i) = \sigma_1\sigma_2\text{-Cl}(V_i)$. Thus, by (4),

$$\begin{aligned}&(\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1))) \cup F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2))))\\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2)).\end{aligned}$$

(5) $\Rightarrow$ (6): Let K_1, K_2 be any $(\sigma_1,\sigma_2)r$-closed sets of Y. Since $\sigma_1\sigma_2\text{-Int}(K_1)$ and $\sigma_1\sigma_2\text{-Int}(K_2)$ are $(\sigma_1,\sigma_2)p$-open in Y, by (5), we have

$$\begin{aligned}&(\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(K_1)) \cup F^+(\sigma_1\sigma_2\text{-Int}(K_2)))\\ &= (\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K_1)))) \cup F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K_2)))))\\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K_1))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K_2)))\\ &= F^-(K_1) \cup F^+(K_2).\end{aligned}$$

(6) $\Rightarrow$ (1): Let V_1, V_2 be any $\sigma_1\sigma_2$-open sets of Y. Then, $\sigma_1\sigma_2\text{-Cl}(V_1)$ and $\sigma_1\sigma_2\text{-Cl}(V_2)$ are $(\sigma_1,\sigma_2)r$-closed in Y. Thus by (6),

$$\begin{aligned}&(\tau_1,\tau_2)\text{-sCl}(F^-(V_1) \cup F^+(V_2))\\ &\subseteq (\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1))) \cup F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2))))\\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2)).\end{aligned}$$

It follows from Theorem 1 that F is weakly quasi (τ_1,τ_2)-continuous.

Theorem 3. *For a multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is weakly quasi (τ_1,τ_2)-continuous;

(2)

$$\begin{aligned}&(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_1))) \cup F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_2))))\\ &\subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1)) \cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))\end{aligned}$$

for every $(\sigma_1,\sigma_2)\beta$-open sets V_1, V_2 of Y;

(3)

$$(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_1)))\cup F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_2))))$$
$$\subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1))\cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))$$

for every $(\sigma_1,\sigma_2)s$-open sets V_1,V_2 of Y;

(4)

$$(\tau_1,\tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_1)))\cup F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V_2))))$$
$$\subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1))\cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))$$

for every $(\sigma_1,\sigma_2)p$-open sets V_1,V_2 of Y.

Proof. (1) ⇒ (2): Let V_1,V_2 be any $(\sigma_1,\sigma_2)\beta$-open sets of Y. Then, we have $V_i\subseteq \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_i)))$ and hence $\sigma_1\sigma_2\text{-Cl}(V_i)=\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_i)))$ for $i=1,2$. Since $\sigma_1\sigma_2\text{-Cl}(V_1)$ and $\sigma_1\sigma_2\text{-Cl}(V_2)$ are $(\sigma_1,\sigma_2)r$-closed sets, by Theorem 2

$$(\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1)))\cup F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2))))$$
$$\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V_1))\cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2)).$$

(2) ⇒ (3): This is obvious since every $(\sigma_1,\sigma_2)s$-open set is $(\sigma_1,\sigma_2)\beta$-open.

(3) ⇒ (4): For any $(\sigma_1,\sigma_2)p$-open set V of Y, $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1,\sigma_2)r$-closed and $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1,\sigma_2)s$-open in Y.

(4) ⇒ (1): Let V_1,V_2 be any $\sigma_1\sigma_2$-open sets of Y. Then, V_1 and V_2 are $(\sigma_1,\sigma_2)p$-preopen in Y. By (4), we have

$$(\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1)))\cup F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2))))$$
$$\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V_1))\cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2)).$$

It follows from Theorem 2 that F is weakly quasi (τ_1,τ_2)-continuous.

Theorem 4. *For a multifunction $F:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) F is weakly quasi (τ_1,τ_2)-continuous;

(2) $\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(F^-(V_1)\cup F^+(V_2)))\subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1))\cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))$ for every $(\sigma_1,\sigma_2)p$-open sets V_1,V_2 of Y;

(3) $(\tau_1,\tau_2)\text{-}sCl(F^-(V_1)\cup F^+(V_2))\subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V_1))\cup F^+(\sigma_1\sigma_2\text{-}Cl(V_2))$ for every $(\sigma_1,\sigma_2)p$-open sets V_1,V_2 of Y;

(4) $F^+(V_1)\cap F^-(V_2)\subseteq(\tau_1,\tau_2)\text{-}sInt(F^+(\sigma_1\sigma_2\text{-}Cl(V_1))\cap F^-(\sigma_1\sigma_2\text{-}Cl(V_2)))$ for every $(\sigma_1,\sigma_2)p$-open sets V_1,V_2 of Y.

Proof. (1) $\Rightarrow$ (2): Let V_1, V_2 be any $(\sigma_1, \sigma_2)p$-open sets of Y. Since F is weakly quasi (τ_1, τ_2)-continuous, by Theorem 2

$$\begin{aligned}&\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(V_1) \cup F^+(V_2)))\\ &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1))) \cup F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2)))))\\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2)).\end{aligned}$$

(2) $\Rightarrow$ (3): Let V_1, V_2 be any $(\sigma_1, \sigma_2)p$-open sets of Y. By (2) and Lemma 2, we have

$$\begin{aligned}(\tau_1, \tau_2)\text{-sCl}(F^-(V_1) \cup F^+(V_2)) &= (F^-(V_1) \cup F^+(V_2)) \cup \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(V_1) \cup F^+(V_2)))\\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(\sigma_1\sigma_2\text{-Cl}(V_2)).\end{aligned}$$

(3) $\Rightarrow$ (4): Let V_1, V_2 be any $(\sigma_1, \sigma_2)p$-open sets of Y. Then by (3), we have

$$\begin{aligned}&X - (\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2)))\\ &= (\tau_1, \tau_2)\text{-sCl}(X - (F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2))))\\ &= (\tau_1, \tau_2)\text{-sCl}((X - F^+(\sigma_1\sigma_2\text{-Cl}(V_1))) \cup (X - F^-(\sigma_1\sigma_2\text{-Cl}(V_2))))\\ &= (\tau_1, \tau_2)\text{-sCl}(F^-(Y - \sigma_1\sigma_2\text{-Cl}(V_1)) \cup F^+(Y - \sigma_1\sigma_2\text{-Cl}(V_2)))\\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V_1))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V_2)))\\ &= (X - F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1)))) \cup (X - F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2))))\\ &= X - (F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_1))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V_2))))\\ &\subseteq X - (F^+(V_1) \cap F^-(V_2))\end{aligned}$$

and hence $F^+(V_1) \cap F^-(V_2) \subseteq (\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V_1)) \cap F^-(\sigma_1\sigma_2\text{-Cl}(V_2)))$.

(4) $\Rightarrow$ (1): Since every $\sigma_1\sigma_2$-open set is $(\sigma_1, \sigma_2)p$-open, this follows from Theorem 1.

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