EUROPEAN JOURNAL OF PURE AND APPLIED MATHEMATICS
Vol. 17, No. 4, 2024, 3915-3931
ISSN 1307-5543 – ejpam.com
Published by New York Business Global

The Impact of White Noise on Chaotic Behavior in a Financial Fractional System with Constant and Variable Order: A Comparative Study

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Abstract. In this study, we investigated the impact of white noise on the chaotic dynamics of a financial system with Caputo fractional derivatives of constant- and variable-order. We solve these fractional financial systems numerically. We analyze the chaotic behavior of these fractional-order hyperchaotic systems through simulations, study the effects of white noise on chaotic dynamics, and provide a comparison between fractional hyperchaotic systems of constant and variable orders. The study reveals that white noise can either amplify or suppress chaos, with significant differences observed between the constant- and variable-order cases. We obtained numerous intriguing graphical findings for the model by evaluating various scenarios. The findings provide useful information for better understanding financial market dynamics in the presence of noise.

2020 Mathematics Subject Classifications: 34C28, 34A08, 65P20

Key Words and Phrases: Constant and Variable-order fractional derivatives, White Noise, hyperchaotic

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1. Introduction

Fractional calculus has garnered significant interest in the modeling of intricate systems, particularly in the field of finance, owing to its adaptable nature and memory characteristics[8, 30].Using the fractional concept, the authors investigated mathematical models and obtained intriguing findings [4, 12]. Furthermore, it is evident that additional research is required to render fractional calculus suitable for researchers worldwide. In response, researchers have been assiduously engaged in the identification and formulation of useful operators, with a particular emphasis on fractional operators. Stochastic non-linear evolution equations play a vital role in a wide range of scientific and engineering fields, allowing us to model and understand complex systems and phenomena. They analyze

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DOI: https://doi.org/10.29020/nybg.ejpam.v17i4.5462

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the impact of white noise on chaotic financial systems, taking into account randomness and nonlinearity[39]. This study investigates the effects of random disturbances on the chaotic dynamics of systems with constant-order and variable-order fractions, employing the principles of fractional calculus. It has become a popular area of research, attracting a lot of attention. Hyperchaotic System (HS) exhibits the remarkable characteristic of displaying multiple chaotic behaviors simultaneously[11, 14]. These types of systems exhibit intricate chaotic dynamics that surpass those of conventional (HS). (HS) exhibit the presence of multiple positive Lyapunov Exponents(LE), which signify a heightened level of unpredictability and complexity in their behavior[10]. The distinct properties of (HS) make them highly beneficial in a diverse range of fields that prioritize elements such as randomness and complexity[42]. A deeper comprehension of processes that depend on numerous variables is also brought about by the expansion of calculus into multivariable functions, opening the door for developments in disciplines like fluid dynamics and thermodynamics[6, 40]. Calculus's perennial relevance in the never-ending quest for knowledge and creativity is shown by the theoretical breakthroughs and real-world applications that scholars discover as they delve further into the subject[5, 13, 21].

Numerous scientific and technical domains have extensive uses for fractional stochastic equations, which combine the ideas of stochastic processes with fractional calculus. These equations are very helpful in domains like biology, physics, and finance since they are used to simulate complicated systems with unpredictability and memory effects[17, 41]. For example, they may represent price dynamics in financial markets, mimic the behavior of biological systems with long-range dependencies, and characterize anomalous diffusion processes in porous surfaces. These equations offer more precise and adaptable models for systems displaying both uncertainty and non-local behavior by fusing the tools of fractional derivatives with stochastic dynamics[3].

Over time, (HS) have made steady progress since they were first introduced by Rossler [37].Recently, researchers have made significant advances in the field of (HS), including the development of various distinct systems. One notable example is the 2D (HS) that has been proposed for image encryption [15].In addition, [45] developed a two-dimensional (HS) using the optimization benchmark function [46]. developed a 2D hyperchaotic map that has proven useful in generating pseudorandom numbers and encrypting color images. In their study, [16] introduced an image encryption algorithm that uses a 2D (HS). An innovative system for secure communications was introduced by[2]. This system utilizes a 2D cosine-sine interleaved chaotic approach. also studied a simple chaotic model with complex chaotic behavior. The number is significant[19]. conducted a study on the advancements in 3D hyperchaotic systems and their potential applications in secure transmission[24]. An ultrawide parameter range was presented by[33] for a nondegenerate hyperchaotic map. In their study, [43] introduced a hyperchaotic system that consists of four dimensions. [26]. It performed a comprehensive investigation on the dynamics of a multi-stable hyperchaotic Lorenz system and examined its several practical uses. [27]. The study conducted by [23].delved into the advancements in dynamical systems with five state variables, specifically focusing on those with multiple positive (LE). The researchers also explored the presence of coexisting hidden attractors [25].

In this paper, the concept of (HS), which is defined by fractional-order derivatives, is introduced, and the impact of white noise is investigated[29]. In addition, we elaborate on certain previous studies, including those detailed in[20]. Furthermore, we present a variety of figures using the MATLAB 2024 software to investigate the influence of noise on hyperchaotic systems (3, 4). Variable-order fractional calculus operators are a valuable mathematical instrument for a more comprehensive examination of dynamical phenomena due to their variable-order fractional order [31]. In reality, the mathematical systems modeled by this innovative concept exhibit greater sensitivity and accuracy. It is important to note that the process of obtaining an analytical solution for issues involving variable-order fractional operators is frequently exceedingly intricate. Therefore, approximative approaches are more likely to be employed to resolve such issues. In [9], the interested reader can find several numerical methods for solving fractional problems of variable order.

Introducing white noise into chaotic systems can effectively decrease their complexity and unpredictability. Studies suggest that noise has a substantial impact on chaotic dynamics, influencing the development of trajectories even when the starting conditions are the same. The unpredictability in chaotic scattering problems is further complicated by stochastic fluctuations, which can make it difficult to anticipate the ultimate states [32]. Nevertheless, research has demonstrated that low-intensity noise can decrease irregularity in chaotic patterns, resulting in a regularization effect that amplifies organization within the system [22]. Moreover, novel techniques for acquiring knowledge about chaotic systems from noise-contaminated data reveal that noise may be effectively controlled, enabling precise forecasts despite intrinsic disorder [44]. Therefore, although noise adds complexity, it might paradoxically enable more predictable and organized behavior in chaotic systems under specific circumstances. This paper presents a novel approach to obtain the constant and variable-order fractional estimates with stochasticity for the Financial Fractional Caputo (HS). In order to derive these Hyperchaotic properties with stochastic properties, we employ a novel numerical technique. Moreover, we build on previous studies, including the findings reported in the reference[7]. The generated solutions play a crucial role in elucidating compelling financial phenomena necessary for characterizing hyperchaotic propagation. Moreover, we investigate the influence of noise on the precise solutions of system (4,5) by displaying several visual representations.

Additionally, white noise is employed in the fields of chemistry and physics. For example, these circuit systems based on memcapacitors are employed in the field of physics to understand intricate systems such as turbulent flow or phase transitions. Scientists can investigate the emergence of ordered structures from random initial conditions by taking into account the stochastic nature of these phenomena. The optimization of reaction conditions and the design of efficient chemical reactors are facilitated by the use of white noise in chemical kinetics to model reaction processes that involve random fluctuations. The peculiarity of this article is the acquisition of the stochastic simulation of the systems (4-5). We employ two distinct methodologies to generate these simulations: the extended fractional constant and varibales orders of the mapping method. Additionally, we elaborate on certain prior studies, including those detailed in [36, 38]. The solutions that were generated are crucial in elucidating certain thrilling physical phenomena, as the Financial Fractional System is necessary to describe marketing propagation. Furthermore, we present a variety of figures to investigate the influence of noise on the precise solutions of the Financial Fractional Systems (4-5).

The article is organized in the following manner: This document presents definitions and notations of variable-order fractional derivatives at the basic level. For section 2. Analysis of the effects of white noise and simulations on the financial system using constant and variable-order fractional methods. An analysis of Caputo hyperchaotic systems is conducted in Section 3. Section 4 of the manuscript focuses on modeling the financial system using white noise. The final conclusion is articulated in Section 5.

2. Preliminaries

This part provides essential definitions of Caputo variable-order fractional derivatives, which will be applied in subsequent sections. These definitions are presented in this section.

Definition 1. *([35]). The beta function is given by $\beta(t)$ is defined as*

$$\beta(\mu, \lambda) = \int_0^1 t^{\mu-1}(1-t)^{\lambda-1}\, dt, \quad \mu > 0, \lambda > 0 \tag{1}$$

It is important to note that the beta function is symmetric, which means that $\beta(\mu, \lambda) = \beta(\lambda, \mu)$. In the interim, the beta function's robust correlation with the gamma function is a noteworthy attribute.

$$\beta(\mu, \lambda) = \frac{\Gamma(\mu)\Gamma(\lambda)}{\Gamma(\mu+\lambda)}$$

where ($\Gamma(.)$) denotes the Gamma function.For the Mittag-Leffler function:

Definition 2. *([35]). The Mittag–Lefer function is given by*

$$E_{\mu,\lambda}(z) = \sum_{j=0}^{\infty} \frac{z^j}{\Gamma(\mu j + \lambda)}, \quad z \in \mathbb{C}^+, \quad t \in \mathbb{R} \tag{2}$$

Note that for $\lambda = 1$, it is abbreviated as $E_\mu(t) = E_{\mu,1}(t)$.

Definition 3. *([28]). The Caputo fractional derivative with a varying order $\beta(t)$ is defined as*

$${}_0^{VO}D_t^{\beta(t)}\{\mathcal{X}(t)\} = \frac{1}{\Gamma(1-\beta(t))} \int_0^t (t-\tau)^{-\beta(t)} \mathcal{X}'(\tau)\, d\tau, \quad 0 < \beta(t) < 1. \tag{3}$$

Definition 4. *([34]). The Caputo derivative with constant order β is given as follows:*

$${}_0^{C}D_t^{\beta}\{\mathcal{X}(t)\} = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{1}{(t-w)^{\beta}} \frac{d}{dw}\mathcal{X}(w)dw, \quad 0 < \beta < 1. \tag{4}$$

Definition 5. *([18]). Hyperchaotic Systems of Constant and Variable Order: White Noise in Fractional CalculusWhite noise is a stochastic process in fractional calculus, especially in constant and variable-order hyperchaotic systems.* $W(t), \mathbb{X}[W(t)] = 0$ *characterized by a mean of zero and a delta-correlated covariance function. Mathematically, it is expressed as:*

$$\mathbb{X}[W(t)W(t')] = \sigma^2\Gamma(t - t'), \tag{5}$$

where $\mathbb{X}$ *denotes the expectation operator,* σ^2 *represents the variance of the noise,* $\delta(\cdot)$ *is the Dirac delta function, and* t *and* t' *are distinct time instances. White noise in fractional-order systems can cause disturbances that shape the dynamic behavior, perhaps resulting in hyperchaotic states, which are determined by the system's order and the strength of the noise.*

3. A Mathematical Model of the Financial Fractional System

This section discusses the hyperchaos and chaos of a new fractional-order system, touching on the effects of white noise on the predictability and stability of constant and variable-order hyperchaotic systems. Through the use of white-noise driven fractional differential equation models, this study examines the financial system[1]:

- fractional constant-order The hyperchaotic financial system of Caputo with σ is the intensity of noise; W(t) is the white noise (Gaussian process).the interest rate $\mathcal{X}(t)$. The investment demand is $\mathcal{Y}(t)$. The exponent $\mathcal{Z}(t)$ represents the price and the rate of investment.

$$\begin{aligned}
{}_0^C D_t^{\beta}\{\mathcal{X}(t)\} &= \mathcal{X}(t)(\mathcal{Y}(t) - a) + \mathcal{Z}(t) + \sigma_{\mathcal{X}} W(t),\\
{}_0^C D_t^{\beta}\{\mathcal{Y}(t)\} &= 1 - (b\mathcal{Z}(t) + \mathcal{Y}^2(t)) + \sigma_{\mathcal{Y}} W(t),\\
{}_0^C D_t^{\beta}\{\mathcal{Z}(t)\} &= -\mathcal{Y}(t) - c\mathcal{Z}(t) + \sigma_{\mathcal{Z}} W(t).
\end{aligned} \tag{6}$$

where $a \geq 0$ is the saving amount, $b \geq 0$ is the cost per investment, and $c \geq 0$ is the elasticity of demand of commercial market.

- fractional variable-order The hyperchaotic financial system of Caputo

$$\begin{aligned}
{}_0^{VO} D_t^{\beta(t)}\{\mathcal{X}(t)\} &= \mathcal{X}(t)(\mathcal{Y}(t) - a) + \mathcal{Z}(t) + \sigma_{\mathcal{X}} W(t),\\
{}_0^{VO} D_t^{\beta(t)}\{\mathcal{Y}(t)\} &= 1 - (b\mathcal{Z}(t) + \mathcal{Y}^2(t)) + \sigma_{\mathcal{Y}} W(t),\\
{}_0^{VO} D_t^{\beta(t)}\{\mathcal{Z}(t)\} &= -\mathcal{Y}(t) - c\mathcal{Z}(t) + \sigma_{\mathcal{Z}} W(t).
\end{aligned} \tag{7}$$

4. A Novel Numerical Method for Stochastic Fractional Financial Systems

Using the starting values of the variables that we will use to derive the phase portraits of the fractional-order chaotic system (4)–(5), this section outlines the numerical scheme for the numerical solution of model. We attempt to apply the standard discretizations found in the literature to our model. Numerous numerical schemes and analytical techniques, including the Chebyshev method, the domain decomposition method, and the homotopy methods, can be applied in fractional calculus. However, because of the difficulties with the stability and convergences of the approximate solutions, many of the drawbacks of the approaches mentioned have yet to be resolved. MATLAB codes, which are essential in chaotic and hyperchaotic systems, are one benefit of using the predictor-corrector approach in our system. The solution of the fractional differential system (4)–(5) can be described as follows in the remaining portion of this section Our study presents a new numerical technique for modeling fractional differential operators of both constant and variable order utilizing stochastic processes.

4.1. A numerical scheme for constant-order fractional financial system.

This study presents a numerical method for a fractional financial system that integrates a stochastic Caputo fractional derivative with a constant-order fractional.

$$ {}^{C}D^{\alpha}_{a^+}\mathcal{S}(t) = \psi_1(t, \mathcal{S}(t)) + \sigma y(t)dB(t) \tag{8}$$

$$ \mathcal{S}(t) = \mathcal{S}(0) + \frac{1}{\Gamma(\beta)}\int_0^t (t-w)^{\beta-1}\psi_1(\mathcal{S}, w)\,dw + \int_0^t \sigma y(t)dB(t) \tag{9}$$

At time $t = t_{n+1}$, Equation 9 is as follows:

$$ \mathcal{S}(t_{n+1}) = \mathcal{S}(0) + \frac{1}{\Gamma(\beta)}\int_0^{t_{n+1}} (t-w)^{\beta-1}\psi_1(\mathcal{S}, w)\,dw + \int_0^{t_{n+1}} \sigma y(t)dB(t) \tag{10}$$

At time $t = t_n$

$$ \mathcal{S}_n = \mathcal{S}(t_n) = \mathcal{S}(0) + \frac{1}{\Gamma(\beta)}\int_0^{t_n} (t-w)^{\beta-1}\psi_1(\mathcal{S}, w)\,dw + \int_0^{t_n} \sigma y(t)dB(t) \tag{11}$$

$$ S(t_{n+1}) - S(t_n) = \frac{1}{\Gamma(\alpha)}\left[\int_0^{t_{n+1}} (t_{n+1}-w)^{\alpha-1}g(S, w)\,dw - \int_0^{t_n} (t_n-w)^{\alpha-1}g(S, w)\,dw\right]. \tag{12}$$

Numerical solution of Equation (10) obtained via Lagrange polynomial interpolation is as follows:

$$ \begin{aligned} \mathcal{S}_{n+1} = \mathcal{S}_0 &+ \frac{\beta h^\beta}{\Gamma(\beta+2)}\sum_{m=0}^{r}\psi_1(t_m, \mathcal{S}_m)\left[(n-m+1)^\beta(n-m+2+2\beta) - (n-m)^\beta(n-m+2+2\beta)\right] \\ &- \frac{h^\beta}{\Gamma(\beta+2)}\sum_{m=0}^{r}\psi_1(t_{m-1}, \mathcal{S}_{m-1})\left[(n-m+1)^{\beta+1} - (n-m)^\beta(n-m+1+\beta)\right] \\ &+ \alpha\sigma y(t)(c_n)(B(t_{n+1}) - B(t_n)). \end{aligned} \tag{13}$$

We therefore present the numerical solution that we have developed for the constant-order fractional(Hs), which is as follows:

$$\begin{aligned}\mathcal{X}_{n+1} = \mathcal{X}_0 + \frac{\beta h^\beta}{\Gamma(\beta+2)} \sum_{m=0}^{r} \psi_1(t_m, \mathcal{X}_m) \Big[(n-m+1)^\beta(n-m+2+2\beta) - (n-m)^\beta(n-m+2+2\beta)\Big] \\ - \frac{h^\beta}{\Gamma(\beta+2)} \sum_{m=0}^{r} \psi_1(t_{m-1}, \mathcal{X}_{m-1}) \Big[(n-m+1)^{\beta+1} - (n-m)^\beta(n-m+1+\beta)\Big] \\ + \alpha\sigma y(t)(c_n)(B(t_{n+1}) - B(t_n)).\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{Y}_{n+1} = \mathcal{Y}_0 + \frac{\beta h^\beta}{\Gamma(\beta+2)} \sum_{m=0}^{r} \psi_1(t_m, \mathcal{Y}_m) \Big[(n-m+1)^\beta(n-m+2+2\beta) - (n-m)^\beta(n-m+2+2\beta)\Big] \\ - \frac{h^\beta}{\Gamma(\beta+2)} \sum_{m=0}^{r} \psi_1(t_{m-1}, \mathcal{Y}_{m-1}) \Big[(n-m+1)^{\beta+1} - (n-m)^\beta(n-m+1+\beta)\Big] \\ + \alpha\sigma y(t)(c_n)(B(t_{n+1}) - B(t_n)).\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{Z}_{n+1} = \mathcal{Z}_0 + \frac{\beta h^\beta}{\Gamma(\beta+2)} \sum_{m=0}^{r} \psi_1(t_m, \mathcal{Z}_m) \Big[(n-m+1)^\beta(n-m+2+2\beta) - (n-m)^\beta(n-m+2+2\beta)\Big] \\ - \frac{h^\beta}{\Gamma(\beta+2)} \sum_{m=0}^{r} \psi_1(t_{m-1}, \mathcal{Z}_{m-1}) \Big[(n-m+1)^{\beta+1} - (n-m)^\beta(n-m+1+\beta)\Big] \\ + \alpha\sigma y(t)(c_n)(B(t_{n+1}) - B(t_n)).\end{aligned} \tag{16}$$

4.2. A numerical scheme for Variable-order fractional financial system.

This study presents a numerical method for a fractional financial system that integrates a stochastic Utilizing a variable-order fractional, the Caputo fractional derivative is employed.

$$ {}^{C}D_{a^+}^{\beta(t)}\mathcal{S}(t) = \psi_1(t, \mathcal{S}(t)) + \sigma y(t)dB(t) \tag{17}$$

$$\mathcal{S}(t) = \mathcal{S}(0) + \frac{1}{\Gamma(\beta)} \int_0^t (t-w)^{\beta(t)-1}\psi_1(\mathcal{S}, w)\,dw + \int_0^t \sigma y(t)dB(t) \tag{18}$$

At time $t = t_{n+1}$, 18 takes the following form:

$$\mathcal{S}(t_{n+1}) = \mathcal{S}(0) + \frac{1}{\Gamma(\beta(t))} \int_0^{t_{n+1}} (t-w)^{\beta(t)-1}\psi_1(\mathcal{S}, w)\,dw + \int_0^{t_{n+1}} \sigma y(t)dB(t) \tag{19}$$

At $t = t_n$

$$\mathcal{S}_n = \mathcal{S}(t_n) = \mathcal{S}(0) + \frac{1}{\Gamma(\beta(t))}\int_0^{t_n}(t-w)^{\beta(t)-1}\psi_1(\mathcal{S},w)\,dw + \int_0^{t_n}\sigma y(t)dB(t) \tag{20}$$

$$\mathcal{S}(t_n+1) = \mathcal{S}(0) + \frac{1}{\Gamma(\beta(t))}\int_0^{t_n}(t-w)^{\beta(t)-1}\psi_1(\mathcal{S},w)\,dw + \int_0^{t_n}\sigma y(t)dB(t) \tag{21}$$

Numerical solution of Equation (9) obtained via Lagrange polynomial interpolation is as follows:

$$\begin{aligned}\mathcal{S}_{n+1} = \mathcal{S}_0 &+ \frac{\beta(t)h^{\beta}(t)}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_m,\mathcal{S}_m)\left[(n-m+1)^{\beta}(t)(n-m+2+2\beta)-(n-m)^{\beta}(t))\right]\\ &- \frac{h^{\beta}}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_{m-1},\mathcal{S}_{m-1})\left[(n-m+1)^{\beta(t)+1}-(n-m)^{\beta}(t)(n-m+1+\beta)\right]\\ &+ \beta(t)\sigma y(t)(c_n)(B(t_{n+1})-B(t_n)).\end{aligned} \tag{22}$$

We therefore present the numerical solution that we have developed for the variable-order fractional hyperchaotic system, which is as follows:

$$\begin{aligned}\mathcal{X}_{n+1} = \mathcal{X}_0 &+ \frac{\beta(t)h^{\beta}(t)}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_m,\mathcal{X}_m)\left[(n-m+1)^{\beta}(t)(n-m+2+2\beta)-(n-m)^{\beta}(t))\right]\\ &- \frac{h^{\beta}}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_{m-1},\mathcal{X}_{m-1})\left[(n-m+1)^{\beta(t)+1}-(n-m)^{\beta}(t)(n-m+1+\beta(t))\right]\\ &+ \alpha\sigma y(t)(c_n)(B(t_{n+1})-B(t_n)).\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{Y}_{n+1} = \mathcal{Y}_0 &+ \frac{\beta(t)h^{\beta}(t)}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_m,\mathcal{Y}_m)\left[(n-m+1)^{\beta}(t)(n-m+2+2\beta(t))-(n-m)^{\beta}(t))\right]\\ &- \frac{h^{\beta}(t)}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_{m-1},\mathcal{Y}_{m-1})\left[(n-m+1)^{\beta(t)+1}-(n-m)^{\beta}(t)(n-m+1+\beta(t))\right]\\ &+ \alpha\sigma y(t)(c_n)(B(t_{n+1})-B(t_n)).\end{aligned} \tag{24}$$

$$\begin{aligned}\mathcal{Z}_{n+1} = \mathcal{Z}_0 &+ \frac{\beta(t)h^{\beta}(t)}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_m,\mathcal{Z}_m)\left[(n-m+1)^{\beta}(t)(n-m+2+2\beta(t))-(n-m)^{\beta}(t))\right]\\ &- \frac{h^{\beta}(t)}{\Gamma(\beta(t)+2)}\sum_{m=0}^{r}\psi_1(t_{m-1},\mathcal{Z}_{m-1})\left[(n-m+1)^{\beta(t)+1}-(n-m)^{\beta}(t)(n-m+1+\beta(t))\right]\\ &+ \beta(t)\sigma y(t)(c_n)(B(t_{n+1})-B(t_n)).\end{aligned} \tag{25}$$

5. Modeling the Financial System with White Noise

In this section, we define the specific financial system under study, introducing the governing equations and parameters. The (HS) is examined both in the absence and presence of white noise. Numerical simulations are conducted to explore how white noise influences the chaotic dynamics for both constant and variable orders. From the previous Figure(1), it is evident that in the absence of noise (i.e., when $\sigma = 0$), there are no distinct types of white noise in systems (4-5). Figure(2) shows that when the noise is absent (i.e., when $\sigma = 0.2$), there are several types of white noise. Figures(3-5) demonstrates that when the noise is absent (i.e., when $\sigma = 0.5$), there are more efficient forms of white noise. Nevertheless, the presence of noise leads to the degradation of all these solutions, causing the hyperchaotic behavior to become flat as the magnitude of the noise increments.

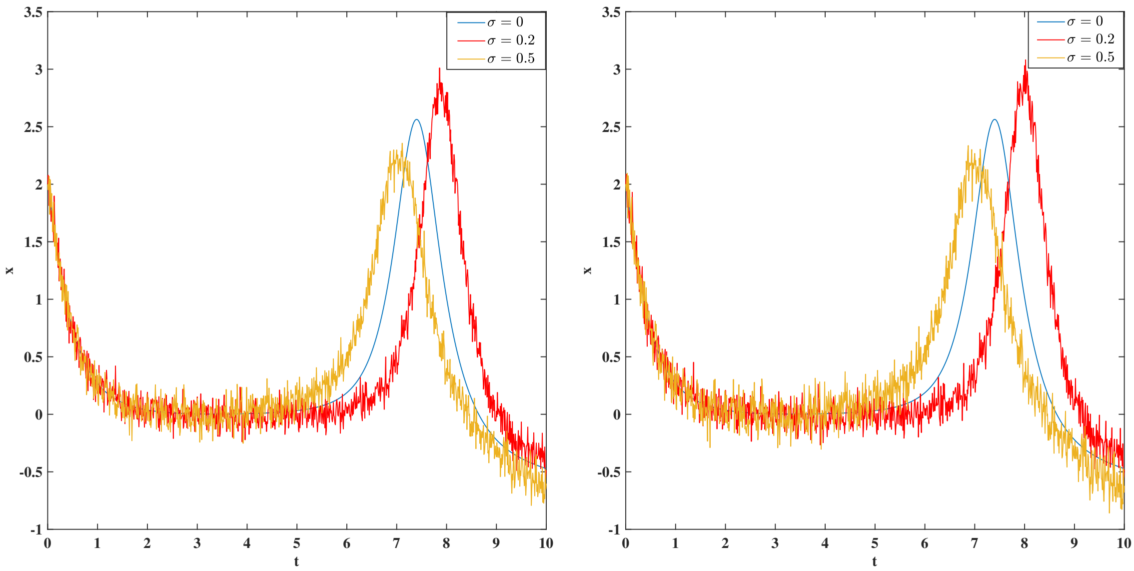

Figure 1: shows a 2D for constant and variable fractional order values for different $\sigma = 0, 0.2, 0.5$.

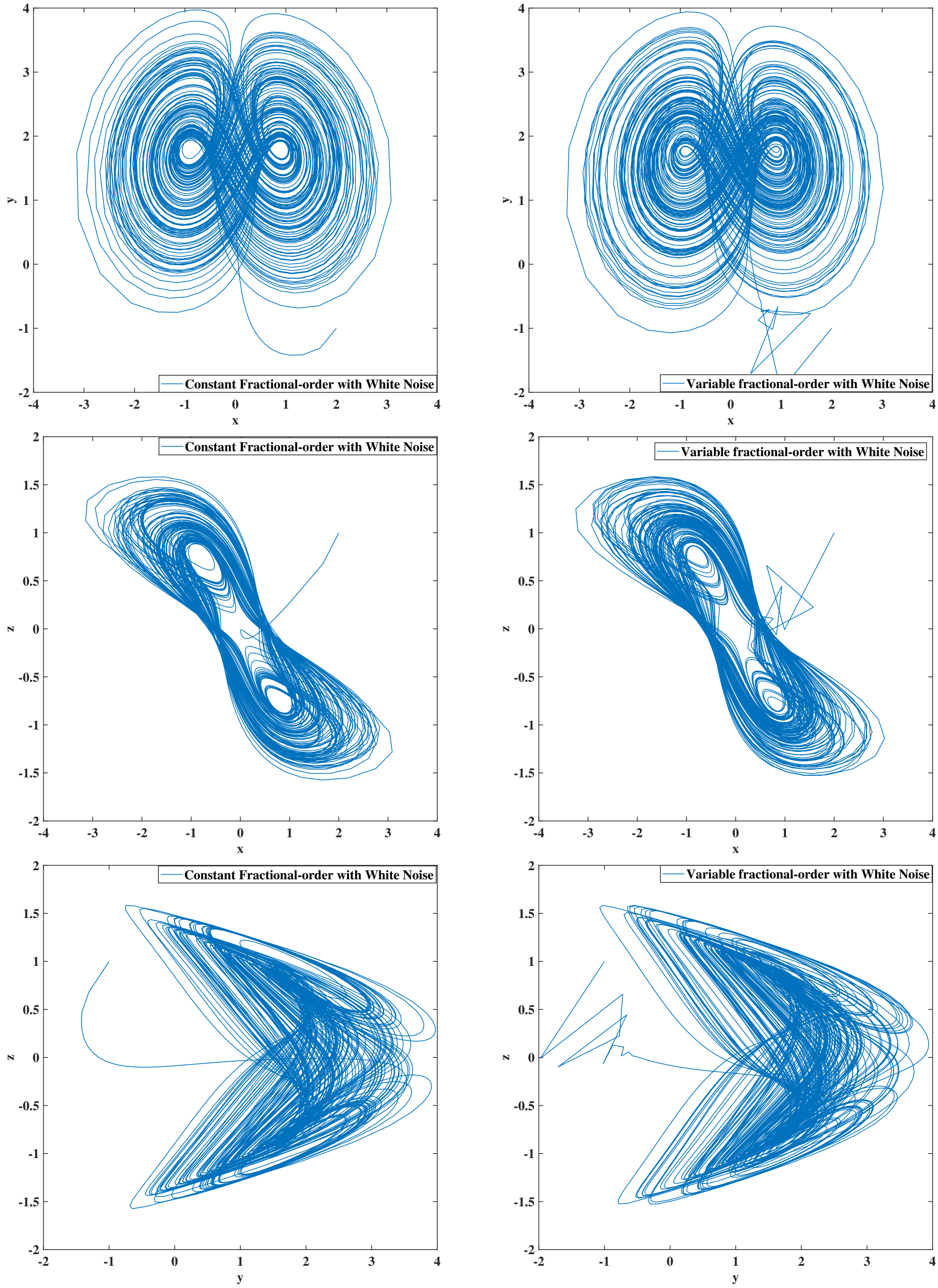

Figure 2: present Comparison between Constant and Variable Fractional Order at $\sigma = 0$.

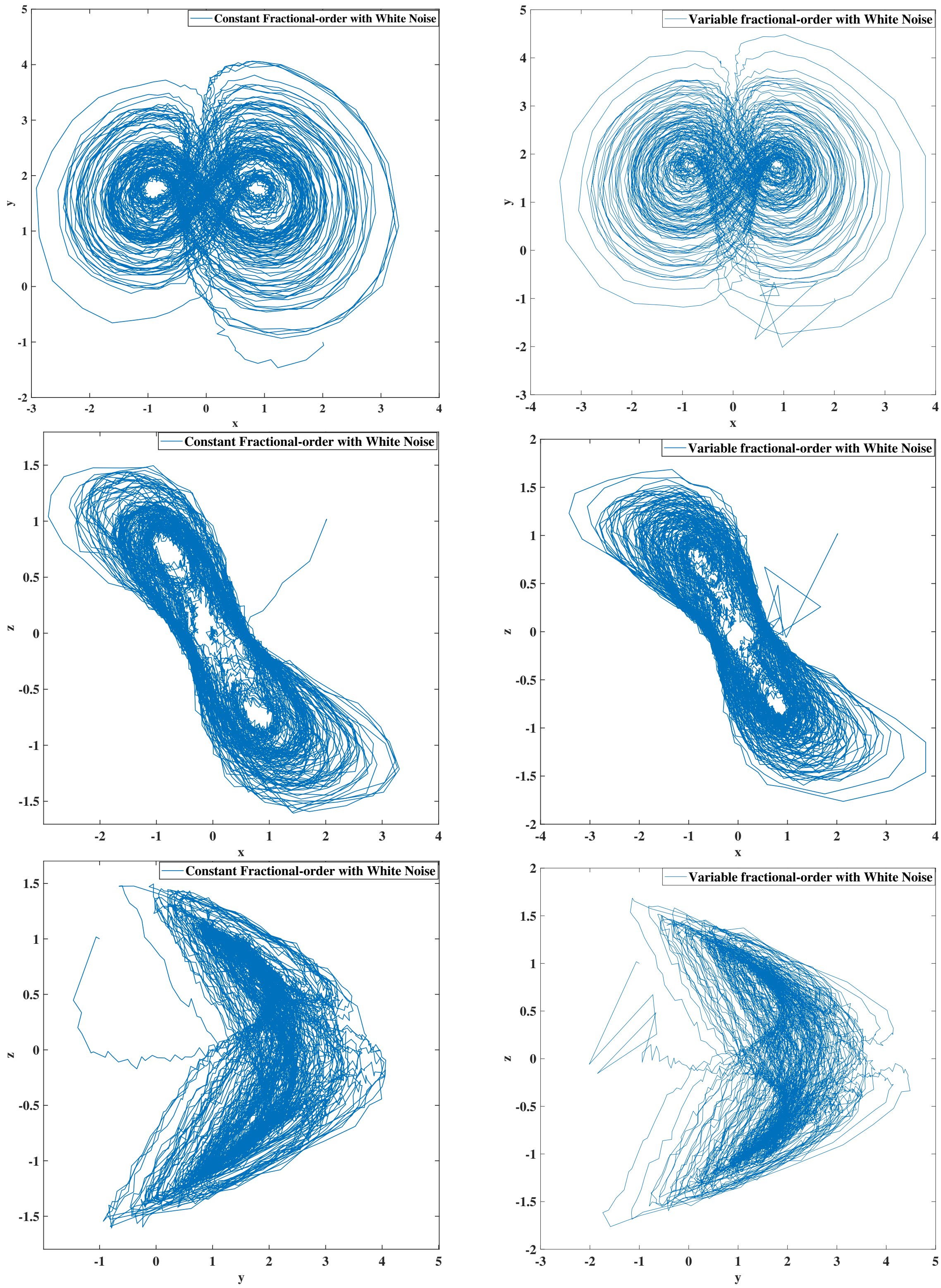

Figure 3: present Comparison between Constant and Variable Fractional Order at $\sigma = 0.2$.

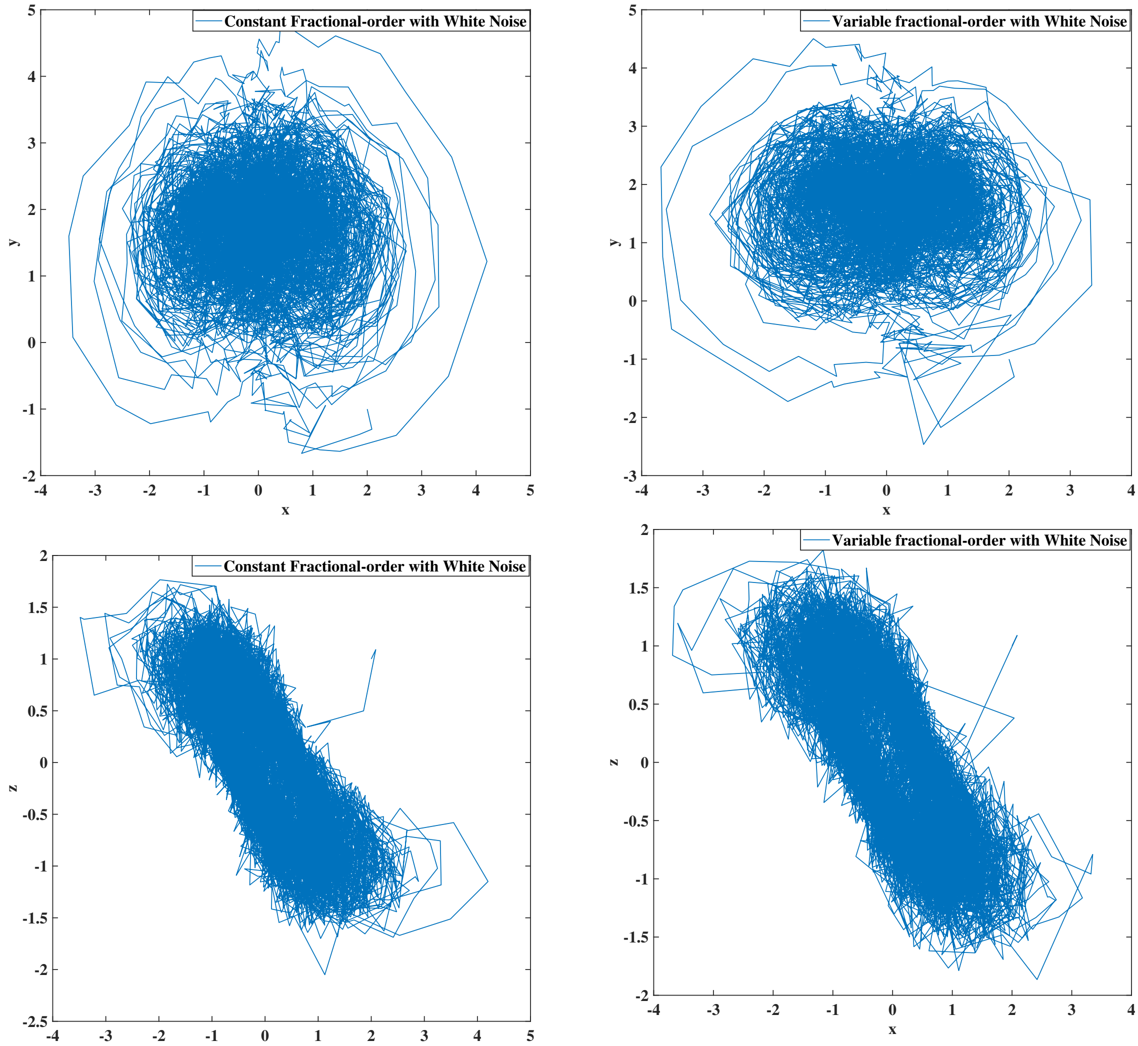

Figure 4:

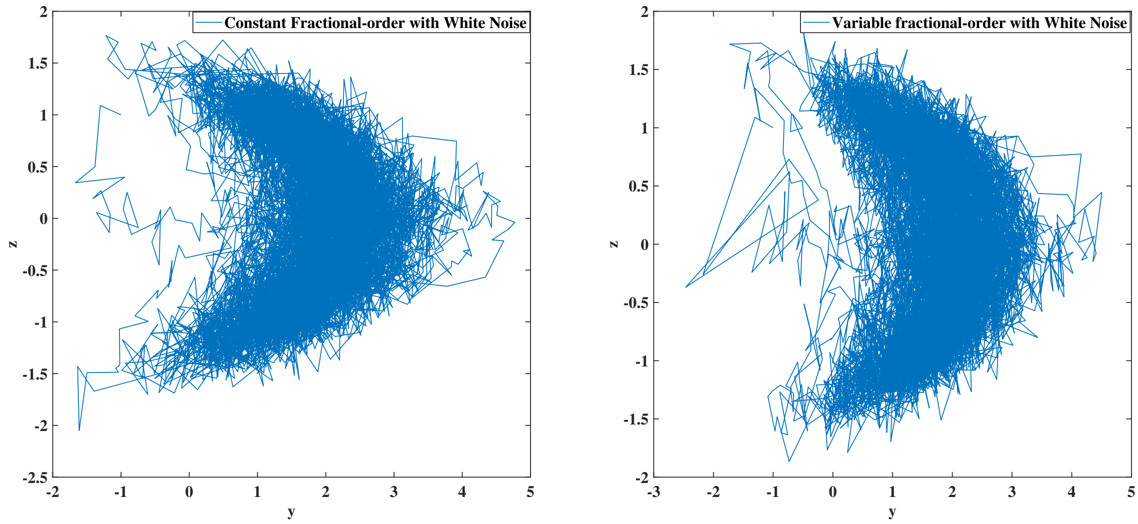

Figure 5: present Comparison between Constant and Variable Fractional Order at $\sigma = 0.5$.

6. Conclusion

In this study, a comparative examination of how white noise affects chaotic behavior in a financial fractional system considering both constant and changing orders. The study demonstrates that systems with changeable orders have intricate dynamics and are highly responsive to stochastic disturbances. This work investigated the control of both constant-order and variable-order fractional (HS) by the use of Caputo derivatives. Figures(1–5) depict the chaotic dynamics of fractional order Caputo fractional (HS) with constant and changing orders. From our analysis of the images, we deduce that the simulations emphasize the distinctions between constant- and variable-order Caputo derivatives. Changes in the fractional order provide intricacy to the chaotic behavior. The Caputo (HS) was examined using fractional derivatives. Our findings indicate that obtaining a comprehensive understanding of the chaotic regions in (HS) with variable-order fractional derivatives is more straightforward compared to (HS) with constant-order fractional derivatives. Therefore, the comprehension of the dynamics of a (HS) enhances when it incorporates a variable-order fractional derivative.We found that the stochastic component stabilizes the chaos of the financial fractional system at zero. In the future, we may investigate the chaotic attractors linked to fractional derivatives or additive noise.In the future, we may conduct an investigation into the financial fractal fractional system that includes additive noise.

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