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Computational Analysis of Reverse Degree-Based Topological Indices in Hex-Derived Networks

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Abstract. Topology is the mathematical study of the geometric and spatial properties that remain unchanged under continuous transformations of a graph's shape and size. In chemical graph theory, topological indices are used to quantify various chemical properties of molecules. These indices are derived from the topological structure of a graph and are crucial in understanding the valency of a chemical substance, which is determined by the number of surrounding atoms in its molecular structure. Topological indices are connected to numerous physicochemical properties, such as vapor pressure, stability, and elastic energy. In molecular structures, topological indices provide a numerical representation of the connections between molecules. In theoretical chemistry, these indices are widely used to simulate the physicochemical characteristics of complex compounds. $QSAR/QSPR$ studies rely heavily on topological indices to predict physical and chemical properties. This article explores the hex-derived network and its first two types, calculating reversed degree-based topological indices for these networks.

2020 Mathematics Subject Classifications: 05C12, 05C90, 92E10

Key Words and Phrases: Chemical graph theory, Topological indices, Molecular structure, Valency, Quantitative analysis, Hex-derived network, Theoretical chemistry, Chemical properties prediction

1. Introduction

Graph theory is the study of graphs and a subfield of combinatorics. It is related with applied mathematics and information technology. It is combination of mathematics, operational research, information technology and electrical engineering. In graph theory, the concept graph doesn't donate the data infect it denotes the structures of molecules.

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Cheminformatics is the combination of mathematics, chemistry and information science. In this subject, we study the $QSAR/QSPR$ relationship, characterisation, physical and bioactivities of chemical compounds [15, 26].

Topological indices basically are the numerical values, polynomials or matrix to represent a chemical graph. Topological indices are assumed to be the building blocks for the prediction of physico-chemical properties of chemical compounds. They based on the topology of chemical networks depend upon the distance, degree and eccentricity.

Graphs discussed in this article are undirected and finite. A graph is a structure made up of vertices which are connected with edges. A graph is a pair of sets $(\mathcal{U}, \mathcal{E})$, where $\mathcal{U}$ is the set of vertices and $\mathcal{E}$ is the set of edges, formed by pair of vertices. Order of $\mathcal{G}$ is represented by $|\mathcal{U}|$ and size of $\mathcal{G}$ represented by $|\mathcal{E}|$. The degree of a vertex $\breve{v}$ is the number of edges incident of that vertex and is denoted by $\mathfrak{d}_{\breve{v}}$. The reverse degree of a vertex $\breve{v}$ is represented by $\mathcal{R}_{\breve{v}}$. It was introduced by Kulli [14]. If Δ is the maximum degree of a graph then reverse degree is defined as $\mathcal{R}_{\breve{v}} = 1 - \mathfrak{d}_{\breve{v}} + \Delta$.

Topological indices are categorised into mainly two types distance based and degree based topological indices [1, 3, 12, 16–20, 28]. Our work is based on degree based topological indices. The theory of topological indices begin with the working of Wiener [30]. It is defined as,

$$W(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} d(\breve{u}, \breve{v}). \tag{1}$$

Randić index is defined in [2, 6, 21], and its reverse Randić index is,

$$\mathcal{R}R_{\alpha}(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} (\mathcal{R}_{\breve{u}} \times \mathcal{R}_{\breve{v}})^{\alpha}, \alpha = -1, 1, \frac{1}{2}, -\frac{1}{2}. \tag{2}$$

ABC index is defined in [8], and its reverse ABC index is,

$$\mathcal{R}ABC(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} \sqrt{\frac{\mathcal{R}_{\breve{u}} + \mathcal{R}_{\breve{v}} - 2}{\mathcal{R}_{\breve{u}} \times \mathcal{R}_{\breve{v}}}} \tag{3}$$

GA index is defined in [29], and its reverse GA index is,

$$\mathcal{R}GA(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} \frac{2\sqrt{\mathcal{R}_{\breve{u}} \times \mathcal{R}_{\breve{v}}}}{\mathcal{R}_{\breve{u}} + \mathcal{R}_{\breve{v}}}. \tag{4}$$

The first Zagreb index is defined in [10], and its reverse Zagreb index is,

$$\mathcal{R}M_1(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} (\mathcal{R}_{\breve{u}} + \mathcal{R}_{\breve{v}}). \tag{5}$$

The hyper Zagrab index is defined in [24], and its reverse hyper Zagrab index is,

$$\mathcal{R}HM(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} (\mathcal{R}_{\breve{u}} + \mathcal{R}_{\breve{v}})^2. \tag{6}$$

The forgotten index is defined in [9], and its reverse forgotten index is,

$$\mathcal{R}F(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} ((\mathcal{R}_{\breve{u}})^2 + (\mathcal{R}_{\breve{v}})^2). \tag{7}$$

The first, second and third redefined Zagreb index is defined in [22, 27], and its reverse redefined Zagreb index is,

$$\mathcal{R}RZ_1(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} \frac{\mathcal{R}_{\breve{u}} + \mathcal{R}_{\breve{v}}}{\mathcal{R}_{\breve{u}} \times \mathcal{R}_{\breve{v}}}. \tag{8}$$

$$\mathcal{R}RZ_2(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} \frac{\mathcal{R}_{\breve{u}} \times \mathcal{R}_{\breve{v}}}{\mathcal{R}_{\breve{u}} + \mathcal{R}_{\breve{v}}}. \tag{9}$$

$$\mathcal{R}RZ_3(\mathcal{G}) = \sum_{(\breve{u},\breve{v})\in\mathcal{E}(\mathcal{G})} (\mathcal{R}_{\breve{u}} + \mathcal{R}_{\breve{v}})(\mathcal{R}_{\breve{u}} \times \mathcal{R}_{\breve{v}}). \tag{10}$$

Hex derived network is derived from hexagonal mesh shown in Figure 1, by adding a layer of triangles around its boundary, after that connecting the faces of $HX(n)$, with the vertices we get $HDN1(n)$ shown in Figure 2 . By connecting the vertices of $HDN1(n)$ with each other we get $HDN2(n)$ shown in Figure 3. For the detail construction of Hex derived network we refer the reader to concern [7, 13, 25].

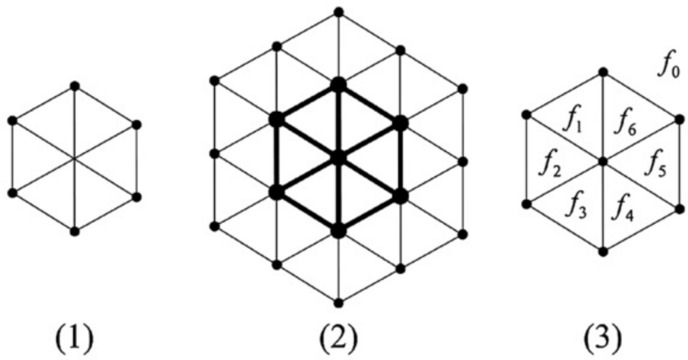

Figure 1: Hexagonal Meshes

2. Main Results

Hex-derived network has a lot of applications in material sciences. Shao Z et al. [23], computed the metric dimensions of hex derived network and Imran et al. [13], computed

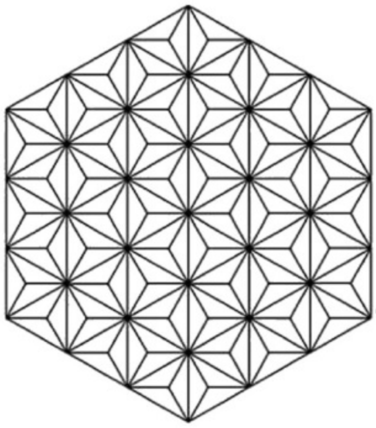

Figure 2: Hex-derived network $HDN1(4)$

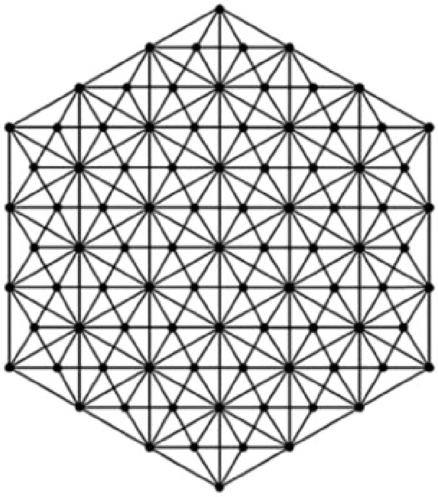

Figure 3: Hex-derived network $HDN2(n)$

the topological indices of it. Here we discuss the first two types of HDN and find the reversed degree based indices of it. The symbols used in this articles is from the book [4, 5, 11].

2.1. Results on Hex-derived network of Type 1

In this section, we will compute reverse randić, ABC, GA, Zagreb, redined Zagreb, hyper Zagreb and forgotten index for hex-derive network of type 1. The reversed edge partition of $HDN1(n)$ is written in Table1.

$(\mathcal{R}_{\breve{u}}, \mathcal{R}_{\breve{v}})$	Number of Edges
$(1,1)$	$9n^2 - 33n + 30$
$(6,1)$	$12n - 24$
$(6,6)$	$6n - 18$
$(8,1)$	6
$(8,6)$	12
$(10,1)$	$18n^2 - 54n + 42$
$(10,6)$	$18n - 36$
$(10,8)$	12

Table 1: Edge Partition of first type of hex-derived network

Theorem 2.1.1. *Let $\mathcal{G}_1$ be the first type of hex-derived network then*

$$\mathcal{R}R_\alpha(HDN1(n)) = \begin{cases} 189n^2 + 795n - 918, & \text{if } \alpha = 1, \\ \frac{54}{5}n^2 - \frac{539}{15}n + \frac{121}{4}, & \text{if } \alpha = -1, \\ 65.920998n^2 + 1.058284n - 75.386629, & \text{if } \alpha = \frac{1}{2}, \\ 14.6921n^2 - 41.85353n + 31.031039, & \text{if } \alpha = -\frac{1}{2}. \end{cases}$$

Proof. Let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 2 and Table 1, we have

$$\begin{aligned} \mathcal{R}R_\alpha(\mathcal{G}_1) &= (1)^\alpha|E_{1,1}(\mathcal{G}_1)| + (6)^\alpha|E_{6,1}(\mathcal{G}_1)| + (36)^\alpha|E_{6,6}(\mathcal{G}_1)| + (8)^\alpha|E_{8,1}(\mathcal{G}_1)| \\ &\quad + (48)^\alpha|E_{8,6}(\mathcal{G}_1)| + (10)^\alpha|E_{10,1}(\mathcal{G}_1)| + (60)^\alpha|E_{10,6}(\mathcal{G}_1)| \\ &\quad + (80)^\alpha|E_{10,8}(\mathcal{G}_1)|, \end{aligned}$$

$$\begin{aligned} \mathcal{R}R_\alpha(\mathcal{G}_1) &= (1)^\alpha(9n^2 - 33n + 30) + (6)^\alpha(12n - 24) + (36)^\alpha(6n - 18) \\ &\quad + (8)^\alpha(6) + (48)^\alpha(12) + (10)^\alpha(18n^2 - 54n + 42) \\ &\quad + (60)^\alpha(18n - 36) + (80)^\alpha(12), \end{aligned}$$

for $\alpha = 1$,

$$\begin{aligned} \mathcal{R}R_1(\mathcal{G}_1) &= (1)(9n^2 - 33n + 30) + (6)(12n - 24) + (36)(6n - 18) \\ &\quad + (8)(6) + (48)(12) + (10)(18n^2 - 54n + 42) \\ &\quad + (60)(18n - 36) + (80)(12), \end{aligned}$$

$$\Rightarrow \mathcal{R}R_1(\mathcal{G}_1) = 189n^2 + 795n - 918.$$

for $\alpha = -1$,

$$\begin{aligned} \mathcal{R}R_{-1}(\mathcal{G}_1) &= (1)^{-1}(9n^2 - 33n + 30) + (6)^{-1}(12n - 24) + (36)^{-1}(6n - 18) \\ &\quad + (8)^{-1}(6) + (48)^{-1}(12) + (10)^{-1}(18n^2 - 54n + 42) \\ &\quad + (60)^{-1}(18n - 36) + (80)^{-1}(12), \end{aligned}$$

$$\Rightarrow \mathcal{R}R_{-1}(\mathcal{G}_1) = \frac{54}{5}n^2 - \frac{539}{15}n + \frac{121}{4}.$$

for $\alpha = \frac{1}{2}$,

$$\begin{aligned}
\mathcal{R}R_{\frac{1}{2}}(\mathcal{G}_1) &= (1)^{\frac{1}{2}}(9n^2 - 33n + 30) + (6)^{\frac{1}{2}}(12n - 24) + (36)^{\frac{1}{2}}(6n - 18) \\
&\quad + (8)^{\frac{1}{2}}(6) + (48)^{\frac{1}{2}}(12) + (10)^{\frac{1}{2}}(18n^2 - 54n + 42) \\
&\quad + (60)^{\frac{1}{2}}(18n - 36) + (80)^{\frac{1}{2}}(12),
\end{aligned}$$

$$\Rightarrow \mathcal{R}R_{\frac{1}{2}}(\mathcal{G}_1) = 65.920998n^2 + 1.058284n - 75.386629.$$

for $\alpha = -\frac{1}{2}$,

$$\begin{aligned}
\mathcal{R}R_{-\frac{1}{2}}(\mathcal{G}_1) &= (1)^{-\frac{1}{2}}(9n^2 - 33n + 30) + (6)^{-\frac{1}{2}}(12n - 24) + (36)^{-\frac{1}{2}}(6n - 18) \\
&\quad + (8)^{-\frac{1}{2}}(6) + (48)^{-\frac{1}{2}}(12) + (10)^{-\frac{1}{2}}(18n^2 - 54n + 42) \\
&\quad + (60)^{-\frac{1}{2}}(18n - 36) + (80)^{-\frac{1}{2}}(12),
\end{aligned}$$

$$\Rightarrow \mathcal{R}R_{-\frac{1}{2}}(\mathcal{G}_1) = 14.6921n^2 - 41.85353n + 31.031039.$$

Theorem 2.1.2. *Let $\mathcal{G}_1$ be the first type of hex-derived network then*

$$\begin{aligned}
\mathcal{R}ABC(\mathcal{G}_1) &= 17.076299n^2 - 28.417343n + 8.03836. \\
\mathcal{R}GA(\mathcal{G}_1) &= 19.349272n^2 - 32.221141n + 12.068803.
\end{aligned}$$

Proof. Let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 3 and Table 1, we have

$$\begin{aligned}
\mathcal{R}ABC(\mathcal{G}_1) &= \sqrt{\frac{1+1-2}{1\times 1}}|E_{1,1}(\mathcal{G}_1)| + \sqrt{\frac{6+1-2}{6\times 1}}|E_{6,1}(\mathcal{G}_1)| \\
&\quad + \sqrt{\frac{6+6-2}{6\times 6}}|E_{6,6}(\mathcal{G}_1)| + \sqrt{\frac{8+1-2}{8\times 1}}|E_{8,1}(\mathcal{G}_1)| \\
&\quad + \sqrt{\frac{8+6-2}{8\times 6}}|E_{8,6}(\mathcal{G}_1)| + \sqrt{\frac{10+1-2}{10\times 1}}|E_{10,1}(\mathcal{G}_1)| \\
&\quad + \sqrt{\frac{10+6-2}{10\times 6}}|E_{10,6}(\mathcal{G}_1)| + \sqrt{\frac{10+8-2}{10\times 8}}|E_{10,8}(\mathcal{G}_1)|,
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}ABC(\mathcal{G}_1) &= \sqrt{\frac{1+1-2}{1\times 1}}(9n^2 - 33n + 30) + \sqrt{\frac{6+1-2}{6\times 1}}(12n - 24) \\
&\quad + \sqrt{\frac{6+6-2}{6\times 6}}(6n - 18) + \sqrt{\frac{8+1-2}{8\times 1}}(6) \\
&\quad + \sqrt{\frac{8+6-2}{8\times 6}}(12) + \sqrt{\frac{10+1-2}{10\times 1}}(18n^2 - 54n + 42)
\end{aligned}$$

$$+\sqrt{\frac{10+6-2}{10\times 6}}(18n-36)+\sqrt{\frac{10+8-2}{10\times 8}}(12),$$

$$\Rightarrow \mathcal{R}ABC(\mathcal{G}_1)=17.076299n^2-28.417343n+8.03836.$$

Now let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 4 and Table 1, we have

$$\begin{aligned}\mathcal{R}GA(\mathcal{G}_1) &= \frac{2\sqrt{1\times 1}}{1+1}|E_{1,1}(\mathcal{G}_1)|+\frac{2\sqrt{6\times 1}}{6+1}|E_{6,1}(\mathcal{G}_1)|\\ &+\frac{2\sqrt{6\times 6}}{6+6}|E_{6,6}(\mathcal{G}_1)|+\frac{2\sqrt{8\times 1}}{8+1}|E_{8,1}(\mathcal{G}_1)|\\ &+\frac{2\sqrt{8\times 6}}{8+6}|E_{8,6}(\mathcal{G}_1)|+\frac{2\sqrt{10\times 1}}{10+1}|E_{10,1}(\mathcal{G}_1)|\\ &+\frac{2\sqrt{10\times 6}}{10+6}|E_{10,6}(\mathcal{G}_1)|+\frac{2\sqrt{10\times 8}}{10+8}|E_{10,8}(\mathcal{G}_1)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}GA(\mathcal{G}_1) &= \frac{2\sqrt{1\times 1}}{1+1}(9n^2-33n+30)+\frac{2\sqrt{6\times 1}}{6+1}(12n-24)\\ &+\frac{2\sqrt{6\times 6}}{6+6}(6n-18)+\frac{2\sqrt{8\times 1}}{8+1}(6)\\ &+\frac{2\sqrt{8\times 6}}{8+6}(12)+\frac{2\sqrt{10\times 1}}{10+1}(18n^2-54n+42)\\ &+\frac{2\sqrt{10\times 6}}{10+6}(18n-36)+\frac{2\sqrt{10\times 8}}{10+8}(12),\end{aligned}$$

$$\Rightarrow \mathcal{R}GA(\mathcal{G}_1)=19.349272n^2-32.221141n+12.068803.$$

Theorem 2.1.3. *Let $\mathcal{G}_1$ be the first type of hex-derived network then*

$$\begin{aligned}\mathcal{R}M_1(\mathcal{G}_1) &= 216n^2-216n\\ \mathcal{R}HM(\mathcal{G}_1) &= 2214n^2-606n-1056\end{aligned}$$

Proof. Let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 5 and Table 1, we have

$$\begin{aligned}\mathcal{R}M_1(\mathcal{G}_1) &= (1+1)|E_{1,1}(\mathcal{G}_1)|+(6+1)|E_{6,1}(\mathcal{G}_1)|+(6+6)|E_{6,6}(\mathcal{G}_1)|\\ &+(8+1)|E_{8,1}(\mathcal{G}_1)|+(8+6)|E_{8,6}(\mathcal{G}_1)|+(10+1)|E_{10,1}(\mathcal{G}_1)|\\ &+(10+6)|E_{10,6}(\mathcal{G}_1)|+(10+8)|E_{10,8}(\mathcal{G}_1)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}M_1(\mathcal{G}_1) &= (1+1)(9n^2-33n+30)+(6+1)(12n-24)+(6+6)(6n-18)\\ &+(8+1)(6)+(8+6)(12)+(10+1)(18n^2-54n+42)\\ &+(10+6)(18n-36)+(10+8)(12),\end{aligned}$$

$$\Rightarrow \mathcal{R}M_1(\mathcal{G}_1) = 216n^2 - 216n.$$

Now let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 6 and Table 1, we have

$$\begin{aligned}\mathcal{R}HM(\mathcal{G}_1) &= (1+1)^2|E_{1,1}(\mathcal{G}_1)| + (6+1)^2|E_{6,1}(\mathcal{G}_1)| + (6+6)^2|E_{6,6}(\mathcal{G}_1)| \\ &+(8+1)^2|E_{8,1}(\mathcal{G}_1)| + (8+6)^2|E_{8,6}(\mathcal{G}_1)| + (10+1)^2|E_{10,1}(\mathcal{G}_1)| \\ &+(10+6)^2|E_{10,6}(\mathcal{G}_1)| + (10+8)^2|E_{10,8}(\mathcal{G}_1)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}HM(\mathcal{G}_1) &= (1+1)^2(9n^2-33n+30) + (6+1)^2(12n-24) + (6+6)^2(6n-18) \\ &+(8+1)^2(6) + (8+6)^2(12) + (10+1)^2(18n^2-54n+42) \\ &+(10+6)^2(18n-36) + (10+8)^2(12),\end{aligned}$$

$$\Rightarrow \mathcal{R}HM(\mathcal{G}_1) = 2214n^2 - 606n - 1056.$$

Theorem 2.1.4. *Let $\mathcal{G}_1$ be the first type of hex-derived network then*

$$\mathcal{R}F(\mathcal{G}_1) = 1836n^2 - 2196n + 780.$$

Proof. Let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 7 and Table 1, we have

$$\begin{aligned}\mathcal{R}F(\mathcal{G}_1) &= ((1)^2+(1)^2)|E_{1,1}(\mathcal{G}_1)| + ((6)^2+(1)^2)|E_{6,1}(\mathcal{G}_1)| \\ &+((6)^2+(6)^2)|E_{6,6}(\mathcal{G}_1)| + ((8)^2+(1)^2)|E_{8,1}(\mathcal{G}_1)| \\ &+((8)^2+(6)^2)|E_{8,6}(\mathcal{G}_1)| + ((10)^2+(1)^2)|E_{10,1}(\mathcal{G}_1)| \\ &+((10)^2+(6)^2)|E_{10,6}(\mathcal{G}_1)| + ((10)^2+(8)^2)|E_{10,8}(\mathcal{G}_1)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}F(\mathcal{G}_1) &= ((1)^2+(1)^2)(9n^2-33n+30) + ((6)^2+(1)^2)(12n-24) \\ &+((6)^2+(6)^2)(6n-18) + ((8)^2+(1)^2)(6) + ((8)^2+(6)^2)(12) \\ &+((10)^2+(1)^2)(18n^2-54n+42) + ((10)^2+(6)^2)(18n-36) \\ &+((10)^2+(8)^2)(12),\end{aligned}$$

$$\Rightarrow \mathcal{R}F(\mathcal{G}_1) = 1836n^2 - 2196n + 780.$$

Theorem 2.1.5. *Let $\mathcal{G}_1$ be the first type of hex-derived network then*

$$\begin{aligned}\mathcal{R}RZ_1(\mathcal{G}_1) &= \frac{189}{5}n^2 - \frac{523}{5}n + \frac{1511}{20} \\ \mathcal{R}RZ_2(\mathcal{G}_1) &= \frac{459}{22}n^2 + \frac{2325}{77}n - \frac{13070}{231} \\ \mathcal{R}RZ_3(\mathcal{G}_1) &= 1998n^2 + 14370n - 12888\end{aligned}$$

Proof. Let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 8 and Table 1, we have

$$\begin{aligned}\mathcal{R}RZ_1(\mathcal{G}_1) &= \left(\frac{1+1}{1\times 1}\right)|E_{1,1}(\mathcal{G}_1)| + \left(\frac{6+1}{6\times 1}\right)|E_{6,1}(\mathcal{G}_1)| + \left(\frac{6+6}{6\times 6}\right)|E_{6,6}(\mathcal{G}_1)| \\ &+\left(\frac{8+1}{8\times 1}\right)|E_{8,1}(\mathcal{G}_1)| + \left(\frac{8+6}{8\times 6}\right)|E_{8,6}(\mathcal{G}_1)| + \left(\frac{10+1}{10\times 1}\right)|E_{10,1}(\mathcal{G}_1)| \\ &+\left(\frac{10+6}{10\times 6}\right)|E_{10,6}(\mathcal{G}_1)| + \left(\frac{10+8}{10\times 8}\right)|E_{10,8}(\mathcal{G}_1)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}RZ_1(\mathcal{G}_1) &= \left(\frac{1+1}{1\times 1}\right)(9n^2-33n+30) + \left(\frac{6+1}{6\times 1}\right)(12n-24) + \left(\frac{6+6}{6\times 6}\right)(6n-18) \\ &+\left(\frac{8+1}{8\times 1}\right)(6) + \left(\frac{8+6}{8\times 6}\right)(12) + \left(\frac{10+1}{10\times 1}\right)(18n^2-54n+42) \\ &+\left(\frac{10+6}{10\times 6}\right)(18n-36) + \left(\frac{10+8}{10\times 8}\right)(12),\end{aligned}$$

$$\Rightarrow \mathcal{R}RZ_1(\mathcal{G}_1) = \frac{189}{5}n^2 - \frac{523}{5}n + \frac{1511}{20}.$$

Now let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 9 and Table 1, we have

$$\begin{aligned}\mathcal{R}RZ_2(\mathcal{G}_1) &= \left(\frac{1\times 1}{1+1}\right)|E_{1,1}(\mathcal{G}_1)| + \left(\frac{6\times 1}{6+1}\right)|E_{6,1}(\mathcal{G}_1)| + \left(\frac{6\times 6}{6+6}\right)|E_{6,6}(\mathcal{G}_1)| \\ &+\left(\frac{8\times 1}{8+1}\right)|E_{8,1}(\mathcal{G}_1)| + \left(\frac{8\times 6}{8+6}\right)|E_{8,6}(\mathcal{G}_1)| + \left(\frac{10\times 1}{10+1}\right)|E_{10,1}(\mathcal{G}_1)| \\ &+\left(\frac{10\times 6}{10+6}\right)|E_{10,6}(\mathcal{G}_1)| + \left(\frac{10\times 8}{10+8}\right)|E_{10,8}(\mathcal{G}_1)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}RZ_2(\mathcal{G}_1) &= \left(\frac{1\times 1}{1+1}\right)(9n^2-33n+30) + \left(\frac{6\times 1}{6+1}\right)(12n-24) + \left(\frac{6\times 6}{6+6}\right)(6n-18) \\ &+\left(\frac{8\times 1}{8+1}\right)(6) + \left(\frac{8\times 6}{8+6}\right)(12) + \left(\frac{10\times 1}{10+1}\right)(18n^2-54n+42) \\ &+\left(\frac{10\times 6}{10+6}\right)(18n-36) + \left(\frac{10\times 8}{10+8}\right)(12),\end{aligned}$$

$$\Rightarrow \mathcal{R}RZ_2(\mathcal{G}_1) = \frac{459}{22}n^2 + \frac{2325}{77}n - \frac{13070}{231}.$$

Again let $\mathcal{G}_1 \cong HDN1(n)$, then by using equation 10 and Table 1, we have

$$\begin{aligned}\mathcal{R}RZ_3(\mathcal{G}_1) &= (1+1)(1\times 1)|E_{1,1}(\mathcal{G}_1)| + (6+1)(6\times 1)|E_{6,1}(\mathcal{G}_1)| \\ &+(6+6)(6\times 6)|E_{6,6}(\mathcal{G}_1)| + (8+1)(8\times 1)|E_{8,1}(\mathcal{G}_1)| \\ &+(8+6)(8\times 6)|E_{8,6}(\mathcal{G}_1)| + (10+1)(10\times 1)|E_{10,1}(\mathcal{G}_1)|\end{aligned}$$

$$+(10+6)(10\times 6)|E_{10,6}(\mathcal{G}_1)| + (10+8)(10\times 8)|E_{10,8}(\mathcal{G}_1)|,$$

$$\begin{aligned}\mathcal{R}RZ_3(\mathcal{G}_1) &= (1+1)(1\times 1)(9n^2-33n+30) + (6+1)(6\times 1)(12n-24)\\ &+(6+6)(6\times 6)(6n-18) + (8+1)(8\times 1)(6)\\ &+(8+6)(8\times 6)(12) + (10+1)(10\times 1)(18n^2-54n+42)\\ &+(10+6)(10\times 6)(18n-36) + (10+8)(10\times 8)(12),\end{aligned}$$

$$\Rightarrow \mathcal{R}RZ_3(\mathcal{G}_1) = 1998n^2 + 14370n - 12888.$$

2.2. Results on Hex-derived network of Type 2

In this section, we will compute reverse randić, ABC, GA, Zagreb, redined Zagreb, hyper Zagreb and forgotten index for hex-derive network of type 2. The reversed edge partition of $HDN2(n)$ is written in Table 2.

$(\mathcal{R}_{\breve{u}}, \mathcal{R}_{\breve{v}})$	Number of Edges
$(1,1)$	$9n^2-33n+30$
$(6,1)$	$12n-24$
$(6,6)$	$6n-18$
$(7,1)$	$18n^2-60n+48$
$(7,6)$	$6n-12$
$(7,7)$	$9n^2-33n+30$
$(8,1)$	$6n$
$(8,6)$	$12n-12$
$(8,7)$	$12n-24$
$(8,8)$	18

Table 2: Edge Partition second type of hex-derived network

Theorem 2.2.1. *Let $\mathcal{G}_2$ be the second type of hex-derived network, then*

$$\mathcal{R}R_\alpha(HDN2(n)) = \begin{cases} 576n^2-234n-228, & \text{if } \alpha = 1,\\ \frac{576}{49}n^2 - \frac{5692}{147}n + \frac{50625}{1568}, & \text{if } \alpha=-1,\\ 119.623524n^2 - 128.557979n + 3.701427, & \text{if } \alpha = \frac{1}{2},\\ 17.089075n^2 - 48.110416n + 35.089224, & \text{if } \alpha = -\frac{1}{2}.\end{cases}$$

Proof. Let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 2 and Table 2, we have

$$\begin{aligned}\mathcal{R}R_\alpha(\mathcal{G}_2) &= (1)^\alpha|E_{1,1}(\mathcal{G}_2)| + (6)^\alpha|E_{6,1}(\mathcal{G}_2)| + (36)^\alpha|E_{6,6}(\mathcal{G}_2)| + (7)^\alpha|E_{7,1}(\mathcal{G}_2)|\\ &+(42)^\alpha|E_{7,6}(\mathcal{G}_2)| + (49)^\alpha|E_{7,7}(\mathcal{G}_2)| + (8)^\alpha|E_{8,1}(\mathcal{G}_2)|\\ &+(48)^\alpha|E_{8,6}(\mathcal{G}_2)| + (56)^\alpha|E_{8,7}(\mathcal{G}_2)| + (64)^\alpha|E_{8,8}(\mathcal{G}_2)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}R_\alpha(\mathcal{G}_2) &= (1)^\alpha(9n^2-33n+30)+(6)^\alpha(12n-24)+(36)^\alpha(6n-18)\\&+(7)^\alpha(18n^2-60n+48)+(42)^\alpha(6n-12)+(49)^\alpha(9n^2-33n+30)\\&+(8)^\alpha(6n)+(48)^\alpha(12n-12)+(56)^\alpha(12n-24)+(64)^\alpha(18),\end{aligned}$$

for $\alpha = 1$,

$$\begin{aligned}\mathcal{R}R_1(\mathcal{G}_2) &= (1)(9n^2-33n+30)+(6)(12n-24)+(36)(6n-18)\\&+(7)(18n^2-60n+48)+(42)(6n-12)+(49)(9n^2-33n+30)\\&+(8)(6n)+(48)(12n-12)+(56)(12n-24)+(64)(18),\end{aligned}$$

$$\Rightarrow \mathcal{R}R_1(\mathcal{G}_2) = 576n^2-234n-228.$$

for $\alpha = -1$,

$$\begin{aligned}\mathcal{R}R_{-1}(\mathcal{G}_2) &= (1)^{-1}(9n^2-33n+30)+(6)^{-1}(12n-24)+(36)^{-1}(6n-18)\\&+(7)^{-1}(18n^2-60n+48)+(42)^{-1}(6n-12)+(49)^{-1}(9n^2-33n+30)\\&+(8)^{-1}(6n)+(48)^{-1}(12n-12)+(56)^{-1}(12n-24)+(64)^{-1}(18),\end{aligned}$$

$$\Rightarrow \mathcal{R}R_{-1}(\mathcal{G}_2) = \frac{576}{49}n^2-\frac{5692}{147}n+\frac{50625}{1568}.$$

for $\alpha = \frac{1}{2}$,

$$\begin{aligned}\mathcal{R}R_{\frac{1}{2}}(\mathcal{G}_2) &= (1)^{\frac{1}{2}}(9n^2-33n+30)+(6)^{\frac{1}{2}}(12n-24)+(36)^{\frac{1}{2}}(6n-18)\\&+(7)^{\frac{1}{2}}(18n^2-60n+48)+(42)^{\frac{1}{2}}(6n-12)+(49)^{\frac{1}{2}}(9n^2-33n+30)\\&+(8)^{\frac{1}{2}}(6n)+(48)^{\frac{1}{2}}(12n-12)+(56)^{\frac{1}{2}}(12n-24)+(64)^{\frac{1}{2}}(18),\end{aligned}$$

$$\Rightarrow \mathcal{R}R_{\frac{1}{2}}(\mathcal{G}_2) = 119.623524n^2-128.557979n+3.701427.$$

for $\alpha = -\frac{1}{2}$,

$$\begin{aligned}\mathcal{R}R_{-\frac{1}{2}}(\mathcal{G}_2) &= (1)^{-\frac{1}{2}}(9n^2-33n+30)+(6)^{-\frac{1}{2}}(12n-24)+(36)^{-\frac{1}{2}}(6n-18)\\&+(7)^{-\frac{1}{2}}(18n^2-60n+48)+(42)^{-\frac{1}{2}}(6n-12)+(49)^{-\frac{1}{2}}(9n^2-33n+30)\\&+(8)^{-\frac{1}{2}}(6n)+(48)^{-\frac{1}{2}}(12n-12)+(56)^{-\frac{1}{2}}(12n-24)+(64)^{-\frac{1}{2}}(18),\end{aligned}$$

$$\Rightarrow \mathcal{R}R_{-\frac{1}{2}}(\mathcal{G}_2) = 17.089075n^2-48.110416n+35.089224.$$

Theorem 2.2.2. *Let $\mathcal{G}_2$ be the second type of hex-derived network then*

$$\begin{aligned}\mathcal{R}ABC(\mathcal{G}_2) &= 21.118607n^2-37.298413n+12.603823.\\ \mathcal{R}GA(\mathcal{G}_2) &= 29.905881n^2-57.684337n+27.164543.\end{aligned}$$

Proof. Let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 3 and Table 2, we have

$$\begin{aligned}
\mathcal{R}ABC(\mathcal{G}_2) &= \sqrt{\frac{1+1-2}{1\times 1}}|E_{1,1}(\mathcal{G}_2)| + \sqrt{\frac{6+1-2}{6\times 1}}|E_{6,1}(\mathcal{G}_2)| \\
&+\sqrt{\frac{6+6-2}{6\times 6}}|E_{6,6}(\mathcal{G}_2)| + \sqrt{\frac{7+1-2}{7\times 1}}|E_{7,1}(\mathcal{G}_2)| \\
&+\sqrt{\frac{7+6-2}{7\times 6}}|E_{7,6}(\mathcal{G}_2)| + \sqrt{\frac{7+7-2}{7\times 7}}|E_{7,7}(\mathcal{G}_2)| \\
&+\sqrt{\frac{8+1-2}{8\times 1}}|E_{8,1}(\mathcal{G}_2)| + \sqrt{\frac{8+6-2}{8\times 6}}|E_{8,6}(\mathcal{G}_2)| \\
&+\sqrt{\frac{8+7-2}{8\times 7}}|E_{8,7}(\mathcal{G}_2)| + \sqrt{\frac{8+8-2}{8\times 8}}|E_{8,8}(\mathcal{G}_2)|,
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}ABC(\mathcal{G}_2) &= \sqrt{\frac{1+1-2}{1\times 1}}(9n^2-33n+30) + \sqrt{\frac{6+1-2}{6\times 1}}(12n-24) \\
&+\sqrt{\frac{6+6-2}{6\times 6}}(6n-18) + \sqrt{\frac{7+1-2}{7\times 1}}(18n^2-60n+48) \\
&+\sqrt{\frac{7+6-2}{7\times 6}}(6n-12) + \sqrt{\frac{7+7-2}{7\times 7}}(9n^2-33n+30) \\
&+\sqrt{\frac{8+1-2}{8\times 1}}(6n) + \sqrt{\frac{8+6-2}{8\times 6}}(12n-12) \\
&+\sqrt{\frac{8+7-2}{8\times 7}}(12n-24) + \sqrt{\frac{8+8-2}{8\times 8}}(18),
\end{aligned}$$

$$\Rightarrow \mathcal{R}ABC(\mathcal{G}_2) = 21.118607n^2 - 37.298413n + 12.603823.$$

Now, let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 3 and Table 2, we have

$$\begin{aligned}
\mathcal{R}GA(\mathcal{G}_2) &= \frac{2\sqrt{1\times 1}}{1+1}|E_{1,1}(\mathcal{G}_2)| + \frac{2\sqrt{6\times 1}}{6+1}|E_{6,1}(\mathcal{G}_2)| \\
&+\frac{2\sqrt{6\times 6}}{6+6}|E_{6,6}(\mathcal{G}_2)| + \frac{2\sqrt{7\times 1}}{7+1}|E_{7,1}(\mathcal{G}_2)| \\
&+\frac{2\sqrt{7\times 6}}{7+6}|E_{7,6}(\mathcal{G}_2)| + \frac{2\sqrt{7\times 7}}{7+7}|E_{7,7}(\mathcal{G}_2)| \\
&+\frac{2\sqrt{8\times 1}}{8+1}|E_{8,1}(\mathcal{G}_2)| + \frac{2\sqrt{8\times 6}}{8+6}|E_{8,6}(\mathcal{G}_2)| \\
&+\frac{2\sqrt{8\times 7}}{8+7}|E_{8,7}(\mathcal{G}_2)| + \frac{2\sqrt{8\times 8}}{8+8}|E_{8,8}(\mathcal{G}_2)|,
\end{aligned}$$

$$\mathcal{R}GA(\mathcal{G}_2) = \frac{2\sqrt{1\times 1}}{1+1}(9n^2-33n+30) + \frac{2\sqrt{6\times 1}}{6+1}(12n-24)$$

$$+\frac{2\sqrt{6\times 6}}{6+6}(6n-18)+\frac{2\sqrt{7\times 1}}{7+1}(18n^2-60n+48)$$
$$+\frac{2\sqrt{7\times 6}}{7+6}(6n-12)+\frac{2\sqrt{7\times 7}}{7+7}(9n^2-33n+30)$$
$$+\frac{2\sqrt{8\times 1}}{8+1}(6n)+\frac{2\sqrt{8\times 6}}{8+6}(12n-12)$$
$$+\frac{2\sqrt{8\times 7}}{8+7}(12n-24)+\frac{2\sqrt{8\times 8}}{8+8}(18),$$
$$\Rightarrow \mathcal{R}GA(\mathcal{G}_2)=29.905881n^2-57.684337n+27.164543.$$

Theorem 2.2.3. *Let $\mathcal{G}_2$ be the second type of hex-derived network then*

$$\begin{aligned}\mathcal{R}M_1(\mathcal{G}_2) &= 288n^2-372n+84\\ \mathcal{R}HM(\mathcal{G}_2) &= 2952n^2-2436n+132\end{aligned}$$

Proof. Let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 5 and Table 2, we have

$$\begin{aligned}\mathcal{R}M_1(\mathcal{G}_2) = {} & (1+1)|E_{1,1}(\mathcal{G}_2)|+(6+1)|E_{6,1}(\mathcal{G}_2)|+(6+6)|E_{6,6}(\mathcal{G}_2)|\\ & +(7+1)|E_{7,1}(\mathcal{G}_2)|+(7+6)|E_{7,6}(\mathcal{G}_2)|+(7+7)|E_{7,7}(\mathcal{G}_2)|\\ & +(8+1)|E_{8,1}(\mathcal{G}_2)|+(8+6)|E_{8,6}(\mathcal{G}_2)|+(8+7)|E_{8,7}(\mathcal{G}_2)|\\ & +(8+8)|E_{8,8}(\mathcal{G}_2)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}M_1(\mathcal{G}_2) = {} & (1+1)(9n^2-33n+30)+(6+1)(12n-24)+(6+6)(6n-18)\\ & +(7+1)(18n^2-60n+48)+(7+6)(6n-12)\\ & +(7+7)(9n^2-33n+30)+(8+1)(6n)+(8+6)(12n-12)\\ & +(8+7)(12n-24)+(8+8)(18),\end{aligned}$$
$$\Rightarrow \mathcal{R}M_1(\mathcal{G}_2)=288n^2-372n+84.$$

Now let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 6 and Table 2, we have

$$\begin{aligned}\mathcal{R}HM(\mathcal{G}_2) = {} & (1+1)^2|E_{1,1}(\mathcal{G}_2)|+(6+1)^2|E_{6,1}(\mathcal{G}_2)|+(6+6)^2|E_{6,6}(\mathcal{G}_2)|\\ & +(7+1)^2|E_{7,1}(\mathcal{G}_2)|+(7+6)^2|E_{7,6}(\mathcal{G}_2)|+(7+7)^2|E_{7,7}(\mathcal{G}_2)|\\ & +(8+1)^2|E_{8,1}(\mathcal{G}_2)|+(8+6)^2|E_{8,6}(\mathcal{G}_2)|+(8+7)^2|E_{8,7}(\mathcal{G}_2)|\\ & +(8+8)^2|E_{8,8}(\mathcal{G}_2)|,\end{aligned}$$

$$\begin{aligned}\mathcal{R}HM(\mathcal{G}_2) = {} & (1+1)^2(9n^2-33n+30)+(6+1)^2(12n-24)+(6+6)^2(6n-18)\\ & +(7+1)^2(18n^2-60n+48)+(7+6)^2(6n-12)\\ & +(7+7)^2(9n^2-33n+30)+(8+1)^2(6n)+(8+6)^2(12n-12)\\ & +(8+7)^2(12n-24)+(8+8)^2(18),\end{aligned}$$
$$\Rightarrow \mathcal{R}HM(\mathcal{G}_2)=2952n^2-2436n+132.$$

Theorem 2.2.4. *Let $\mathcal{G}_2$ be the second type of hex-derived network then*

$$\mathcal{R}F(\mathcal{G}_2) = 1800n^2 - 1968n + 588.$$

Proof. Let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 7 and Table 2, we have

$$\begin{aligned}
\mathcal{R}F(\mathcal{G}_2) &= ((1)^2+(1)^2)|E_{1,1}(\mathcal{G}_2)| + ((6)^2+(1)^2)|E_{6,1}(\mathcal{G}_2)| \\
&\quad +((6)^2+(6)^2)|E_{6,6}(\mathcal{G}_2)| + ((7)^2+(1)^2)|E_{7,1}(\mathcal{G}_2)| \\
&\quad +((7)^2+(6)^2)|E_{7,6}(\mathcal{G}_2)| + ((7)^2+(7)^2)|E_{7,7}(\mathcal{G}_2)| \\
&\quad +((8)^2+(1)^2)|E_{8,1}(\mathcal{G}_2)| + ((8)^2+(6)^2)|E_{8,6}(\mathcal{G}_2)| \\
&\quad +((8)^2+(7)^2)|E_{8,7}(\mathcal{G}_2)| + ((8)^2+(8)^2)|E_{8,8}(\mathcal{G}_2)|,
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}F(\mathcal{G}_2) &= ((1)^2+(1)^2)(9n^2-33n+30) + ((6)^2+(1)^2)(12n-24) \\
&\quad +((6)^2+(6)^2)(6n-18) + ((7)^2+(1)^2)(18n^2-60n+48) \\
&\quad +((7)^2+(6)^2)(6n-12) + ((7)^2+(7)^2)(9n^2-33n+30) \\
&\quad +((8)^2+(1)^2)(6n) + ((8)^2+(6)^2)(12n-12) \\
&\quad +((8)^2+(7)^2)(12n-24) + ((8)^2+(8)^2)(18),
\end{aligned}$$

$$\Rightarrow \mathcal{R}F(\mathcal{G}_2) = 1800n^2 - 1968n + 588.$$

Theorem 2.2.5. *Let $\mathcal{G}_2$ be the second type of hex-derived network then*

$$\begin{aligned}
\mathcal{R}RZ_1(\mathcal{G}_2) &= \frac{288}{7}n^2 - \frac{3155}{28}n + \frac{562}{7} \\
\mathcal{R}RZ_2(\mathcal{G}_2) &= \frac{207}{4}n^2 - \frac{124361}{2730}n - \frac{4588}{455} \\
\mathcal{R}RZ_3(\mathcal{G}_2) &= 7200n^2 - 1116n - 1800
\end{aligned}$$

Proof. Let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 8 and Table 2, we have

$$\begin{aligned}
\mathcal{R}RZ_1(\mathcal{G}_2) &= \left(\frac{1+1}{1\times 1}\right)|E_{1,1}(\mathcal{G}_2)| + \left(\frac{6+1}{6\times 1}\right)|E_{6,1}(\mathcal{G}_2)| + \left(\frac{6+6}{6\times 6}\right)|E_{6,6}(\mathcal{G}_2)| \\
&\quad + \left(\frac{7+1}{7\times 1}\right)|E_{7,1}(\mathcal{G}_2)| + \left(\frac{7+6}{7\times 6}\right)|E_{7,6}(\mathcal{G}_2)| + \left(\frac{7+7}{7\times 7}\right)|E_{7,7}(\mathcal{G}_2)| \\
&\quad + \left(\frac{8+1}{8\times 1}\right)|E_{8,1}(\mathcal{G}_2)| + \left(\frac{8+6}{8\times 6}\right)|E_{8,6}(\mathcal{G}_2)| + \left(\frac{8+7}{8\times 7}\right)|E_{8,7}(\mathcal{G}_2)| \\
&\quad + \left(\frac{8+8}{8\times 8}\right)|E_{8,8}(\mathcal{G}_2)|,
\end{aligned}$$

$$\mathcal{R}RZ_1(\mathcal{G}_2) = \left(\frac{1+1}{1\times 1}\right)(9n^2-33n+30) + \left(\frac{6+1}{6\times 1}\right)(12n-24)$$

$$
\begin{aligned}
&+\left(\frac{6+6}{6\times 6}\right)(6n-18)+\left(\frac{7+1}{7\times 1}\right)(18n^2-60n+48)\\
&+\left(\frac{7+6}{7\times 6}\right)(6n-12)+\left(\frac{7+7}{7\times 7}\right)(9n^2-33n+30)\\
&+\left(\frac{8+1}{8\times 1}\right)(6n)+\left(\frac{8+6}{8\times 6}\right)(12n-12)\\
&+\left(\frac{8+7}{8\times 7}\right)(12n-24)+\left(\frac{8+8}{8\times 8}\right)(18),
\end{aligned}
$$

$$\Rightarrow \mathcal{R}RZ_1(\mathcal{G}_2)=\frac{288}{7}n^2-\frac{3155}{28}n+\frac{562}{7}.$$

Now let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 9 and Table 2, we have

$$
\begin{aligned}
\mathcal{R}RZ_2(\mathcal{G}_2) \quad = \quad & \left(\frac{1\times 1}{1+1}\right)|E_{1,1}(\mathcal{G}_2)|+\left(\frac{6\times 1}{6+1}\right)|E_{6,1}(\mathcal{G}_2)|+\left(\frac{6\times 6}{6+6}\right)|E_{6,6}(\mathcal{G}_2)|\\
&+\left(\frac{7\times 1}{7+1}\right)|E_{7,1}(\mathcal{G}_2)|+\left(\frac{7\times 6}{7+6}\right)|E_{7,6}(\mathcal{G}_2)|+\left(\frac{7\times 7}{7+7}\right)|E_{7,7}(\mathcal{G}_2)|\\
&+\left(\frac{8\times 1}{8+1}\right)|E_{8,1}(\mathcal{G}_2)|+\left(\frac{8\times 6}{8+6}\right)|E_{8,6}(\mathcal{G}_2)|+\left(\frac{8\times 7}{8+7}\right)|E_{8,7}(\mathcal{G}_2)|\\
&+\left(\frac{8\times 8}{8+8}\right)|E_{8,8}(\mathcal{G}_2)|,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}RZ_2(\mathcal{G}_2) \quad = \quad & \left(\frac{1\times 1}{1+1}\right)(9n^2-33n+30)+\left(\frac{6\times 1}{6+1}\right)(12n-24)\\
&+\left(\frac{6\times 6}{6+6}\right)(6n-18)+\left(\frac{7\times 1}{7+1}\right)(18n^2-60n+48)\\
&+\left(\frac{7\times 6}{7+6}\right)(6n-12)+\left(\frac{7\times 7}{7+7}\right)(9n^2-33n+30)\\
&+\left(\frac{8\times 1}{8+1}\right)(6n)+\left(\frac{8\times 6}{8+6}\right)(12n-12)\\
&+\left(\frac{8\times 7}{8+7}\right)(12n-24)+\left(\frac{8\times 8}{8+8}\right)(18),
\end{aligned}
$$

$$\Rightarrow \mathcal{R}RZ_2(\mathcal{G}_2)=\frac{207}{4}n^2-\frac{124361}{2730}n-\frac{4588}{455}.$$

Again let $\mathcal{G}_2 \cong HDN2(n)$, then by using equation 10 and Table 2, we have

$$
\begin{aligned}
\mathcal{R}RZ_3(\mathcal{G}_2) \quad = \quad & (1+1)(1\times 1)|E_{1,1}(\mathcal{G}_2)|+(6+1)(6\times 1)|E_{6,1}(\mathcal{G}_2)|\\
&+(6+6)(6\times 6)|E_{6,6}(\mathcal{G}_2)|+(7+1)(7\times 1)|E_{7,1}(\mathcal{G}_2)|\\
&+(7+6)(7\times 6)|E_{7,6}(\mathcal{G}_2)|+(7+7)(7\times 7)|E_{7,7}(\mathcal{G}_2)|\\
&+(8+1)(8\times 1)|E_{8,1}(\mathcal{G}_2)|+(8+6)(8\times 6)|E_{8,6}(\mathcal{G}_2)|
\end{aligned}
$$

$$+(8+7)(8\times 7)|E_{8,7}(\mathcal{G}_2)|+(8+8)(8\times 8)|E_{8,8}(\mathcal{G}_2)|,$$

$$\begin{aligned}\mathcal{R}RZ_3(\mathcal{G}_2) &= (1+1)(1\times 1)(9n^2-33n+30)+(6+1)(6\times 1)(12n-24)\\ &+(6+6)(6\times 6)(6n-18)+(7+1)(7\times 1)(18n^2-60n+48)\\ &+(7+6)(7\times 6)(6n-12)+(7+7)(7\times 7)(9n^2-33n+30)\\ &+(8+1)(8\times 1)(6n)+(8+6)(8\times 6)(12n-12)\\ &+(8+7)(8\times 7)(12n-24)+(8+8)(8\times 8)(18),\end{aligned}$$

$$\Rightarrow \mathcal{R}RZ_3(\mathcal{G}_2)=7200n^2-1116n-1800.$$

n	$\mathcal{R}R_1$	$\mathcal{R}R_{-1}$	$\mathcal{R}R_{\frac{1}{2}}$	$\mathcal{R}R_{-\frac{1}{2}}$	$\mathcal{R}ABC$	$\mathcal{R}GA$
1	66	—	−8.41	3.87	−3.30	−0.80
2	1428	1.58	190.41	6.09	19.51	25.02
3	3168	19.65	521.07	37.69	76.47	89.55
4	5286	59.32	983.58	98.69	167.59	192.77
5	7782	120.58	1577.93	189.06	292.86	334.69
6	10656	203.45	2304.12	308.83	452.28	515.32
7	13908	307.92	3162.15	457.97	645.85	734.64
8	17538	433.98	4152.02	636.49	873.58	992.65
9	21546	581.65	5273.74	844.41	1135.46	1289.37
10	25932	750.92	6527.29	1081.71	1431.49	1624.78

Table 3: Comparison Table for $HDN1(n)$

n	$\mathcal{R}M_1$	$\mathcal{R}HM$	$\mathcal{R}F$	$\mathcal{R}RZ_1$	$\mathcal{R}RZ_2$	$\mathcal{R}RZ_3$
1	0	552	420	8.75	−5.52	3480
2	432	6588	3732	17.55	87.26	23844
3	1296	17052	10716	101.95	221.77	48204
4	2592	31944	21372	261.95	398.02	76560
5	4320	51264	35700	497.55	615.98	108912
6	6480	75012	53700	808.75	875.68	145260
7	9072	303188	75372	1195.55	1177.10	185604
8	12096	135792	100716	1657.95	1520.25	229944
9	15552	172824	129732	2195.95	1905.13	278280
10	19440	214284	162420	2809.55	2331.73	330612

Table 4: Comparison Table for $HDN1(n)$

3. Discussion

While this study demonstrates the potential of reversed degree-based topological indices in understanding molecular structures and predicting chemical properties, limitations

n	$\mathcal{R}R_1$	$\mathcal{R}R_{-1}$	$\mathcal{R}R_{\frac{1}{2}}$	$\mathcal{R}R_{-\frac{1}{2}}$	$\mathcal{R}ABC$	$\mathcal{R}GA$
1	114	—	−5.23	4.07	−3.57	−0.61
2	1608	1.86	225.08	7.22	22.48	31.42
3	4254	21.91	694.64	44.56	90.77	123.26
4	8052	65.48	1403.45	116.07	201.31	274.92
5	13002	132.56	2351.50	221.76	354.07	486.39
6	19104	223.14	3538.80	361.63	549.08	757.67
7	26358	337.24	4965.35	535.68	786.33	1088.76
8	34764	474.84	6631.14	743.96	1065.81	1479.66
9	44322	635.96	8536.18	986.31	1387.53	1930.38
10	55032	820.58	10680.47	1262.89	1751.48	2440.91

Table 5: Comparison Table for $HDN2(n)$

n	$\mathcal{R}M_1$	$\mathcal{R}HM$	$\mathcal{R}F$	$\mathcal{R}RZ_1$	$\mathcal{R}RZ_2$	$\mathcal{R}RZ_3$
1	0	648	420	8.75	−3.88	4284
2	492	7068	3852	19.5	105.81	24768
3	1560	19392	10884	112.54	319.01	59652
4	3204	37620	21516	287.86	635.70	108936
5	5424	61752	35748	545.46	1055.89	172620
6	8220	91788	53580	885.36	1579.59	250704
7	11592	127728	75012	1307.54	2206.79	343188
8	15540	169572	100044	1812	2937.48	450072
9	20064	217320	128676	2398.75	3771.68	571356
10	25164	270972	160908	3067.78	4709.38	707040

Table 6: Comparison Table for $HDN2(n)$

include a focus on hex-derived networks, computational method limitations for large or complex structures, and the need for experimental validation. Future directions include applying topological indices to pharmacokinetics and pharmacodynamics, developing new indices incorporating electronic or steric properties, using machine learning to improve predictive power, and designing novel materials with specific properties, such as high-temperature superconductors or nanomaterials, to further unlock the potential of topological indices in chemical graph theory and drive innovation in molecular design and property prediction.

4. Conclusion

This study successfully computed reversed degree-based topological indices for hex-derived networks of type 1 and 2, contributing to the advancement of quantitative structure-property relationships (QSPRs) and quantitative structure-activity relationships (QSARs). The findings of this research have significant implications for understanding the physical, biomedical, and molecular properties of chemicals, as well as their biological activities.

Given the diverse applications of hex-derived networks in fields such as networking, pharmacy, electronics, and data analysis, this study's results can be applied to optimize molecular structures and predict desired properties, ultimately driving innovation and discovery in various scientific domains. We obtain some closed formulas for reversed Randić, ABC, GA, first Zagreb, hyper Zagreb, forgotten, first, second, third redefined Zagreb index of Hex-derived network. The numerical behavior of these indices are shown in Table 3, 4, 5, 6, for $HDN1(n)$ and $HDN2(n)$. It is clear that the values of these indices are directly proportional to the value of n, as n increases the values of these indices also increases, which is beneficial for the researchers. These results are helpful in the field of computer science and chemistry.

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