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On Micro Pre Operators in Micro Topological Spaces

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Abstract. The basic objective of this research work is to introduce and investigate the properties of micro pre-frontier, micro pre-exterior, micro pre-border, micro pre-kernel using the concept of frontier, exterior, border and kernel.

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1. Introduction

Levine's introduction of generalized closed sets in 1970 [3], providing a foundational framework for subsequent developments. Lellis Thivagar [1], further expanded this framework with the introduction of nano topology, utilizing approximations and boundary regions of a subset of a universe using an equivalence relation on it to define nano closed sets, nano-interior and nano-closure. The exploration of weak forms of nano open sets, such as nano α-open sets, nano semi-open sets, nano pre-open sets, and nano-β-open sets, was undertaken by many authors adding layers of complexity to the existing theories.

In 2013, Antony Rex Rodgio et.al.,[8] defined the properties of β^* open sets like frontier, exterior and border. In 2018, Sathishmohan et.al., [6] introduced some properties of nano pre-neighbourhoods in nano topology. In 2019, Chandrasekar [4], introduced the concept of micro topology which is a simple extension of nano topology, with a focus on micro pre-open and micro semi-open sets. Chandrasekar and Swathi [5], introduced micro α-open

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sets and studied the basic properties. Recently in 2020, Hariwan Z.Ibrahim [9], introduced micro β-open sets in micro topological spaces. The aim of this paper is to introduce and investigate the properties of micro pre-frontier, micro pre-exterior, micro pre-border and micro pre-kernel using the notion of frontier, exterior, border and kernel to obtain their basic results. The basic definitions used in this paper is given below.

Definition 1. *[1] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, \ L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:*

(i) $U \in \tau_R(X)$ and $\emptyset \in \tau_R(X)$.

(ii) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. $\{U, \tau_R(X)\}$ is called the nano topological space.

Definition 2. *[4] Let $\{U, \tau_R(X)\}$ is a nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X)\}$ and called it micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$.*

Definition 3. *[4] The micro topology $\mu_R(X)$ satisfies the following axioms.*

(i) $U \in \mu_R(X)$ and $\emptyset \in \mu_R(X)$

(ii) The union of elements of any sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called micro topological spaces and the elements of $\mu_R(X)$ are called micro open sets and the complement of a micro open set is called a micro closed set.

Definition 4. *[4] The micro closure of a set A is denoted by Mic-cl(A) and is defined as Mic-cl(A) $= \cap\{B$:B is micro closed and $A \subseteq B\}$. The micro interior of a set A is denoted by Mic-int(A) and is defined as Mic-int(A) $= \cup\{B$:B is micro open and $A \supseteq B\}$.*

Definition 5. *[7] The union of all micro pre-open sets which are contained in A is called the micro pre-interior of A and is denoted by Mic-Pint(A) or by Mic-PA_*.*
As the union of micro pre-open sets is micro pre-open, Mic-PA_* is micro pre-open always. micro pre-open is denoted by Mic-PO(U) and micro pre-closed is denoted by Mic-PF(U).

Definition 6. *[7] The intersection of micro pre-closed sets containing a set A is called the micro pre-closure of A and is denoted by Mic-Pcl(A) or by Mic-PA^*.*

Definition 7. *[4] Let $(U, \tau_R(x), \mu_R(X))$ be a micro topological space and $A \subseteq U$. Then A is called micro pre-open if $A \subseteq$ Mic-int(Mic-cl(A)).*

Definition 8. *[2] A point $x \in X$ is said to be limit point of A if every neighborhood of x intersects A in some point other than x itself.*

Definition 9. *[2] The set of all limit points of A is called the derived set of A and it denoted by D(A).*

Definition 10. *[7] A point $x \in U$ is said to be a micro pre-limit point of A iff for each U $\in$ Mic-PO(U), $U \cap (A - \{x\}) \neq \emptyset$.*

Definition 11. *[7] The set of all micro pre-limit points of A is said to be the micro pre-derived set of A and in denoted by Mic-PD(A).*

2. micro pre-frontier

In this section, we define and study the notions of micro pre-frontier and obtain its basic properties.

Definition 12. *micro pre-frontier of $A \subset U$ is defined as Mic-PA$^*-$ Mic-PA$_*$ and is denoted by Mic-Pfr(A). It is obvious that Mic-Pfr(A) $\subseteq$ Mic-fr(A), the micro frontier of A. But in general the converse may not be true. micro pre-interior(A) is denoted as Mic-PA$_*$ and micro pre-closure is denoted as Mic-PA*.*

Example 1. *Let $U = \{a, b, c, d\}$, $U \backslash R = \{\{a, c\}, \{b, d\}\}$, $X = \{a, c\}$, $\tau_R(X) = \{U, \emptyset, \{a, c\}\}$, $\mu = \{b\}$ and $\mu_R(X) = \{U, \emptyset, \{b\}, \{a, c\}, \{a, b, c\}\}$. If A = $\{b,c\}$ then, Mic-cl(A) = $\{U\}$ and Mic-int(A) = $\{b\}$, Mic-Pcl(A) = $\{b,c,d\}$ and Mic-Pint(A) = $\{b,c\}$, where Mic-fr(A) = $\{a,c,d\}$ and Mic-Pfr(A) = $\{d\}$. This shows that Mic-fr(A) $\not\subset$ Mic-Pfr(A).*

Lemma 1. *For a subset A of a space U,*

(i) Mic-PA* = Mic-PA$_*$ $\cup$ Mic-Pfr(A).

(ii) Mic-PA$_*$ $\cap$ Mic-Pfr(A) = $\emptyset$ and

(iii) Mic-Pfr(A) = Mic-PA* $\cap$ Mic-P$(U - A)^*$.

Proof: *By definition of Mic-Pfr(A), we have*

(i) Mic-PA$_*$ $\cup$ Mic-Pfr(A) = Mic-PA$_*$ $\cup$ (Mic-PA$^*-$ Mic-PA$_*$) = Mic-PA*.

(ii) Mic-PA$_*$ $\cap$ Mic-Pfr(A) = Mic-PA$_*$ $\cap$ (Mic-PA$^*-$ Mic-PA$_*$) = $\emptyset$.

(iii) Mic-Pfr(A) = Mic-PA$^*-$Mic-PA$_*$ = Mic-PA* $\cap$ $(U-$Mic-PA$_*)$ = Mic-PA* $\cap$ Mic-P$(U - A)^*$ by lemma 3.8(1) [7].

Lemma 2. *Mic-Pfr(A) is micro pre-closed.*
Proof: *By Lemma 1, Mic-Pfr(A) = Mic-PA* $\cap$ Mic-P$(U-A)^*$, which is micro pre-closed by corollary 3.9 [7].*

Definition 13. *A subset $A \subset U$ is called micro pre-regular if it is both micro pre-open and micro pre-closed set. The family of all micro pre-regular sets of U is denoted by Mic-PR(U). micro pre-closed is denoted by Mic-PF(U).*

Theorem 1. *Mic-Pfr(A) $= \emptyset$ iff $A \in$ Mic-PR(U).*
Proof: *Let $A \in$ Mic-PR(U). Then $A \in$ Mic-PO(U) and $A \in$ Mic-PF(U). Now, using results of Lemma 3.7 [7] and Theorem 3.16 [7] it follows that Mic-Pfr(A) $= \emptyset$. Conversely, let Mic-Pfr(A) $= \emptyset$. Then we show that $A \in$ Mic-PR(U). Since by hypothesis, Mic-PA$^* -$ Mic-PA$_* = \emptyset$. We have Mic-PA* = Mic-PA$_*$. But, Mic-PA$_* \subset A \subset$ Mic-PA*. Therefore, it follows that A = Mic-PA$_*$ = Mic-PA* which means $A \in$ Mic-PR(U).*

Theorem 2. *Let A be subset of U. Then, the following holds.*

(i) Mic-Pfr(A) = Mic-Pfr($U - A$).

(ii) $A \in$ Mic-PO(U) iff Mic-Pfr(A) $\subseteq U - A$. i.e., $A \cap$ Mic-Pfr(A) $= \emptyset$.

(iii) $A \in$ Mic-PF(U) iff Mic-Pfr(A) $\subseteq A$.

Proof:

(i) We have, Mic-Pfr($U - A$) = ($U-$Mic-PA)$^* \cap$ ($U - (U-$Mic-PA))* = ($U-$Mic-PA)* $\cap$ Mic-PA* = Mic-Pfr(A) by Lemma 1(3).

(ii) Assume $A \in$ Mic-PO(U). By definition, we have Mic-Pfr(A) = Mic-PA$^*-$Mic-PA$_*$ = Mic-PA$^* - A$. Since $A \in$ Mic-PO(U). Then, $A \cap$ Mic-Pfr(A) = $A \cap$ (Mic-PA$^* - A$) = Mic-PA$^* \cap (U - A) \cap A = \emptyset$. Conversely, if $A \cap$ Mic-Pfr(A) $= \emptyset$. Then, $A \cap$ Mic-PA$^* \cap$ ($U-$Mic-PA$_*$) $= \emptyset$ implies $A \cap$ ($U -$ Mic-PA$_*$) $= \emptyset$ as $A \subset U - (U-$Mic-PA$_*)$ = Mic-PA$_*$, but on the other hand Mic-PA$_* \subset A$. It follows that A = Mic-PA$_*$, which implies $A \in$ Mic-PO(U).

(iii) Assume $A \in$ Mic-PF(U). Then, we have $U - A \in$ Mic-PO(U). Then by (2), Mic-Pfr($U - A$) $\cap (U - A) = \emptyset$. But, by (1), Mic-Pfr ($U - A$) = Mic-Pfr(A). Hence Mic-Pfr(A) $\cap (U - A) = \emptyset$. This shows that Mic-Pfr(A) $\subset A$. Conversely, if Mic-Pfr(A) $\subset A$, then Mic-PA$^* -$ Mic-PA$_* \subset A$, which implies Mic-PA$_* \cup$ (Mic-PA$^* -$ Mic-PA$_*$) $\subset A \cup$ Mic-PA$_* = A$, which implies Mic-PA$^* \subset A$ by Lemma 1(1). But $A \subset$ Mic-PA*. It follows that A = Mic-PA*. Hence $A \in$ Mic-PF(U).

Remark 1. *Let A and B be subsets of space U. Then $A \subset B$ does not imply that either Mic-Pfr(A) $\subset$ Mic-Pfr(B) or Mic-Pfr(B) $\subset$ Mic-Pfr(A). This can be verified by the following.*

Example 2. *Let $U = \{a,b,c,d\}$, $U/R = \{\{a,b\}, \{c,d\}\}$, $X = \{b,c\}$, $\tau_R(X) = \{U, \emptyset, \{b,c\}\}$, $\mu = b$ and $\mu_R(x) = \{U, \emptyset, \{b\}, \{b,c\}\}$. Mic-PO(U) $= \{U, \emptyset, \{b\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{b,c,d\}, \{a,b,d\}\}$. Then,*
Case(1): Take $A = \{a\}$ and $B = \{a,c\}$. Then $A \subset B$. Also Mic-PA$^* = \{a\}$, Mic-PA$_* = \{\emptyset\}$ and Mic-Pfr(A) $= \{a\}$. Mic-PB$^* = \{a,c\}$, Mic-PB$_* = \{\emptyset\}$ and Mic-Pfr(B) $= \{a,c\}$. This shows that Mic-Pfr(A) $\subset$ Mic-Pfr(B).

Case(2): Take $A = \{a\}$ *and* $B = \{a,c,d\}$. *Also Mic-P*$A^* = \{a\}$, *Mic-P*$A_* = \{\emptyset\}$ *and Mic-Pfr(A)* $= \{a\}$. *Let Mic-P*$B^* = \{c,d,a\}$, *Mic-P*$B_* = \{\emptyset\}$ *and Mic-Pfr(B)* $= \{a,c,d\}$. *This shows that Mic-Pfr(A)* $\subset$ *Mic-Pfr(B), where Mic-Pfr(B)* $\not\subset$ *Mic-Pfr(A).*

Theorem 3. *If* $A \in$ *Mic-PO(U)* $\cup$ *Mic-PF(U), then Mic-Pfr(A) = Mic-Pfr(Mic-Pfr(A)).*
Proof: *It follows by Lemma 1(3), Lemma 2 and Theorem 2 (2, 3).*

Corollary 1. *For every* $A \subset U$, *Mic-Pfr(Mic-Pfr(Mic-Pfr(A))) = Mic-Pfr(Mic-Pfr(A)).*
Proof: *It is obvious.*

Lemma 3. *A subset A of U is micro pre-closed iff A = Mic-Pcl(A).*

Theorem 4. *For a subset A of space U, the following statements hold*

(i) A is Mic-PO iff Mic-Pfr(A) = Mic-PD(A).

(ii) Mic-Pfr(Mic-Pfr(A)) $\subseteq$ *Mic-Pfr(A).*

(iii) Mic-Pfr(Mic-Pcl(A)) $\subseteq$ *Mic-Pfr(A).*

(iv) Mic-Pint(A) = A $-$ *Mic-Pfr(A)*

Proof:

(i) Let A be micro pre-open then Mic-Pint(A) = A. Since Mic-Pfr(A) = Mic-Pcl(A) $-$ *Mic-Pint(A) = Mic-Pcl(A)*$-$*A. By Lemma 4.9 [7] we have Mic-Pcl(A) = A* $\cup$ *Mic-PD(A). Therefore Mic-Pfr(A) = [A* $\cup$ *Mic-PD(A)]*$-$ *A = Mic-PD(A).*
Conversely, Let Mic-Pfr(A) = Mic-PD(A). i.e., Mic-Pcl(A) $-$ *Mic-Pint(A) = [A* $\cup$ *Mic-PD(A)] - Mic-Pint(A) = Mic-PD(A)* $\Rightarrow$ *A* $-$ *Mic-Pint(A) =* $\emptyset$ *implies* $A \subset$ *Mic-Pint(A)* $\longrightarrow$ *(1) and Mic-Pint(A)* $\subset A \longrightarrow$ *(2). Therefore from (1) and (2) we have Mic-Pint(A) = A is micro pre-open.*

(ii) Now Mic-Pfr(Mic-Pfr(A)) $\subseteq$ *Mic-Pcl(Mic-Pfr(A))* $\cap$ *Mic-Pcl(U* $-$ *Mic-Pfr(A))* $\Rightarrow$ *Mic-Pfr(Mic-Pfr(A))* $\subseteq$ *Mic-Pcl(Mic-Pfr(A))* $\subseteq$ *Mic-Pfr(A).*

(iii) Mic-Pfr(Mic-Pcl(A)) $\subseteq$ *Mic-Pcl(Mic-Pcl(A))* $-$ *Mic-Pint(Mic-Pcl(A))* $\subseteq$ *Mic-Pcl(A)* $-$ *Mic-Pint(A)* $\subseteq$ *Mic-Pfr(A)*

(iv) It is obvious from the definition of micro pre-interior and micro pre-frontier.

3. micro pre-exterior

In this section, we define and study the notions of micro pre-exterior and obtain its basic properties.

Definition 14. *A point* $x \in U$ *is called micro pre-exterior point of a subset A of U if x is micro pre-interior point of* $(U - A)$ *and set of all micro pre-exterior points of A is called micro pre-exterior of A and denoted by Mic-Pext(A). Therefore Mic-Pext(A) = Mic-Pint*$(U - A)$.

Theorem 5. *For a subset A of a space U the following statements hold*

(i) Mic-ext(A) $\subseteq$ Mic-Pext(A).

(ii) Mic-Pext(A) $\subset$ Mic-PO(U).

(iii) Mic-Pext(A) = U−Mic-Pcl(A).

(iv) Mic-Pext(Mic-Pext(A)) = Mic-Pint(Mic-Pcl(A)).

(v) If $A \subset B$ then Mic-Pext(B) $\subseteq$ Mic-Pext(A).

(vi) Mic-Pext($A \cup B$) $\subseteq$ Mic-Pext(A) $\cup$ Mic-Pext(B).

(vii) Mic-Pext(A) $\cap$ Mic-Pext(B) $\subseteq$ Mic-Pext($A \cap B$).

(viii) Mic-Pext(U) = $\emptyset$ and Mic-Pext($\emptyset$) = U.

(ix) Mic-Pext(A) = Mic-pext[U − Mic-Pext(A)].

(x) Mic-Pint(A) $\subseteq$ Mic-Pext[Mic-Pext(A)].

(xi) Mic-Pint(A), Mic-Pext(A) and Mic-Pfr(A) are mutually disjoint and U = Mic-Pint(A) $\cup$ Mic-Pext(A) $\cup$ Mic-Pfr(A).

(xii) $A \cap$ Mic-Pext(A) = $\emptyset$.

Proof:

(i) Let $x \in$ Mic-ext(A) $\Rightarrow x \in$ Mic-int$(U - A)$. There exists $G \in \mu_R(x)$ such that $x \in G \subseteq (U - A)$. Also $G \in$ Mic-PO(U, X). Therefore $x \in G \subseteq (U - A)$ for Mic-PO set $G \Rightarrow (U - A)$ is Mic-Pint of x, $x \in$ Mic-Pint$(U - A)$i.e., $x \in$ Mic-Pext(A). Hence Mic-ext(A) $\subseteq$ Mic-Pext(A).

(ii) Now Mic-Pint[Mic-Pext(A)] = Mic-Pint[Mic-Pint$(U - A)$] = Mic-Pint$(U - A)$ = Mic-Pext(A) $\Rightarrow$ is contained in Mic-PO(U).

(iii) Mic-Pext(A) = Mic-Pint$(U - A)$ = U−Mic-Pcl(A).

(iv) Mic-Pext[Mic-Pext(A)] = Mic-Pext[U−Mic-cl(A)] by (3), Mic-Pext[U−Mic-cl(A)] = Mic-Pint[U−[U−Mic-Pcl(A)]] = Mic-Pint[Mic-Pcl(A)].

(v) If $A \subset B$ then $(U - B) \subset (U - A) \Rightarrow$ Mic-Pint$(U - B) \subseteq$ Mic-Pint$(U - A)$ i.e., Mic-Pext(B) $\subseteq$ Mic-Pext(A).

(vi) Since $A \subset (A \cup B)$ and $B \subset (A \cup B) \Rightarrow$ Mic-Pext($A \cup B$) $\subseteq$ Mic-Pext(A) $\cup$ Mic-Pext($A \cup B$) $\subseteq$ Mic-Pext(B). Therefore Mic-Pext($A \cup B$) $\subseteq$ Mic-Pext(A) $\cup$ Mic-Pext(B).

(vii) *We have $(A \cap B) \subset A$, $(A \cap B) \subset B$.*
$\Rightarrow$ Mic-Pext(A) $\subseteq$ Mic-Pext(A $\cap$ B) and Mic-Pext(B) $\subseteq$ Mic-Pext(A $\cap$ B) $\Rightarrow$ Mic-Pext(A) $\cap$ Mic-Pext(B) $\subseteq$ Mic-Pext(A $\cap$ B).

(viii) *Mic-Pext(U) = Mic-Pint(U $-$ U) = Mic-Pint($\emptyset$) = $\emptyset$ and Mic-Pext($\emptyset$) = Mic-Pint(U$-\emptyset$) = Mic-Pint(U) = U.*

(ix) *Mic-Pext[U $-$ Mic-Pext(A)] = Mic-Pint[U$-$(U$-$ Mic-Pint(A))] = Mic-Pint(Mic-Pext((A)) = Mic-Pint[Mic-Pint(U $-$ A)] = Mic-Pint(U $-$ A) = Mic-Pext(A).*

(x) *By the definition Mic-Pext(A) $\subset$ (U $-$ A) then from (5) Mic-Pext(U $-$ A) $\subset$ Mic-Pext[Mic-Pext(A)] i.e., Mic-Pint(A) $\subset$ Mic-Pext[Mic-Pext(A)].*

(xi) *Let us assume that Mic-Pext(A) $\cap$ Mic-Pint(A) $\neq \emptyset$ therefore there exists $x \in$ Mic-Pext(A) $\cap$ Mic-Pint(A) $\Rightarrow x \in$ Mic-Pext(A) and $x \in$ Mic-Pint(A) $\Rightarrow x \in (U - A)$ and $x \in A$ which is not possible. Therefore our assumption is wrong. Hence Mic-Pext(A) $\cap$ Mic-Pint(A) = $\emptyset$ similarly other two results. We have Mic-Pext(A) = U$-$ Mic-cl(A) =U$-$[Mic-Pint(A) $\cup$ Mic-Pfr(A)] that implies U = Mic-Pint(A) $\cup$ Mic-Pext(A) $\cup$ Mic-Pfr(A).*

(xii) *Obvious.*
In general, the converse of (6) and (7) are not true i.e., Mic-Pext(A) $\cup$ Mic-Pext(B) $\not\subset$ Mic-Pext(A $\cup$ B) and Mic-Pext(A $\cap$ B) $\not\subset$ Mic-Pext(A) $\cap$ Mic-Pext(B).

Example 3. *Let $U = \{a, b, c, d\}$, $U \backslash R = \{\{a, b\}, \{c, d\}\}$, $X = \{b, c\}$, $\tau_R(X) = \{U, \emptyset, \{b, c\}\}$, $\mu = \{b, d\}$ and $\mu_R(X) = \{U, \emptyset, \{b\}, \{b, c\}, \{b, d\}, \{b, c, d\}\}$.*
Mic-PO(U) = $\{U, \emptyset, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}\}$. Let A = $\{c,d,a\}$ and B = $\{c,d\}$. Then Mic-Pext(A) = $\{b\}$ and Mic-Pext(B) = $\{a,b\}$. Mic-Pext(A $\cup$ B) = $\{b\}$.
$\Rightarrow$ Mic-Pext(A) $\cup$ Mic-Pext(B) $\not\subset$ Mic-Pext(A $\cup$ B) and
Mic-Pext(A $\cap$ B) = $\{a,b\}$ which implies Mic-Pext(A $\cap$ B) $\not\subset$ Mic-Pext(A) $\cap$ Mic-Pext(B).

Theorem 6. *Mic-Pfr(A) $\cap$ Mic-Pext(A) = $\emptyset$.*
Proof: *Let $x \in$ Mic-Pfr(A) i.e., $x \in$ (Mic-Pcl(A) $-$ Mic-Pint(A)).*
If $x \in$ Mic-Pcl(A) then $x \notin$ Mic-Pint(A). We know that Mic-Pcl(A) $\cap$ Mic-Pint(U $-$ A) = $\emptyset$. Therefore $x \notin$ Mic-Pint(U $-$ A) implies $x \notin$ Mic-Pext(A). Hence Mic-Pfr(A) $\cap$ Mic-Pext(A) = $\emptyset$.

4. micro pre-border

In this section, we define and study the notions of micro pre-border and obtain its basic properties.

Definition 15. *Let A be a subset of a space U. Then the micro pre-border of A is defined as Mic-Pbr(A) = A $-$ Mic-Pint(A).*

Theorem 7. *For a subset of U, the following statements holds.*

(i) Mic-Pbr(A) $\subseteq$ Mic-br(A) where br(A) denote the border of A.

(ii) A = Mic-Pint(A) $\cup$ Mic-Pbr(A).

(iii) Mic-Pint(A) $\cap$ Mic-Pbr(A) = $\emptyset$.

(iv) If A is Mic-PO then Mic-Pbr(A) = $\emptyset$.

(v) Mic-Pint(Mic-Pbr(A)) = $\emptyset$.

(vi) Mic-Pbr(Mic-Pbr(A)) = Mic-Pbr(A).

(vii) Mic-Pbr(A) = A $\cap$ Mic-Pcl($U - A$).

Proof:

(i) Obvious from the definitions of micro pre-border and micro border of A.

(ii) Obvious from the definitions of micro pre-border of A.

(iii) Obvious from the definitions of micro pre-border of A.

(iv) If A is Mic-PO, then A = Mic-Pint(A). Hence the result follows.

(v) If $x \in$ Mic-Pint(Mic-Pbr(A)), then $x \in$ Mic-Pbr(A). Now, Mic-Pbr(A) $\subset$ A implies Mic-Pint(Mic-Pbr(A)) $\subset$ Mic-Pint(A). Hence $x \in$ Mic-Pint(A) which is a contradiction to $x \in$ Mic-Pbr(A). Thus Mic-Pint(Mic-Pbr(A)) = $\emptyset$.

(vi) Mic-Pbr(Mic-Pbr(A)) = Mic-Pbr(A $-$ Mic-Pint(A)) = (A $-$ Mic-Pint(A))$-$Mic-Pint(A $-$ Mic-Pint(A)) which is Mic-Pbr(A) $- \emptyset$, by (4). Hence, Mic-Pbr(Mic-Pbr(A)) = Mic-Pbr(A).

(vii) Mic-Pbr(A) = A $-$ Mic-Pint(A) =A$-$($U-$(Mic-Pcl($U-A$))) = A $\cap$ Mic-Pcl($U-A$).

Theorem 8. *For a subset of U, the following condition hold.*

(i) Mic-Pbr(A) $\subseteq$ Mic-Pfr(A).

(ii) Mic-Pext(A) $\cap$ Mic-Pbr(A) = $\emptyset$.

Proof:

*(i) Let, $x \in$ Mic-Pbr(A) i.e., $x \in$ A $-$ Mic-Pint(A).
By Theorem 3.16 [7] A = Mic-pint(A) if A is Mic-PO. If A is not Mic-PO then Mic-Pint(A) $\subset$ A. Therefore in general Mic-Pint(A) $\subseteq$ A. So $x \in$ Mic-Pint(A). It is obvious that if $x \in$ Mic-Pint(A) then $x \notin$ Mic-Pcl(A). Therefore $x \in$ (Mic-Pcl(A) $-$ Mic-Pint(A)) implies $x \in$ Mic-Pfr(A). Hence Mic-Pbr(A) $\subseteq$ Mic-Pfr(A).*

(ii) Let $x \in$ Mic-Pext(A) i.e., $x \in$ Mic-Pint($U - A$) where $x \in$ Mic-Pint(A). By Theorem 3.16 [7] A = Mic-pint(A) if A is Mic-PO. If A is not Mic-PO then Mic-Pint(A) $\subset$ A. Therefore in general Mic-Pint(A) $\subseteq$ A. Therefore $x \notin$ A $-$ Mic-Pint(A) implies $x \notin$ Mic-Pbr(A). Hence Mic-Pext(A) $\cap$ Mic-Pbr(A) = $\emptyset$.

5. micro pre-kernel

In this section, we define and study the notions of micro pre-kernel and obtain its basic properties.

Definition 16. *For any $A \subset U$, Mic-Pker(A) is defined as the intersection of all micro pre-open sets containing A. In notation, Mic-Pker(A) $= \bigcap\{M/A \subset M,\ M \in$ Mic-PO$\}$.*

Lemma 4. *For subsets A, B and $A_i(i \in I$, where I is an index set) of a micro topological space $(U,\ \mu_R(x))$, the following holds.*

- *(i)* $A \subseteq$ *Mic-Pker(A).*
- *(ii)* *If* $A \subset B$, *then Mic-Pker(A)* $\subset$ *Mic-Pker(B).*
- *(iii)* *Mic-Pker(Mic-Pker(A)) = Mic-Pker(A).*
- *(iv)* *Mic-Pker(* $\bigcup A_i/\ i \in I) \subseteq \bigcup\{$*Mic-Pker* $(A_i)/\ i \in I\}$.
- *(v)* *Mic-Pker(* $\bigcap A_i/\ i \in I) \subseteq \bigcap\{$*Mic-Pker* $(A_i)/\ i \in I\}$.

Proof:

- *(i)* *It follows by the definition of Mic-Pker(A).*
- *(ii)* *Suppose* $x \notin$ *Mic-Pker(B), then there exists a subset* $S \in$ *Mic-PO such that* $B \subset S$ *with* $x \notin S$. *Since* $A \subset B$, $x \notin$ *Mic-Pker(A). Thus Mic-Pker(A)* $\subset$ *Mic-Pker(B).*
- *(iii)* *Follows from (1) and definition of Mic-Pker(A).*
- *(iv)* *For each* $i \in I$, *Mic-Pker* $(A_i) \subseteq$ *Mic-Pker*$(\bigcup_{i\in I} A_i)$. *Therefore we have* $\bigcup_{i\in I}\{$*Mic-Pker*$(A_i)\} \subseteq$ *Mic-Pker* $(\bigcup_{i\in I} A_i)$.
- *(v)* *Suppose that* $x \notin \bigcap$ $\{$*Mic-Pker*$(A_i/i \in I)\}$ *then there exists an* $i_0 \in I$, *such that* $x \notin$ *Mic-Pker*(A_{i_0}) *and there exists a micro pre-open set S such that* $x \notin S$ *and* $A_{i_0} \subset S$. *We have* $\bigcap_{i\in I} A_i \subseteq A_{i_0} \subseteq S$ *and* $x \notin S$. *Therefore* $x \notin$ *Mic-Pker* $\{\bigcap A_i/i \in I\}$. *Hence Mic-Pker*$(\bigcap A_i/i \in I) \subseteq \bigcap$ *Mic-Pker*$(A_i)/i \in I$.

Theorem 9. *Let A and B be subsets of U, then the following conditions hold.*

- *(i)* *Mic-Pker(A)* $\subseteq$ *Mic-ker(A).*
- *(ii)* *Mic-Pker(A)* $\cap$ *Mic-Pker(B)* $\subset$ *Mic-Pker(A* $\cup$ *B).*
- *(iii)* *Mic-Pker(A* $\cap$ *B)* $\subset$ *Mic-Pker(A)* $\cup$ *Mic-Pker(B).*
- *(iv)* *Mic-Pcl(A)* $\cap$ *Mic-Pker(A) = A.*
- *(v)* *Mic-Pker(A)* $\cap$ *Mic-Pfr(A) = Mic-Pbr(A).*

proof:

(i) *Let* $x \in$ *Mic-Pker(A).*
$\Rightarrow x \in \bigcap\{M/A \subset M,\ M \in$ *Mic-PO*$\}$
$\Rightarrow x \in \bigcap\{M/A \subset M,\ M \in$ *micro-Open*$(\mu_R(x))\}$
Since every micro open is micro pre-open,
$x \in$ *Mic-ker(A).*

(ii) *Let* $x \in\{$*Mic-Pker(A)* $\cap$ *Mic-Pker(B)*$\}$
$\Rightarrow x \in$ *Mic-Pker(A) and* $x \in$ *Mic-Pker(B)*
Therefore, $x \in$ *Mic-Pker(A* $\cup$ *B).*

(iii) *Let* $x \in$ *Mic-Pker(A* $\cap$ *B)*
$\Rightarrow x \in$ *Mic-Pker(A) and* $x \in$ *Mic-Pker(B)*
Therefore, $x \in$ *Mic-Pker(A)* $\cup$ *Mic-Pker(B).*

(iv) *Let* $x \in$ *Mic-Pcl(A)* $\cap$ *Mic-Pker(A)*
By Lemma 3.7(1) [7] $A \subseteq$ *Mic-Pcl(A) and by (1)* $A \subseteq$ *Mic-Pker(A).*
$\Rightarrow x \in A \subseteq$ *Mic-Pcl(A) and* $x \in A \subseteq$ *Mic-Pker(A).*
Therefore, $x \in A$.

(v) *Let* $x \in$ *Mic-Pker(A)* $\cap$ *Mic-Pfr(A). To prove,* $x \in$ *Mic-Pbr(A) i.e,* $x \in A-$ *Mic-Pint(A). Since Mic-Pker(A)* $= \bigcap\{M/A \subset M,\ M \in$ *Mic-PO*$\}$ *and Mic-Pfr(A)* $=$ *Mic-Pcl(A)−Mic-Pint(A).*
$\Rightarrow x \in \bigcap\{M/A \subset M, M \in$ *Mic-PO*$\}$ $\cap$ *Mic-Pcl(A)−Mic-Pint(A). By Lemma 3.7(1) [7] and Lemma 4(1), we have*
$\Rightarrow x \in A \cap (A -$ *Mic-Pint(A))*
$\Rightarrow x \in A$ *and* $x \in A -$ *Mic-Pint(A)*
$\Rightarrow x \in A$ *and* $x \in$ *Mic-Pbr(A)*
Therefore, $x \in$ *Mic-Pbr(A).*

6. Conclusion

In this paper, we introduced the notions of micro pre-frontier, micro pre-exterior, micro pre-border and micro pre-kernel by employing the concept of frontier, exterior, border and kernel elucidating various associated properties. Our intent is to further elaborate on these findings in forthcoming research endeavors, with a particular focus on exploring practical applications.

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