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Site Selection for Thermal Power Plant Based on Sombor Index in Neutrosophic Graphs

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Abstract. A graph or molecular graph's overall resilience and connectedness are gauged by the Sombor Index ($\mathcal{SOI}$). It is an improved version of the conventional $\mathcal{SOI}$ that is designed to capture ambiguity in the relationships between nodes or atoms. This indicator aids in assessing the network's resilience and stability in erratic circumstances. We have investigated the characteristics of the $\mathcal{SOI}$ of neutrosophic graphs ($\mathcal{NG}$s) in this study. The association between the neutrosophic first Zagreb Index ($\mathcal{NFZI}$) of $\mathcal{NG}$s and the neutrosophic Sombor Index ($\mathcal{NSI}$) was revealed in the study. The application of $\mathcal{SOI}$ based on Site Selection for Thermal Power Plants in $\mathcal{NG}$s is finally covered.

2020 Mathematics Subject Classifications: 05C72, 05C09, 03B52

Key Words and Phrases: Fuzzy logic, Neutrosophic logic, $\mathcal{NSOI}$, $\mathcal{NFZI}$, $\mathcal{NSZI}$

1. Introduction

1.1. Fuzzy Graphs($\mathcal{FG}$s)

Real-world problems are rarely strict and often involve fuzziness and roughness. With the use of FG theory, real-world problems may be efficiently and understandably mathematically explained. Zadeh used this collection's membership function in the work that was given in [59]. This function assigns a membership value($\mathcal{MV}$), from zero to one, to each member. Expanding the traditional knowledge of set theory was his goal. Human views, judgment and assessment, according to Zadeh and Goguen, reduce fuzziness. Fuzzy sets ($\mathcal{FS}$s) are useful for solving situations where the fault is caused by random variables rather than class membership. Scientists can study ambiguous conceptual issues with the use of this mathematical technique. The 1960s and 1970s saw

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a modest increase in the acceptance of this notion. Researchers were interested in this area when fuzzy systems built around rules were used to control technological processes in the late 1970s. Its effective application in washing machines, video cameras, subway trains and other places inspired many to carry out study in this field. Prior to 2000, there were more than 30,000 publications on $\mathcal{FS}$ theory. It has expanded over the last 10 years to encompass both an application-focused theory and an actual concept. The writers of [62] concluded after analyzing $\mathcal{FS}$ theory that it may be used to bridge the gap between formal and natural models and explain deterministic uncertainty. According to the author of this page, a $\mathcal{FS}$ is defined as follows: $M = \{(\gamma, \lambda_M(\gamma)), \gamma \in M\}$ is the collection of ordered pairs and is denoted as such if M is the collection of objects represented by γ. In this case, $\lambda_M(\gamma)$ denotes the $\mathcal{MV}$, and $0 \leq \lambda_M(\gamma) \leq 1$. A variable can be connected to both the possibility distribution and the probability distribution, according to research published in 1978 by authors of [60]. In [19], the relationship between the finite valued $\mathcal{FS}$, the Zadeh $\mathcal{FS}$ and the n-dimensional $\mathcal{FS}$ is clarified. The approaches and methods of $\mathcal{FG}$ theory were presented by the authors of [53] for the analysis of multi-species fishing dynamics. The patterns of data can be obtained by using the above work. Fuzzy refers to not being able to hear or see clearly. $\mathcal{FG}$s are useful in modeling most of the difficulties that arise in our daily lives discussed in [46] along with an introduction to some of their characteristics. Few rooted trees that encapsulate a particular $\mathcal{FG}$, an algorithm and complexity analysis are presented [11]. In [12] and [14] discusses $\mathcal{FG}$s that account for the fuzziness of vertex and edge existence, edge weight and connectedness. The hyper-wiener Index's associated with various of graph has been given in [29]. The amazing notion of fuzzy cognitive map structure is developed by researcher in [44] through establishing a concept of output issues and lowering the amount of concepts and connections between them.

1.2. Intuitionistic Fuzzy Graphs ($\mathcal{IFG}$s)

The traditional $\mathcal{FS}$ theory and graphs are extended into $\mathcal{IFS}$s and $\mathcal{IFG}$s. In order to handle circumstances like ambiguity and hesitation as well as the need for a more accommodating model of the degree of $\mathcal{MV}$, non-membership value ($\mathcal{NMV}$) and reluctance, Atanassov originally proposed them in the 1980s. Developing an uncertain model: $\mathcal{IFS}$s allow for a more complex depiction of uncertainty by incorporating the concept of resistance. To better reflect confusing or poorly understood information, $\mathcal{IFS}$ adds $\mathcal{NMV}$ and objects membership to $\mathcal{MV}$s [37]. Making decisions in ambiguous circumstances: $\mathcal{IFS}$s provide a framework for decision-making in ambiguous circumstances [[54], [55]]. Making decisions based on both $\mathcal{MV}$ and $\mathcal{NMV}$ helps decision-makers assess the level of resistance associated with various choices and reach more precise conclusions. Handling ambiguous and precise data: Graphs and $\mathcal{IFS}$s can be useful in modelling ambiguous or imprecise data. By addressing both the degree of $\mathcal{MV}$ and the degree of ambiguity or uncertainty associated with the details, they increase the flexibility of information characterisation [[32], [36]].

Formal methodologies and procedures for the research and implementation of $\mathcal{IFS}$s and $\mathcal{IFG}$s are made possible by the robust mathematical underpinnings of the intuitionistic fuzzy objects. For use with $\mathcal{IFS}$s and graphs, a wide range of operations, aggregation techniques and algorithms have been created. Applications encompass networking, control systems, image recognition, clustering, and decision-making. Fuzzy sets and graphs with intuitive properties are employed in these and other domains. It has been demonstrated that they increase the precision and efficacy of decision-making processes and offer a helpful toolkit for handling ambiguous or incomplete data in a variety of sectors. Thus, by adding the concept of hesitation or indeterminacy, $\mathcal{IFS}$s and $\mathcal{IFG}$s enhance conventional $\mathcal{FS}$ theory and graphs. For managing ambiguity and incomplete information, they offer a more sophisticated framework that facilitates more precise analysis and taking decisions in a variety of domains [8]. The phrase strong $\mathcal{IFG}$ graph is used by the authors in [3], who also discuss various postulations about line graphs and self-complimentary. The scientists in [42] looked at $\mathcal{IFG}$ elements and used these concepts to look at other kinds of $\mathcal{IFG}$ elements.

In [41] discusses an enhanced technique for identifying dominant vertex set $\mathcal{IFG}$s. $\mathcal{IFG}$ theory is used to explain and evaluate the connectivity of uncertain networks; in addition, [10] looks at the vertex connectivity inside an $\mathcal{IFG}$. In [49], various product operations are defined on $\mathcal{IFG}$ and some key concepts are illustrated on these graphs. In [45], the fuzzy graph energy idea is expanded to include $\mathcal{IFG}$. The clustering of fuzzy and $\mathcal{IFG}$ vertices is the topic of the essay [34]. The same page also introduces a few $\mathcal{IFG}$-related parameters. One can analyze [48] to get a sense of how connected $\mathcal{IFG}$ is. One can review [16] in order to analyze Index concurrently with connection Index. An intuitionistic fuzzy model has been used to analyze and make decisions for several individuals based on multiple factors. The two main pieces of information employed in [7] are the expert dependability ratings and the assessments of their methods.

In [30], scientists computed the third and fourth iterations of the $\mathcal{SOI}$ for various graph families inside an $\mathcal{IFG}$ setting. They then provided an application that makes use of these indices to enhance the efficiency of immunization facilities. The kinds of intuitionistic fuzzy rough graphs are specified in [61]. Nodes and links are the representation of physical networks, such as those found in circuits in electronics, biological intricate systems, digital networks and social networks. In these networks, things are represented by nodes and the relationships between them are shown by links. Cities may be seen as nodes in the transportation system, for instance and the routes connecting them as connections. Harry Wiener first used topological indices in 1947 when he examined how pure structural change affected paraffin's boiling temperature in [58]. The paraffin boiling temperatures are found using the linear formula $s_\gamma = p\alpha + q\beta + r$, where α is the sum of the distances between any two carbon atoms in a molecule. This was how the Wiener Index was first presented. Numerous requirements are met in [17] in terms of various graph features as measured by the Wiener and Harary Indices. The $\mathcal{SOI}$, a novel graph constant, is defined in [25]. The relations between the $\mathcal{SOI}$ and topological indices is discussed in the paper [18]. [13] Discusses the features of Sombor Indices, examines the extreme values of many graphical networks and suggests possible uses. The $3^{rd}, 4^{th}, 5^{th}$ and 6^{th} iterations of $\mathcal{SOI}$ were established by Ivan Gutman in [26].

In [27], the geometric arithmetic Index and the atomic bond connectivity Index for various graph networks are calculated. Using indices of topology is one method of determining the correlation between a chemical compound's structure and its characteristics [57]. A number of features for various graph families and applications pertaining to these indices are described in [[24]-[9]], which also define Sombor Index and topological indices of degree-two.

A $\mathcal{FG}$which makes it simple to define the fuzzy relationship between any item, is one of the most versatile mathematical tools available. In [33], a number of fundamental topological metrics are established in the context of $\mathcal{FG}$s, this study covers limits of indices and its applications. In reference [1], the fuzzy Randic Index and fuzzy harmonic Index are introduced, their maximum values are derived and an application pertaining to cybercrime showcased. The novel fuzzy Wiener Index and connection Index for bibolar fuzzy incidence graphs are defined and their interrelationship s are examined in research article [23]. In the fuzzy structure, a few topological indices are specified and in [38], the corresponding properties were discussed for pizza-graph. Together with a cybercrime application shown. The intersection, union and extremals of bibolar $\mathcal{FG}$ indices are computed, established and analyzed in [47]. The distance between the $\mathcal{IFS}$s is a well-known regularly utilized information metric to enhance decision-making performance. Conversely, using different distance metrics produces different numerical results. As a result, it is worthwhile to thoroughly research the procedure for selecting an appropriate formula for calculating distance. Two types of connection indices are defined by the $\mathcal{IFG}$ design and [40] shows examples of their use on the transport network and internet routing system.

In reference [43], the complements of an $\mathcal{IFG}$ and a self-complimentary $\mathcal{IFG}$ are delineated, their characteristics are examined and some functions are also implemented for these graphs. In [20], a number of degree, order and size attributes are presented; the identical article also defines full and regular $\mathcal{IFG}$s. It is confirmed that operations on a strong $\mathcal{IFG}$ produce another strong $\mathcal{IFG}$s in [51], where three products are specified. Illustrations are used to define and describe a few products and references [[52], [2]] provide calculations for the degree of vertices of $\mathcal{IFG}$s that resulted from this process. [[15], [56]] provide the notions of constant $\mathcal{IFG}$, the 2^{nd} type of $\mathcal{IFG}$ and generalized $\mathcal{IFG}$. In [21], a few different kinds of irregular graphs are defined and some findings on completely irregular $\mathcal{IFG}$s are also covered.

1.3. Neutrosophic Graphs($\mathcal{NG}s$)

Regarding the neutrosophic sets ($\mathcal{NS}s$), Smarandache gave them the go-ahead. The truth membership value ($\mathcal{TMV}$), falsity membership value ($\mathcal{FMV}$) and indeterminacy membership value ($\mathcal{IMV}$) are all included. By this work, some of the features of the strong $\mathcal{NG}$ proposed in [39] have been examined and an example is shown. Establishments are made about definitions, propositions, homomorphism and isomorphism theorems in strong neutrosophic graphs. [22] Investigated the Wiener Index in neutrosophic graphs by Masoud Ghods and Zahra Rostami. A significant topological Index is the Wiener Index. With reference to the geodesic distance between two vertices, this Index is a distance-based Index. Specifically, following the definition of the Wiener Index in neutrosophic graphs. [35] In addition to providing some applications for computer networks, highway systems and transport network flow, this author employed the notion of connectedness indices in $\mathcal{NG}$s. The first six Sombor numbers for single-valued neutrosophic fuzzy networks are defined by Anwar et al. in [6]. The six Sombor numbers for these basic graph families are then calculated for single-valued neutrosophic fuzzy graphs and the degree of vertices of various graph families is established in a single-valued neutrosophic fuzzy framework. AL-Omeri and Kaviyarasu used the $\mathcal{NG}$ idea in [4] to locate the Japan Earthquake Response Center. Complex $\mathcal{NG}$s are constructed by fusing concepts from graph theory with complex $\mathcal{NS}$s, as stated Alqahtani et al. in [5]. This offers an adaptable framework for handling challenging situations involving the solution of problems. A number of procedures, including union, join and composition are studied in detail to enhance the management of complex $\mathcal{NG}$s.

1.4. Research Gaps

The $\mathcal{SOI}$ was developed to extend the analysis toward molecular descriptors' calculation while the prior strategies pay their most attention to crisp graphs; however, crisp graphs are not well suited in uncertain or fuzzy scenarios.

This gap emerges since, in real-world graphically modeled systems e.g. for decision-making in molecular chemistry and urban planning, there are inevitable uncertainties and ambiguities that crisp graph theory does not consider or handle well enough.

The research question to address this gap could be: As a contribution to answering these questions, this paper aims to demonstrate the potential of certain formalisms in dealing with uncertainty in networks-neutrosophic graph theory and $\mathcal{NSOI}$ in particular.

1.5. Motivation of Study

This research is motivated by the increasing demand for sophisticated methodologies for system analysis where formal uncertainty prevails. Sombor Indices and $\mathcal{FG}s$ have some limitations in describing the indeterminacy, therefore, the proposed neutrosophic Sombor Indices expand a range of Indices and contribute to the investigation of the graph properties. Extending the Sombor Indices of $\mathcal{IFG}s$, it provides fresh approaches toward uncertainty management in networks and can serve as a base for potential future studies in a variety of diverse disciplines ranging from thermal power plant site selection to brand optimization.

1.6. Objectives of the Study

For crisp graphs, a wide range of topological indices have been investigated and shown to have several uses. However, it is seen in many real-world applications that many scenarios cannot be represented by crisp graphs and hence $\mathcal{FG}s$ can only handle $\mathcal{MV}$. Determining a $\mathcal{IFG}s$ is necessary. To respond to this query, a $\mathcal{IFG}s$ must be defined. The definition of the Sombre index for $\mathcal{IFG}s$ and certain findings pertaining to vertex and edge critical $\mathcal{IFG}s$ are presented in this work. The association between $\mathcal{SOI}$ of $\mathcal{IFG}s$ and the first Zagreb Index was also found. We used the first full $\mathcal{SOI}$ in site selection for thermal power plants at the conclusion of this work.

1.7. Structure of Article

The following describes the structure of this research article: Section 2: Given the fundamental ideas behind neutrosophic graphs, Section 3 proposes a framework for $\mathcal{SOI}$ of $\mathcal{NG}$ and discusses their theorems and example. Applications for site selection for thermal power plant utilizing $\mathcal{SOI}$ of $\mathcal{NG}$ are described in Section 4. Section 5 provides findings and suggests ideas for further research at the end.

Several symbols and their associated meanings are often used in this text. Table 1 below provides a summary of these symbols together with their explanations:

Symbols	Abbreviations	Symbols	Abbreviations
$\mathcal{SOI}$	Sombor Index	$\mathcal{NG}s$	Neutrosophic Graphs
$\mathcal{NFZI}$	Neutrosophic First Zagreb Index	$\mathcal{NSZI}$	Neutrosophic Second Zagreb Index
$\mathcal{FS}$s	Fuzzy sets	$\mathcal{FG}$s	Fuzzy Graph
$\mathcal{MV}$	Membership Value	$\mathcal{IFG}$s	Intuitionistic Fuzzy Graphs
$\mathcal{IFS}$s	Intuitionistic Fuzzy Sets	$\mathcal{NMV}$	Non Membership Value
$\mathcal{NS}s$	Neutrosophic sets	$\mathcal{NG}s$	Neutrosophic Graphs
$\mathcal{TMN}$	Truth Membership Value	$\mathcal{IMN}$	Indeterminacy Membership Value
$\mathcal{FMN}$	Falsity Membership Value	$\mathcal{NSOI}$	Neutrosophic Sombor Index
$\mathcal{NSOI}$	Neutrosophic Sombor Index	TSOI	Truth Sombor Index
ISOI	Indeterminacy Sombor Index	FSOI	Falsity Sombor Index
TFZI	Truth First Zagreb Index	IFZI	Indeterminacy First Zagreb Index
FFZI	Falsity First Zagreb Index		

Table 1: List of symbols and abbreviations.

2. Preliminaries

Definition 1. *[26] Consider a simple graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Let $đ_\alpha$ represent the degree of the vertex $\alpha \in \mathcal{V}(\mathcal{G})$. I.Gutman developed a new vertex-degree-based topological Index called $\mathcal{SOI}$, which is defined as follows:*

$$\mathcal{SOI}(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} = \sqrt{(đ_\alpha)^2 + (đ_\beta)^2}.$$

Definition 2. *[59] Assume Đ is a universal set. A fuzzy set S on Đ is a mapping $\sigma :$ Đ $\to [0, 1]$. σ represents the fuzzy set S membership function. $S = (u, \sigma)$ represents a $\mathcal{FS}$.*

Definition 3. *[50] Given a $\mathcal{FG}$ such that $(\mathcal{G}, \sqsupseteq, \sigma)$, the $\mathcal{SOI}$ of $\mathcal{FG}$ is stated that by*

$$\mathcal{SOI}(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\varpi(\alpha)đ_G(\alpha))^2 + (\varpi(\beta)đ_G(\beta))^2}.$$

Definition 4. *[28] If $\mathcal{G} = (\mathcal{V}, \mathcal{W}, \mathcal{P})$ is a $\mathcal{FG}$, the first Zagreb Index for $\mathcal{FG}$s is stated below:*

$$\mathcal{NFZI}(\mathcal{G}) = \sum_{i=1}^{m}[\varpi(\beta_i)đ_{\mathcal{G}}(\beta_i)]^2.$$

Definition 5. *[31] Let's call $\mathcal{G}$ a $\mathcal{FG}$. The fuzzy entire Zagreb Index of $\mathcal{G}$ is denoted by M_1^z, which is defined as*

$$M_1^z : \sum_{\beta\in V(\mathcal{G})}(\varpi(\beta)d(\beta))^2 + \sum_{\varrho\in\mathcal{E}(\mathcal{G})}(e(\varrho)d(\varrho))^2.$$

Definition 6. *[31] Let $(\mathcal{G}, \sqsupseteq, \sigma)$ be an $\mathcal{FG}$. The second entire Zagreb Index of $\mathcal{G}$ is indicated by M_2^z and its defined by*

$$M_2^z : \sum_{\alpha,\beta \text{ is adjacent to eachother}}(w(\alpha)d(\alpha)w(\beta)d(\beta)) + \sum_{\beta \text{ is incidente to each orther}}(w(\beta)d(\beta)e(\varrho)d(\varrho)),$$

where w is the $\mathcal{MV}$ of a vertex and e is the $\mathcal{MV}$ of an edge.

Definition 7. *[4] 1. A $\mathcal{NG}$ indicated as $\mathcal{G} = ((\tau\alpha_1, \imath\alpha_2, F\alpha_3), (\tau\beta_1, \imath\beta_2, F\beta_3))$ is symbolized as $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices and $\mathcal{E}$is the collection of edges. The functions $\tau\alpha_1, \imath\alpha_2$ and $F\alpha_3$ are mappings from $\mathcal{V}$ to the closed interval $[0, 1]$, various degrees of $\mathcal{MV}$, $\mathcal{IV}$ and $\mathcal{NMV}$,accordingly, for each element $y_i \in \mathcal{V}$. It holds that $0 \leq \tau\alpha_1(y_i) + \imath\alpha_2(y_i) + F\alpha_3(y_i) \leq 3$ for all $y_i \in \mathcal{V}$.*
2. Furthermore in the framework of $\mathcal{G}^*$, the functions $\tau_{\beta_1}, \imath_{\beta_2}$ and F_{β_3} are mappings from $\mathcal{V}\times\mathcal{V}$ to the closed interval $[0, 1]$, representing the degrees of $\mathcal{MV}$, $\mathcal{IV}$ and $\mathcal{NMV}$, accordingly for each edge. $(y_i, y_j) \in \mathcal{E}$.

$$\begin{gathered}\tau\beta_1(y_i, y_j) \leq \tau\alpha_1(y_i) \wedge \tau\alpha_1(y_j),\\ \imath\beta_1(y_i, y_j) \leq \imath\alpha_1(y_i) \wedge \imath\alpha_1(y_j),\\ F\beta_1(y_i, y_j) \geq F\alpha_1(y_i) \wedge F\alpha_1(y_j),\end{gathered}$$

$0 \leq \tau\beta_1(y_i, y_j) + \imath\beta_2(y_i, y_j) + F\beta_3(y_i, y_j) \leq 3.$

3. $\mathcal{SOI}$ of Neutrosophic Graphs

Definition 8. *Let $\mathcal{G} = (\mathcal{V}, \varpi, \rho)$ be a $\mathcal{NG}$ then the $\mathcal{SOI}$ of neutrosophic graphs is define as, follows*

$$\begin{aligned} NSOI(\mathcal{G}) = &\sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_\varpi(\alpha)\tau_{d\mathcal{G}}(\alpha))^2 + (\tau_\varpi(\beta)\tau_{d\mathcal{G}}(\beta))^2} \\ &+ \sqrt{(\imath_\varpi(\alpha)\imath_{d\mathcal{G}}(\alpha))^2 + (\imath\varpi(\beta)\imath_{d\mathcal{G}}(\beta))^2} + \sqrt{(F_\varpi(\alpha)F_{d\mathcal{G}}(\alpha))^2 + (F\varpi(\beta)F_{d\mathcal{G}}(\beta))^2} \end{aligned} \tag{3.1}$$

Example 1. *Let $\mathcal{G}$ be a $\mathcal{NG}$ with $\mathcal{V}(\mathcal{G}) = \{a, b, c, d\}$ such that $\varpi(a) = (0.7, 0.5, 0.3), \varpi(b) = (0.6, 0.4, 0.5), \varpi(c) = (0.5, 0.4, 0.2), \varpi(d) = (0.3, 0.2, 0.1), \rho(ab) = (0.6, 0.4, 0.5), \rho(bc) = (0.5, 0.4, 0.5), \rho(cd) = (0.3, 0.2, 0.2), \rho(ac) = (0.4, 0.5, 0.4), \rho(ad) = (0.3, 0.1, 0.4)$ $đ_{\mathcal{G}}(a) =$*

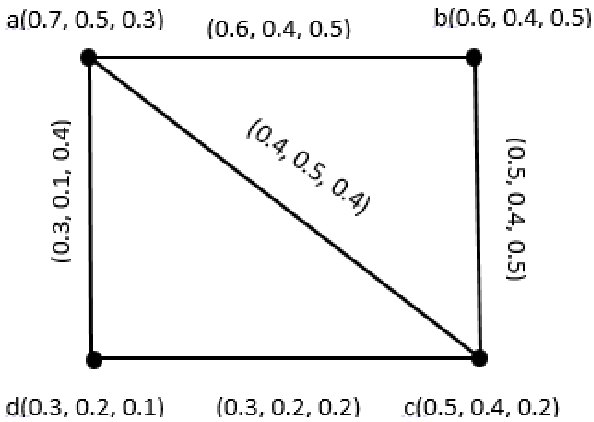

Figure 1: Neutrosophic graph

$(1.3, 1, 1.3), đ_{\mathcal{G}}(b) = (1.1, 0.8, 1), đ_{\mathcal{G}}(c) = (1.2, 1.1, 1.1), đ_{\mathcal{G}}(d) = (0.6, 0.3, 0.6)$.

$$
\begin{aligned}
NSOI(\mathcal{G}) = & \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{dG}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{dG}(\beta))^2} \\
& + \sqrt{(\imath_{\varpi}(\alpha)\imath_{dG}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta))^2} + \sqrt{(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha))^2 + (F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta))^2} \\
NSOI(\mathcal{G}) = & \sqrt{((0.7)(1.3))^2 + ((0.6)(1.1))^2 + ((0.5)(1))^2 + ((0.4)(0.8))^2 + ((0.3)(1.3))^2 + ((0.5)(1.3))^2} \\
& + \sqrt{((0.7)(1.3))^2 + ((0.5)(1.2))^2 + ((0.5)(1))^2 + ((0.4)(1.1))^2 + ((0.3)(1.1))^2 + ((0.2)(1.1))^2} \\
& + \sqrt{((0.7)(1.3))^2 + ((0.3)(0.6))^2 + ((0.5)(1))^2 + ((0.2)(0.3))^2 + ((0.3)(1.3))^2 + ((0.1)(0.6))^2} \\
& + \sqrt{((0.6)(1.1))^2 + ((0.5)(1.2))^2 + ((0.4)(0.8))^2 + ((0.4)(1.1))^2 + ((0.5)(1))^2 + ((0.2)(1.1))^2} \\
& + \sqrt{((0.5)(1.2))^2 + ((0.3)(0.6))^2 + ((0.4)(1.1))^2 + ((0.2)(0.3))^2 + ((0.2)(1.1))^2 + ((0.1)(0.6))^2} \\
NSOI(\mathcal{G}) = & \ 6.5653.
\end{aligned}
$$

Theorem 1. *Let the neutrosophic path graph be denoted by $P_{\hat{m}}$. Consequently,* $NSOI(\rho_{\hat{m}}) \le 6(\sqrt{2}(\hat{m} - 3) + \sqrt{5})$.

Proof. A neutrosophic path $\mathcal{G} = P_{\hat{m}}$ has $\mathcal{V}(P_{\hat{m}}) = \{\beta_1, \beta_2, \beta_3, ..\beta_{\hat{m}}\}$ and $\mathcal{E}(P_{\hat{m}}) = \{\varrho_1, \varrho_2, \varrho_3, ..\varrho_{\hat{m}-1}\}$. Assume that $\varpi_1, \varpi_2, \varpi_3, ..., \varpi_{\hat{m}}$ and $\rho_1, \rho_2, \rho_3, \rho_{\hat{m}-1}$, respectively, denote the type of $\mathcal{MV}$s of $P_{\hat{m}}$'s vertices and edges. Then clearly $đ_{\mathcal{G}}(\beta_1) = \rho(\varrho_1), đ_{\mathcal{G}}(\beta_{\hat{m}}) = \rho(\varrho_{\hat{m}-1})$ and $đ_{\mathcal{G}}(\beta_i) = \rho(\varrho_i) + \rho(\varrho_{i+1})$,for $2 \le i \le \hat{m} - 2$. Therefore,

$$
NSOI(P_{\hat{m}}) = TSOI(P_{\hat{m}}) + ISOI(P_{\hat{m}}) + FSOI(P_{\hat{m}}) \tag{3.2}
$$

$$
TSOI(P_{\hat{m}}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta))^2}
$$

$$
\begin{aligned}
&= \sqrt{(\tau_{\varpi}(\beta_1)\tau_{d\mathcal{G}}(\beta_1))^2 + (\tau_{\varpi}(\beta_2)\tau_{d\mathcal{G}}(\beta_2))^2} \\
&+ \sqrt{(\tau_{\varpi}(\beta_{\hat{m}-1})\tau_{d\mathcal{G}}(\beta_{\hat{m}-1}))^2 + (\tau_{\varpi}(\beta_{\hat{m}})\tau_{d\mathcal{G}}(\beta_{\hat{m}}))^2} \\
&+ \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}} \sqrt{(\tau_{\varpi}(\beta_i)\tau_{d\mathcal{G}}(\beta_i))^2 + (\tau_{\varpi}(\beta_j)\tau_{d\mathcal{G}}(\beta_j))^2} \\
&= \sqrt{(\tau_{\varpi}(\beta_1)\tau_{\rho}(\varrho_1))^2 + (\tau_{\varpi}(\beta_2)(\tau_{\rho}(\varrho_1) + \tau_{\rho}(\varrho_2)))^2} \\
&+ \sqrt{(\tau_{\varpi}(\beta_{\hat{m}})\tau_{\rho}(\varrho_{\hat{m}-1}))^2 + (\tau_{\varpi}(\beta_{\hat{m}-1})(\tau_{\rho}(\varrho_{\hat{m}-1}) + \tau_{\rho}(\varrho_{\hat{m}-2})))^2}
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}} \sqrt{(\tau_{\varpi}(\beta_i)(\tau_{\rho}(\varrho_i) + \tau_{\rho}(\varrho_{j+1})))^2 + (\tau_{\varpi}(\beta_j)(\tau_{\rho}(\varrho_j) + \rho(\varrho_{j+1})))^2} \\
&= \sqrt{(\tau_{\varpi}(\beta_1)(\tau_{\rho}(\varrho_1))^2 + (\tau_{\varpi}(\beta_2)^2(\tau_{\rho}(\varrho_1)^2 + \tau_{\rho}(\varrho_2)^2) + 2\tau_{\rho}(\varrho_1)\tau_{\rho}(\varrho_2))} \\
&+ \sqrt{\tau_{\varpi}(\beta_{\hat{m}})^2\tau_{\rho}(\varrho_{\hat{m}-1})^2 + (\tau_{\varpi}(\beta_{\hat{m}-1})^2(\tau_{\rho}(\varrho_{\hat{m}-1})^2 + \tau_{\rho}(\varrho_{\hat{m}-2})^2 + 2\tau_{\rho}(\varrho_{\hat{m}-1})\tau_{\rho}(\varrho_{\hat{m}-2})))} \\
&+ \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}} \sqrt{\begin{aligned}&(\tau_{\varpi}(\beta_i)^2(\tau_{\rho}(\varrho_i)^2 + \tau_{\rho}(\varrho_{i+1})^2 + 2\tau_{\rho}(\varrho_i)\tau_{\rho}(\varrho_{i+1}))) \\ &+ (\tau_{\varpi}(\beta_j)^2(\tau_{\rho}(\varrho_j)^2 + \tau_{\rho}(\varrho_{j+1})^2 + 2\tau_{\rho}(\varrho_j)\tau_{\rho}(\varrho_{j+1}))),\end{aligned}}
\end{aligned}
$$

since $0 \leq \tau_{\varpi}(\beta) \leq 1$ and $0 \leq \tau_{\rho}(\varrho) \leq 1$. Therefore,

$$
\begin{aligned}
TSOI(P_{\hat{m}}) &= \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{G})} \sqrt{1^2.1^2 + 1^2(1^2+1^2) + 2(1))(1).1} + \sqrt{1^2.1^2 + 1^2.1^2 + 1^2.1^2 + 2(1^2).1.1} \\
&= (\hat{m}-3)\sqrt{1^2.1^2 + 1^2.1^2 + 2(1^2).1.1 + 1^2.1^2 + 1^2.1^2 + 2(1^2).1.1} \\
TSOI(P_{\hat{m}}) &= 2\sqrt{2}(\hat{m}-3) + 2\sqrt{5}
\end{aligned}
$$

$$
TSOI(P_{\hat{m}}) = 2(\sqrt{2}(\hat{m}-3) + \sqrt{5}) \tag{3.3}
$$

$$
\begin{aligned}
ISOI(P_{\hat{m}}) &= \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{dG}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta))^2} \\
&= \sqrt{(\imath_{\varpi}(\beta_1)\imath_{d\mathcal{G}}(\beta_1))^2 + (\imath_{\varpi}(\beta_2)\imath_{d\mathcal{G}}(\beta_2))^2} \\
&+ \sqrt{(\imath_{\varpi}(\beta_{\hat{m}-1})\imath_{d\mathcal{G}}(\beta_{\hat{m}-1}))^2 + (\imath_{\varpi}(\beta_{\hat{m}})\imath_{d\mathcal{G}}(\beta_{\hat{m}}))^2} \\
&+ \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}} \sqrt{(\imath_{\varpi}(\beta_i)\imath_{d\mathcal{G}}(\beta_i))^2 + (\imath_{\varpi}(\beta_j)\imath_{d\mathcal{G}}(\beta_j))^2} \\
&= \sqrt{(\imath_{\varpi}(\beta_1)\imath_{\rho}(\varrho_1))^2 + (\imath_{\varpi}(\beta_2)(\imath_{\rho}(\varrho_1) + \imath_{\rho}(\varrho_2)))^2} \\
&+ \sqrt{(\imath_{\varpi}(\beta_{\hat{m}})\imath_{\rho}(\varrho_{\hat{m}-1}))^2 + (\imath_{\varpi}(\beta_{\hat{m}-1})(\imath_{\rho}(\varrho_{\hat{m}-1}) + \imath_{\rho}(\varrho_{\hat{m}-2})))^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}}\sqrt{(\imath_{\varpi}(\beta_i)(\imath_{\rho}(\varrho_i)+\imath_{\rho}(\varrho_{j+1})))^2+(\imath_{\varpi}(\beta_j)(\imath_{\rho}(\varrho_j)+\rho(\varrho_{j+1})))^2}\\
&=\sqrt{(\imath_{\varpi}(\beta_1)(\imath_{\rho}(\varrho_1))^2+(\imath_{\varpi}(\beta_2)^2(\imath_{\rho}(\varrho_i)^2+\imath_{\rho}(\varrho_2)^2)+2\imath_{\rho}(\varrho_1)\imath_{\rho}(\varrho_2))}\\
&+\sqrt{\imath_{\varpi}(\beta_{\hat{m}})^2\imath_{\rho}(\varrho_{\hat{m}-1})^2+(\imath_{\varpi}(\beta_{\hat{m}-1})^2(\imath_{\rho}(\varrho_{\hat{m}-1})^2+\imath_{\rho}(\varrho_{\hat{m}-2})^2+2\imath_{\rho}(\varrho_{\hat{m}-1})\imath_{\rho}(\varrho_{\hat{m}-2})))}\\
&+\sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}}\sqrt{\begin{array}{l}(\imath_{\varpi}(\beta_i)^2(\imath_{\rho}(\varrho_i)^2+\imath_{\rho}(\varrho_{i+1})^2+2\imath_{\rho}(\varrho_i)\imath_{\rho}(\varrho_{i+1})))\\ +(\imath_{\varpi}(\beta_j)^2(\imath_{\rho}(\varrho_j)^2+\imath_{\rho}(\varrho_{j+1})^2+2\imath_{\rho}(\varrho_j)\imath_{\rho}(\varrho_{j+1}))),\end{array}}
\end{aligned}
$$

since $0\le \imath_{\varpi}(\alpha)\le 1$ and $0\le \imath_{\rho}(\varrho)\le 1$. Therefore,

$$
\begin{aligned}
ISOI(P_{\hat{m}})&=\sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})}\sqrt{1^2.1^2+1^2(1^2+1^2)+2(1))(1).1}+\sqrt{1^2.1^2+1^2.1^2+1^2.1^2+2(1^2).1.1}\\
&=(\hat{m}-3)\sqrt{1^2.1^2+1^2.1^2+2(1^2).1.1+1^2.1^2+1^2.1^2+2(1^2).1.1}\\
ISOI(P_{\hat{m}})&=2\sqrt{2}(\hat{m}-3)+2\sqrt{5}
\end{aligned}
$$

$$ISOI(P_{\hat{m}})=2(\sqrt{2}(\hat{m}-3)+\sqrt{5}) \tag{3.4}$$

and

$$
\begin{aligned}
FSOI(P_{\hat{m}})&=\sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})}\sqrt{(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha))^2+(F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta))^2}\\
&=\sqrt{(F_{\varpi}(\beta_1)F_{d\mathcal{G}}(\beta_1))^2+(F_{\varpi}(\beta_2)F_{d\mathcal{G}}(\beta_2))^2}\\
&+\sqrt{(F_{\varpi}(\beta_{\hat{m}-1})F_{d\mathcal{G}}(\beta_{\hat{m}-1}))^2+(F_{\varpi}(\beta_{\hat{m}})F_{d\mathcal{G}}(\beta_{\hat{m}}))^2}\\
&+\sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}}\sqrt{(F_{\varpi}(\beta_i)F_{d\mathcal{G}}(\beta_i))^2+(F_{\varpi}(\beta_j)F_{d\mathcal{G}}(\beta_j))^2}\\
&=\sqrt{(F_{\varpi}(\beta_1)F_{\rho}(\varrho_1))^2+(F_{\varpi}(\beta_2)(F_{\rho}(\varrho_1)+F_{\rho}(\varrho_2)))^2}\\
&+\sqrt{(F_{\varpi}(\beta_{\hat{m}})F_{\rho}(\varrho_{\hat{m}-1}))^2+(F_{\varpi}(\beta_{\hat{m}-1})(F_{\rho}(\varrho_{\hat{m}-1})+F_{\rho}(\varrho_{\hat{m}-2})))^2}\\
&+\sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}}\sqrt{(F_{\varpi}(\beta_i)(F_{\rho}(\varrho_i)+F_{\rho}(\varrho_{j+1})))^2+(F_{\varpi}(\beta_j)(F_{\rho}(\varrho_j)+\rho(\varrho_{j+1})))^2}\\
&=\sqrt{(F_{\varpi}(\beta_1)(F_{\rho}(\varrho_1))^2+(F_{\varpi}(\beta_2)^2(F_{\rho}(\varrho_i)^2+F_{\rho}(\varrho_2)^2)+2F_{\rho}(\varrho_1)F_{\rho}(\varrho_2)}\\
&+\sqrt{F_{\varpi}(\beta_{\hat{m}})^2F_{\rho}(\varrho_{\hat{m}-1})^2+(F_{\varpi}(\beta_{\hat{m}-1})^2(F_{\rho}(\varrho_{\hat{m}-1})^2+F_{\rho}(\varrho_{\hat{m}-2})^2+2F_{\rho}(\varrho_{\hat{m}-1})F_{\rho}(\varrho_{\hat{m}-2})))}\\
&+\sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})-\{\varrho_1,\varrho_{\hat{m}-1}\}}\sqrt{\begin{array}{l}(F_{\varpi}(\beta_i)^2(F_{\rho}(\varrho_i)^2+F_{\rho}(\varrho_{i+1})^2+2F_{\rho}(\varrho_i)F_{\rho}(\varrho_{i+1})))\\ +(F_{\varpi}(\beta_j)^2(F_{\rho}(\varrho_j)^2+F_{\rho}(\varrho_{j+1})^2+2F_{\rho}(\varrho_j)F_{\rho}(\varrho_{j+1}))),\end{array}}
\end{aligned}
$$

since $0 \leq F_{\varpi}(\beta) \leq 1$ and $0 \leq F_{\rho}(\varrho) \leq 1$. Therefore,

$$\begin{aligned}
FSOI(P_{\hat{m}}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{1^2.1^2 + 1^2(1^2+1^2) + 2(1))(1).1} + \sqrt{1^2.1^2 + 1^2.1^2 + 1^2.1^2 + 2(1^2).1.1} \\
&= (\hat{m}-3)\sqrt{1^2.1^2 + 1^2.1^2 + 2(1^2).1.1 + 1^2.1^2 + 1^2.1^2 + 2(1^2).1.1} \\
FSOI(P_{\hat{m}}) &= 2\sqrt{2}(\hat{m}-3) + 2\sqrt{5}
\end{aligned}$$

$$FSOI(P_{\hat{m}}) = 2(\sqrt{2}(\hat{m}-3) + \sqrt{5}). \tag{3.5}$$

Substitute (3.3), (3.4) and (3.5) in (3.2),

$$\begin{aligned}
NSOI(P_{\hat{m}}) &= 2(\sqrt{2}(\hat{m}-3) + \sqrt{5}) + 2(\sqrt{2}(\hat{m}-3) + \sqrt{5}) + 2(\sqrt{2}(\hat{m}-3) + \sqrt{5}) \\
&= 6(\sqrt{2}(\hat{m}-3) + \sqrt{5}).
\end{aligned}$$

Hance proof is compete.

Theorem 2. *Let $C_{\hat{m}}$ denote a neutrosophic cycle graph respectively, then $NSOI(C_{\hat{m}}) \leq 6\sqrt{2}\hat{m}$.*

Proof. Let $G = C_{\hat{m}}$ be a neutrosophic cycle with $\mathcal{V}(C_{\hat{m}}) = \{v_1, v_2, v_3, ..., v_{\hat{m}}\}$ and $E(C_{\hat{m}}) = \{\varrho_1, \varrho_2, \varrho_3, ..., \varrho_{\hat{m}-1}\}$. Let $\varpi_1, \varpi_2, \varpi_3, ..., \varpi_{\hat{m}}$ and $\rho_1, \rho_2, \rho_3, ..., \rho_{\hat{m}-1}$ be the there of type of $\mathcal{MV}$s of vertices and edges of $C_{\hat{m}}$ respectively. Then, clearly $đ_G(\beta_i) = \rho(\varrho_i) + \rho(\varrho_{i+1})$. Therefore,

$$NSOI(C_{\hat{m}}) = TSOI(C_{\hat{m}}) + ISOI(C_{\hat{m}}) + FSOI(C_{\hat{m}}) \tag{3.6}$$

$$\begin{aligned}
TSOI(C_{\hat{m}}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{đ\mathcal{G}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{đ\mathcal{G}}(\beta))^2} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\beta_i)(\tau_{\rho}(\varrho_i) + \tau_{\rho}(\varrho_{i+1})))^2} + \sqrt{(\tau_{\varpi}(\beta_j)(\tau_{\rho}(\varrho_j) + \tau_{\rho}(\varrho_{j+1})))^2} \\
&+ \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{\begin{array}{l}(\tau\varpi(\beta_i)^2(\tau_{\rho}(\varrho_i)^2 + \tau_{\rho}(\varrho_{i+1})^2 + 2\tau_{\rho}(\varrho_i)\tau_{\rho}(\varrho_{i+1}))) \\ + (\tau\varpi(\beta_j)^2(\tau_{\rho}(\varrho_j)^2 + \tau_{\rho}(\varrho_{j+1})^2 + 2\tau_{\rho}(\varrho_j)\tau_{\rho}(\varrho_{j+1}))),\end{array}}
\end{aligned}$$

since $0 \leq \tau_{\varpi}(\beta) \leq 1$ and $0 \leq \tau_{\rho}(\varrho) \leq 1$. Therefore,

$$TSOI(C_{\hat{m}}) \leq n\sqrt{1^2.1^2 + 1^2.1^2 + 2(1^2).1.1 + 1^2.1^2 + 1^2.1^2 + 2(1^2).1.1}$$

$$TSOI(C_{\hat{m}}) \leq 2\sqrt{2}\hat{m} \tag{3.7}$$

$$ISOI(C_{\hat{m}}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{đ\mathcal{G}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{đ\mathcal{G}}(\beta))^2}$$

$$= \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})} \sqrt{(\imath_\varpi(\beta_i)(\imath_\rho(\varrho_i)+\imath_\rho(\varrho_{i+1})))^2} + \sqrt{(\imath_\varpi(\beta_j)(\imath_\rho(\varrho_j)+\imath_\rho(\varrho_{j+1})))^2}$$

$$+ \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})} \sqrt{\begin{array}{l}(\imath_\varpi(\beta_i)^2(\imath_\rho(\varrho_i)^2+\imath_\rho(\varrho_{i+1})^2+2\imath_\rho(\varrho_i)\imath_\rho(\varrho_{i+1})))\\ +(\imath_\varpi(\beta_j)^2(\imath_\rho(\varrho_j)^2+\imath_\rho(\varrho_{j+1})^2+2\imath_\rho(\varrho_j)\imath_\rho(\varrho_{j+1}))),\end{array}}$$

since $0 \le \imath_\varpi(\beta) \le 1$ and $0 \le \imath_\rho(\varrho) \le 1$. Therefore,

$$ISOI(C_{\hat{m}}) \le n\sqrt{1^2.1^2+1^2.1^2+2(1^2).1.1+1^2.1^2+1^2.1^2+2(1^2).1.1}$$

$$ISOI(C_{\hat{m}}) \le 2\sqrt{2}\hat{m}. \tag{3.8}$$

$$FSOI(C_{\hat{m}}) = \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{G})} \sqrt{(F_\varpi(\alpha)F_{d\mathcal{G}}(\alpha))^2+(F_\varpi(\beta)F_{d\mathcal{G}}(\beta))^2}$$

$$= \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})} \sqrt{(F_\varpi(\beta_i)(F_\rho(\varrho_i)+F_\rho(\varrho_{i+1})))^2} + \sqrt{(F_\varpi(\beta_j)(F_\rho(\varrho_j)+F_\rho(\varrho_{j+1})))^2}$$

$$+ \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})} \sqrt{\begin{array}{l}(F_\varpi(\beta_i)^2(F_\rho(\varrho_i)^2+F_\rho(\varrho_{i+1})^2+2F_\rho(\varrho_i)F_\rho(\varrho_{i+1})))\\ +(F_\varpi(\beta_j)^2(F_\rho(\varrho_j)^2+F_\rho(\varrho_{j+1})^2+2F_\rho(\varrho_j)F_\rho(\varrho_{j+1}))),\end{array}}$$

since , $0 \le F_\varpi(\beta) \le 1$ and $0 \le F_\rho(\varrho) \le 1$. Therefore,

$$FSOI(C_{\hat{m}}) \le n\sqrt{1^2.1^2+1^2.1^2+2(1^2).1.1+1^2.1^2+1^2.1^2+2(1^2).1.1}$$

$$FSOI(C_{\hat{m}}) \le 2\sqrt{2}m. \tag{3.9}$$

Substitute (3.7), (3.8) and (3.9) in (3.6).

$$NSOI(\rho_{\hat{m}}) \le 2\sqrt{2}m + 2\sqrt{2}m + 2\sqrt{2}m$$
$$NSOI(\rho_{\hat{m}}) \le 6\sqrt{2}m.$$

Hance proof is compete.

Theorem 3. *Let $K_{\hat{m}}$ denotes neutrosophic complete graph respectively, then* $NSOI(K_{\hat{m}}) \le 3(\frac{\hat{m}(\hat{m}-1)}{2}\sqrt{2\hat{m}-2})$.

Proof. Let $\mathcal{G} = K_{\hat{m}}$ be a neutrosophic complete graph with $\mathcal{V}(K_{\hat{m}}) = \{\beta_1, \beta_2, \beta_3, ..., \beta_{\hat{m}}\}$ and $E(K_{\hat{m}}) = \{\varrho_1, \varrho_2, \varrho_3, .., \varrho_{\hat{m}-1}\}$.
Let $\varpi_1, \varpi_2, \varpi_3, ..., \varpi_{\hat{m}}$ and $\rho_1, \rho_2, \rho_3, ..., \rho_{\hat{m}}$ be the there of type of $\mathcal{MV}$s of vertices and edges of $K_{\hat{m}}$ respectively. Then clearly, $d_G(\beta_i) = \sum_{\beta_i \sim \varrho_j} \rho(\varrho_j)$. Therefore,

$$NSOI(K_{\hat{m}}) = TSOI(K_{\hat{m}}) + ISOI(K_{\hat{m}}) + FSOI(K_{\hat{m}}) \tag{3.10}$$

$$\begin{aligned}
TSOI(K_{\hat{m}}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{\bar{d}\mathcal{G}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{\bar{d}\mathcal{G}}(\beta))^2} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\beta_i)^2(\sum_{\beta_i\sim\varrho_j}\tau_{\rho}(\varrho_j)^2)) + (\tau_{\varpi}(\beta_j)^2(\sum_{\beta_j\sim\varrho_i}\tau_{\rho}(\varrho_i)^2))} \\
&\leq \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{\tau_{\varpi}(\beta_i)^2(\hat{m}-1)\tau_{\rho}(\varrho_j)^2 + \tau_{\varpi}(\beta_j)^2(\hat{m}-1)\tau_{\rho}(\varrho_i)^2},
\end{aligned}$$

since, $0 \leq \tau_{\varpi}(\beta) \leq 1$ and $0 \leq \tau_{\rho}(\varrho) \leq 1$. Therefore,

$$TSOI(K_{\hat{m}}) \leq \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\hat{m}-1)+(\hat{m}-1)}$$

$$TSOI(K_{\hat{m}}) = \frac{\hat{m}(\hat{m}-1)}{2}\sqrt{2\hat{m}-2}. \tag{3.11}$$

$$\begin{aligned}
ISOI(k_{\hat{m}}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{\bar{d}\mathcal{G}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{\bar{d}\mathcal{G}}(\beta))^2} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\beta_i)^2(\sum_{\beta_i\sim\varrho_j}\imath_{\rho}(\varrho_j)^2)) + (\imath_{\varpi}(\beta_j)^2(\sum_{\beta_j\sim\varrho_i}\imath_{\rho}(\varrho_i)^2))} \\
&\leq \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{\imath_{\varpi}(\beta_i)^2(\hat{m}-1)\imath_{\rho}(\varrho_j)^2 + \imath_{\varpi}(\beta_j)^2(\hat{m}-1)\imath_{\rho}(\varrho_i)^2},
\end{aligned}$$

since, $0 \leq \imath_{\varpi}(\beta) \leq 1$ and $0 \leq \imath_{\rho}(\varrho) \leq 1$. Therefore,

$$ISOI(K_{\hat{m}}) \leq \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\hat{m}-1)+(\hat{m}-1)}$$

$$ISOI(K_{\hat{m}}) = \frac{\hat{m}(\hat{m}-1)}{2}\sqrt{2\hat{m}-2} \tag{3.12}$$

and

$$\begin{aligned}
FSOI(K_{\hat{m}}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\digamma_{\varpi}(\alpha)\digamma_{\bar{d}\mathcal{G}}(\alpha))^2 + (\digamma_{\varpi}(\beta)\digamma_{\bar{d}\mathcal{G}}(\beta))^2} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\digamma_{\varpi}(\beta_i)^2(\sum_{\beta_i\sim\varrho_j}\digamma_{\rho}(\varrho_j)^2)) + (\digamma_{\varpi}(\beta_j)^2(\sum_{\beta_j\sim\varrho_i}\digamma_{\rho}(\varrho_i)^2))} \\
&\leq \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{\digamma_{\varpi}(\beta_i)^2(\hat{m}-1)\digamma_{\rho}(\varrho_j)^2 + \digamma_{\varpi}(\beta_j)^2(\hat{m}-1)\digamma_{\rho}(\varrho_i)^2},
\end{aligned}$$

since, $0 \leq \digamma_{\varpi}(\beta) \leq 1$ and $0 \leq \digamma_{\rho}(\varrho) \leq 1$. Therefore,

$$FSOI(K_{\hat{m}}) \leq \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})} \sqrt{(\hat{m}-1)+(\hat{m}-1)}$$

$$FSOI(K_{\hat{m}}) = \frac{\hat{m}(\hat{m}-1)}{2}\sqrt{2\hat{m}-2}. \tag{3.13}$$

Substitute (3.11), (3.12) and (3.13) in (3.10), we get

$$NSOI(K_{\hat{m}}) \leq \frac{\hat{m}(\hat{m}-1)}{2}\sqrt{2\hat{m}-2} + \frac{\hat{m}(\hat{m}-1)}{2}\sqrt{2\hat{m}-2} + \frac{\hat{m}(\hat{m}-1)}{2}\sqrt{2\hat{m}-2}$$
$$NSOI(K_{\hat{m}}) \leq 3(\frac{\hat{m}(\hat{m}-1)}{2})\sqrt{2\hat{m}-2}.$$

Lemma 1. *Let $\mathcal{G} = K_{1,\hat{m}-1}$ be a neutrosophic star and statisties the condition $\tau_{\varpi}(0) \leq \tau_{\varpi}(\beta), \imath_{\varpi}(0) \leq \imath_{\varpi}(\beta)$ and $\digamma_{\varpi}(0) \leq \digamma_{\varpi}(\beta)$ where 0 is the center of the neurotrophic star, then, $NSOI(K_1, \hat{m}-1) \leq 3(\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2}$.*

Proof. Let $\mathcal{G} = K_{1,\hat{m}-1}$ be a neutrosophic star graph with $\mathcal{V}(K_{1,\hat{m}-1}) = \{\beta_1, \beta_2, \beta_3, ...\beta_{\hat{m}}\}$ and $\mathcal{E}(K_{1,\hat{m}-1}) = \{\varrho_1, \varrho_2, \varrho_3, ...\varrho_{\hat{m}-1}\}$. Let $\varpi_1, \varpi_2, \varpi_3, ...\varpi_{\hat{m}}$ and $\rho_1, \rho_2, \rho_3, ...\rho_{\hat{m}}$ be the $\mathcal{MV}$s of vertices and edges of $K_{\hat{m}}$.
Let $\beta_1 = 0$ be the center of the star. It is given that $\varpi(0) \leq \varpi(\beta)$. Then, clearly $\text{đ}_G(\beta_i) = \varpi(0)$ and $\text{đ}_G(0) = (\hat{m}-1)\varpi(0)$. Therefor,

$$NSOI(K_1, \hat{m}-1) = TSOI(K_1, \hat{m}-1) + ISOI(K_1, \hat{m}-1) + FSOI(K_1, \hat{m}-1) \tag{3.14}$$

$$\begin{aligned} TSOI(K_1, \hat{m}-1) &= \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{\text{đ}\mathcal{G}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{\text{đ}\mathcal{G}}(\beta))^2} \\ &= \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\beta_i)^2(\tau_{\text{đ}\mathcal{G}}(0))^2) + (\tau_{\varpi}(\beta_j)^2(\tau_{\text{đ}\mathcal{G}}(\beta_j))^2)} \\ &= \sum_{\beta_i,\beta_j \in \mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\beta_i)^2((\hat{m}-1)\tau_{\varpi}(0))^2) + (\tau_{\varpi}(\beta_j)^2(\tau_{\varpi}(0)))^2}, \end{aligned}$$

since , $0 \leq \tau_{\varpi}(\beta) \leq 1$ and $0 \leq \tau_{\rho}(\varrho) \leq 1$. Therefore,

$$TSOI(K_1, \hat{m}-1) \leq (\hat{m}-1)[\sqrt{(1^2.((\hat{m}-1)^2.1^2)) + (1^2.1^2)}]$$

$$= (\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2} \tag{3.15}$$

$$
\begin{aligned}
ISOI(K_1,\hat{m}-1) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta))^2} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\beta_i)^2(\imath_{d\mathcal{G}}(0))^2) + (\imath_{\varpi}(\beta_j)^2(\imath_{d\mathcal{G}}(\beta_j))^2)} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\beta_i)^2((\hat{m}-1)\imath_{\varpi}(0))^2) + (\imath_{\varpi}(\beta_j)^2(\imath_{\varpi}(0)))^2},
\end{aligned}
$$

since $0 \leq \imath_{\varpi}(\beta) \leq 1$ and $0 \leq \imath_{\rho}(\varrho) \leq 1$. Therefore,

$$ISOI(K_1,\hat{m}-1) \leq (\hat{m}-1)[\sqrt{(1^2.((\hat{m}-1)^2.1^2)) + (1^2.1^2)}]$$

$$ISOI(K_1,\hat{m}-1) = (\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2} \tag{3.16}$$

$$
\begin{aligned}
FSOI(K_1,\hat{m}-1) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha))^2 + (F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta))^2} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\beta_i)^2(F_{d\mathcal{G}}(0))^2) + (F_{\varpi}(\beta_j)^2(F_{d\mathcal{G}}(\beta_j))^2)} \\
&= \sum_{\beta_i,\beta_j\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\beta_i)^2((\hat{m}-1)F_{\varpi}(0))^2) + (F_{\varpi}(\beta_j)^2(F_{\varpi}(0)))^2},
\end{aligned}
$$

since $0 \leq F_{\varpi}(\beta) \leq 1$ and $0 \leq F_{\rho}(\varrho) \leq 1$. Therefore,

$$FSOI(K_1,\hat{m}-1) \leq (\hat{m}-1)[\sqrt{(1^2.((\hat{m}-1)^2.1^2)) + (1^2.1^2)}]$$

$$FSOI(K_1,\hat{m}-1) = (\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2} \tag{3.17}$$

Substitute (3.15), (3.16) and (3.17) in (3.14), we get

$$NSOI(K_1,\hat{m}-1) = (\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2} + (\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2} + (\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2}$$
$$NSOI(K_1,\hat{m}-1) = 3((\hat{m}-1)\sqrt{\hat{m}^2-2\hat{m}+2}).$$

Hence the proof.

Definition 9. *Let $\mathcal{G} = (\mathcal{V},\varpi,\rho)$ be a $\mathcal{NG}$ then the first Zagreb Index of neutrosophic graph is defined as follows:*

$$
\begin{aligned}
NFZI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} &[(\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha)) + (\tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta))] + [(\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha)) + (\imath\varpi(\beta)\imath_{d\mathcal{G}}(\beta))] \\
&+ [(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha)) + (F\varpi(\beta)F_{d\mathcal{G}}(\beta))]
\end{aligned}
\tag{3.18}
$$

Theorem 4. *Let $\mathcal{G} = (\mathcal{V}, \varpi, \rho)$ be a $\mathcal{NFZI}$ of graph denoted the first Zagreb Index for neurotrophic graphs. Then $\mathcal{NSOI}(\mathcal{G}) \leq \mathcal{NFZI}(\mathcal{G})$.*

Proof. Let $\mathcal{G} = (\mathcal{V}, \mathcal{W}, \mathcal{P})$ be a $\mathcal{NG}$. By the definition of $\mathcal{SOI}$ for $\mathcal{NG}$s we have,

$$NSOI(\mathcal{G}) = TSOI(\mathcal{G}) + ISOI(\mathcal{G}) + FSOI(\mathcal{G}) \tag{3.19}$$

$$\begin{aligned} TSOI(\mathcal{G}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta))^2} \\ &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha)) + (\tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta))^2 - 2\tau_{\varpi}(\alpha)\tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\alpha)\tau_{d\mathcal{G}}(\beta)} \\ &\leq \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha) + \tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta))^2} \end{aligned}$$

$$TSOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} (\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha) + \tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta)) \tag{3.20}$$

$$\begin{aligned} ISOI(\mathcal{G}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta))^2} \\ &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha)) + (\imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta))^2 - 2\imath_{\varpi}(\alpha)\imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\alpha)\imath_{d\mathcal{G}}(\beta)} \\ &\leq \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha) + \imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta))^2} \end{aligned}$$

$$ISOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} (\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha) + \imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta)) \tag{3.21}$$

$$\begin{aligned} FSOI(\mathcal{G}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha))^2 + (F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta))^2} \\ &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha)) + (F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta))^2 - 2F_{\varpi}(\alpha)F_{\varpi}(\beta)F_{d\mathcal{G}}(\alpha)F_{d\mathcal{G}}(\beta)} \\ &\leq \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha) + F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta))^2} \end{aligned}$$

$$FSOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} (F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha) + F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta)) \tag{3.22}$$

Substitute (3.20), (3.21) and (3.22) in (3.19), we get

$$NSOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} (\tau_{\varpi}(\alpha)\tau_{\text{đ}\mathcal{G}}(\alpha) + \tau_{\varpi}(\beta)\tau_{\text{đ}\mathcal{G}}(\beta)) + \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} (\imath_{\varpi}(\alpha)\imath_{\text{đ}\mathcal{G}}(\alpha) + \imath_{\varpi}(\beta)\imath_{\text{đ}\mathcal{G}}(\beta))$$
$$+ \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} (F_{\varpi}(\alpha)F_{\text{đ}\mathcal{G}}(\alpha) + F_{\varpi}(\beta)F_{\text{đ}\mathcal{G}}(\beta))$$
$$NSOI(\mathcal{G}) = NFZI(\mathcal{G}).$$

Example 2. *Let $\mathcal{G}$ be a $\mathcal{NG}$ with $\mathcal{V}(\mathcal{G}) = \{p, q, r, s, t\}$ such that $\varpi(p) = (0.7, 0.6, 0.4), \varpi(q) = (0.6, 0.5, 0.4), \varpi(s) = (0.5, 0.4, 0.3), \varpi(r) = (0.5, 0.4, 0.3), \varpi(t) = (0.8, 0.7, 0.4)$ and $\varpi(pq) = (0.6, 0.4, 0.5), \varpi(qr) = (0.4, 0.4, 0.5), \varpi(rs) = (0.4, 0.3, 0.5), \varpi(st) = (0.5, 0.4, 0.5), \varpi(tq) = (0.6, 0.5, 0.5)$. $\text{đ}_G(p) = (0.6, 0.4, 0.5), \text{đ}_G(q) = (1, 0.9, 1), \text{đ}_G(r) = (0.8, 0.7, 1), \text{đ}_G(s) = (0.9, 0.7, 0.5),$*

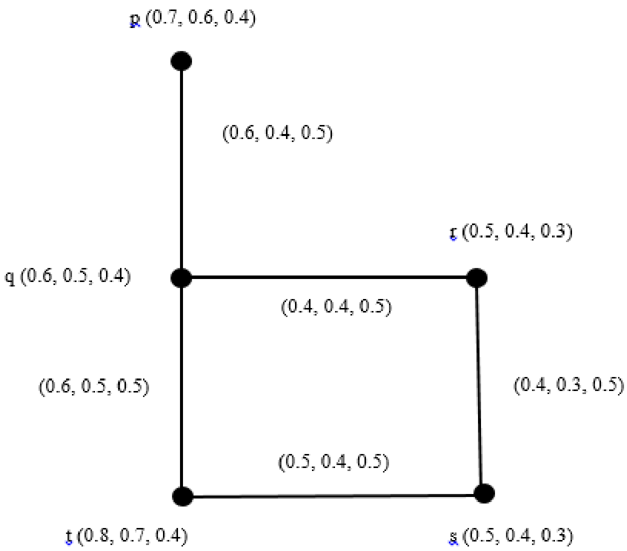

Figure 2: Neutrosophic Graph

$\text{đ}_G(t) = (1.1, 0.9, 1)$.

$$NSOI(\mathcal{G}) = TSOI(\mathcal{G}) + ISOI(\mathcal{G}) + FSOI(\mathcal{G}) \tag{3.23}$$

$$TSOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{\text{đ}\mathcal{G}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{\text{đ}\mathcal{G}}(\beta))^2}$$
$$= \sqrt{(\tau_{\varpi}(p)\tau_{\text{đ}\mathcal{G}}(p))^2 + (\tau_{\varpi}(q)\tau_{\text{đ}\mathcal{G}}(q))^2} + \sqrt{(\tau_{\varpi}(q)\tau_{\text{đ}\mathcal{G}}(q))^2 + (\tau_{\varpi}(r)\tau_{\text{đ}\mathcal{G}}(r))^2}$$
$$+ \sqrt{(\tau_{\varpi}(r)\tau_{\text{đ}\mathcal{G}}(r))^2 + (\tau_{\varpi}(s)\tau_{\text{đ}\mathcal{G}}(s))^2} + \sqrt{(\tau_{\varpi}(s)\tau_{\text{đ}\mathcal{G}}(s))^2 + (\tau_{\varpi}(t)\tau_{\text{đ}\mathcal{G}}(t))^2}$$
$$+ \sqrt{(\tau_{\varpi}(t)\tau_{\text{đ}\mathcal{G}}(t))^2 + (\tau_{\varpi}(q)\tau_{\text{đ}\mathcal{G}}(q))^2}$$

$$TSOI(\mathcal{G}) = \sqrt{((0.7)(0.6))^2 + ((0.6)(1))^2} + \sqrt{((0.6)(1))^2 + ((0.5)(0.8))^2} + \sqrt{((0.5)(0.8))^2 + ((0.5)(0.9))^2}$$
$$+ \sqrt{((0.5)(0.9))^2 + ((0.8)(1.1))^2} + \sqrt{((0.8)(1.1))^2 + ((0.6)(1))^2}$$

$$TSOI(\mathcal{G}) = 4.6069 \tag{3.24}$$

$$ISOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta))^2}$$
$$= \sqrt{(\imath_{\varpi}(p)\imath_{d\mathcal{G}}(p))^2 + (\imath_{\varpi}(q)\imath_{d\mathcal{G}}(q))^2} + \sqrt{(\imath_{\varpi}(q)\imath_{d\mathcal{G}}(q))^2 + (\imath_{\varpi}(r)\imath_{d\mathcal{G}}(r))^2}$$
$$+ \sqrt{(\imath_{\varpi}(r)\imath_{d\mathcal{G}}(r))^2 + (\imath_{\varpi}(s)\imath_{d\mathcal{G}}(s))^2} + \sqrt{(\imath_{\varpi}(s)\imath_{d\mathcal{G}}(s))^2 + (\imath_{\varpi}(t)\imath_{d\mathcal{G}}(t))^2}$$
$$+ \sqrt{(\imath_{\varpi}(t)\imath_{d\mathcal{G}}(t))^2 + (\imath_{\varpi}(q)\imath_{d\mathcal{G}}(q))^2}$$
$$ISOI(\mathcal{G}) = \sqrt{((0.6)(0.4))^2 + ((0.5)(0.6))^2} + \sqrt{((0.5)(0.7))^2 + ((0.4)(0.8))^{0.7}} + \sqrt{((0.4)(0.7))^2 + ((0.4)(0.7))^2}$$
$$+ \sqrt{((0.4)(0.7))^2 + ((0.7)(0.9))^2} + \sqrt{((0.7)(0.9))^2 + ((0.5)(0.9))^2}$$

$$ISOI(\mathcal{G}) = 2.7387 \tag{3.25}$$

and

$$FSOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha))^2 + (F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta))^2}$$
$$= \sqrt{(F_{\varpi}(p)F_{d\mathcal{G}}(p))^2 + (F_{\varpi}(q)F_{d\mathcal{G}}(q))^2} + \sqrt{(F_{\varpi}(q)F_{d\mathcal{G}}(q))^2 + (F_{\varpi}(r)F_{d\mathcal{G}}(r))^2}$$
$$+ \sqrt{(F_{\varpi}(r)F_{d\mathcal{G}}(r))^2 + (F_{\varpi}(s)F_{d\mathcal{G}}(s))^2} + \sqrt{(F_{\varpi}(s)F_{d\mathcal{G}}(s))^2 + (F_{\varpi}(t)F_{d\mathcal{G}}(t))^2}$$
$$+ \sqrt{(F_{\varpi}(t)F_{d\mathcal{G}}(t))^2 + (F_{\varpi}(q)F_{d\mathcal{G}}(q))^2}$$
$$FSOI(\mathcal{G}) = \sqrt{((0.4)(0.5))^2 + ((0.4)(1))^2} + \sqrt{((0.4)(1))^2 + ((0.3)(1))^{0.7}} + \sqrt{((0.3)(1))^2 + ((0.3)(0.5))^2}$$
$$+ \sqrt{((0.3)(0.5))^2 + ((0.4)(0.1))^2} + \sqrt{((0.4)(1))^2 + ((0.4)(1))^2}$$

$$FSOI(\mathcal{G}) = 2.7387 \tag{3.26}$$

Substitute (3.24), (3.25) and (3.26) in (3.23), we get

$$NSOI(\mathcal{G}) = 4.6069 + 2.7387 + 2.7387$$
$$NSOI(\mathcal{G}) = 9.2297.$$

Now consider the first Zagreb Index of neutrosophic graphs.

$$NFZI(G) = TFZI(G) + IFZI(G) + FFZI(G) \tag{3.27}$$

$$TFZI(G) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} [\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha) + \tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta)]$$
$$= (\tau_{\varpi}(p)\tau_{d\mathcal{G}}(p) + \tau_{\varpi}(q)\tau_{d\mathcal{G}}(q)) + (\tau_{\varpi}(q)\tau_{d\mathcal{G}}(q) + \tau_{\varpi}(r)\tau_{d\mathcal{G}}(r)) + (\tau_{\varpi}(r)\tau_{d\mathcal{G}}(r)$$

$$+\tau_{\varpi}(s)\tau_{d\mathcal{G}}(s)) + (\tau_{\varpi}(s)\tau_{d\mathcal{G}}(s) + \tau_{\varpi}(t)\tau_{d\mathcal{G}}(t)) + (\tau_{\varpi}(t)\tau_{d\mathcal{G}}(t) + \tau_{\varpi}(q)\tau_{d\mathcal{G}}(q))$$
$$= ((0.7)(0.6) + (0.6)(1)) + ((0.6)(1) + (0.5)(0.8)) + ((0.5)(0.8) + (0.5)(0.9))$$
$$+ ((0.5)(0.9) + (0.8)(1.1)) + ((0.8)(1.1) + (0.6)(1))$$

$$TFZI(G) = 6.8 \tag{3.28}$$

$$\begin{aligned} IFZI(G) = &\sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} [\imath_{\varpi}(\alpha)\imath_{d\mathcal{G}}(\alpha) + \imath_{\varpi}(\beta)\imath_{d\mathcal{G}}(\beta)] \\ &= (\imath_{\varpi}(p)\imath_{d\mathcal{G}}(p) + \imath_{\varpi}(q)\imath_{d\mathcal{G}}(q)) + (\imath_{\varpi}(q)\imath_{d\mathcal{G}}(q) + \imath_{\varpi}(r)\imath_{d\mathcal{G}}(r)) + (\imath_{\varpi}(r)\imath_{d\mathcal{G}}(r) \\ &+ \imath_{\varpi}(s)\imath_{d\mathcal{G}}(s)) + (\imath_{\varpi}(s)\imath_{d\mathcal{G}}(s) + \imath_{\varpi}(t)\imath_{d\mathcal{G}}(t)) + (\imath_{\varpi}(t)\imath_{d\mathcal{G}}(t) + \imath_{\varpi}(q)\imath_{d\mathcal{G}}(q)) \\ &= ((0.6)(0.4) + (0.5)(0.6)) + ((0.5)(0.7) + (0.4)(0.8)) + ((0.4)(0.7) + (0.4)(0.7)) \\ &+ ((0.4)(0.7) + (0.7)(0.9)) + ((0.7)(0.9) + (0.5)(0.9)) \end{aligned}$$

$$IFZI(G) = 3.87 \tag{3.29}$$

$$\begin{aligned} FFZI(G) = &\sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} [F_{\varpi}(\alpha)F_{d\mathcal{G}}(\alpha) + F_{\varpi}(\beta)F_{d\mathcal{G}}(\beta)] \\ &= (F_{\varpi}(p)F_{d\mathcal{G}}(p) + F_{\varpi}(q)F_{d\mathcal{G}}(q)) + (F_{\varpi}(q)F_{d\mathcal{G}}(q) + F_{\varpi}(r)F_{d\mathcal{G}}(r)) + (F_{\varpi}(r)F_{d\mathcal{G}}(r) \\ &+ F_{\varpi}(s)F_{d\mathcal{G}}(s)) + (F_{\varpi}(s)F_{d\mathcal{G}}(s) + F_{\varpi}(t)F_{d\mathcal{G}}(t)) + (F_{\varpi}(t)F_{d\mathcal{G}}(t) + F_{\varpi}(q)F_{d\mathcal{G}}(q)) \\ FFZI(G) &= ((0.4)(0.5) + (0.4)(1)) + ((0.4)(1) + (0.3)(1)) + ((0.3)(1) + (0.3)(0.5)) + ((0.3)(0.5) \\ &+ (0.4)(0.1)) + ((0.4)(1) + (0.4)(1)) \end{aligned}$$

$$FFZI(G) = 3.04 \tag{3.30}$$

Substitute (3.28), (3.29) and (3.30) in (3.27), we get

$$NFZI(G) = 6.8 + 3.87 + 3.04$$
$$NFZI(G) = 13.71.$$

Thus, cleary $NSOI(\mathcal{G}) < NFZI(G)$.

Theorem 5. *let $\mathcal{G} = (\mathcal{V}, \varpi, \rho)$ be a n-vertex $\mathcal{NG}$ with m-edges. Then $NSOI(\mathcal{G}) \geq NSOI(\mathcal{G} - e)$, where $e \in \mathcal{E}(\mathcal{G})$.*

Proof. Let $\mathcal{G} = (\mathcal{V}, \mathcal{W}, \mathcal{P})$ be a $\mathcal{NG}$ and $\mathcal{H} = \mathcal{G} - e$ is a graph obtained by removing an edge $e \in \mathcal{E}(\mathcal{G})$. The $\mathcal{MV}$s in $\mathcal{G}$ and H are given by the relationship.
$\tau_{\varpi G}(\beta) \geq \tau_{\varpi H}(\beta), \imath_{\varpi G}(\beta) \geq \imath_{\varpi H}(\beta)$ and $F_{\varpi G}(\beta) \geq F_{\varpi H}(\beta)$ and $\tau_{\rho\mathcal{G}}(\varrho) \geq \tau_{\rho\mathcal{H}}(\varrho), \imath_{\rho\mathcal{G}}(\varrho) \geq \imath_{\rho\mathcal{H}}(\varrho)$ and $F_{\rho\mathcal{G}}(\varrho) \geq F_{\rho\mathcal{H}}(\varrho)$. This show that $\tau_{d\mathcal{G}}(\beta) \geq \tau_{d\mathcal{H}}(\beta), \imath_{d\mathcal{G}}(\beta) \geq \imath_{d\mathcal{H}}(\beta)$ and $F_{d\mathcal{G}}(\beta) \geq F_{d\mathcal{H}}(\beta)$. Now,

$$NSOI(\mathcal{G}) = TSOI(\mathcal{G}) + ISOI(\mathcal{G}) + FSOI(\mathcal{G}) \tag{3.31}$$

$$TSOI(\mathcal{G}) = \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{d\mathcal{G}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{d\mathcal{G}}(\beta))^2}$$

$$\geq \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{đ\mathcal{H}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{đ\mathcal{H}}(\beta))^2}$$
$$= TSOI(\mathcal{H})$$

$$= TSOI(\mathcal{G} - e) \tag{3.32}$$

$$\begin{aligned} ISOI(\mathcal{G}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{đ\mathcal{G}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{đ\mathcal{G}}(\beta))^2} \\ &\geq \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{đ\mathcal{H}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{đ\mathcal{H}}(\beta))^2} \\ &= ISOI(\mathcal{H}) \end{aligned}$$

$$ISOI(\mathcal{G}) = ISOI(\mathcal{G} - e) \tag{3.33}$$

$$\begin{aligned} FSOI(\mathcal{G}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{đ\mathcal{G}}(\alpha))^2 + (F_{\varpi}(\beta)F_{đ\mathcal{G}}(\beta))^2} \\ &\geq \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{đ\mathcal{H}}(\alpha))^2 + (F_{\varpi}(\beta)F_{đ\mathcal{H}}(\beta))^2} \\ &= FSOI(\mathcal{H}) \end{aligned}$$

$$FSOI(\mathcal{G}) = FSOI(\mathcal{G} - e) \tag{3.34}$$

Substitute (3.32), (3.33) and (3.34) in (3.31), we get
$NSOI(\mathcal{G}) \geq NSOI(\mathcal{G} - \varrho)$.

Example 3. *Form the Example 2, we can obtained,* $\mathcal{NSPI}(\mathcal{G})$= *9.2297.*

Now, Let $\mathcal{H} = \mathcal{G} - \{bc\}$ be a graph obtained by removing an edge $bc \in \mathcal{E}(\mathcal{G})$. The $\mathcal{MV}$s of the vertices of $\mathcal{H}$ will remain same as in $\mathcal{G}$ but there is a change in degree of the b and c in $\mathcal{H}$. Then $đ_G(b) = (1.2, 0.9, 0.9)$ and then $đ_G(c) = (0.4, 0.3, 0.5)$. Now,

$$NSOI(\mathcal{G}) = TSOI(\mathcal{H}) + ISOI(\mathcal{H}) + FSOI(\mathcal{H}) \tag{3.35}$$

$$\begin{aligned} TSOI(\mathcal{G}) &= \sum_{\alpha,\beta\in\mathcal{E}(\mathcal{H})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{đ\mathcal{H}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{đ\mathcal{H}}(\beta))^2} \\ &= \sqrt{(\tau_{\varpi}(a)\tau_{đ\mathcal{H}}(a))^2 + (\tau_{\varpi}(b)\tau_{đ\mathcal{H}}(b))^2} + \sqrt{(\tau_{\varpi}(b)\tau_{đ\mathcal{H}}(b))^2 + (\tau_{\varpi}(e)\tau_{đ\mathcal{H}}(e))^2} \\ &+ \sqrt{(\tau_{\varpi}(\varrho)\tau_{đ\mathcal{H}}(\varrho))^2 + (\tau_{\varpi}(d)\tau_{đ\mathcal{H}}(d))^2} + \sqrt{(\tau_{\varpi}(d)\tau_{đ\mathcal{H}}(d))^2 + (\tau_{\varpi}(c)\tau_{đ\mathcal{H}}(c))^2} \\ TSOI(\mathcal{G}) &= \sqrt{((0.7)(0.6))^2 + ((0.6)(1.2))^2} + \sqrt{((0.6)(1.2))^2 + ((0.8)(1.1))^2} \end{aligned}$$

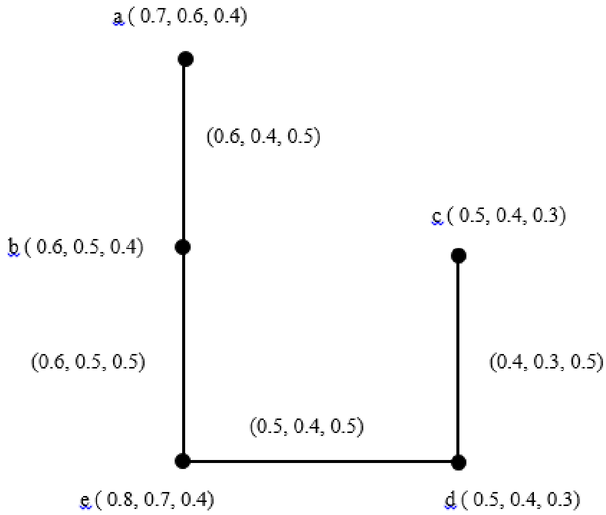

Figure 3: $\mathcal{NG}$ on 5-vertices

$$+\sqrt{((0.8)(1.1))^2+((0.5)(0.9))^2}+\sqrt{((0.5)(0.9))^2+((0.5)(0.4))^2}$$
$$=0.833+1.137+0.988+0.4924$$

$$TSOI(\mathcal{G})=3.4504 \tag{3.36}$$

$$ISOI(\mathcal{G})=\sum_{\alpha,\beta\in\mathcal{E}(\mathcal{H})}\sqrt{(\imath_{\varpi}(\alpha)\tau_{\text{đ}\mathcal{H}}(\alpha))^2+(\imath_{\varpi}(\beta)\imath_{\text{đ}\mathcal{H}}(\beta))^2}$$
$$=\sqrt{(\imath_{\varpi}(a)\imath_{\text{đ}\mathcal{H}}(a))^2+(\imath_{\varpi}(b)\imath_{\text{đ}\mathcal{H}}(b))^2}+\sqrt{(\imath_{\varpi}(b)\imath_{\text{đ}\mathcal{H}}(b))^2+(\imath_{\varpi}(e)\imath_{\text{đ}\mathcal{H}}(e))^2}$$

$$+\sqrt{(\imath_{\varpi}(e)\imath_{\text{đ}\mathcal{H}}(e))^2+(\imath_{\varpi}(d)\imath_{\text{đ}\mathcal{H}}(d))^2}+\sqrt{(\imath_{\varpi}(d)\imath_{\text{đ}\mathcal{H}}(d))^2+(\imath_{\varpi}(c)\imath_{\text{đ}\mathcal{H}}(c))^2}$$
$$ISOI(\mathcal{G})=\sqrt{((0.6)(0.4))^2+((0.5)(0.9))^2}+\sqrt{((0.5)(0.9))^2+((0.7)(0.9))^2}$$
$$+\sqrt{((0.7)(0.9))^2+((0.4)(0.7))^2}+\sqrt{((0.4)(0.7))^2+((0.4)(0.3))^2}$$
$$=0.51+0.774+0.689+0.3046$$

$$ISOI(\mathcal{G})=2.2776 \tag{3.37}$$

$$FSOI(\mathcal{G})=\sum_{\alpha,\beta\in\mathcal{E}(\mathcal{H})}\sqrt{(F_{\varpi}(\alpha)\tau_{\text{đ}\mathcal{H}}(\alpha))^2+(F_{\varpi}(\beta)F_{\text{đ}\mathcal{H}}(\beta))^2}$$
$$=\sqrt{(F_{\varpi}(a)F_{\text{đ}\mathcal{H}}(a))^2+(F_{\varpi}(b)F_{\text{đ}\mathcal{H}}(b))^2}+\sqrt{(F_{\varpi}(b)F_{\text{đ}\mathcal{H}}(b))^2+(F_{\varpi}(e)F_{\text{đ}\mathcal{H}}(e))^2}$$
$$+\sqrt{(F_{\varpi}(e)F_{\text{đ}\mathcal{H}}(e))^2+(F_{\varpi}(d)F_{\text{đ}\mathcal{H}}(d))^2}+\sqrt{(F_{\varpi}(d)F_{\text{đ}\mathcal{H}}(d))^2+(F_{\varpi}(c)F_{\text{đ}\mathcal{H}}(c))^2}$$

$$\begin{aligned}FSOI(\mathcal{G}) &= \sqrt{((0.4)(0.5))^2 + ((0.4)(1.0))^2} + \sqrt{((0.4)(1.0))^2 + ((0.4)(1))^2} \\ &\quad + \sqrt{((0.4)(1))^2 + ((0.3)(1))^2} + \sqrt{((0.3)(1))^2 + ((0.3)(0.5))^2} \\ &= 0.447 + 0.565 + 0.5 + 0.334\end{aligned}$$

$$FSOI(\mathcal{G}) = 1.846 \tag{3.38}$$

Substitute (3.36), (3.37) and (3.38) in (3.35), we get

$$\begin{aligned}NZI(G) &= 3.4504 + 2.2776 + 1.846 \\ NZI(G) &= 7.574.\end{aligned}$$

Thus clearly $NSOI(\mathcal{G}) > NSOI(\mathcal{G}) - e$.

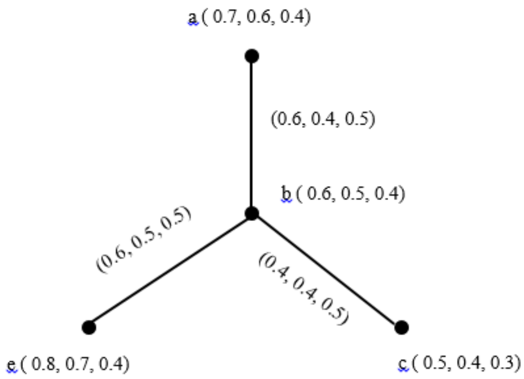

Figure 4: Neutrosophic Graph $\mathcal{G} - \{bc\}$

Theorem 6. *Let $\mathcal{G} = (\mathcal{V}, \varpi, \rho)$ be a n-vertex neutrosophic graph. Then $NSOI(\mathcal{G}) \geq NSOI(\mathcal{G} - \beta)$, where $\beta \in \mathcal{V}(\mathcal{G})$.*

Proof. Let $\mathcal{G} = (\mathcal{V}, \varpi, \rho)$ be a $\mathcal{NG}$ and $H = \mathcal{G} - \beta$ is a graph obtained by removing an edge $\beta \in \mathcal{V}(\mathcal{G})$. The $\mathcal{MV}$s in $\mathcal{G}$ and $\mathcal{H}$ are given by the relationship.
$\tau_{\varpi G}(\beta) \geq \tau_{\varpi H}(\beta), \imath_{\varpi G}(\beta) \geq \imath_{\varpi H}(\beta)$ and $\digamma_{\varpi G}(\beta) \geq \digamma_{\varpi H}(\beta)$ and $\tau_{\rho\mathcal{G}}(\varrho) \geq \tau_{\rho\mathcal{H}}(\varrho), \imath_{\rho\mathcal{G}}(\varrho) \geq \imath_{\rho\mathcal{H}}(\varrho)$ and $\digamma_{\rho\mathcal{G}}(\varrho) \geq \digamma_{\rho\mathcal{H}}(\varrho)$. Now, This show that $\text{đ}_{\mathcal{G}}(\beta) \geq \text{đ}_{\mathcal{H}}(\beta)$ and $\text{đ}_{\mathcal{G}}(\rho) \geq \text{đ}_{\mathcal{H}}(\rho)$

$$NSOI(\mathcal{G}) = TSOI(\mathcal{H}) + ISOI(\mathcal{H}) + FSOI(\mathcal{H}) \tag{3.39}$$

$$\begin{aligned}TSOI(\mathcal{G}) &= \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{G})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{\text{đ}\mathcal{H}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{\text{đ}\mathcal{H}}(\beta))^2} \\ &\geq \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{H})} \sqrt{(\tau_{\varpi}(\alpha)\tau_{\text{đ}\mathcal{H}}(\alpha))^2 + (\tau_{\varpi}(\beta)\tau_{\text{đ}\mathcal{H}}(\beta))^2} \\ &= TSOI(\mathcal{H})\end{aligned}$$

$$TSOI(\mathcal{G}) = TSOI(\mathcal{G} - \beta) \tag{3.40}$$

$$\begin{aligned}
ISOI(\mathcal{G}) &= \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{G})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{\text{đ}\mathcal{H}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{\text{đ}\mathcal{H}}(\beta))^2} \\
&\geq \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{H})} \sqrt{(\imath_{\varpi}(\alpha)\imath_{\text{đ}\mathcal{H}}(\alpha))^2 + (\imath_{\varpi}(\beta)\imath_{\text{đ}\mathcal{H}}(\beta))^2} \\
&= ISOI(\mathcal{H})
\end{aligned}$$

$$ISOI(\mathcal{G}) = ISOI(\mathcal{G} - \beta) \tag{3.41}$$

$$\begin{aligned}
FSOI(\mathcal{G}) &= \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{G})} \sqrt{(F_{\varpi}(\alpha)F_{\text{đ}\mathcal{H}}(\alpha))^2 + (F_{\varpi}(\beta)F_{\text{đ}\mathcal{H}}(\beta))^2} \\
&\geq \sum_{\alpha,\beta \in \mathcal{E}(\mathcal{H})} \sqrt{(F_{\varpi}(\alpha)F_{\text{đ}\mathcal{H}}(\alpha))^2 + (F_{\varpi}(\beta)F_{\text{đ}\mathcal{H}}(\beta))^2} \\
&= FSOI(\mathcal{H})
\end{aligned}$$

$$FSOI(\mathcal{G}) = FSOI(\mathcal{G} - \beta) \tag{3.42}$$

Substitute (3.40), (3.41) and (3.42) in (3.39), we get

$$\begin{aligned}
&NSOI(\mathcal{G}) = TSOI(\mathcal{G} - \beta) + ISOI(\mathcal{G} - \beta) + FSOI(\mathcal{G} - \beta) \\
&NSOI(\mathcal{G}) > NSOI(\mathcal{G} - \beta)
\end{aligned}$$

Thus clearly $NSOI(\mathcal{G}) > NSOI(\mathcal{G} - \beta)$.

Example 4. *Let $\mathcal{H} = \mathcal{G} - \{d\}$ be a $\mathcal{NG}$ obtained by removing a vertex đ from figure 4. Clearly, the memberships values of $\mathcal{H}$ remains same for the vertices and there is a change in degree of vertices e and c. Then $\text{đ}_{\mathcal{H}}(c) = (0.4, 0.4, 0.5)$ and $\text{đ}_{\mathcal{H}}(\varrho) = (0.6, 0.7, 0.4)$.*

$$NSOI(\mathcal{G}) = TSOI(\mathcal{H}) + ISOI(\mathcal{H}) + FSOI(\mathcal{H}) \tag{3.43}$$

$$\begin{aligned}
TSOI(\mathcal{H}) &= \sqrt{(\tau_{\varpi}(a)\tau_{\text{đ}\mathcal{H}}(a))^2 + (\tau_{\varpi}(b)\tau_{\text{đ}\mathcal{H}}(b))^2} + \sqrt{(\tau_{\varpi}(b)\tau_{\text{đ}\mathcal{H}}(b))^2 + (\tau_{\varpi}(c)\tau_{\text{đ}\mathcal{H}}(c))^2} \\
&+ \sqrt{(\tau_{\varpi}(b)\tau_{\text{đ}\mathcal{H}}(b))^2 + (\tau_{\varpi}(e)T\text{đ}_{\mathcal{H}}(e))^2} \\
&= \sqrt{((0.7)(0.6))^2 + ((0.6)(1.6))^2} + \sqrt{((0.6)(1.6))^2 + ((0.6)(1.6))^2} + \sqrt{((0.5)(0.4))^2 + ((0.8)(0.6))^2} \\
&= 1.047 + 0.9806 + 0.52
\end{aligned}$$

$$TSOI(\mathcal{H}) = 2.5476 \tag{3.44}$$

$$
\begin{aligned}
ISOI(\mathcal{H}) &= \sqrt{(\imath_{\varpi}(a)\imath_{\text{đ}\mathcal{H}}(a))^2 + (\imath_{\varpi}(b)\imath_{\text{đ}\mathcal{H}}(b))^2} + \sqrt{(\imath_{\varpi}(b)\imath_{\text{đ}\mathcal{H}}(b))^2 + (\imath_{\varpi}(c)\imath_{\text{đ}\mathcal{H}}(c))^2} \\
&\quad + \sqrt{(\imath_{\varpi}(b)\imath_{\text{đ}\mathcal{H}}(b))^2 + (\imath_{\varpi}(e)\imath_{\text{đ}\mathcal{H}}(e))^2} \\
&= \sqrt{((0.6)(0.4))^2 + ((0.5)(1.3))^2} + \sqrt{((0.5)(1.3))^2 + ((0.4)(0.4))^2} + \sqrt{((0.5)(1.3))^2 + ((0.7)(0.5))^2} \\
&= 0.6928 + 0.6694 + 0.7382
\end{aligned}
$$

$$ISOI(\mathcal{H}) = 2.1004 \tag{3.45}$$

$$
\begin{aligned}
FSOI(\mathcal{H}) &= \sqrt{(F_{\varpi}(a)F_{\text{đ}\mathcal{H}}(a))^2 + (F_{\varpi}(b)F_{\text{đ}\mathcal{H}}(b))^2} + \sqrt{(F_{\varpi}(b)F_{\text{đ}\mathcal{H}}(b))^2 + (F_{\varpi}(c)F_{\text{đ}\mathcal{H}}(c))^2} \\
&\quad + \sqrt{(F_{\varpi}(b)F_{\text{đ}\mathcal{H}}(b))^2 + (F_{\varpi}(e)F_{\text{đ}\mathcal{H}}(e))^2} \\
&= \sqrt{((0.4)(0.5))^2 + ((0.4)(1.4))^2} + \sqrt{((0.4)(1.4))^2 + ((0.3)(0.5))^2} + \sqrt{((0.4)(1.4))^2 + ((0.4)(0.5))^2} \\
&= 0.5946 + 0.5797 + 0.5946
\end{aligned}
$$

$$FSOI(\mathcal{H}) = 1.7689.............(44). \tag{3.46}$$

Substitute (3.44), (3.45) and (3.46) in (3.43), we get

$$
\begin{aligned}
NSOI(\mathcal{G}) &= 2.5476 + 2.1004 + 1.7689 \\
&= 6.4169 \\
NSOI(\mathcal{G}) &> NSOI(\mathcal{G} - \alpha)
\end{aligned}
$$

Thus clearly $NSOI(\mathcal{G}) > NSOI(\mathcal{G} - \alpha)$.

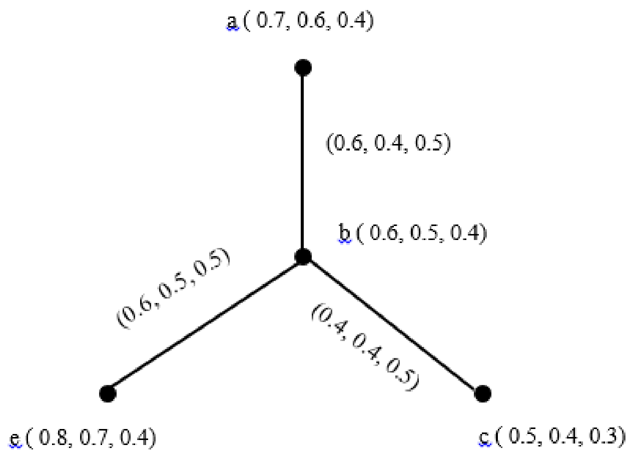

Figure 5: Neutrosophic Graph $\mathcal{G} - \{d\}$

4. Site Selection for Thermal Power Plant by using $\mathcal{SOI}$ in $\mathcal{NG}s$

Now a day without power plants, people wouldn't have reliable access to electricity, which would make it difficult to live and work in the ways we're accustomed to today. Power plants help distribute energy to large populations, making it possible for society to function smoothly and efficiently.

Selecting an appropriate location for a thermal power plant is a crucial choice that affects the facility's overall performance, operational expenses, and efficiency over time. To make sure that the chosen site satisfies all technical and financial needs while also taking social and environmental aspects into account, a thorough examination of many different elements must be conducted. Finding a site that fully satisfies every ideal criteria is difficult, but the objective is to choose a location that gives the best possible balance of these characteristics, assuring the plant's long-term survival and financial rationale by using neutrosophic graph.

4.1. Essential Conditions for Building and Functioning:

Several essential requirements must be met for a thermal power plant to be built and operated. The soil stability at the location and the availability of water supplies are two of the most crucial elements.

1. Fuel Supply(F): One of the most important considerations when choosing a location is its closeness to a consistent fuel source. Fossil fuels including oil, gas, and coal are usually used in thermal power plants. Long-distance fuel transportation can result in considerable cost increases, which lower the plant's overall productivity and profitability. Thus, it is best to locate the facility near important fuel supplies or transit corridors like pipelines or railroads. This close proximity guarantees a consistent supply of fuel to the plant while reducing the risk of supply disruptions and transportation expenditures.

2. Soil type and geology(SG): The construction and stability of the power plant depend heavily on the site's geology and soil composition. For the plant's safety and to prevent structural problems, the foundation needs to be placed on solid ground. It is best to stay away from areas with unstable soils, such as those that are vulnerable to landslides, erosion, or subsidence. To reduce the danger of earthquakes, the location should also have little seismic risk or be devoid of seismic activity. To determine if the soil and underlying rock formations are suitable for sustaining large buildings like cooling towers, boilers and turbines, a thorough geotechnical assessment is necessary.

3. Water Availability(WA): For thermal power plants, where it is mostly utilized for steam generation and cooling, water is an essential resource. A steady and sufficient supply of water is essential to the plant's productivity and stability of operations. Thus, locations close to big bodies of water, such lakes, rivers, or reservoirs, are frequently optimal. Water availability must be weighed against environmental factors, such as the effect on aquatic ecosystems and water rights, though. Alternative cooling techniques or water sources, including seawater or treated wastewater, may be required in desert places where water is scarce.

4. Land Availability(LA): A thermal power plant needs land for auxiliary infrastructure, such as fuel storage, water treatment facilities and waste disposal sites, in addition to the primary facilities. The location should have enough room for upcoming improvements or expansions. For equipment placement and to save building expenses, the terrain should also be level or moderately sloping. The ownership and present usage of the property should also be taken into account, as clearing and acquiring land may be expensive and time-consuming.

5. Facilities for Transportation(FT): Building and running a thermal power plant requires an effective transportation infrastructure. Heavy equipment, building supplies, and plant parts need to be carried to the construction site. Fuel and other required supplies must be supplied on a regular basis once the plant is operating. Being close to ports, railroads and highways may greatly lower logistical difficulties and transportation costs. Additionally, efficient transportation networks are essential for plant staff mobility and for enabling emergency services.

In the process we can identify the order of key components by using neutrosophic $\mathcal{SOI}$ of thermal thermal power plant. The relationship among the key components like Fuel Supply, Soil type and geology, Water Availability, Land Availability and Facilities for Transportation of the thermal power plant is constructed as Neutrosophic Graphs and it is given below.

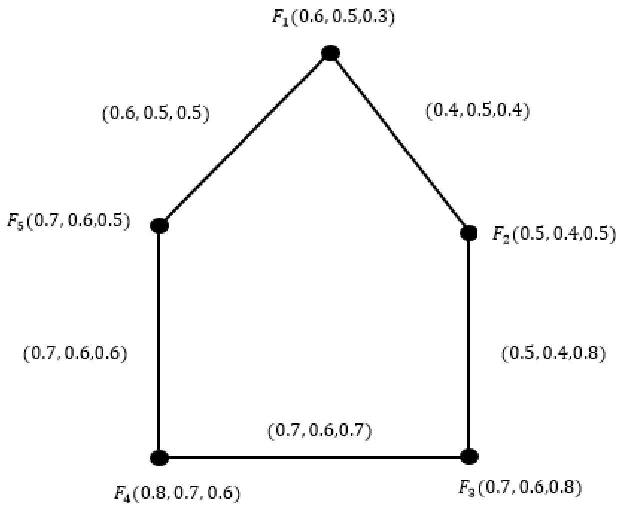

Figure 6: $\mathcal{NG}$ based on Fuel Supply(F)

$$\begin{aligned}
d(F_1) &= (1.0, 1.0, 0.9), d(F_2) = (0.9, 0.9, 1.2), d(F_3) = (1.2, 1, 1.5), d(F_4) = (1.4, 1.2, 1.3),\\
d(F_5) &= (1.3, 1.1, 1.1).\\
NSOI(\mathcal{G}) &= \sqrt{[((0.6)(1.0))^2 + ((0.5)(0.9))^2] + [((0.5)(1.0))^2 + ((0.4)(0.9))^2] + [((0.3)(0.9))^2 + ((0.5)(1.2))^2]}\\
&= \sqrt{[((0.5)(0.9))^2 + ((0.7)(1.2))^2] + [((0.4)(0.9))^2 + ((0.6)(1))^2] + [((0.5)(1.2))^2 + ((0.8)(1.5))^2]}\\
&= \sqrt{[((0.7)(1.2))^2 + ((0.8)(1.4))^2] + [((0.6)(1))^2 + ((0.7)(1.2))^2] + [((0.8)(1.5))^2 + ((0.6)(1.3))^2]}\\
&= \sqrt{[((0.8)(1.4))^2 + ((0.7)(1.3))^2] + [((0.7)(1.2))^2 + ((0.6)(1.1))^2] + [((0.6)(1.3))^2 + ((0.5)(1.1))^2]}\\
&= \sqrt{[((0.7)(1.3))^2 + ((0.6)(1.0))^2] + [((0.6)(1.1))^2 + ((0.5)(1.0))^2] + [((0.5)(1.1))^2 + ((0.3)(0.9))^2]}\\
NSOI(\mathcal{G}) &= 8.747.
\end{aligned}$$

$$\begin{aligned}
d(W_1) &= (1.5, 1.3, 1.2), d(W_2) = (1.9, 1.6, 1.6), d(W_3) = (1.2, 1, 0.9), d(W_4) = (2.0, 1.8, 1.5),\\
d(W_5) &= (1.2, 1.1, 1.0).\\
NSOI(\mathcal{G}) &= \sqrt{[((0.8)(1.5))^2 + ((0.7)(1.9))^2] + [((0.7)(1.3))^2 + ((0.8)(1.6))^2] + [((0.6)(1.2))^2 + ((0.5)(1.6))^2]}\\
&= \sqrt{[((0.7)(1.9))^2 + ((0.6)(1.9))^2] + [((0.8)(1.6))^2 + ((0.5)(1.6))^2] + [((0.5)(1.6))^2 + ((0.4)(1.6))^2]}\\
&= \sqrt{[((0.6)(1.9))^2 + ((0.9)(2))^2] + [((0.5)(1.6))^2 + ((0.7)(1.8))^2] + [((0.4)(1.6))^2 + ((0.5)(1.5))^2]}\\
&= \sqrt{[((0.9)(2))^2 + ((0.6)(1.2))^2] + [((0.7)(1.8))^2 + ((0.6)(1.1))^2] + [((0.5)(1.5))^2 + ((0.4)(1.0))^2]}\\
&= \sqrt{[((0.9)(2))^2 + ((0.8)(1.5))^2] + [((0.7)(1.8))^2 + ((0.7)(1.3))^2] + [((0.5)(1.5))^2 + ((0.6)(1.2))^2]}
\end{aligned}$$

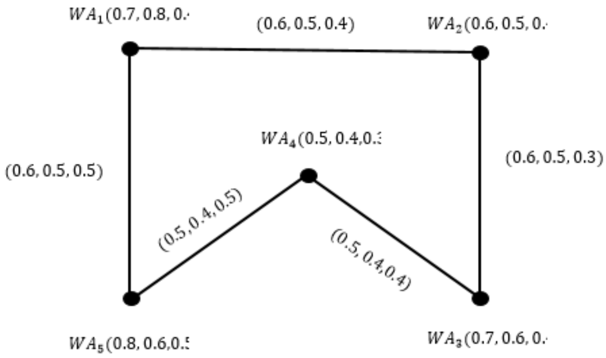

Figure 7: $\mathcal{NG}$ basesd on Water Availability(WA)

$$= \sqrt{[((0.6)(1.2))^2 + ((0.7)(1.9))^2] + [((0.7)(1.8))^2 + ((0.8)(1.6))^2] + [((0.4)(1))^2 + ((0.5)(1.6))^2]}$$

$$NSOI(\mathcal{G}) = 15.847$$

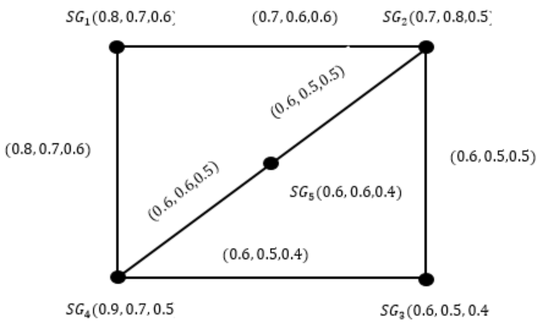

Figure 8: $\mathcal{NG}$ basesd on Soil type and geology(SG)

$$d(S_1) = (1.2, 1.0, 0.9), d(S_2) = (1.2, 1, 1.7), d(S_3) = (1.1, 0.9, 0.7), d(S_4) = (1.0, 0.8, 0.9),$$
$$d(S_5) = (1.1, 0.9, 1.0).$$

$$\begin{aligned} NSOI(\mathcal{G}) &= \sqrt{[((0.7)(1.2))^2 + ((0.6)(1.2))^2] + [((0.8)(1))^2 + ((0.5)(1))^2] + [((0.4)(0.9))^2 + ((0.4)(0.7))^2]} \\ &= \sqrt{[((0.6)(1.2))^2 + ((0.7)(1.1))^2] + [((0.5)(1))^2 + ((0.6)(0.9))^2] + [((0.4)(0.7))^2 + ((0.4)(0.7))^2]} \\ &= \sqrt{[((0.7)(1.1))^2 + ((0.5)(1))^2] + [((0.6)(0.9))^2 + ((0.4)(0.8))^2] + [((0.4)(0.7))^2 + ((0.3)(0.9))^2]} \\ &= \sqrt{[((0.5)(1))^2 + ((0.8)(1.1))^2] + [((0.4)(0.8))^2 + ((0.6)(0.9))^2] + [((0.3)(0.9))^2 + ((0.5)(1.0))^2]} \\ &= \sqrt{[((0.8)(1.1))^2 + ((0.7)(1.2))^2] + [((0.6)(0.9))^2 + ((0.8)(1))^2] + [((0.5)(1))^2 + ((0.4)(0.9))^2]} \end{aligned}$$

$$NSOI(\mathcal{G}) = 7.0375$$

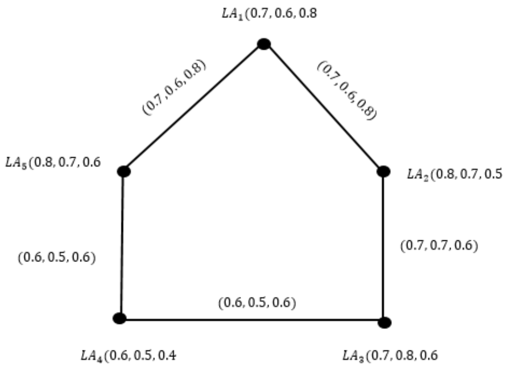

Figure 9: $\mathcal{NG}$ basesd on Land Availability(LA)

$$\begin{aligned}
&d(L_1) = (1.4, 1.2, 1.6), d(L_2) = (1.4, 1.3, 1.4), d(L_3) = (1.3, 1.2, 1.2), d(L_4) = (1.2, 1, 1.2),\\
&d(L_5) = (1.3, 1.1, 1.4).\\
NSOI(\mathcal{G}) &= \sqrt{[((0.7)(1.4))^2 + ((0.8)(1.4))^2] + [((0.6)(1.2))^2 + ((0.7)(1.3))^2] + [((0.8)(1.6))^2 + ((0.5)(1.4))^2]}\\
&= \sqrt{[((0.8)(1.4))^2 + ((0.7)(1.3))^2] + [((0.7)(1.3))^2 + ((0.8)(1.2))^2] + [((0.5)(1.4))^2 + ((0.6)(1.2))^2]}\\
&= \sqrt{[((0.7)(1.3))^2 + ((0.6)(1.2))^2] + [((0.8)(1.2))^2 + ((0.5)(1.0))^2] + [((0.6)(1.2))^2 + ((0.4)(1.2))^2]}\\
&= \sqrt{[((0.6)(1.2))^2 + ((0.8)(1.3))^2] + [((0.5)(1.0))^2 + ((0.7)(1.1))^2] + [((0.4)(1.2))^2 + ((0.6)(1.4))^2]}\\
&= \sqrt{[((0.8)(1.3))^2 + ((0.7)(1.4))^2] + [((0.7)(1.1))^2 + ((0.7)(1.3))^2] + [((0.6)(1.4))^2 + ((0.8)(1.6))^2]}\\
NSOI(\mathcal{G}) &= 10.6409
\end{aligned}$$

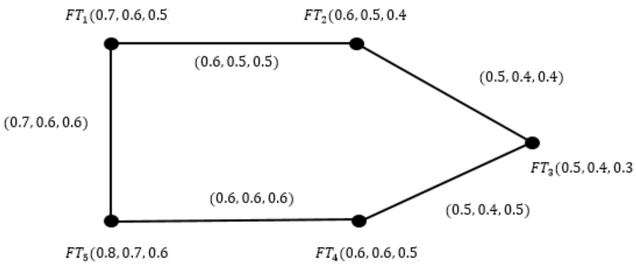

Figure 10: $\mathcal{NG}$ basesd on Facilities for Transportation(FT)

$$\begin{aligned}
&d(FT_1) = (1.3, 1.1, 1.1), d(FT_2) = (1.1, 0.9, 0.9), d(FT_3) = (1.0, 0.9, 0.7), d(FT_4) = (1.3, 1.2, 1.2),\\
&d(FT_5) = (1.3, 1.2, 1.2).\\
NSOI(\mathcal{G}) &= \sqrt{[((0.7)(1.3))^2 + ((0.6)(1.1))^2] + [((0.6)(1.1))^2 + ((0.5)(0.9))^2] + [((0.5)(1.1))^2 + ((0.4)(0.9))^2]}\\
&= \sqrt{[((0.6)(1.1))^2 + ((0.5)(1.0))^2] + [((0.5)(0.9))^2 + ((0.4)(0.9))^2] + [((0.4)(0.9))^2 + ((0.3)(0.7))^2]}\\
&= \sqrt{[((0.5)(1.0))^2 + ((0.6)(1.3))^2] + [((0.4)(0.9))^2 + ((0.6)(1.2))^2] + [((0.3)(0.7))^2 + ((0.5)(1.2))^2]}
\end{aligned}$$

$$= \sqrt{[((0.6)(1.3))^2 + ((0.8)(1.3))^2] + [((0.6)(1.2))^2 + ((0.7)(1.2))^2] + [((0.5)(1.2))^2 + ((0.6)(1.2))^2]}$$
$$= \sqrt{[((0.8)(1.3))^2 + ((0.7)(1.3))^2] + [((0.7)(1.2))^2 + ((0.6)(1.1))^2] + [((0.6)(1.2))^2 + ((0.5)(1.1))^2]}$$
$NSOI(\mathcal{G}) = 7.9165$

4.2. Decision Making

Key Components	**NSOI Valus**
Fuel Supply(F)	8.747
Water Availability(WA)	15.847
Soil type and geology(SG)	7.0375
Land Availability(LA)	10.6409
Facilities for Transportation(FT)	7.9165

Table 2: Neutrosophic $\mathcal{SOI}$ Values

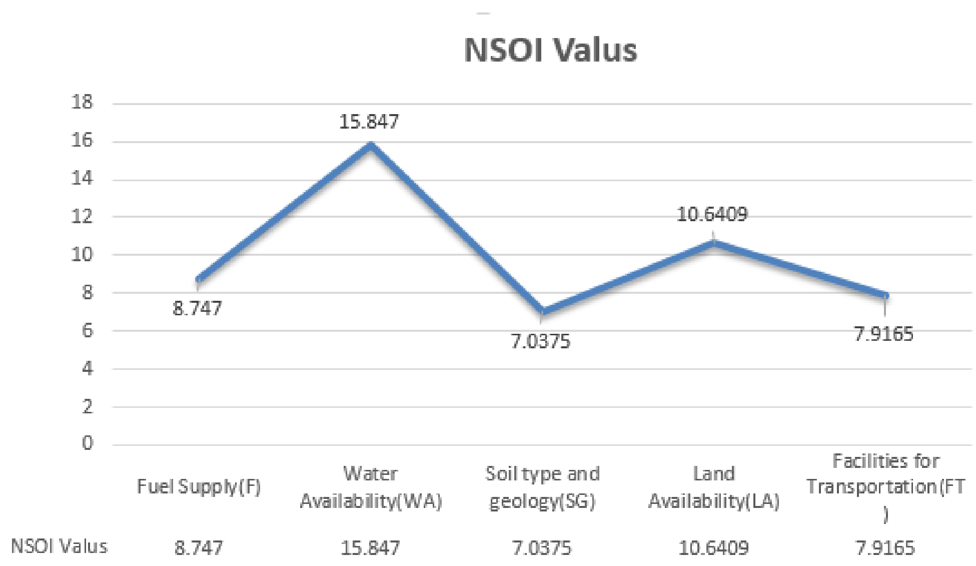

Figure 11: Graphical Representation of $\mathcal{SOI}$ Values

The Neutrosophic $\mathcal{SOI}$ values presented in table 1 Water Availability(WA) plays a vital role in selecting suitable place to establish thermal power plant. The various attributes taken into considerations to calculate $\mathcal{SOI}$ are graphically represented in Fig 10.

Since the graphical networks are selected through defining the connection between elements of the system for fuel supply, water and the kind of the soil. These factors are crucial because the firm's power plant operations depend on them as well as the structure's stability. Experiences indicate that geotechnical factors such as the type of Water Availability(WA) features are critical in the design of foundations required to support the construction of the plant in the long run.

In practical use, unsound ground presents a variety of structural hazards which is why they pose critical importance in large-scale power plant construction such as thermal power plants.

The dependencies can be represented using neutrosophic graphs and particularly when dealing with uncertainties.

4.3. Comparative Analysis

The $\mathcal{SOI}$s for the fuzzy and $\mathcal{IFG}$s have been defined, the usage is more limited, as compared to the critical path, strong, complete, complementary, wheel and star graphs. Weight for $\mathcal{MV}s$, $\mathcal{IMV}s$ and $\mathcal{NMV}s$ assigned for vertices, can be from 0 to 1 as the domain of X is defined more thoroughly over closed interval $[0, 1]$. This is similar to the Neutrosophic framework which has not been designed in this manner but is based on capturing the value of the unknown component of an expression as well as the truth value. Sombor topological Indices in $\mathcal{NG}$ theory are somewhat less formalized than crisp graph theory since each vertex can be only one value, and yet they are more versatile and can be applied wherever some decision has to be made, from fighting cyber-crime to diagnosing diseases and planning for roads. We cannot do that in crisp graph theory because vertices can have only one value assigned to them in this mathematical concept. It is also more all-encompassing from this perspective, for our work is wider and more inclusive.

4.4. Sensitivity Analysis

The evaluation of $\mathcal{NSOI}$ values, through sensitivity analysis, determines the significance of multiple factors involved in decision making for the selection of an ideal site for the establishment of thermal power plant. From Table 2, Fuel Supply (F) has $\mathcal{NSOI}$ of 8.747, Water Availability (WA)15.847, Soil Type and Geology (SG) 7.0375, Land Availability (LA) 10.6409 and Facilities for Transportation (FT) 7.9165. Due to the rather high NSOI value with respect to the factors of Soil Type and Geology (SG), it can be stated that these factors have a strong influence on selecting a site for genset construction, pointing to the fact that these conditions are critical for obtaining the required thermal power plant efficiency. The various attributes that have been used to develop the $\mathcal{NSOI}$ model have been illustrated in Fig. 11 in order to create a more informative visual context for decision makers. This analysis would put forward that SG has to be considered side by side some other significant aspects like Land Availability or Fuel Supply while screening potential locations for thermal power plant. Due to this, the decision-makers can develop solutions that can improve the energy infrastructure's robustness and endurance under conditions of uncertainty and variability of environmental factors reducing the vulnerability as noted in the research. Consequently, it enables comprehensive decision-making leading to the development of thermal power plants.

4.5. Advantages and limitations

As a result of our examination, the following are the main benefits and Limitations:

- The $\mathcal{NSOI}$ is very effective in a real-life application since it allows the use of imprecise and uncertain data.
- However, some practitioners might notice that $\mathcal{NSOI}$ is less friendly, especially in the sense that the computations may require some understanding.
- The support of multiple parameters ($\mathcal{MV}, \mathcal{IMV}$ and $\mathcal{NMV}$) enables $\mathcal{NSOI}$ to provide a more refined understanding of relations between characteristics.
- In contrast, numbers that may be objectively obtained may significantly influence the results which in turn will significantly vary from one analysis to another.

- However, its cross-disciplinary usefulness means that the $\mathcal{NSOI}$ is important; however, the lack of substantial empirical research to support the validity of the technique means that there may be worries over its reliability in terms of real-life implementation.

5. Conclusion

In this research, we proposed the $\mathcal{SOI}$ for neutrosophic graphs to measure the overall structural and communication resilience of networks under uncertainty. Apart from setting up constraints for specific neutrosophic graphs, the research posited the neutrosophic first and second Zagreb Indices. Based on our main findings, $\mathcal{SOI}$ can serve as a helpful instrument to reveal structural characteristics in conditions when their nature remains unclear. The numerical analysis revealed that the maximum value of the $\mathcal{SOI}$ corresponds to the critical components that should be given more focus when choosing the right location of thermal power plants taking into account the former's operation efficiency with the later's operational constraints. $\mathcal{NG}s$ could be employed to achieve rational criteria in the decision-making processes, particularly in complex ones. This research is important because it bridges the gap between works proposing such Indices and their feasible realizations.

5.1. Future Work

Wiener Index, Zagreb Indices, Randic type Indices, Schult type Indices, Dominating Indice of neutrosophic graph and its applications.

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