EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 17, No. 3, 2024, 1727-1736
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

On The Picture Fuzzy-TOPSIS

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Abstract. Research related to the Technique for Order Preference by Similarity to Ideal Solotion (TOPSIS) method have been developed using fuzzy or intuitionistic fuzzy sets. In this article, we provide the development of the topsis method based on the generalization of fuzzy sets. We present a new method of topsis based on picture fuzzy sets. In this article, we propose steps of TOPSIS picture fuzzy sets method. Finally, we presented the illustrative example of this method.

2020 Mathematics Subject Classifications: 03E72, 08A72, 94D05

Key Words and Phrases: TOPSIS, Picture Fuzzy Sets

1. Introduction

In the development of set theory, classical sets were expanded into fuzzy sets by Zadeh [18]. He has generalized classical sets theory to fuzzy sets by allowing intermediate situations between the whole and nothing. In fuzzy sets, the membership function replaced the characteristic function in classical sets. Then Atanassov [3, 4] expanded again concept of fuzzy sets into intuitionistic fuzzy sets. Due to the limitations of membership values in intuitionistic fuzzy sets, Yunianti [16] developed the concept of intuitionistic fuzzy set into collection of intuitionistic fuzzy set, Yager [15] introduced a general class of intuitionistic fuzzy sets. Then the researchers developed the concept of picture-fuzzy sets. Cuong and Kreinovich [6] introduced the concept of picture-fuzzy sets. Then Dinh and Thao [7], and Dutta [8] did further research. There has been a lot of research on the application of the fuzzy concept of fuzzy TOPSIS. In previous studies, the intuitive fuzzy topsis method was introduced by Boran [5], Rouyendegh [12, 13], Tlig and Rebai [14], and was used by Astuti et al [2] who used Intuitionistic Fuzzy Topsis with Euclidean distance to determine the dominant factors that influence the resilience of COVID-19 patients. Meanwhile, Ashraf et al. [1] developed the method for multiple criteria decision making (MCDM) problem

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DOI: https://doi.org/10.29020/nybg.ejpam.v17i3.5225

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for picture fuzzy environment. Some researchers have written the application of picture fuzzy and TOPSIS. Jiulin et.al [10] developed Picture Fuzzy-TOPSIS method. This method based on covering-based picture fuzzy rough set (CPRS). Turdan and Kahraman [11] proposed method that combined concepts of the picture fuzzy, Z-AHP, and TOPSIS. While Zeng et.al [19] developed application of extended version of linguistic picture fuzzy-TOPSIS method in Enterprise Resource Planning Systems. Somewhat different from all the methods above, we developed a method which is a combination of picture fuzzy and TOPSIS. This article proposed the new method in MCDM, namely picture fuzzy-TOPSIS method. This method simpler and gives good result in making decisions. Therefore, the construction of the TOPSIS method on the Picture-Fuzzy set is very important to do.

2. Preliminaries

2.1. Picture Fuzzy Set (PFS)

In this part, we gave some basic definitions and results which will be used later on. We wrote again preliminary the concepts of intuitionistic fuzzy sets (IFS), and picture fuzzy sets (PFS).

Every crisp set X can be represented as fuzzy set $A = \{(x, 1), x \in$X$\}$. The definition of fuzzy set the first introduces by Zadeh [17] in 1965. Then in 1986 Atanassov [3] generalized the concept of fuzzy set by introducing the concept of intuitonistic fuzzy set.

Definition 1. *Let X be a crisp set. Intuitionistic Fuzzy Set (IFS) $\mathcal{A}$ of X is defined as $\mathcal{A} = \{(x, \mu_A(x), v_A(x)) : x \in X\}$,where:*
$0 \leq \mu_A(x) \leq 1, 0 \leq v_A(x) \leq 1,$ *and* $0 \leq \mu_A(x) + v_A(x) \leq 1$, *for all* $x \in X$.

The function μ_A is called the membership function and the the function v_A is called non-membership function. The number $\mu_A(x)$ is called the degree of membership of x to the set $\mathcal{A}$, while the $v_A(x)$ is called the degree of non-membership of x to the set $\mathcal{A}$. The amount $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of indeterminacy or hesitation part, which may cater to either membership value or non-membership value or both.

Every fuzzy set $A = \{(x, \mu_A(x)) : x \in X\}$ can be viewed as intuitionistic fuzzy set $\mathcal{A} = \{(x, \mu_A(x), 1 - \mu_A(x)) : x \in X\}$. In 2017, Yanger [15] introduced the picture fuzzy set (PFS) as an extension of FS and IFS.

Definition 2. *(see [6])Let X be a crisp set. Picture fuzzy set (PFS) $\mathcal{A}$ of X is defined as $\mathcal{A} = \{(x, \mu_A(x), v_A(x), \gamma_A(x)) : x \in X\}$,where:*
$0 \leq \mu_A(x) \leq 1, 0 \leq v_A(x) \leq 1, 0 \leq \gamma_A(x) \leq 1$ *and* $0 \leq \mu_A(x) + v_A(x) + \gamma_A(x) \leq 1$, *for all* $x \in X$.

Example 1. *Let $X = \{x_1, x_2, x_3\}$. The following is an example of PFS of X.*
$\mathcal{A} = \{(x_1, 0.6, 0.2, 0.1), (x_2, 0.5, 0.3, 0.2), (x_3, 0.6, 0.2, 0.1)\}$,

Every intuitionistic fuzzy set $A = \{(x, \mu_A(x), v_A(x)) : x \in X\}$ can be viewed as PFS $\mathcal{A} = \{(x, \mu_A(x), v_A(x), 1 - \mu_A(x)) - v_A(x) : x \in X\}$. The number $\mu_A(x)$ is called the degree of membership of x to the set $\mathcal{A}$, the number $v_A(x)$ is called the degree of non-membership of x to the PFS $\mathcal{A}$, while $\gamma_A(x)$ is called the degree of neutral membership of x in the PFS $\mathcal{A}$. The amount $1 - \mu_A(x) - v_A(x) - \gamma_A(x)$ is called the degree of refusal membership of x in the PFS $\mathcal{A}$.

It can be seen that FS is an extension of classical sets, IFS is an extension of FS, and PFS is an extension of IFS.

2.2. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method

The developing method that provides a solution to a given multiple criteria decision making (MCDM) problem is always a challenging endeavor. The one method that has been developed is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). In general, MCDM problem has the objective of assessing and ranking alternative A_i based on certain attributes or criteria C_j. Every alternative A_i represents the available options for the decision maker which requires to be ranked. While every criteria C_j represents the factors influencing the decision maker's choice while ranking the alternative A_i. The weights indicating the relative significance of the criteria C_j were represented by C_j.

In this section, we present a literature review of existing the classical TOPSIS method. This method proposed by Hwang and Yoon [9] as a simple and useful MCDM method, is a distance-based method aiming to choose the best alternative with the farthest distance from the negative ideal solution and the shortest distance from the positive ideal solution. Decision-makers express their opinions by assigning crisp values in the classical TOPSIS method.

Below, is shown the five steps of the TOPSIS method.

Step 1: Normalize the decision matrix as follows

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1j} \\ y_{21} & y_{22} & \cdots & y_{2j} \\ \cdots & \cdots & \cdots & \cdots \\ y_{i1} & y_{i2} & \cdots & y_{ij} \end{bmatrix} \tag{1}$$

where

$$y_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{I} x_{ij}^2}} \tag{2}$$

and $\boldsymbol{x}_{ij}$ is the performance of every alternative $\boldsymbol{A_i}$ for every criteria $\boldsymbol{C}_j$.

Step 2: Aggregate the criteria weights to the normalized matrix as follows

$$V = \begin{bmatrix} .v_{11} & v_{12} & \cdots & v_{1j} \\ v_{21} & v_{22} & \cdots & v_{2j} \\ \cdots & \cdots & \cdots & \cdots \\ v_{i1} & v_{i2} & \cdots & v_{ij} \end{bmatrix} \tag{3}$$

where $\boldsymbol{v}_{ij} = \boldsymbol{W}_j \boldsymbol{y}_{ij}$.

Step 3: Find positive ideal solution $\boldsymbol{A}^{+}$and negative ideal solution $\boldsymbol{A}^{-}$as follows

$$\begin{aligned} A^{+} &= \left[\begin{array}{cccc} v_1^{+} & v_2^{+} & \dots & v_J^{+} \end{array} \right], \\ A^{-} &= \left[\begin{array}{cccc} v_1^{-} & v_2^{-} & \dots & v_J^{-} \end{array} \right], \end{aligned} \tag{4}$$

where

$$v_j^{+} = \begin{cases} \max v_{ij}, \text{ if } C_j \text{ is beneficial criteria,} \\ \min v_{ij}, \text{ if } C_j \text{ is non beneficial criteria,} \end{cases} \tag{5}$$

and

$$v_j^{-} = \begin{cases} \min v_{ij}, & \text{if } C_j \text{ is beneificial criteria,} \\ \max v_{ij}, & \text{if } C_j \text{ is non beneficial criteria,} \end{cases} . \tag{6}$$

Step 4: Compute the separation measure for each alternative as follows

$$S_i^{+} = \sqrt{\sum_{j=1}^{J} \left(v_{ij} - v_j^{+} \right)^2}, \tag{7}$$

and

$$S_i^{-} = \sqrt{\sum_{j=1}^{J} \left(v_{ij} - v_j^{-} \right)^2}. \tag{8}$$

Step 5: Compute the closeness coefficient of each alternative to the ideal solution as follows

$$V_i = \frac{S_i^{-}}{S_i^{-} + S_i^{+}}, \tag{9}$$

where alternatives are descending ordered by value of $\boldsymbol{V}_i$.

3. Proposed Method

3.1. Picture Fuzzy-TOPSIS method

In this section, we provide the picture fuzzy-TOPSIS method. It is a modification of TOPSIS method that consist of seven step. The seven setps of picture fuzzy-TOPSIS method is shown below.

Step 1: Compute the weight of every decision makers

$$\lambda_k = \frac{\left(\mu_k + v_k \left(\frac{\mu_k}{\mu_k + \gamma_k} \right) \right)}{\sum_{k=1}^{l} \left(\mu_k + v_k \left(\frac{\mu_k}{\mu_k + \gamma_k} \right) \right)} \tag{10}$$

where
λ_k = weight of k-th decision maker,
μ_k = membership degree of k-th decision maker,
v_k = nonmembership degree of k-th decision maker,
γ_k = neutral degree of k-th decision maker,
and

$$\sum_{k=1}^{l} \lambda_k = 1. \tag{11}$$

Step 2: Compute the weight of each criterion as follows

$$\boldsymbol{W} = (\boldsymbol{W}_1, \boldsymbol{W}_2, \ldots, \boldsymbol{W}_m) \tag{12}$$

where

$$\begin{aligned} W_j &= IFWAr_\lambda \left(W_j^{(1)}, W_j^{(2)}, W_j^{(3)}, \ldots, W_j^{(l)} \right) \\ &= \lambda_1 W_j^{(1)} \oplus \lambda_2 W_j^{(2)} \oplus \lambda_3 W_j^{(3)} \oplus \cdots \oplus \lambda_l W_j^{(l)} \\ &= \left(\mathbf{1} - \prod_{k=1}^{l} \left(\mathbf{1} - \boldsymbol{\mu}_j^{(k)} \right)^{\lambda_k}, \prod_{k=1}^{l} \left(\boldsymbol{v}_j^{(k)} \right)^{\lambda_k}, \prod_{k=1}^{l} \left(\boldsymbol{\gamma}_j^{(k)} \right)^{\lambda_k} \right). \end{aligned} \tag{13}$$

Step 3 : Construct the picture fuzzy decision matrix $\boldsymbol{R}$ as follows

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & \cdots & r_{1m} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & r_{n3} & r_{n4} & \cdots & r_{mn} \end{bmatrix}, \tag{14}$$

where

$$\begin{aligned} r_{ij} &= IFWAr_\lambda \left({r_{ij}}^{(1)}, {r_{ij}}^{(2)}, {r_{ij}}^{(3)}, \ldots, {r_{ij}}^{(l)} \right) \\ &= \lambda_1 {r_{ij}}^{(1)} \oplus \lambda_2 {r_{ij}}^{(2)} \oplus \lambda_3 {r_{ij}}^{(3)} \oplus \cdots \oplus \lambda_l {r_{ij}}^{(l)} \\ &= \left(1 - \prod_{k=1}^{l} \left(1 - {\mu_{ij}}^{(k)} \right)^{\lambda_k}, \prod_{k=1}^{l} \left({v_{ij}}^{(k)} \right)^{\lambda_k}, \prod_{k=1}^{l} \left({\gamma_{ij}}^{(k)} \right)^{\lambda_k} \right) \end{aligned} \tag{15}$$

Step 4 : Construct the weighted fuzzy picture decision aggregate matrix

The aggregate weighted fuzzy picture decision matrix (R') is obtained from multiplying the picture fuzzy decision matrix (R) in Step 3 and the weight matrix (W) in Step 2 as follows.

$$R' = R \otimes W = \left[r'_{ij} \right] \tag{16}$$

where

$$\begin{aligned} r'_{ij} &= \left(\mu'_{ij}, v'_{ij}, \gamma'_{ij}\right) \\ \mu'_{ij} &= \mu_{ij} * \mu_j \\ v'_{ij} &= v_{ij} + v_j - v_{ij} \cdot v_j \\ \gamma'_{ij} &= \gamma_{ij} + \gamma_j - \gamma_{ij} * \gamma_j \end{aligned} \tag{17}$$

Step 5 : Find positive ideal solution $\boldsymbol{A}^{+}$and negative ideal solution $\boldsymbol{A}^{-}$as follows

$$\begin{aligned} A^{*} &= \left(r_1'^{+}, r_2'^{+}, \ldots, r_n'^{+}\right) \\ A^{-} &= \left(r_1'^{-}, r_2'^{-}, \ldots, r_n'^{-}\right) \end{aligned} \tag{18}$$

where

$$\begin{aligned} r_j'^{+} &= \left(\mu_j'^{+}, v_j'^{+}, \gamma_j'^{+}\right), j = 1, 2, \ldots, n, \\ r_j'^{-} &= \left(\mu_j'^{-}, v_j'^{-}, \gamma_j'^{-}\right), j = 1, 2, \ldots, n, \\ \mu_j'^{+} &= \left(\left(\max_i \mu'_{ij} \mid j \in J_1\right), \left(\min_i \mu'_{ij} \mid j \in J_2\right)\right), \\ v_j'^{+} &= \left(\left(\min_i v'_{ij} \mid j \in J_1\right), \left(\max_i v'_{ij} \mid j \in J_2\right)\right), \\ \gamma_j'^{+++} &= \left(\left(\min_i \gamma'_{ij} \mid j \in J_1\right), \left(\max_i \gamma'_{ij} \mid j \in J_2\right)\right), \\ \mu_j'^{-} &= \left(\left(\min_i \mu'_{ij} \mid j \in J_1\right), \left(\max_i \mu'_{ij} \mid j \in J_2\right)\right), \\ v_j'^{-} &= \left(\left(\max_i v'_{ij} \mid j \in J_1\right), \left(\min_i v'_{ij} \mid j \in J_2\right)\right), \\ \gamma_j'^{-} &= \left(\left(\max_i \gamma'_{ij} \mid j \in J_1\right), \left(\min_i \gamma'_{ij} \mid j \in J_2\right)\right), \end{aligned} \tag{19}$$

J_1 is beneficial criteria, J_2 is non beneficial criteria.

Step 6: Compute the separation measure for each alternative as follows

$$\begin{aligned} S^{*} &= \sqrt{\frac{1}{2n} \sum_{j=1}^{n} \left(\mu'_{ij} - \mu_j'^{+}\right)^2 + \left(v'_{ij} - v_j'^{+}\right)^2 + \left(\gamma'_{ij} - \gamma_j'^{+}\right)^2}, \\ S^{-} &= \sqrt{\frac{1}{2n} \sum_{j=1}^{n} \left(\mu'_{ij} - \mu_j'^{-}\right)^2 + \left(v'_{ij} - v_j'^{-}\right)^2 + \left(\gamma'_{ij} - \gamma_j'^{-}\right)^2}. \end{aligned} \tag{20}$$

Step 7: Compute the closeness coefficient of each alternative to the ideal solution as follows

$$C_{i^{*}} = \frac{s_{i^{-}}}{s_{i^{+}} + s_{i^{-}}}, 0 \leq C_{i^{*}} \leq 1. \tag{21}$$

3.2. Illustrative Example

In this example (for the simulation), we will determine the type of educational YouTube content that viewers are most interested in. There are 3 alternatives, namely: A_1 =school/ college subject matter. A_2 =discussion/solving questions, and A_3 =enrichment materials. Meanwhile, the criteria used are: C_1 =accuracy of the content, C_2 =suitability of the title to the content, C_3 = attractiveness of the introductory video, C_4 = existence of related video links, and C_5 =attractiveness of the material presented.

Step 1: Compute the weight of every decision makers.
To obtain the data, we conducted interviews or distributed questionnaires to several respondents. In this example, the respondent are the decision makers and the number of decision makers are 5: R_1, R_2, R_3, R_{41}, and R_5. The importance levels/rating of the decision makers are considered based on linguistic term, that are: Very Very Important (VVI), Very Important (VI), Important (I), Medium (M), and Unimportant (UI). The linguistic terms were assigned Picture Fuzzy Number (PFN) and we write them as Dk $= (\mu_A(x), v_A(x), \gamma_A(x))$. The importance levels of the decision-makers in the PFN as shown in in Table 1 below.

Table 1: Criteria Important level of the decision-makers and PFN's.

Criteria/rating Important Level	PFN's (D_k)
VVI	$(0.9, 0.1, 0)$
VI	$(0.7, 0.2, 0.1)$
I	$(0.6, 0.3, 0.1)$
M	$(0.5, 0.4, 0.1)$
UI	$(0.3, 0.5, 0.2)$

By using the formula (10) we get the weight of every decision-makers as Table 2 below.

Table 2: Criteria and weight decision makers.

Decision maker	weight
R_1	0.25773195
R_2	0.18556701
R_3	0.18556701
R_4	0.18556701
R_5	0.18556701

Step 2: Compute the weight of each criterion.
The results of data from respondents obtained the data of criteria as in Table 3 below.

By using the formula (12) we get the weight of each criterion as Table 4 below.

Step 3: Based on data from respondents regarding alternative ratings, and by using formula (15) the following results of matrix R were obtained.

Table 3: The rating of each criterion.

Decision Maker	C_1	C_2	C_3	C_4	C_5
R_1	VVI	VVI	M	Ui	VVI
R_2	VVI	VI	VVI	VVI	VVI
R_3	VVI	VI	VVI	VVI	VVI
R_4	VI	VI	UI	M	VI
R_5	M	VI	VI	UI	VVI

Table 4: The weight each criterion.

Weight of the j^{th} criteria (W_j)	$PFN's$
W_1	$(0.01566, 0.98247, 0)$
W_2	$(0.01566, 0.98247, 0)$
W_3	$(0.011006, 0.98635, 0)$
W_4	$(0.008299, 0.989428, 0)$
W_5	$(0.018566, 0.979975, 0)$

$$\text{R}=\begin{bmatrix} (0.65, 0.14, 0.00077) & (0.48, 0.23, 0.00219) & (0, 62, 0.16, 0.00178) \\ (0.65, 0.14, 0.000776) & (0.48, 0.23, 0.00219) & (0, 62, 0.16, 0.00178) \\ (0.65, 0.14, 0.000776) & (0.48, 0.23, 0.00219) & (0, 62, 0.16, 0.00178) \\ (0.65, 0.14, 0.00077) & (0.48, 0.23, 0.00219) & (0, 62, 0.16, 0.00178) \\ (0.65, 0.14, 0.00077) & (0.48, 0.23, 0.00219) & (0, 62, 0.16, 0.00178) \end{bmatrix}$$

Step 4: By using the formula (17) the following results of matrix R' were obtained.

$$R'=\begin{bmatrix} (0.0102, 0.98, 0.0007) & (0.0076, 0.98, 0.0021) & (0.0098, 0.98, 0.0017) \\ (0.0102, 0.98, 0.0007) & (0.0076, 0.98, 0.0021) & (0.0098, 0.98, 0.0017) \\ (0, 0.14, 0.0117) & (0, 0.239, 0.013) & (0, 0.16, 0.012) \\ (0.64, 0.14, 0.0007) & (0.48, 0.23, 0.002) & (0.62, 0.16, 0.001) \\ (0.005, 0.99, 0.0007) & (0.004, 0.99, 0.002) & (0.005, 0.99, 0.001) \end{bmatrix}$$

Step 5 & 6, and 7: By using the formula (19) we have the positive ideal solution and negative ideal solution as Table 5 follow.

Table 5: Ideal solution picture fuzzy.

Criteria	A^+	A^-
C_1	$(0.0102, 0.985, 0.0007)$	(0.0076,0.98,0.0021)
C_2	$(0.0102, 0.985, 0.0007)$	(0.0076,0.98,0.0021)
C_3	$(0, 0.1429, 0.0117)$	(0,0.239,0.0131)
C_4	$(0.64, 0.14, 0.0007)$	(0.482,0.239,0.0021)
C_5	$(0.004, 0.99, 0.002)$	(0.0054,0.99,0.00077)

Then, by using the formula (20) we have:
S^* for $A_1 = 0.000702$, S^* for $A_2 = 0.067083$, S^* for $A_3 = 0.013221$.

S^- for $A_1 = 0.9962$,S^- for $A_2 = 1.0000$, S^- for $A_3 = 0.9975$.
Finally, by applying the formula (21) we have:
C_{i^*} for $A_1 = 0.999296$, C_{i^*} for $A_2 = 0.937135$, C_{i^*} for $A_3 = 0.986919$. It means, we rank according to the descending order of C_{i^*} to show the measure of relative closeness of each alternative that is A_1, A_3, A_2.

4. Conclusion

In the classical TOPSIS method, the decision-makers express their opinions by assigning crisp values. However, these crisp values are often insufficient and inadequate for the solution of real decision-making problems when uncertain and vague information is taken into account in decision-making. The picture-fuzzy TOPSIS method involves the membership degree, the non-membership degree, and the neutral degree. Therefore, this method that we provide is better to capture the uncertainty in the evaluations of decision-makers.

Acknowledgement

We sincerely acknowledge the anonymous reviewers for their suggestions which improve the quality of this article.

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