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The Learning Skills and Level of Understanding of Grade 9 Students on a Mathematics Lesson on Sequences and Series

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Abstract. This study assesses the Algebra skills of Grade 9 students post-pandemic, using an open-ended mathematics questionnaire during the first week of the second quarter. It evaluates students' critical and creative thinking and problem-solving abilities. Results show that while students have made progress in creative thinking and can generate original ideas, their understanding of Algebra remains at a basic level. Critical thinking and problem-solving skills are also at a benchmark level, indicating that students often replicate information without thorough analysis or questioning. To address these issues, the study recommends targeted remediation to enhance students' comprehension and skills in Algebra. It also suggests conducting a comparative qualitative study and exploring how different skill levels relate to overall understanding.

2020 Mathematics Subject Classifications: 97-11

Key Words and Phrases: Conceptual Understanding, Creative Thinking Skill, Problem Solving Skill and Reasoning Skills

1. Front Matter

In the Philippines, distance learning began on October 6, 2020, due to the pandemic, transitioning to modular and online formats. Dangle and Somaoang [1] highlighted challenges such as students struggling with module directions, late submissions, and lack of resources for teachers. Many students faced additional difficulties due to family responsibilities and inadequate parental support. Additionally, technical issues like poor cellphone

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reception and power outages further hindered learning. Stephen Sawchuk and Sarah D. Sparks (2020) noted that math education was particularly impacted by the pandemic because it relies heavily on formal instruction and parents often lack the skills to assist with math. Stress and trauma from the pandemic exacerbated math anxiety and made effective remote instruction more challenging. Post-pandemic, schools began reopening in August 2022, adopting various modalities to balance safety and learning. Teachers faced challenges ensuring understanding in classroom discussions. Modular instruction was recommended to promote independence and self-confidence among students.

The Philippines' low performance in PISA assessments, particularly in Mathematics, has raised concerns. Bingölbali et al. [2] discussed the diverse definitions of open-ended problems, which are used to assess mathematical creativity. Rahayuningsih et al. [13] found that open-ended problem-solving tests effectively measure students' mathematical creativity and problem-solving skills. The current focus is on assessing Grade 9 students' performance in Math, specifically their understanding of algebraic concepts like patterns, sequences, and series. The goal is to evaluate their creative thinking, problem-solving abilities, and reasoning skills as they near the end of Junior High School.

The restated research problems are "what is the level of conceptual understanding, as well as the creative thinking, problem solving, and reasoning skills of the learners in the concepts of patterns, sequences, and series?" and "as these learners use critical and creative thinking and reasoning skills, do they increase their mathematical knowledge in the following six categories: identifying patterns, constructing sequences, ordering sequences, reasoning about patterns-number series?"

The research questions are (1) If generalization is possible, what is the level of understanding of the learners in the concepts of patterns, sequences, and series? (2) What is the creative thinking skill level of the learners in the concepts of patterns, sequences, and series? The results that answer this research question would give the instructors some directions in terms of the levels of mathematical understanding and creative thinking skills of the learners. (3) Are the learners at benchmarking level, milestone level, or capstone level? The result of this research would give the instructor some directions in terms of the levels of mathematical understanding and creative thinking skills of the learners. (4) What is the problem solving skill level of the learners in the concepts of patterns, sequences, and series? Are the learners at benchmarking level, milestone level, or capstone level? (5) What is the reasoning skill level of the learners in the concepts of patterns, sequences, and series? Are the learners at benchmarking level, milestone level, or capstone level?

The study aims to measure the level of understanding of learners in select topics in Mathematics 9 which is covered in the first quarter based on the K-12 Curriculum of the Department of Education, specifically Identifying Patterns, Sequences and Series- through an open-ended test. Furthermore, through the same test, the researcher aims to assess the creative thinking, problem solving and reasoning skills of the learners which are really important for a sustainable, life long learning of the learners. Rubrics are used as reference for the evaluation of their level of understanding, creative thinking, problem solving and reasoning skills.

Patterns, Sequences and Series, one of the basic concepts in Algebra that requires

creative thinking, problem solving and reasoning skills are the most appropriate topics to cover in measuring such skills of the learners. Also, measuring the learners level of understanding in such topics using open-ended questions gives the researchers, as well as the teachers, the idea of how far the learners have understood the basic concepts in arithmetic and algebra. And for these particular intentions, it is important that the researchers use a quantitative method in this study. Results of this study will also lead to the idea of how to design a remediation program or a learning material that will assist the learners in fully understanding and internalizing the mathematical concepts.

This study's results only focus on the generalization of the level of understanding and learning skills- creative thinking, critical thinking and problem solving skills of the Grade 9 students of the science laboratory high school. Also, the study uses a researcher made tool which is an open-ended 23-item test questionnaire, covering the first quarter lesson in algebra "Identifying Patterns, Sequences and Series".

2. Constitutive Definitions

2.1. Open Ended Questions

According to [12], asking open ended questions is one such method in which to determine how and what our students are thinking when solving a problem or answering a question. Open ended questions can and should be asked in every subject area from history to science and physical education to mathematics. An open ended question is one in which there is no one correct answer; there are multiple right answers depending on the way in which you approach the problem. This is different from an open-routed question, which has one correct answer but many ways to get there. In the last few years, there has been a lot of emphasis and importance placed on open ended questions because they are designed in such a way as to uncover the level of student understanding (e.g., procedural vs. conceptual) and misconceptions of mathematical concepts.

2.2. Conceptual Understanding

Conceptual understanding, as defined by, Jean Piaget [11], through his theories on cognitive development. He described it as the ability to develop and refine mental frameworks, or schema, which help individuals grasp and manipulate abstract concepts. This understanding evolves as children progress through stages of cognitive development—sensorimotor, preoperational, concrete operational, and formal operational—by using processes called assimilation and accommodation to adapt their thinking.

Hiebert and Carpenter, [6], supports this by highlighting that the conceptual understanding improves long-term math performance, application, and motivation. They emphasize that active engagement in learning enhances achievement and that observing students' reasoning skills is key to assessing understanding. Their work supports the benefits of deep conceptual learning and active involvement in math.

2.3. Creative Thinking Skill

The creative thinking skill, according to Gick [5], involves three processes: recognizing the problem and generating alternative solutions (alternative thinking); assessing each solution and selecting a solution that solves the problem (critical thinking); and effectively communicating one's ideas to others. They organize their knowledge into a logical structure that allows them to learn new concepts by connecting them to what they already know. Among other kinds of intelligence, Gardner's theory of multiple intelligences [4], there are also two categories: linguistic intelligence and logical, mathematical, or computational intelligence. Creative thinking is the ability to look at a problem and come up with an innovative solution, which is why it is so important for students to be creative thinkers. Creative thinking is an essential skill that students need to develop in order to succeed in their math classes. It will help them solve problems and make better use of their time. Students who have problem solving skills are able to think through problems logically and solve them using the correct methods. They are able to work out solutions quickly and come up with logical steps that lead them to their final conclusion.

2.4. Critical Thinking Skill

The mathematical critical-thinking skill is a process of thinking systematically to develop logical and critical thinking on mathematical problems, which characterize and demand to learn in the 21st century. In [7], it is stated that Critical thinking is clear, reasonable, reflective thinking focused on deciding what to believe or do. It means asking probing questions like, "How do we know?" or "Is this true in every case or just in this instance?" It involves being skeptical and challenging assumptions, rather than simply memorizing facts or blindly accepting what you hear or read. Who are critical thinkers, and what characteristics do they have in common? Critical thinkers are usually curious and reflective people. They like to explore and probe new areas and seek knowledge, clarification, and new solutions. They ask pertinent questions, evaluate statements and arguments, and they distinguish between facts and opinion. They are also willing to examine their own beliefs, possessing a manner of humility that allows them to admit lack of knowledge or understanding when needed. They are open to changing their mind. Perhaps most of all, they actively enjoy learning, and seeking new knowledge is a lifelong pursuit. Facione state that, [3] Critical thinking and problem-solving are often discussed together as interconnected skills. They involve reasoning, questioning, analyzing perspectives, and reflecting on decisions

2.5. Problem Solving Skill

Problem-solving skills help students understand and choose effective methods for various situations. Reasoning skills are essential for grasping mathematical concepts like variables and functions. Problem-solving involves skills such as observation and critical thinking:

(i) Observation: Gathering and interpreting information through pattern recognition and sensory input.

(ii) Critical Thinking:

- Conceptualizing: Understanding and identifying problems and topics.
- Logical Reasoning: Making inferences and forming arguments based on gathered information.
- Application: Using knowledge to resolve problems in different contexts.
- Analytical Thinking: Breaking down complex problems, recognizing patterns, and drawing conclusions.
- Decision-Making: Evaluating options and choosing the best course of action.
- Synthesizing: Combining information in innovative ways to generate new solutions.
- These skills are vital for solving both simple and complex problems effectively.

3. Methodology

3.1. Research Design

This study is to determine the students' level of understanding and learning skills such as creative and critical thinking skills, as well as problem solving skills in the topic identifying patterns, sequences and series in algebra. This research study is quantitative in nature. From the data gathered from the learners, the researcher is able to check for the instrument's (questionnaire's) internal validity, consistency and reliability using the Cronbach's Alpha.

3.2. Research Setting

The study is conducted within a laboratory school in Mindanao, Philippines at the start of the second quarter of the academic year 2022-2023. The school is selected by convenience, since the researcher is a faculty of the said school. Also, the researcher purposely chose to conduct the study after the first quarter since the students are already through with the lesson at the time.

3.3. Subjects and Sampling

The study was participated by a total of 123 out of the 128 Grade 9 students of a laboratory school in Mindanao, Philippines who have taken the topic during the first quarter of the academic year 2022-2023. Following the passage of RA 10173, the Data Privacy Act of 2012, research ethics are considered. For the permit to conduct the study, the researcher visited the school on a weekday and scheduled the study sessions. In order to make sure of the conduct the study in a way that is comfortable for both teachers and

students, the researcher sought permission from the school's administrator (principal) to be able to conduct the study in the school. Once permission granted to conduct the study in a classroom, the arrangement with the teacher concerned was made. Also, consent letter in the cover page of the test questionnaire is provided to each student. For the selection of participants, convenience sampling is used. The grade level of the participants, as well as the school's curriculum, is being considered. The school where the study is conducted only has 4 sections of grade 9 students, with each section having at least 30 students. The researcher opted to conduct the study on all the grade 9 sections. Though they are all requested to participate, they were informed that they were not and could not be required to participate via cover letter and verbally. Thus, some declined the invitation to participate in the study.

3.4. Instrumentation

In the conduct of the study, the learners are given a content-based questionnaire which construct is based on the PISA Mathematics Framework. It is composed of 24 open-ended questions, focusing on five scenarios, namely Writing Patterns- a sequence of integers with the same second difference is provided, Piling Unit Cubes-unit cubes are provided to form a bigger cube, Rubik's Cube- A $3 \times 3 \times 3$ cube is created using 27 unit cubes (a unit cube has a length, width and height of 1 unit), and only the faces of each unit cube that are visible are painted blue, Seating Capacity- an auditorium has 30 rows of seats, 20 seats in the first row, 24 seats in the second row, and so on, Number Patterns-students instructed to choose two numbers of which difference is more than 20, and they're going to form arithmetic sequence, as well as a geometric sequence, out of the numbers between their chosen numbers, and Folding a Sheet of Paper- students will take a sheet of paper and estimate its thickness, they will then fold the paper repeatedly recording the change in its thickness every time it is being folded. All of these comprises the 24-item test questionnaire which is to be answered within an hour. The instrument was pilot-tested which result yearned to a Cronbach alpha of 0.73, which is highly acceptable. Learners as participants are expected to provide varied answers to each of the given problems, and rubrics for each item are created as basis for the credits scoring. In each item, a student gets a full credit, a partial credit or no credit on their answers and solutions to the problems which will be based on the rubric used by [14] in 2013.

Table 1: Algebra Open-Ended Questions Credit Score Rubric [14]

Column	Criteria
Full Credit	Response is correct and the underlying reasoning process is appropriate and clearly communicated.
Partial Credit	Response indicates substantial and appropriate mathematical reasoning but is lacking in some minor way(s).
No Credit	Response indicates some mathematical reasoning but fails to address the item's main mathematical ideas or no answer

3.5. Data Gathering Procedure

Each of their answers for each item is reviewed and credit scores are tabulated. Tabulated results will be used and prepared for a descriptive data analysis. The researcher used this method due to limited time constraint and the need to re-evaluate students' understanding on basic concepts in Math in preparation for next quarter and Math courses

3.6. Data Analysis

In order to analyze the data gathered, descriptive statistics were used. The analysis of the data can be of great help for the concerned teachers to decide whether the topic needs more emphasis for the next lesson discussions. As for the creative and critical thinking, and problem solving skills, factor analysis in PSPP is used to categorize items that measure the said skills. As for the analysis of their learning skill levels, their credit scores for the items belonging to each of the measure of learning skills and Appendices A, B and C are used. Skill levels are categorized as Benchmark, Milestones and Capstone, each of the answers is scored with 4 points as full credit and 0 for no credit. The average credit score of each participant , given by

$$\frac{\text{Total Credit}}{\text{Number of Questions}}$$

is tabulated to identify their skill level. Average credit score that is higher than 3 means the learner's skill is in Capstone, between 3 and 2 (inclusive) as Milestones level and less than 2 is in Benchmark level.

3.7. Validity and Reliability

In conducting the research, the validity and reliability of the instrument is of utmost priority. The instrument being the source of all the data needed for the success of a research study can only be valid and reliable if it has undergone validity and reliability tests.

To ensure the validity of the designed instrument, the researcher had it validated by select three Mathematics education experts under the College of Education of the Mindanao State University- Iligan Institute of Technology. Evaluation and validation sheets were provided for their reference. The validators comments, suggestions and corrections, were reflected on the revision of the instrument. The revised version was again checked by the validators and has been approved for implementation. Aside from the validators, the researcher also had the instrument pilot tested to Grade 10 students with the consent from the school's principal and the concerned mathematics teacher to view some other errors or concerns that may arise during the actual implementation of the instrument to the grade 9 students.

4. Results and Discussions

4.1. Level of Understanding

The level of understanding of the learners is based on their total score from the open-ended test. The scores of the students ranges from 0 to 34. Those which scores fall under the attempted level scored less than 10 $x < 10$, under the familiar level scored between 10 and 19 $10 \leq x < 19$, under the proficient level are those who scored between 20 and 28 $20 \leq x < 28$, and lastly those under the mastery level are those who scored 28 and above $x \geq 28$.

Table 2: Level of Understanding of Grade 9 Students on Identifying Patterns and Series

Level of Understanding	Scores	Frequency	Percentage
Attempted	Less than 10	12	9.67%
Familiar	Between 10 and 19	63	51.22%
Profecient	Between 20 and 28	37	30.08%
Mastered	28 and above	11	8.94%

As shown in Table 2, the students' level of understanding on the topic identifying patterns, sequences and series is in familiar level with a mean of 17.54 with a standard deviation of 6.80. Out of the 123 participants, 9.76% (or 12 participants) are in the attempted level, 51.22% (or 63 participants) are in the familiar level, 30.08% (or 37 participants) are in the proficient level, and 8.94% (or 11 participants) are in the mastery level of understanding on the topic. This implies that students have not yet reached the proficiency level of understanding the concepts they are being taught. From this, the demand for remediation to assist learners in gearing towards proficiency and mastery in the said mathematical concept is deemed necessary

4.2. Creative Thinking Skill

The creative thinking skill of the learners is measured and interpreted using the rubric in [8] and their solutions and answers to writing patterns Q1, piling unit cubes Q5, a cube of unit cubes Q8 and Q9, seating capacity Q12, number patterns Q15, Q17, Q18 and Q19, and folding a sheet of paper Q21.

Table 3: Creative Thinking Skill of Grade 9 Students on Identifying Patterns and Series

Level of Understanding	Scores	Frequency	Percentage
Benchmark	Less than 2	44	32.58%
Milestones	Between 2 and 3	70	56.91%
Capstone	More than 3	9	7.32%

As shown in Table 3, the students' creative skill level is in Milestone with a mean of 2.18 and standard deviation of 0.48. Out of the 123 participants, 32.58% (or 44 participants) are in benchmark creative thinking skill level, 56.91% (or 70 participants) are

in milestones and 7.32% (or 9 participants) are in capstone skill level. This implies that students are creative enough in such math area. This only means that students are highly capable of learning and can generate their own ideas out of their understanding of the concept. Students have somehow acquired strategies and skills within this specific area in math- students can create and entirely new solution or idea that is appropriate to this area in math. Also, this views that learners may take risks which may include personal risk or risk of failure in completing assignment, i.e. going beyond the original parameters of the assignment, introducing new materials and forms, relating to or tackling controversial topics, advocating unpopular ideas or solutions- students incorporates new directions or approaches to the any assigned math task. This result also indicates that learners have selected alternatives, and developed a logical, consistent plan to solve the problem. They can also incorporate alternate, divergent, or contradictory perspective or ideas in an exploratory way. When it comes to innovative thinking, students can extend the unique idea, question, format, or product to create new knowledge or knowledge that cross boundaries, and can synthesize ideas or solutions into a coherent whole.

4.3. Critical Thinking Skill

The critical thinking skill of the learners is measured and interpreted using a rubric in [9] and their solutions and answers to writing pattern Q2 and Q3, piling unit cubes Q5, Q6 and Q7, a cube of unit cubes Q8, Q10 and Q11, number patterns Q15, Q18 and Q19, and folding a sheet of paper Q22 and Q24. As shown in table 4.3.1, the critical thinking skill level of all the 123 participating students is in benchmark level with a mean of 1.08 and standard deviation of 0.26.

Table 4: Critical Thinking Skill of Grade 9 Students on Identifying Patterns and Series

Level of Understanding	Scores	Frequency	Percentage
Benchmark	Less than 2	123	100%
Milestones	Between 2 and 3	0	0%
Capstone	More than 3	0	0%

This implies that students, when it comes to explaining problems to be considered critically, can only state without clarification nor description. Also, students take information from sources without any interpretation/evaluation, and the viewpoints of experts are taken as fact without question. The result also indicates that students show an emerging awareness of present assumptions (sometimes label assertions as assumptions) and begin to identify some contexts when presenting a position. Regarding conclusions and related outcomes, students in this level of critical thinking is inconsistently tied to some of the information discussed; related outcomes (consequences and implications) are overly simplified.

4.4. Problem Solving Skill

The Problem solving skill of the learners is measured and interpreted using a rubric in [10] and their solutions and answers to writing patterns Q3 and Q4, piling unit cubes Q7, a cube of unit cubes Q11, seating capacity Q13 and Q14, number patterns Q15, Q16, Q19 and Q20, and folding a sheet of paper Q21, Q22 and Q24.

Table 5: Problem Solving Skill of Grade 9 Students on Identifying Patterns and Series

Level of Understanding	Scores	Frequency	Percentage
Benchmark	Less than 2	64	52.03%
Milestones	Between 2 and 3	54	43.90%
Capstone	More than 3	5	4.07%

As shown in Table 5, the problem solving skill of the learners is also in benchmark skill level with a mean of 1.97 and standard deviation of 0.57. This implies that learners demonstrate limited ability to identify a problem statement or related contextual factors. Students can also identify one or more approaches for solving the problem that do not apply within the specific context, proposes a solution/hypothesis that is difficult to evaluate because it is vague or only indirectly addresses the problem statement, and their evaluation of solutions is superficial (for example: contains cursory, surface level explanation) and includes the following: considers history of the problem, reviews logic/reasoning examines feasibility of the solution, and weighs impacts of solution. Aside from these, the result entails that students can implement the solution in a manner that does not directly address the problem statement, and student capability to review results superficially in terms of the problem defined with no consideration of the need for further work.

5. Conclusion

Based on the results of the study, the grade 9 students have low level of understanding and familiarity of the concept sequences and series, same as their critical thinking and problem solving skill levels. This is due to the fact that in order to be capable of thinking critically and solve problems on sequences and series, a student must have deeper understanding of the concept. To improve the students' level of understanding, critical thinking abilities and problem solving skills of the students, they may be provided with an instrument, a webpage or a module, to assist them through self-paced learning. This study can also be used as a basis in planning for a remediation program or instrument in algebra as it is deemed important since the learners are still adjusting to the new teaching and learning modalities. The author highly recommends that a relative qualitative research study be done to support the claims of this study. For further study, interested researcher may proceed to study the relationship of the level of understanding to the learning skills, as well as the skills from each other.

6. Future Direction

It is important to note that the results of this study focus only on the performance of the Grade 9 Junior High School students of a laboratory school in Mindanao, Philippines of the academic year 2022-2023, just right after the pandemic during which the learners had their classes in a distant learning modality, online or modular, or that is the start of the blended (online and face-to-face) modalities. For further studies, researchers may study the level of understanding and learning skills of the learners in other topics they had in their Algebra course, or that of learners from other science curriculum secondary schools in Iligan City for generalizability. Since the study is conducted in a face-to-face modality, one may also conduct study via online to determine whether the modality of which the test is given affects the students' performance.

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