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A Novel Forced Scrambling Model Under Time-Scaled Surveys for Estimation of Population Variance

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Abstract. In survey sampling, forced randomized response models are special variants of traditional models employed by researchers in sensitive surveys. The existing forced models are based on traditional one-time surveys where the respondents are interviewed only at a single time-point. These existing one-time survey-based models suffer from a serious drawback - the scrambling process is performed only once by each respondent. The lack of replication usually results in measurement errors which often have a negative influence on the estimators of population parameters. This study reveals that time-scaled surveys provide more efficient estimates of the population variance of sensitive variables than the traditional one-time surveys, under forced randomized response models. An Exponentially Weighted Moving Average (EWMA) estimator is used to estimate the population variance, based on the responses obtained at different time points. Further, a new forced randomized scrambling model is also proposed and the improvement over the available models is observed.

2020 Mathematics Subject Classifications: 62D05, 62F10

Key Words and Phrases: Forced model, EWMA estimator, scrambled response, time-scaled surveys, traditional surveys

1. Introduction

In sample surveys, Warner [27] presented the idea of getting scrambled responses in sensitive sample surveys. Gjestvang and Singh [7] introduced a forced scrambling model, using fixed responses. Later on, Diana and Perri [4] presented a linear model, achieving further improvement over the previous scrambling models. Murtaza et al. [17] analyzed the use of correlated variables under randomized response techniques. Azeem and Salam [3] recently presented a new model based on the mixture of direct response and scrambled response. Gupta et al. [10] suggested an optional scrambling technique for application in sensitive surveys. Pushadapu et al. [21] and Narjis and Shabbir [18] also explored different

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aspects of randomized scrambling models.

The available scrambling models are all based on one-time surveys, where the data is obtained at one time-point. In practice, some survey situations may require data collection repeatedly at different points in time. Time-scaled surveys can provide estimates of the parameters at different time points. Further, time-scaled surveys are also useful in measurement of the frequency and/or duration of different events of interest. For details about the different applications of time-scaled surveys, readers may refer to the studies of Duncan and Kalton [5] and Fuller [6]. More recently, Alomair and Iftikhar [1] also discussed different real-world uses of time-based sample surveys. Qureshi et al. [22] also used time-scaled surveys to develop efficient estimators.

The exponential weighted moving average (EWMA) technique was first suggested by Roberts [23] for monitoring the process mean using control charts. The study of Haq [11] improved the EWMA method by developing Hybrid EWMA (abbreviated as HEWMA) control charts. These methods assign weights to the data obtained at different time-points in such a manner that the recent-most data set gets the highest weightage. The weightage decreases as the time points get older. The data gathered at the oldest time-point gets the smallest weightage. As far as estimation of parameters is concerned under time-scaled surveys, the studies of Noor-ul-Amin [19, 20] analyzed novel memory-type estimators under time-scaled sample surveys.

In addition to central tendency metrics, survey researchers often need to analyze the variability among population units, e.g. the variation in the monthly income of individuals, the variation in the taxes paid, and the variation in the number of cigarettes consumed, etc. Since these variables are sensitive, so a randomized response data collection method may be more appropriate than direct questioning. Once the data collection process is completed, a suitable variance estimator can be used to estimate the population variance. Over the past fifty years, many variance estimators have been proposed by researchers under different sampling methods. Isaki [12] introduced a ratio-type variance estimator using non-sensitive variable. Gupta et al. [9] analyzed variance estimation of sensitive variables under the existence of auxiliary information. Recently, Saleem et al. [24], Kumar et al. [14], and Azeem et al. [2] also suggested variance estimators under one-time sample surveys. Shahzad et al. [25] studied variance estimation using calibration-based estimation methods. Jewsbury [13] introduced new variance estimation techniques and showed the validity of the proposed methods using simulation. Mahdizadeh and Zamanzade [15, 16], also studied variance estimation under ranked set sampling design. Sluijterman et al. [26] studied optimal implementation approaches of a mean variance estimation network.

In the traditional randomized response methods, each survey participants performs the scrambling process at one particular time-point to report his/her scrambled response. The numerical value of the reported response may be smaller or larger than the true response, depending on the random number chosen by the respondent. The use of random numbers in the scrambling process may result in large differences between the reported response and the true response. The lack of replication in one-time surveys may make the measurement errors even worse. To overcome this issue, time-scaled surveys may be employed where the scrambling process is replicated at many time points. Such replication of scrambling

process may balance out the under-reporting and over-reporting over time. This process ensures that reliable data is obtained from the survey respondents yet protecting their privacy.
This manuscript is outlined as follows:
An introduction to the notations has been given in Section 2 along with the assumptions used in the current study. Section 3 introduces EWMA estimators, with Section 4 providing some available scrambling models. The proposed new forced response model has been presented in Section 5. The results of a real-word survey have been presented in Section 6. The findings of the performance-evaluation have been presented in Section 7. A detailed discussion about the results of the current study has been presented in Section 8. Finally, some recommendations for future researchers have been provided in Section 9.

2. Notations and Assumptions

Before proceeding further, let us introduce some notations. Let the population and sample size be denoted by N and n, respectively. Let Y be the main variable of interest and let the additive and multiplicative random variables be denoted by S and T, respectively. Moreover, we use the notations μ_Y, μ_S, and μ_T to denote the means of Y, S, and T, respectively, for the population data. Likewise, σ_Y^2, σ_S^2, and σ_T^2 are the notations for the population variance of Y, S, and T, respectively, in the subsequent sections. Let us also employ some assumptions in line with those used by the previous researchers. These assumptions include: $\mu_S = 0$, and $\mu_T = 1$. Further, to facilitate privacy protection, we use the assumption of independence among all possible combinations of the variables.
In addition to the main variable, this paper also analyzes ratio estimators of variance under forced response models. For this, we use an auxiliary variable denoted by X which is assumed to have correlation with the main variable. Let the sample and population mean of the auxiliary variable X be denoted by $\bar{x}$ and μ_X, respectively, with s_X^2 and σ_X^2 denoting the variances for the sample and population data, respectively. Further, for positive numbers, 'r' and 's', we can define the moment ratio as:

$$\lambda_{TS} = \frac{\eta_{TS}}{\eta_{20}^{\frac{r}{2}}\eta_{02}^{\frac{s}{2}}}, \tag{1}$$

$$\eta_{TS} = \frac{\sum_{i=1}^{N}(x_i - \mu_X)^s(y_i - \mu_Y)^r}{N-1}. \tag{2}$$

3. EWMA-type Estimators

In time-scaled surveys, we can compute Exponentially Weighted Moving Average (EWMA) type estimators for estimating the population parameters. Consider a smoothing constant 'c' which is predefined by the researcher, we can define an EWMA-type estimator of the mean of variable Y as follows:

$$\hat{U}_t = c\bar{z} + (1-c)\hat{U}_{t-1}, \tag{3}$$

where $0 < c < 1$, and $\bar{z}$ denotes the sample mean calculated from the observed scrambled response. An EWMA-type variance estimator is expressed as follows:

$$\hat{V}_t = cs_Z^2 + (1-c)\hat{V}_{t-1}. \tag{4}$$

In (4), s_Z^2 denotes the sample variance computed from the scrambled response. An EWMA estimator of the finite population variance of variable X is presented as:

$$\hat{W}_t = cs_X^2 + (1-c)\hat{W}_{t-1}. \tag{5}$$

An EWMA ratio-type variance estimator of the variance of Y can be written as:

$$R_t = \hat{V}_t \frac{\sigma_X^2}{\hat{W}_t}. \tag{6}$$

In the subsequent sections, different estimators of the population variance of the sensitive main variable Y have been discussed.

4. Available Scrambling Models

Let us revisit two of the existing randomized response scrambling models and the underlying estimators.

4.1. Gjestvang and Singh [7] Forced Model (R1)

$$Z = \begin{cases} Y, & \text{with probability } P_1, \\ TY, & \text{with probability } P_2, \\ F, & \text{with probability } P_3 = 1 - P_1 - P_2, \end{cases} \tag{7}$$

where Z denotes the scrambled response.
Under traditional one-time sample surveys, a simple unbiased variance estimator of Y is:

$$s_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2. \tag{8}$$

An unbiased variance estimator of the variance of Z is as follows:

$$s_Z^2 = \frac{1}{n-1}\sum_{i=1}^{n}(z_i - \bar{z})^2. \tag{9}$$

An unbiased variance estimator of the variance of X is as follows:

$$s_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2. \tag{10}$$

In traditional sample surveys, a simple variance estimator of a sensitive variable using the model (R1) is given by:

$$s_Y^2(GS) = \frac{s_Z^2 - (p_1+p_2)(1-p_1-p_2)(\bar{z}-F)^2 - p_2\sigma_T^2\bar{z}^2}{p_1+p_2(\sigma_T^2+1)}. \tag{11}$$

In traditional sample surveys, a ratio-type estimator of Y is:

$$R_{GS} = \frac{s_Z^2 - (p_1+p_2)(1-p_1-p_2)(\bar{z}-F)^2 - p_2\sigma_T^2\bar{z}^2}{p_1+p_2(\sigma_T^2+1)}\frac{\sigma_X^2}{s_X^2}. \tag{12}$$

In time-scaled surveys, a ratio-type EWMA estimator of Y model is expressed as follows:

$$R_{t(GS)} = \frac{\hat{V}_t - (p_1+p_2)(p_1-p_2)(\hat{U}_t-F)^2 - p_2\sigma_T^2\hat{U}_t^2}{p_1+p_2(\sigma_T^2+1)}\frac{\sigma_X^2}{\hat{W}_t}. \tag{13}$$

4.2. Saleem et al. [24] Model (R2)

The Saleem et al. [24] scrambling model is given by the following equation:

$$Z = \gamma(Y+\alpha S) + (1-\gamma)T(Y+\alpha S), \tag{14}$$

where $0 < \gamma < 1$, with α denoting some pre-defined constant.
In order to simplify the comparison, we use $\alpha = 1$ in equation (14) to get:

$$Z = \gamma(Y+S) + (1-\gamma)T(Y+S). \tag{15}$$

In traditional sample surveys, a simple variance estimator based on the Saleem et al. [24] model is expressed as:

$$s_Y^2(S) = \frac{s_Z^2 - (1-\gamma)^2\sigma_T^2(\sigma_S^2+\bar{z}^2) + \bar{z}^2}{(1-\gamma^2)\sigma_T^2}. \tag{16}$$

Under traditional one-time sample surveys, a ratio-type estimator of Y is as follows:

$$R_S = \frac{s_Z^2 - (1-\gamma)^2\sigma_T^2(\sigma_S^2+\bar{z}^2) + \bar{z}^2}{(1-\gamma^2)\sigma_T^2}\frac{\sigma_X^2}{s_X^2}. \tag{17}$$

Under time-scaled sample surveys, the ratio-type EWMA variance estimator of Y using the Saleem et al. [24] model is expressed as follows:

$$R_{t(S)} = \frac{\hat{V}_t - (1-\gamma)^2\sigma_T^2(\sigma_S^2+\hat{U}_t^2) + \hat{U}_t^2}{(1-\gamma^2)\sigma_T^2}\frac{\sigma_X^2}{\hat{W}_t}. \tag{18}$$

5. Proposed Forced Scrambling Model (R3)

Motivated by the study of Saleem et al. [24], the following forced scrambling model is proposed.

$$Z = \begin{cases} \gamma(Y+TS)+(1-\gamma)(TY+TS), & \text{with probability } P \\ F, & \text{with probability } 1-P, \end{cases} \tag{19}$$

where $0 < \gamma < 1$, and F denotes a pre-determined forced response, such that $E(F) = F$, and $Var(F) = 0$. A simple mean estimator under the proposed model is given as follows:

$$\hat{\mu}_P = \frac{\bar{z} - (1-P)F}{P}. \tag{20}$$

In order to derive variance of Z, we take expectation on equation (19) to get:

$$E(Z) = P\left[\gamma E(Y+TS) + (1-\gamma)E(TY+TS)\right] + (1-P)E(F).$$

We can use the assumptions of Section 2 to get:

$$E(Z) = P\mu_Y + (1-P)F. \tag{21}$$

Using the result of equation (21), we can easily show that:

$$E(\hat{\mu}_P) = \mu_Y. \tag{22}$$

Also,

$$E(Z^2) = P\left(E\left[\gamma(Y+TS)+(1-\gamma)(TY+YS)\right]^2\right) + (1-P)E(F^2).$$

Using the assumptions discussed in Section 2, the variance of Z can be obtained as:

$$\begin{aligned} Var(Z) = {} & P\sigma_Y^2\left[\gamma^2 + (1-\gamma)^2(\sigma_T^2+1) + (1-\gamma)^2\sigma_S^2 + 2\gamma(1-\gamma)\right] + \gamma^2 P\left[(\sigma_T^2+1)\sigma_S^2 + \mu_Y^2\right] \\ & + (1-\gamma)^2 P[(\sigma_T^2+1)\mu_Y^2 + \mu_Y^2\sigma_S^2] + 2\gamma(1-\gamma)P[\mu_Y\sigma_S^2 + \mu_Y^2] + (1-P)F^2 \\ & - [P\mu_Y + (1-P)F]^2, \end{aligned}$$

or,

$$\begin{aligned} \sigma_Z^2 = {} & P\sigma_Y^2\left[\gamma^2 + (1-\gamma)^2(\sigma_T^2+\sigma_S^2+1) + 2\gamma(1-\gamma)\right] + \gamma^2 P\left[(\sigma_T^2+1)\sigma_S^2 + \mu_Y^2\right] \\ & + (1-\gamma)^2 P[(\sigma_T^2+1)\mu_Y^2 + \mu_Y^2\sigma_S^2] + 2\gamma(1-\gamma)P[\mu_Y\sigma_S^2 + \mu_Y^2] \\ & - P^2\mu_Y^2 + P(1-P)(F-2\mu_Y)F. \end{aligned} \tag{23}$$

Applying variance on equation (20) leads to:

$$Var(\hat{\mu}_P) = Var\left[\frac{\frac{1}{n}\sum_{i=1}^n Z_i - (1-P)F}{P}\right] = \frac{1}{n^2P^2}\sum_{i=1}^n Var(Z_i). \tag{24}$$

Using equation (23) in equation (24) gives:

$$\begin{aligned} Var(\hat{\mu}_P) = \frac{1}{nP}[&\sigma_Y^2\{\gamma^2 + (1-\gamma)^2(\sigma_T^2 + \sigma_S^2 + 1) + 2\gamma(1-\gamma)\} \\ &+ \gamma^2\{(\sigma_T^2 + 1)\sigma_S^2 + \mu_Y^2\} + (1-\gamma)^2\{(\sigma_T^2 + 1)\mu_Y^2 + \mu_Y^2\sigma_S^2\} \\ &+ 2\gamma(1-\gamma)\{\mu_Y\sigma_S^2 + \mu_Y^2\} - P\mu_Y^2 + (1-P)(F - 2\mu_Y)F]. \end{aligned} \tag{25}$$

Solving equation (23) for σ_Y^2 yields:

$$\begin{aligned} \sigma_Y^2 = \frac{1}{PA}[&\sigma_Z^2 - \gamma^2 P\{(\sigma_T^2 + 1)\sigma_S^2 + \mu_Y^2\} - (1-\gamma)^2 P\{(\sigma_T^2 + 1)\mu_Y^2 + \mu_Y^2\sigma_S^2\} \\ &- 2\gamma(1-\gamma)P(\mu_Y\sigma_S^2 + \mu_Y^2) + P^2\mu_Y^2 - P(1-P)(F - 2\mu_Y F)], \end{aligned} \tag{26}$$

where,

$$A = \gamma^2 + (1-\gamma)^2(\sigma_T^2 + \sigma_S^2 + 1) + 2\gamma(1-\gamma). \tag{27}$$

A basic variance estimator under traditional surveys may be obtained by using estimators in place of population parameters to yield:

$$\begin{aligned} s_Y^2(R) = \frac{1}{PA}[&s_Z^2 - \gamma^2 P\{(\sigma_T^2 + 1)\sigma_S^2 + \bar{z}^2\} - (1-\gamma)^2 P\{(\sigma_T^2 + 1)\bar{z}^2 + \bar{z}^2\sigma_S^2\} \\ &- 2\gamma(1-\gamma)P(\bar{z}\sigma_S^2 + \bar{z}^2) + P^2\bar{z}^2 - P(1-P)(F - 2\bar{z})F)]. \end{aligned} \tag{28}$$

In the case of traditional one-time surveys, a ratio-type estimator of the variance of Y is:

$$\begin{aligned} R_P = \frac{1}{PA}[&s_Z^2 - \gamma^2 P\{(\sigma_T^2 + 1)\sigma_S^2 + \bar{z}^2\} - (1-\gamma)^2 P\{(\sigma_T^2 + 1)\bar{z}^2 + \bar{z}^2\sigma_S^2\} \\ &- 2\gamma(1-\gamma)P(\bar{z}\sigma_S^2 + \bar{z}^2) + P^2\bar{z}^2 - P(1-P)(F - 2\bar{z})F]\frac{\sigma_X^2}{s_X^2}. \end{aligned} \tag{29}$$

In the case of time-scaled surveys, a ratio-type EWMA estimator of Y using the proposed model is:

$$\begin{aligned} R_{t(P)} = \frac{1}{PA}[&\hat{V}_t - \gamma^2 P\{(\sigma_T^2 + 1)\sigma_S^2 + \hat{U}_t^2\} - (1-\gamma)^2 P\{(\sigma_T^2 + 1)\hat{U}_t^2 + \hat{U}_t^2\sigma_S^2\} \\ &- 2\gamma(1-\gamma)P(\hat{U}_t\sigma_S^2 + \hat{U}_t^2) + P^2\hat{U}_t^2 - P(1-P)(F - 2\hat{U}_t)F]\frac{\sigma_X^2}{\hat{W}_t}. \end{aligned} \tag{30}$$

Theorem 1. *Using the proposed forced model, as $t \to \infty$, the mean squared error of $R_{t(P)}$ may be obtained as:*

$$\begin{aligned} MSE(R_{t(P)}) = \frac{\theta}{P^2A^2}(\frac{c}{2-c})[&\sigma_Z^4(\lambda_{40} - 1) + 4B^2C_Z^2 + D^2(\lambda_{04} - 1) - 4B\sigma_Z^2\lambda_{12}C_Z \\ &- 2D\sigma_Z^2(\lambda_{22} - 1) + 4BD\lambda_{30}C_Z], \end{aligned} \tag{31}$$

where, $\theta = \frac{1}{n}$, and

$$C_Z^2 = C_Y^2\sigma_T^2 + \frac{\sigma_S^2}{\bar{Y}^2}, \tag{32}$$

$$A = \gamma^2 + (1-\gamma)^2(\sigma_T^2 + \sigma_S^2 + 1) + 2\gamma(1-\gamma), \tag{33}$$

$$B = \gamma^2\bar{Z}^2 + (1-\gamma)^2 P\bar{Z}^2(\sigma_T^2+\sigma_S^2+1) + \gamma(1-\gamma)P(\bar{Z}\sigma_S^2 + 2\bar{Z}^2) - P^2\bar{Z}^2 - P(1-P)\bar{Z}F, \tag{34}$$

and,

$$\begin{aligned} D = \sigma_Z^2 - \gamma^2 P\{(\sigma_T^2+1)\sigma_S^2 + \bar{Z}^2\} - (1-\gamma)^2 P(\sigma_T^2+\sigma_S^2+1)\bar{Z}^2 \\ - 2\gamma(1-\gamma)P(\bar{Z}^2\sigma_S^2 + \bar{Z}^2) + P^2\bar{Z}^2 - P(1-P)(F-2\bar{Z})F. \end{aligned} \tag{35}$$

Proof. Define:

$$\begin{aligned} \hat{V}_t &= \sigma_Z^2(1+d_{Zt}), \\ \hat{W}_t &= \sigma_X^2(1+d_{Xt}), \\ \hat{U}_t &= \bar{Z}(1+e_{Zt}), \end{aligned} \tag{36}$$

so that,
$d_{Zt} = \frac{\hat{V}_t}{\sigma_Z^2} - 1$, $d_{Xt} = \frac{\hat{W}_t}{\sigma_X^2} - 1$, and, $e_{Zt} = \frac{\hat{U}_t}{\bar{Z}} - 1$.

Applying expectation yields, $E(d_{Zt}) = E(d_{Xt}) = E(e_{Zt}) = 0$
Further,

$$\begin{aligned} E(d_{Zt}^2) &= \theta(\frac{c}{2-c})[1-(1-c)^{2t}](\lambda_{40}-1), \\ E(d_{Xt}^2) &= \theta(\frac{c}{2-c})[1-(1-c)^{2t}](\lambda_{04}-1), \\ E(e_{Zt}^2) &= \theta(\frac{c}{2-c})[1-(1-c)^{2t}]C_Z^2, \\ E(d_{Zt}d_{Xt}) &= \theta(\frac{c}{2-c})[1-(1-c)^{2t}](\lambda_{22}-1), \\ E(d_{Zt}e_{Zt}) &= \theta(\frac{c}{2-c})[1-(1-c)^{2t}]\lambda_{30}C_Z, \\ E(d_{Xt}e_{Zt}) &= \theta(\frac{c}{2-c})[1-(1-c)^{2t}]\lambda_{12}C_Z. \end{aligned} \tag{37}$$

As $t \xrightarrow{\infty}$, $1-(1-c)^{2t} \to 1$ equation (37) tends to:

$$\begin{aligned} E(d_{Zt}^2) &= \theta(\frac{c}{2-c})(\lambda_{40}-1), \\ E(d_{Xt}^2) &= \theta(\frac{c}{2-c})(\lambda_{04}-1), \\ E(e_{Zt}^2) &= \theta(\frac{c}{2-c})C_Z^2, \\ E(d_{Zt}d_{Xt}) &= \theta(\frac{c}{2-c})(\lambda_{22}-1), \\ E(d_{Zt}e_{Zt}) &= \theta(\frac{c}{2-c})\lambda_{30}C_Z, \\ E(d_{Xt}e_{Zt}) &= \theta(\frac{c}{2-c})\lambda_{12}C_Z. \end{aligned} \tag{38}$$

The ratio-type EWMA estimator may be written as follows:

$$\begin{aligned}R_{t(P)} = &\frac{1}{PA}[\sigma_Z^2(1+d_{Zt}) - \gamma^2 P\{(\sigma_T^2+1)\sigma_S^2 + \bar{Z}^2(1+e_{Zt})^2\} \\ &- (1-\gamma)^2 P\{(\sigma_T^2+\sigma_S^2+1)\bar{Z}^2(1+e_{Zt})^2\} \\ &- 2\gamma(1-\gamma)P\{\bar{Z}(1+e_{Zt})\sigma_S^2 + (\bar{Z}^2(1+e_{Zt})^2\} + P^2\bar{Z}^2(1+e_{Zt})^2 \\ &- P(1-P)\{F - 2\bar{Z}(1+e_{Zt})\}F]\frac{\sigma_X^2}{\sigma_X^2(1+d_{Xt})}.\end{aligned} \tag{39}$$

Further simplification yields:

$$R_{t(P)} - \sigma_Y^2 = \frac{1}{PA}[\sigma_Z^2 d_{Zt} - 2Be_{Zt} - Dd_{Xt}]. \tag{40}$$

Squaring equation (40), applying expectation, and after simplification, we get the required result as:

$$\begin{aligned}MSE(R_{t(P)}) = &\frac{\theta}{P^2A^2}(\frac{c}{2-c})[\sigma_Z^4(\lambda_{40}-1) + 4B^2C_Z^2 + D^2(\lambda_{04}-1) - 4B\sigma_Z^2\lambda_{12}C_Z \\ &- 2D\sigma_Z^2(\lambda_{22}-1) + 4BD\lambda_{30}C_Z].\end{aligned}$$

Remark 1: The mean square error of the ratio estimator using the proposed forced model under traditional one-time surveys may be written as:

$$\begin{aligned}MSE(R_{(P)}) = &\frac{\theta}{A^2}[\sigma_Z^4(\lambda_{40}-1) + 4B^2C_Z^2 + D^2(\lambda_{04}-1) - 4B\sigma_Z^2\lambda_{12}C_Z \\ &- 2D\sigma_Z^2(\lambda_{22}-1) + 4BD\lambda_{30}C_Z].\end{aligned} \tag{41}$$

Remark 2: Using equation (36) and equation (38), the mean square error of the ratio-type estimator using the Saleem et al. [24] model under traditional one-time surveys can be obtained as:

$$\begin{aligned}MSE(R_{(S)}) = &\frac{\theta}{(1-\gamma)^4\sigma_T^4}[\sigma_Z^4(\lambda_{40}-1) + G^2C_Z^2 + H^2(\lambda_{04}-1) - 2G\sigma_Z^2\lambda_{30}C_Z \\ &- 2H\sigma_Z^2(\lambda_{22}-1) + 2GH\lambda_{12}C_Z],\end{aligned} \tag{42}$$

where,

$$G = 2\bar{Z}^2\{(1-\gamma)^2\sigma_T^2 - 1\}, \tag{43}$$

and,

$$H = \sigma_Z^2 - (1-\gamma)^2\sigma_T^2(\bar{Z}^2+\sigma_S^2) + \bar{Z}^2. \tag{44}$$

A time-scaled variant of the mean square error given in equation (42) can be obtained as follows:

$$\begin{aligned}MSE(R_{t(S)}) = &\frac{\theta}{(1-\gamma)^4\sigma_T^4}(\frac{c}{2-c})[\sigma_Z^4(\lambda_{40}-1) + G^2C_Z^2 + H^2(\lambda_{04}-1) - 2G\sigma_Z^2\lambda_{30}C_Z \\ &- 2H\sigma_Z^2(\lambda_{22}-1) + 2GH\lambda_{12}C_Z].\end{aligned} \tag{45}$$

Remark 3: Using equation (36) and equation (38), the mean square error of the ratio-type estimator using the Gjestvang and Singh [7] forced model under traditional one-time surveys can be obtained as:

$$MSE(R_{G(S)}) = \frac{\theta}{[P_1 + P_2(\sigma_T^2 + 1)]^2}[\sigma_Z^4(\lambda_{40} - 1) + J^2C_Z^2 + K^2(\lambda_{04} - 1) - 2K\sigma_Z^2(\lambda_{22} - 1) \\ - 2J\sigma_Z^2\lambda_{30}C_Z + 2JK\lambda_{12}C_Z], \tag{46}$$

where,

$$J = 2\{P_2\sigma_T^2\bar{Z}^2 + (P_1 + P_2)(1 - P_1 - P_2)\bar{Z}(F - \bar{Z})\}, \tag{47}$$

$$K = \sigma_Z^2 - P_2\sigma_T^2\bar{Z}^2 - (P_1 + P_2)(1 - P_1 - P_2)(F - \bar{Z})^2. \tag{48}$$

A time-scaled variant of the mean square error given in equation (46) can be obtained as follows:

$$MSE(R_{t(GS)}) = \frac{\theta}{[P_1 + P_2(\sigma_T^2 + 1)]^2}(\frac{c}{2 - c})[\sigma_Z^4(\lambda_{40} - 1) + J^2C_Z^2 + K^2(\lambda_{04} - 1) \\ - 2K\sigma_Z^2(\lambda_{22} - 1) - 2J\sigma_Z^2\lambda_{30}C_Z + 2JK\lambda_{12}C_Z]. \tag{49}$$

Remark 4: The variance of the simple unbiased variance estimator can be obtained by simply ignoring the terms containing λ_{04}, λ_{22}, and λ_{12} in equation (46). Thus, under the proposed model, the variance of the simple unbiased variance estimator, $\hat{V}_t$, may be derived as:

$$VAR(\hat{V}_t) = \frac{\theta}{P^2A^2}(\frac{c}{2 - c})[\sigma_Z^4(\lambda_{40} - 1) + 4B^2C_Z^2 + 4BD\lambda_{30}C_Z]. \tag{50}$$

Remark 5: The bias of $R_{t(P)}$ under the proposed model in time-scaled surveys can be derived as:

$$Bias(R_{t(P)}) \approx \frac{-\theta}{PA}(\frac{c}{2 - c})[P(1 - P)\bar{Z}^2C_Z^2 + \sigma_Z^2(\lambda_{22} - 1) - 2G\lambda_{12}C_Z],$$

where,

$$G = P[\bar{Z}^2\{\gamma^2 + (1 - \gamma)^2(\sigma_T^2 + \sigma_S^2 + 1)\} + \{\gamma(1 - \gamma)(\bar{Z}\sigma_S^2 + 2\bar{Z}^2) - 2P\bar{Z}^2 - (1 - P)F\bar{Z}\}].$$

6. A Real-World Survey using the Proposed Method

We applied the proposed time-based randomized survey method to obtain data from a sample of 50 undergraduate students. Data was collected on the number of times the students cheated in last examination. Using our suggested model, we collected the scrambled responses on three different days (three time-points). The interviewer guided the respondents in calculating their scrambled responses. Using calculators, the respondents calculated and reported their scrambled responses using the proposed model. The observed responses at different time-points have been displayed in Table 1.

Table 1: Responses of Survey Participants at Different Time-Points

Responses on Day 1($t=1$)					Responses on Day 2($t=2$)					Responses on Day 3($t=3$)				
1.4	9.6	-7.1	2.8	-0.6	2.1	-1.6	7.4	-2.0	-3.9	-7.3	-1.5	5.0	2.1	-6.3
7.3	-6.1	2.4	0.4	4.5	-3.4	3.1	-6.2	0.1	3.0	1.8	4.7	0.4	-4.0	4.1
-2.6	3.0	8.3	-6.9	3.0	-2.9	-7.3	3.0	-4.5	-2.6	-2.5	-0.4	-2.0	3.0	-2.5
0.3	-1.8	-4.2	-1.0	5.8	9.1	-1.1	4.8	-7.1	5.0	3.0	0.3	-0.2	-1.1	-1.6
-5.2	-7.4	5.8	3.0	-3.7	-3.3	5.4	-4.2	4.2	-2.9	-1.4	-3.5	1.3	8.1	3.0
0.8	0.3	1.5	4.2	-1.2	3.0	1.5	1.1	0.5	8.6	0.6	1.0	3.1	0.6	-4.7
4.9	-4.7	-0.6	7.1	4.3	2.7	3.0	-6.6	-3.5	2.4	-1.0	-0.5	-4.8	0.1	4.0
-3.5	8.4	5.9	3.7	8.0	-1.8	0.0	-0.9	1.4	-6.7	8.2	0.1	6.4	-3.8	7.8
-6.4	-3.1	3.4	-1.8	-5.1	1.4	-1.9	3.2	-4.2	0.2	-3.9	3.0	-2.5	3.0	-2.1
2.7	4.6	-6.1	2.1	0.1	-5.7	-4.2	-2.6	2.0	3.0	-0.4	-4.6	3.0	-0.9	-5.3

It should be noted that the number of times a candidate cheated in an examination is a positive integer value, however, Table 1 shows that many of the observed responses are negative real numbers. This is due to the scrambling process employed by the proposed model which helps in protecting the privacy of the respondents.

Using the responses given in Table 1, we can calculate the estimator of population variance as:

$\hat{V}_t = Cs_Z^2 + (1-t)\hat{V}_{t-1}$.

The value of $\hat{V}_t$ can be chosen by the researcher based on some prior knowledge about the population variance. Alternatively, a pilot survey may also be conducted to get the value of $\hat{V}_t$. In this example, we choose at time $t=0$. Let $c=0.3$, so the variance at time $t=1$ may be calculated by the formula:

$$\hat{V}_1 = 0.3s_Z^2 + 0.7(10).$$

At time-point $t=1$ (Day 1), the sample variance of the 50 observed responses is $s_Z^2 = 21.75$, therefore:

$$\hat{V}_1 = 0.3(21.75) + 0.7(10) = 13.525.$$

At time-point $t=2$ (Day 1), the sample variance of the 50 observed responses is $s_Z^2 = 16.95$, therefore:

$$\hat{V}_1 = 0.3(16.95) + 0.7(10) = 14.553.$$

At time-point $t=3$ (Day 1), the sample variance of the 50 observed responses is $s_Z^2 = 13.69$, therefore:

$$\hat{V}_1 = 0.3(13.69) + 0.7(10) = 14.294.$$

In this example, we collected the data at three different time-points. Researchers may collect the data at more than three time-points, depending on the availability of time and other resources to conduct the survey.

7. Efficiency Comparison

The efficiency condition for the ratio-type EWMA variance estimator and traditional ratio estimator using the proposed model may be derived as:

$$MSE(R_{t(P)}) < MSE(R_P).$$

After simplification, the efficiency condition reduces to the simple form:

$$\frac{c}{2-c} < 1,$$

or,

$c < 1$.

Yan et al. [28] developed a model-evaluation metric as follows:

$$\Delta = E(U - Y)^2. \tag{51}$$

Gupta et al. [8] suggested a better evaluation measure which can be expressed as:

$$\delta = \frac{Var(\hat{\mu})}{\Delta}. \tag{52}$$

Table 2 displays the empirical results of the efficiency comparison of the simple variance estimator between traditional and time-scaled sample surveys, under different scrambling models. Table 3 shows the results of the corresponding Δ and δ values based on simple variance estimator under different models. Likewise, Table 4 presents the efficiency comparison results for the ratio type variance estimators under various models. The Δ and δ values based on ratio estimator have been provided in Table 5 using different values of the smoothing constant c. The superiority of the EWMA estimators over the traditional survey-based estimators can be clearly observed from tables. Further, we see that the proposed forced model provides the most efficient estimates of variance under both traditional and time-scaled sample surveys.

8. Results and Discussion

Survey researchers employ one-time sample surveys for use with randomized response models. This research study analyzed the efficiency of simple and ratio-type variance estimators using time-scale surveys under different randomized response scrambling models. Additionally, a new forced randomized response technique was also presented in Section

Table 2: Efficiency comparison of simple estimators for $\mu_Y = 1.2, \sigma_Y^2 = 0.8$, $\sigma_T^2 = 0.5$, $\sigma_S^2 = 5$, $\sigma_Z^2 = 5, n = 500$

c	γ	EWMA Estimators (Time-Scaled Surveys)			One-Time based Estimators (Traditional Surveys)		
		$Var(\hat{V}_{t(GS)})$	$Var(\hat{V}_{t(S)})$	$Var(\hat{V}_{t(P)})$	$s^2_{Y(GS)}$	$s^2_{Y(S)}$	$s^2_{Y(P)}$
0.1	0.2	0.04	0.03	0.01	0.67	0.61	0.16
	0.4	0.04	0.04	0.01	0.67	0.67	0.17
	0.6	0.04	0.04	0.01	0.67	0.72	0.21
0.2	0.2	0.07	0.07	0.02	0.67	0.61	0.16
	0.4	0.07	0.07	0.02	0.67	0.67	0.17
	0.6	0.07	0.08	0.02	0.67	0.72	0.21
0.3	0.2	0.12	0.11	0.03	0.67	0.61	0.16
	0.4	0.12	0.12	0.03	0.67	0.67	0.17
	0.6	0.12	0.13	0.04	0.67	0.72	0.21
0.4	0.2	0.17	0.15	0.04	0.67	0.61	0.16
	0.4	0.17	0.17	0.04	0.67	0.67	0.17
	0.6	0.17	0.18	0.05	0.67	0.72	0.21
0.5	0.2	0.22	0.20	0.05	0.67	0.61	0.16
	0.4	0.22	0.22	0.06	0.67	0.67	0.17
	0.6	0.22	0.24	0.07	0.67	0.72	0.21
0.6	0.2	0.29	0.26	0.07	0.67	0.61	0.16
	0.4	0.29	0.29	0.07	0.67	0.67	0.17
	0.6	0.29	0.31	0.09	0.67	0.72	0.21
0.7	0.2	0.36	0.33	0.09	0.67	0.61	0.16
	0.4	0.36	0.36	0.09	0.67	0.67	0.17
	0.6	0.36	0.39	0.11	0.67	0.72	0.21
0.8	0.2	0.45	0.41	0.11	0.67	0.61	0.16
	0.4	0.45	0.45	0.11	0.67	0.67	0.17
	0.6	0.45	0.48	0.14	0.67	0.72	0.21
0.9	0.2	0.55	0.50	0.13	0.67	0.61	0.16
	0.4	0.55	0.55	0.14	0.67	0.67	0.17
	0.6	0.55	0.59	0.17	0.67	0.72	0.21

5. Results of the comparative analysis have been provided in Table 2-5 based on different performance-evaluation measures.

In Table 2, the values of the sampling variance of the simple variance estimator under various scrambling models have been presented. The results for both traditional surveys and time-scaled surveys have been shown. Observing Table 2, we find that the EWMA variance estimators are much more efficient than the traditional survey-based variance estimators. Further, we also observe that the basic variance estimator under the proposed forced model is more efficient than the estimator under the Gjestvang and Singh [7] forced scrambling model and the Saleem et al. [24] model. This result holds not only in traditional surveys but also in time-scaled surveys. Table 3 displays the values of pri-

Table 3: Comparison of Δ and δ values based on simple estimators under different models for $\mu_Y = 1.2$, $\sigma_Y^2 = 0.8$, $\sigma_T^2 = 0.5$, $\sigma_S^2 = 5$, $\sigma_Z^2 = 5$,$n = 500$

c	γ	Privacy Measure (Δ)			Unified Measure (δ)		
		Gjestvang & Singh	Saleem et al.	Proposed Model	Gjestvang & Singh	Saleem et al.	Proposed Model
0.1	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001
0.2	0.2	113.552	8.917	201.639	0.006	0.068	0.001
	0.4	113.552	7.903	201.163	0.006	0.085	0.001
	0.6	113.552	7.179	200.873	0.006	0.101	0.001

vacy measure Δ and the unified measure δ under various models for different values of the smoothing constant. The table shows that the Δ values under the proposed forced scrambling model are higher than those based on the existing scrambling models. This implies that the proposed forced model offers the best level of privacy protection of all three competitor models. Moreover, Table 3 also reveals that the δ values are the smallest under the suggested forced scrambling model.

In Table 4, the values of mean squared errors of the ratio-type variance estimators have been presented under various models. Results of mean square error have been provided under both traditional and time-scaled sample surveys. From Table 4, we observe that

Table 4: Efficiency comparison of ratio-type estimators under different models for $\mu_Y = 1.2, \sigma_Y^2 = 0.8$, $\sigma_T^2 = 0.5$, $\sigma_S^2 = 5$, $\sigma_Z^2 = 5, n = 500$

c	γ	EWMA Estimators (Time-Scaled Surveys)			One-Time based Estimators (Traditional Surveys)		
		$MSE(R_{t(DP)})$	$MSE(R_{t(S)})$	$MSE(R_{t(P)})$	$MSE(R_{DP})$	$MSE(R_S)$	$MSE(R_P)$
0.1	0.2	1.18	0.02	0.02	22.33	0.47	0.29
	0.4	1.18	0.03	0.01	22.33	0.51	0.18
	0.6	1.18	0.03	0.01	22.33	0.55	0.22
0.2	0.2	2.48	0.05	0.03	22.33	0.47	0.29
	0.4	2.48	0.06	0.02	22.33	0.51	0.18
	0.6	2.48	0.06	0.02	22.33	0.55	0.22
03	0.2	3.94	0.08	0.05	22.33	0.47	0.29
	0.4	3.94	0.09	0.03	22.33	0.51	0.18
	0.6	3.94	0.10	0.04	22.33	0.55	0.22
0.4	0.2	5.58	0.12	0.07	22.33	0.47	0.29
	0.4	5.58	0.13	0.05	22.33	0.51	0.18
	0.6	5.58	0.14	0.05	22.33	0.55	0.22
0.5	0.2	7.44	0.16	0.10	22.33	0.47	0.29
	0.4	7.44	0.17	0.06	22.33	0.51	0.18
	0.6	7.44	0.18	0.07	22.33	0.55	0.22
0.6	0.2	9.57	0.20	0.13	22.33	0.47	0.29
	0.4	9.57	0.22	0.08	22.33	0.51	0.18
	0.6	9.57	0.23	0.09	22.33	0.55	0.22
0.7	0.2	12.02	0.25	0.16	22.33	0.47	0.29
	0.4	12.02	0.28	0.10	22.33	0.51	0.18
	0.6	12.02	0.29	0.12	22.33	0.55	0.22
0.8	0.2	14.89	0.31	0.19	22.33	0.47	0.29
	0.4	14.89	0.34	0.12	22.33	0.51	0.18
	0.6	14.89	0.36	0.14	22.33	0.55	0.22
0.9	0.2	18.27	0.39	0.24	22.33	0.47	0.29
	0.4	18.27	0.42	0.15	22.33	0.51	0.18
	0.6	18.27	0.45	0.18	22.33	0.55	0.22

the EWMA ratio estimators perform more efficiently than the traditional survey-based estimators. Moreover, under both surveys, the ratio-type variance estimator under the proposed forced model is observed to be more efficient than the available variance estimators. Table 5 displays the computed values of the privacy measure Δ and the unified measure δ based on the ratio-type variance estimators. The results have been provided under different scrambling models for different values of the constant 'c'. We find that the Δ values under the proposed forced model are the highest of all three models. This implies that the proposed forced response model offers the best level of privacy protection of all three competitor models. Additionally, Table 5 also indicates that the δ values un-

Table 5: Comparison of Δ and δ values based on ratio-type estimators under different models for $\mu_Y = 1.2$, $\sigma_Y^2 = 0.8$, $\sigma_T^2 = 0.5$, $\sigma_S^2 = 5$, $\sigma_Z^2 = 5$,$n = 500$

c	γ	Privacy Measure (Δ)			Unified Measure (δ)		
		Diana & Perri	Saleem et al.	Proposed Model	Diana & Perri	Saleem et al.	Proposed Model
0.1	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.2	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.3	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.4	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.5	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.6	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.7	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.8	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001
0.9	0.2	113.552	8.917	201.639	0.197	0.053	0.001
	0.4	113.552	7.903	201.163	0.197	0.065	0.001
	0.6	113.552	7.179	200.873	0.197	0.076	0.001

der the proposed forced model are the smallest of all three models. These results make the proposed forced model the most suitable model of all three models for application in real-world sample surveys.

It may be noted that the mean square error presented in equation (31) depends on population parameters. This makes it difficult to construct confidence intervals and to compare the efficiency of estimators as the information about the values of parameters is rarely available. However, using computer programs, Monte-Carlo simulations can be performed in time-scaled surveys to estimate the sampling variance of the variance estimators. These simulated sampling variances can be used to compare the efficiency of variance estimators.

Likewise, simulation-based analyses also help the researchers to find estimates of standard errors for construction of confidence intervals.

9. Future Research Suggestions

For future researchers, we make the following recommendations:

- Future researchers may evaluate the performance of the variance estimators in the case of response and non-response errors under time-scaled surveys.
- Besides simple unified measures, researchers can also evaluate the performance of variance estimators using weighted unified measures, by using different weights for efficiency and privacy.
- Complex sampling designs can be used with time-scaled surveys to develop new forced response models and new variance estimators.

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Conflict of Interest
The authors declare no conflict of interest.

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The data associated with this study has not been deposited into a publicly available repository. All relevant data are available within the article and its references.

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