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A Comparative Analysis of Four Group Decision-Making Techniques: KEMIRA G-I, KEMIRA G-II, Lon-Zo, and MACASP

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Abstract. Most selection problems are multi-decision and multi-criteria in nature. The group decision (GD) literature presents several methods for solving them. Most of them belong to utilities functions based class. However, the use of any one group decision method of this class for a specific problem is often not appropriate, given the characteristics of the latter. In this the present work is to compare four GD utility functions based methods, two of which are classical (Lon-Zo and MACASP) and two new (KEMIRA G-I and KEMIRA G-II), by examining their suitability for solving two multi-criteria choice problems, namely the selection of a crop variety adapted to the Centre-Est region of Burkina Faso and the selection of a site for the implementation of a waste incineration plant in the city of Vilnius in Lithuania. The results show that group decision methods based on aggregation utility functions are most suitable when the criteria are homogeneous (*i.e.* when criteria can compensate naturally). However, when the criteria are heterogeneous (*i.e.* when there is no natural compensation between criteria), these methods can still be successfully applied when the heterogeneous nature of the criteria is taken into account. This explains the good performance of the KEMIRA G-I and KEMIRA G-II methods, which take into account the heterogeneous nature of the criteria, compared with the Lon-Zo and MACASP methods, which do not.

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Key Words and Phrases: Group decision, KEMIRA G-I, KEMIRA G-II, Lon-Zo, MACASP, Borda method

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1. Introduction

Selection problems are generally multi-criteria and multi-decision-maker in nature. To solve them, the literature on group decision support generally presents three categories of methods: Those based on outranking relations[1, 4], multi-attribute utility theory[11, 12] and interactive methods[2]. In this work, we are interested in the class of methods based on multi-attribute utility theory. More specifically in this paper we focus on the classical Lon-Zo [11] and MACASP [11] methods and two new methods KEMIRA G-I[10] and KEMIRA G-II, both extensions of the KEMIRA method[7]. The classical Lon-Zo and MACASP methods are based on the harmonic and arithmetic mean, respectively.

The two new methods are based on the KEmeny Median Indicator Ranks Accordance (KEMIRA[7]) method and the Borda [8] method. All these methods have their own advantages and disadvantages.

The Lon-Zo and MACASP methods are often used for ranking problems, where alternatives are ranked from best to worst[11, 15]. The KEMIRA G-I and KEMIRA G-II methods are suitable for solving, in general, the multi-criteria problem when the set of criteria is divided into a few homogeneous sub-groups of criteria, i.e. criteria between which compensation is naturally possible[5, 7, 10]. For example, with two criteria such as average annual economic profit and average annual wage, we could naturally accept compensation between them when aggregating them using a utility function because they are naturally expressed in the same monetary unit. However, if we consider two criteria such as average annual economic profit and average annual level of education, aggregating the latter two using a utility function would naturally not be acceptable. which is why the last two criteria are referred to as heterogeneous.

The KEMIRA G-I and KEMIRA G-II methods can be used simultaneously to elicit criteria weights and to select the best alternatives by eliminating certain alternatives according to predefined performance thresholds.

Through two case studies, we propose to compare the two methods of group decision, Lon-Zo and MACASP with the two new methods KEMIRA G-I and KEMIRA G-II

The rest of our paper is organised as follows. In Section 2, we present the classic Lon-Zo and MACASP methods. Section 3 describes the new KEMIRA G-I and KEMIRA G-II methods. Section 4 is dedicated to the results of applying the four methods to the two case studies. In section 5 we discuss the different results obtained in section 4. We conclude our work and open the door to future work in section 6.

2. Presentation of Lon-Zo and MACASP method

In what follows, we adopt the following notations:

- $D = \{d_1, d_2, \cdots, d_L\}$ the set of Decision Makers (DMs), $(L \geq 2)$ with L the number of DMs;
- $A = \{a^1, a^2, ..., a^K\}$ the set of alternatives, $(K \geq 2)$ and K is the total number of alternatives;

- $G = \{g_1, g_2, \cdots, g_m\}$ the set of criteria;
- $w_j^{d_l}$ the weight assigned to criterion j par by Decision Maker d_l;
- $g_j^{d_l}(a^k)$ the partial evaluation of the alternative a^k w.r.t. criterion g_j by the DM d_l.

2.1. Lon-Zo method

The Lon-Zo method uses the weighted sum and harmonic mean as aggregation functions. By weighted sum, the overall performance $g^{d_l}(a^k)$ given to each alternative by the decision-maker d_l is determined by equation (1):

$$g^{d_l}(a^k) = \sum_{j=1}^{j=m} w_j^{d_l}.g_j^{d_l}(a^k), k = 1, \cdots, K, j = 1, \cdots, m, \tag{1}$$

where m is the total number of criteria.

The harmonic mean is used to determine the overall evaluation (or performance) $g(a^k)$ of the action a^k. It is defined by equation (2):

$$g(a^k) = \frac{L}{\sum_{l=1}^{L} \frac{1}{g^{d_l}(a^k)}}. \tag{2}$$

2.2. MACASP method

MACASP[11] uses the weighted sum and arithmetic mean as aggregation functions. The evaluation that decision-makers give by consensus on alternative a^k with regard to criterion j is $g_j(a^k)$ and it is defined by equation (3):

$$g_j(a^k) = \sum_{l=1}^{L} w_j^{d_l}.g_j^{d_l}(a^k), k = 1, \cdots, K, j = 1, \cdots, m. \tag{3}$$

The overall performance $g(a^k)$ of the alternative a^k is obtained according to equation (4) :

$$g(a^k) = \frac{1}{m}\sum_{j=1}^{m} g_j(a^k) = \frac{1}{m}\sum_{j=1}^{m}(\sum_{l=1}^{L} w_j^{d_l}.g_j^{d_l}(a^k)). \tag{4}$$

3. Description of KEMIRA G-I and KEMIRA G-II methods

In this section we introduce the following notations:

- G denotes the set of criteria that can be partitioned into S groups G_i such that $G_i = \{(i,1),(i,2),\cdots(i,n_i)\}$ with $i \in \{1,\cdots,S\}$, where n_i is the number of criteria inside the group G_i, and (i,j) denotes the criterion g_j inside the group G_i. So $G = G_1 \cup G_2 \cup \cdots \cup G_S$.
- $a^{k,d_l}_{i,j}$ denotes the performance of the alternative a^k with respect to criterion g_j inside the group G_i given by the DM d_l and $w^{d_l}_{i,j}$ the weight of criterion g_j inside the group G_i, given by the DM d_l.

We assume that each decision-maker is able to rank the criteria in each group G_i from most to least important[3, 9, 10], as specified by relation (5). Without loss of generality, we also assume that all criteria are to be maximized.

$$\begin{array}{l} (i,1)^{d_1} \succsim (i,2)^{d_1} \succsim \cdots \succsim (i,n_1)^{d_1} \\ (i,1)^{d_2} \succsim (i,2)^{d_2} \succsim \cdots \succsim (i,n_i)^{d_2} \\ \vdots \qquad \vdots \qquad \qquad \vdots \\ (i,1)^{d_L} \succsim (i,2)^{d_L} \succsim \cdots \succsim (i,n_i)^{d_L}. \end{array} \tag{5}$$

3.1. KEMIRA G-I method

In what follows, we present the main stages of KEMIRA G-I method[10].

3.1.1. Step 1: median ranking of criteria in descending order of preference

Applying Borda's voting method [8] based on relation (5), we obtain the median ranking of criteria for the set D of decision-makers in each group G_i as specified in equation (6) and of course its corresponding weights ranking in equation (7) such that relation (8) holds.

$$(i,1) \succsim (i,2) \succsim \cdots \succsim (i,n_i), \forall i \in \{1,2,\ldots,S\} \tag{6}$$

$$w_{i,1} \succsim w_{i,2} \succsim \cdots \succsim w_{i,n_i}, \forall i \in \{1,2,\ldots,S\} \tag{7}$$

$$\sum_{j=1}^{n_i} w_{i,j} = 1, \forall i \in \{1,\ldots,S\}. \tag{8}$$

3.1.2. Step 2: calculating average performance

The average performance $W_i(a^k)$ of each alternative a^k with respect to each group of criteria G_i is then determined according to equation (9).

$$W_i(a^k) = \sum_{j=1}^{n_i} {a^*}^k_{i,j}.w_{i,j}, \tag{9}$$

where ${a^*}^k_{i,j}$ is the normalized performance of $a^k_{i,j}$ obtained following the equation (10)

$${a^*}^k_{i,j} = \frac{a^k_{i,j} - \min_j a^k_{i,j}}{\max_j a^k_{i,j} - \min_j a^k_{i,j}}, \forall i \in \{1,2,\ldots,S\}. \tag{10}$$

3.1.3. Step 3: optimization problem

This stage consists of formulating an optimization problem that elicits the decision-makers' preferences, *i.e.*, the weights of the criteria, and also determine the best alternatives for the decision-makers as a whole, by first setting the performance thresholds for each group G_i of criteria. This optimization problem is defined by the equation (11).

$$\begin{aligned} &\max_{w_{i,j}} f_{opt} = |B| \\ &s.t. \begin{cases} w_{i,1} \geq w_{i,2} \geq \cdots \geq w_{i,n_i}, \forall i \in \{1,2,\cdots,S\}, \\ \sum_{j=1}^{n_i} w_{i,j} = 1, \quad \forall i \in \{1,2,\cdots,S\}, \\ W_i(a^k) > \alpha_i, \quad \forall i \in \{1,2,\cdots,S\}, \end{cases} \end{aligned} \tag{11}$$

where

- f_{opt} is the value of the objective function;
- α_i is the performance threshold associated to the group G_i, set by the decision-maker;
- $B = \{a^k : W_i(a^k) > \alpha_i, i \in \{1,2,\cdots,S\}\}$ denotes the set of best alternatives;
- $|B|$ denotes the number of elements of B.

3.1.4. Step 4: Choosing best alternative(s)

The choice of a best alternative(s) is based on the following two conditions:

- if for the highest possible threshold we have a single alternative, it will be considered the best alternative;
- if for the highest possible threshold, we have at least two alternatives, each decision-maker is asked to rank the alternatives according to his preferences. Then the Borda method is used to obtain a median ranking. This median ranking gives the best alternative(s).

3.2. KEMIRA G-II method

In contrast to KEMIRA G-I, in KEMIRA G-II each decision-maker completes the process of choosing the best alternative(s). The intersection and reunion of the sets of best solutions found by each decision-maker is then exploited. Formally, under the hypothesis stipulated by relation (5), the main stages of the KEMIRA G-II method are as follows.

3.2.1. Step 1: Calculate average performance

For each decision-maker d_l the average performance $W_i^{d_l}(a^k)$ of each alternative a^k is determined according to equation (12):

$$W_i^{d_l}(a^k) = \sum_{j=1}^{n_i} a_{i,j}^{*k,d_l}.w_{i,j}^{d_l} \tag{12}$$

where $a_{i,j}^{*k,d_l}$ is the normalized performance of $a_{i,j}^{k,d_l}$ obtained following the equation (13)

$$a_{i,j}^{*k,d_l} = \frac{a_{i,j}^{k,d_l} - \min_j a_{i,j}^{k,d_l}}{\max_j a_{i,j}^{k,d_l} - \min_j a_{i,j}^{k,d_l}}, \forall i \in \{1, 2, \ldots, S\}, \forall l \in \{1, 2, \ldots, L\}. \tag{13}$$

3.2.2. Step 2: optimization problem

The optimization problem used to elicit the weights of the criteria and to select the best alternatives according to the preferences of each decision-maker is defined by equation (14). The performance thresholds α_i for each group of G_i must first be set by each decision-maker d_l:

$$\begin{array}{l} max_{w_{i,j}} f_{opt} = |B^{d_l}| \\ s.t. \begin{cases} w_{i,1}^{d_l} \geq w_{i,2}^{d_l} \geq \cdots \geq w_{i,n_i}^{d_l}, \forall i \in \{1, 2, \cdots, S\}, \\ \sum_{j=1}^{n_i} w_{i,j}^{d_l} = 1, \quad \forall i \in \{1, 2, \cdots, S\}, \\ W_i^{d_l}(a^k) > \alpha_i, \quad \forall i \in \{1, 2, \cdots, S\}, \end{cases} \end{array} \tag{14}$$

where

- α_i denotes the performance threshold of the group G_i;
- $B^{d_l} = \{a^k : W_i^{d_l}(a^k) > \alpha_i, i \in \{1, 2, \cdots, S\}\}$ denotes the set of best alternatives;
- $|B^{d_l}|$ denotes the number of elements of the set B^{d_l}.

3.2.3. Step 3: determining the best compromise alternative(s)

To choose the best alternative(s), we use intersection and/or reunion and afterwards Borda's voting method:

- for each decision-maker d_l, we determine the set B^{d_l} of the best alternatives according to his preferences;
- if $B^{d_1} \cap B^{d_2} \cap \cdots \cap B^{d_L} \neq \varnothing$: each decision-maker ranks the alternatives obtained in the intersection, and the Borda method is applied to select the best alternative(s);
- if $B^{d_1} \cap B^{d_2} \cap \cdots \cap B^{d_L} = \varnothing$: we consider the reunion $B^{d_1} \cup B^{d_2} \cup \cdots \cup B^{d_L}$; each decision-maker ranks the alternatives inside the reunion $B^{d_1} \cup B^{d_2} \cup \cdots \cup B^{d_L}$ and the Borda method is applied to select the best alternative(s).

4. Case studies: application to two group decision-making case studies

4.1. First case study

The first case study concerns the selection of the best cowpea varieties (a bean species) adapted to the Centre-North region of Burkina Faso. The team of stakeholders involved in this research study includes breeders, producers and processors[9, 10]. The structuring phase enabled fifteen (15) cowpea crop varieties to be identified and twelve (12) evaluation criteria in interaction with the stakeholders. The evaluation criteria were divided into three group (see [10] for more details).

4.1.1. Groups of criteria

The twelve criteria were divided into three groups of eight, three and one criteria respectively.

- **Group 1 (Production criteria):** Type of plant habit $(1,1)$, Cycle-semi-maturity $(1,2)$, Yield potential $(1,3)$, Disease resistance $(1,4)$, Striga resistance $(1,5)$, Drought resistance $(1,6)$, Insect resistance $(1,7)$, forage potential $(1,8)$.
- **Group 2 (Quality criteria):** seed size $(2,1)$, seed color $(2,2)$, seed taste $(2,3)$. **Group 3 (Processing criteria):** cooking time $(3,1)$.

4.1.2. Decision matrix

The normalized decision matrix or evaluation matrix is unique for all decision-makers and is the one obtained from the evaluations of domain experts and the result synthesised in Table 1.

4.1.3. Using KEMIRA G-I and KEMIRA G-II methods

As regards the application of the KEMIRA G-I and KEMIRA G-II methods to this first case study, we highlight the following elements.

- Firstly the four decision-makers were able to express their preferences on the criteria by ranking them through the respective groups as showed in Table 2.
- Secondly, for the KEMIRA G-I method, the median ranking is obtained by applying the Borda voting method algorithm [8] and the result presented in Table 3.
- Thirdly, the KEMIRA[7] algorithm is implemented iteratively with the parameters indicated in relations (15),(16),(17). The different results obtained using the KEMIRA G-I and KEMIRA G-II methods, including execution times in second (s), are presented in Table 4 and Table 5 respectively.

$$max_iter = 10000 \qquad (15)$$

Table 1: Normalized Evaluation Matrix

Names of the varieties		**(1,3)** $a^k_{1,1}$	**(1,2)** $a^k_{1,2}$	**(1,5)** $a^k_{1,3}$	**(1,8)** $a^k_{1,4}$	**(1,4)** $a^k_{1,5}$	**(1,7)** $a^k_{1,6}$	**(1,6)** $a^k_{1,7}$	**(1,1)** $a^k_{1,8}$	**(2,2)** $a^k_{2,1}$	**(2,1)** $a^k_{2,2}$	**(2,3)** $a^k_{2,3}$	**(3,1)** $a^k_{3,1}$
KVx442-3-25SH(Komcallé)	a^1	0.32	0.71	0.11	0.07	0.23	0	1	1	1	1	1	1
KVx61-1(Bengsiido)	a^2	0.25	0.42	1	0.34	0.23	0	1	1	1	0	1	0.57
KVx745-11P	a^3	0	0.42	1	0.82	0.12	0	1	0	1	1	1	0.07
KVx771-10G(Nafi)	a^4	0.25	0.42	1	0.04	0.23	1	1	1	1	1	1	0.78
KVx775-33-2G(Tiligré)	a^5	0.36	0.42	1	0.46	1	0	0	1	1	1	1	0.57
Moussa Local	a^6	0.25	0	0.11	0	0	1	0	1	1	0.5	1	0.14
Teeksongo	a^7	0.25	0.57	1	1	1	1	1	0.5	1	0.5	1	0.5
Yipoussi(KVx780-1)	a^8	0.41	1	0.11	0.29	0.23	0	0	0.5	1	0	1	0.07
Niizwe	a^9	0.11	0.71	1	0.09	1	0	0	1	1	1	1	0.07
Yiss-Yande	a^{10}	0.36	0.71	1	0.14	0.23	0	1	0.5	1	1	1	0.14
Gorom local	a^{11}	0.17	0.28	0	0.09	0.12	0	0	0	0	0	0	0.14
Makoyin(KVx780-4)	a^{12}	0.7	0.57	1	0.34	0.23	0	1	0.5	1	1	0.9	0.07
Issa-Sosso(KVx780-3)	a^{13}	0.7	0.71	1	0.34	0.23	0.16	1	0.5	1	1	0.9	0.14
Neerwaya(KVx780-6)	a^{14}	0.85	0.57	1	0.46	0.23	0.16	0	0.5	1	1	0.9	0
Gourgou(TZ1-GOURGOU)	a^{15}	1	0.28	1	0.58	0.23	0	0	0.5	1	0.5	0.9	0.14

$$\alpha_i = p_i\% max_{k=1}^{15} W_i(a^k), i \in \{1,2,3\}, p_i \in \{10,20,30,40,50,60,70,75,80\} \quad (16)$$

$$p_1 = p_2 = p_3 \quad (17)$$

where max_iter is the maximum number of iterations.

- Looking KEMIRA G-I result as showed in Table 4, we have a single best variety for the highest threshold: KVx771-10G(Nafi) (a^4). So we did not need to ask decision-

Table 2: Criteria ranking by Decision-Makers

	Goup 1				Goup 2				Goup 3			
Rank	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4
1^{st}	(1, 3)	(1, 3)	(1, 3)	(1, 3)	(2, 2)	(2, 1)	(2, 2)	(2, 2)	(3, 1)	(3, 1)	(3, 1)	(3, 1)
2^{nd}	(1,2)	(1,2)	(1,2)	(1,2)	(2,3)	(2,3)	(2,1)	(2,1)				
3^{th}	(1,1)	(1,5)	(1,4)	(1,8)	(2,1)	(2,2)	(2,3)	(2,3)				
4^{th}	(1,8)	(1,6)	(1,5)	(1,7)								
5^{th}	(1,5)	(1,8)	(1,7)	(1,5)								
6^{th}	(1,6)	(1,7)	(1,8)	(1,4)								
7^{th}	(1,4)	(1,4)	(1,6)	(1,6)								
8^{th}	(1,7)	(1,1)	(1,1)	(1,1)								

Table 3: Median criteria ranking

Group 1 median ranking	Group 2 median ranking	Group 3 median ranking
$(1,3) \succsim (1,2) \succsim (1,5) \succsim (1,8) \succsim (1,4) \succsim (1,7) \succsim (1,6) \succsim (1,1)$	$(2,2) \succsim (2,1) \succsim (2,3)$	$(3,1)$

Table 4: KEMIRA G-I results

p_i	Criteria weights												best varieties	Time (s)
	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)	(2,1)	(2,2)	(2,3)	(3,1)		
	$w_{1,1}$	$w_{1,2}$	$w_{1,3}$	$w_{1,4}$	$w_{1,5}$	$w_{1,6}$	$w_{1,7}$	$w_{1,8}$	$w_{2,1}$	$w_{2,2}$	$w_{2,3}$	$w_{3,1}$		
10	0.19	0.17	0.15	0.15	0.13	0.13	0.05	0.02	0.66	0.25	0.09	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}, a^{15}\}$	19.9298
20	0.2	0.16	0.15	0.13	0.11	0.11	0.1	0.04	0.5	0.36	0.14	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}\}$	23.5859
30; 40;50;60	0.26	0.22	0.20	0.10	0.09	0.02	0.02	0.02	0.42	0.32	0.25	1.0	$\{a^1, a^2, a^4, a^5, a^7\}$	20.5216
70	0.31	0.29	0.06	0.06	0.05	0.04	0.04	0.04	0.37	0.37	0.25	1.0	$\{a^1, a^4\}$	21.2741
75;80	0.18	0.18	0.12	0.08	0.08	0.06	0.04	0.04	0.56	0.23	0.20	1.0	$\{a^4\}$	20.8315

makers to rank the best varieties.

Table 5: KEMIRA G-II results

DMs	p_i	Criteria weights												Best varieties	Time (s)
		(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)	(2,1)	(2,2)	(2,3)	(3,1)		
		$w_{1,1}$	$w_{1,2}$	$w_{1,3}$	$w_{1,4}$	$w_{1,5}$	$w_{1,6}$	$w_{1,7}$	$w_{1,8}$	$w_{2,1}$	$w_{2,2}$	$w_{2,3}$	$w_{3,1}$		
	10	0.18	0.18	0.17	0.13	0.1	0.09	0.09	0.06	0.39	0.35	0.26	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}, a^{15}\}$	21.30
	20	0.22	0.22	0.19	0.13	0.1	0.06	0.05	0.05	0.38	0.35	0.27	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}\}$	19.8321
d_1	30; 40;50;60	0.23	0.22	0.17	0.15	0.12	0.05	0.00	0.00	0.37	0.33	0.28	1.01	$\{a^1, a^2, a^4, a^5, a^7\}$	20.0806
	70;75;80	0.21	0.2	0.17	0.14	0.1	0.09	0.05	0.04	0.55	0.31	0.13	1.0	$\{a^1, a^4\}$	20.3378
	90	0.24	0.23	0.21	0.14	0.09	0.05	0.05	0.0	0.64	0.32	0.03	1.0	$\{a^4\}$	
	10	0.19	0.18	0.16	0.12	0.12	0.09	0.07	0.06	0.52	0.31	0.17	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}, a^{15}\}$	20.0149
	20	0.21	0.21	0.13	0.13	0.1	0.08	0.07	0.06	0.58	0.38	0.04	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}\}$	20.0825
d_2	30; 40;50;60	0.24	0.20	0.17	0.08	0.08	0.04	0.04	0.02	0.23	0.19	0.19	1.	$\{a^1, a^2, a^4, a^5, a^7\}$	20.2741
	70	0.24	0.19	0.13	0.13	0.06	0.05	0.05	0.05	0.59	0.39	0.02	1.01	$\{a^1, a^4\}$	19.9214
	75;80	0.17	0.17	0.16	0.13	0.12	0.11	0.09	0.06	0.5	0.31	0.19	1.0	$\{a^4\}$	19.8384
	90													$\emptyset$	
	10	0.18	0.18	0.17	0.16	0.16	0.06	0.05	0.03	0.99	0.0	0.0	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}, a^{15}\}$	20.4354
	20	0.25	0.16	0.15	0.12	0.08	0.05	0.05	0.04	0.43	0.38	0.17	1.	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}\}$	20.2265
	30; 40;50;60	0.27	0.22	0.15	0.13	0.11	0.05	0.05	0.01	0.54	0.37	0.07	1	$\{a^1, a^2, a^4, a^5, a^7\}$	20.0531
d_3	70	0.25	0.24	0.19	0.06	0.06	0.04	0.03	0.02	0.52	0.44	0.04	1.01	$\{a^1, a^4\}$	20.0882
	75;80	0.22	0.15	0.14	0.12	0.12	0.09	0.08	0.06	0.38	0.33	0.27	1	$\{a^4\}$	20.0554
	90													$\emptyset$	
	10	0.23	0.18	0.16	0.13	0.1	0.08	0.07	0.06	0.44	0.32	0.25	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}, a^{15}\}$	20.0593
	20	0.18	0.17	0.17	0.16	0.14	0.11	0.04	0.03	0.58	0.39	0.03	1.0	$\{a^1, a^2, a^4, a^5, a^7, a^6, a^{10}, a^{13}\}$	20.1220
d_4	30; 40;50;60	0.27	0.15	0.11	0.09	0.05	0.05	0.05	0.05	0.68	0.18	0.14	0.99	$\{a^1, a^2, a^4, a^5, a^7\}$	20.0279
	70	0.24	0.23	0.09	0.07	0.07	0.07	0.07	0.04	0.72	0.16	0.11	0.99	$\{a^1, a^4\}$	20.2027
	75;80	0.15	0.14	0.14	0.14	0.10	0.07	0.07	0.03	0.54	0.24	0.20	1.	$\{a^4\}$	20.0774
	90													$\emptyset$	

- Looking KEMIRA G-II result as showed in Table 5 we note that:
  - for $p_i = 80$ we have $B^{d_1} \cap B^{d_2} \cap B^{d_3} \cap B^{d_4} = \{a^4\}$;
  - for $p_i = 70$ we have $B^{d_1} \cap B^{d_2} \cap B^{d_3} \cap B^{d_4} = \{a^1, a^4\}$;
  - for $p_i = 90$ we have $B^{d_1} = \{a^1\}$ and $B^{d_2} = B^{d_3} = B^{d_4} = \emptyset$.

Considering particularly the case where $p_i = 70$, the set of the best varieties is $B = \{a^1, a^4\}$. To choose the best alternative among the element of B, we also use the Borda's rule as illustrated in Table 6. These results show that Vx771-10G(Nafi) a^4 is the best variety for KEMIRA G-II method, as for the results obtained with $p_i = 80$.

4.1.4. Resolution using Lon-Zo and MACASP methods

The application of these two methods requires the weights of the criteria to be determined. To do this, we used the revised Simos[13, 14] card method, called SFR[3]. The different

Table 6: KEMIRA G-II results on the set of best alternatives

Rank	d_1	d_2	d_3	d_4
1^{st}	a^1	a^4	a^4	a^4
2^{nd}	a^4	a^1	a^1	a^1

weight values found are shown in Table 7.

Table 7: Criteria weights for Lon-Zo and MACASP methods in study case 1

Criteria	g_1 $w_1^{d_i}$	g_2 $w_2^{d_i}$	g_3 $w_3^{d_i}$	g_4 $w_4^{d_i}$	g_5 $w_5^{d_i}$	g_6 $w_6^{d_i}$	g_7 $w_7^{d_i}$	g_8 $w_8^{d_i}$	g_9 $w_9^{d_i}$	g_{10} $w_{10}^{d_i}$	g_{11} $w_{11}^{d_i}$	g_{12} $w_{12}^{d_i}$
d_1	5.25	6.13	7.30	2.04	3.79	2.92	1.45	4.08	10.99	10.99	10.99	33
d_2	1.03	7.21	6.89	2.97	3.30	2.65	3.63	5.27	10.99	10.99	10.99	33
d_3	0.83	6.54	7.53	4.80	4.55	1.58	4.31	2.82	13.34	16.84	2.80	33
d_4	2.00	5.47	6.01	2.90	5.29	4.36	3.10	3.82	14.13	7.06	11.80	33

- ***Resolution using Lon-Zo method:*** Applying the Lon-Zo method to the first case study gives the results shown in Table 8. Looking the overall performance, the

Table 8: Lon-Zo results

	$g^{d_1}(a^k) = \sum_{j=1}^{j=12} w_j^{d_1}.g_j^{d_1}(a^k)$	$g^{d_2}(a^k) = \sum_{j=1}^{j=12} w_j^{d_2}.g_j^{d_2}(a^k)$	$g^{d_3}(a^k) = \sum_{j=1}^{j=12} w_j^{d_3}.g_j^{d_3}(a^k)$	$g^{d_4}(a^k) = \sum_{j=1}^{j=12} w_j^{d_4}.g_j^{d_4}(a^k)$	$g(a^k) = \frac{4}{\sum_{l=1}^{4} \frac{1}{g^{d_l}(a^k)}}$	Time (s)
a^1	82.0401	78.4368	77.2826	79.7055	79.3273	
a^2	59.0424	55.0413	52.1441	55.1176	55.2289	
a^3	29.9938	29.0018	34.5060	25.5707	29.4315	
a^4	77.2033	75.0291	75.8826	78.1417	76.5455	
a^5	69.9877	67.0653	68.7719	68.2416	68.5005	
a^6	41.0759	38.8767	38.4859	37.7534	39.0096	
a^7	66.2537	68.2096	66.9445	67.5662	67.2357	
a^8	38.1381	37.4442	34.4434	32.4802	35.4773	0.004998
a^9	51.9282	48.9857	51.2404	50.4098	50.6171	
a^{10}	58.9875	53.1307	53.0384	55.5417	54.1520	
a^{11}	8.1923	8.6440	8.5635	7.8684	8.3053	
a^{12}	54.0195	52.1071	52.6589	54.0951	53.2062	
a^{13}	57.4217	56.0099	56.5747	57.6689	56.9111	
a^{14}	50.6085	49.3857	50.9241	49.2745	50.0376	
a^{15}	49.3026	47.4969	47.7555	46.1038	47.6377	

  KVx442-3-25SH(Komcallé (a^1) variety is considered the best.

- ***Resolution using the MACASP method:*** Applying the MACASP method to the first case study gives the results shown in Table 9.

  Like the Lon-Zo method, the MACASP method proposes KVx442-3-25SH(Komcallé (a^1) as the best variety.

4.2. Second case study

The second case study concerns the choice of a site for a non-hazardous waste incineration plant in the Lithuanian capital Vilnius[6, 7]. In this study, five experts were involved in

Table 9: MACASP results

Varieties	$g(a^k) = \frac{\sum_{j=1}^{j=12}\sum_{l=1}^{j=4} w_j^{d_l} j.g_j^{d_l}(a^k)}{4}$	Time(s)
a^1	79.366	
a^2	55.3364	
a^3	29.7681	
a^4	76.5642	
a^5	68.5166	
a^6	39.0480	
a^7	67.2435	
a^8	35.6265	
a^9	50.6410	0.004994
a^{10}	54.1746	
a^{11}	8.3171	
a^{12}	53.2201	
a^{13}	56.9188	
a^{14}	50.0482	
a^{15}	47.6647	

the decision-making process. They also played the role of decision-makers. Seven potential sites were selected on the basis of seven criteria, divided into two groups of criteria.

- Group 1 representing criteria related to different engineering infrastructures: (1,1): Distance en km au réseaux de chauffage centralisé; (1,2):Distance in km to power supply networks of 110 kW; (1,3): Distance in km to high-pressure gas pipeline (12 bar); (1,4): Distance in km to water supply networks

- Group 2 representing urban planning and social criteria: (2,1):Distance in km to Vilnius city center; (2,2):Average number of people living in the territory within a radius of 1 km^2; (2,3):Usable surface owned by people living in the project area in m^2.

The normalized evaluation matrix is given in Table 10.

Table 10: Normalized Evaluation matrix

Sites	(1, 1) $a^k_{1,1}$	(1, 2) $a^k_{1,2}$	(1, 3) $a^k_{1,3}$	(1, 4) $a^k_{1,4}$	(2, 1) $a^k_{2,1}$	(2, 2) $a^k_{2,2}$	(2, 3) $a^k_{2,3}$
a^1	1.5	0.6	2.5	1.37	9.26	3188.6	55,269
a^2	3.5	1.2	4.5	0.5	8.64	497.5	9,327
a^3	0.8	0.5	3	0.1	6.44	2.484	50.798
a^4	4.8	1.2	1.6	2	11.19	2.676	56.206
a^5	5.5	1	1.6	0.3	5.9	3.291	66.807
a^6	0.6	0.7	2	0.6	6.09	6.490	132.136
a^7	0.3	0.4	2	0.6	5.72	5946.7	123.314

4.2.1. Resolution using the KEMIRA G-I and KEMIRA G-II methods

- First, the five decision-makers (Experts) express their preferences by ranking the criteria as showed in the Table 11.

- In a second stage, the median ranking is determined using the Borda voting method and the result presented in Table 12.

Table 11: Ranking of criteria by the five DMs

	Group 1					Group 2				
Rank	d_1	d_2	d_3	d_4	d_5	d_1	d_2	d_3	d_4	d_5
1^{st}	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(2,2)	(2,1)	(2,3)	(2,3)	(2,1)
2^{nd}	(1,4)	(1,4)	(1,2)	(1,2)	(1,2)	(2,3)	(2,2)	(2,1)	(2,1)	(2,3)
3^{rd}	(1,2)	(1,2)	(1,3)	(1,4)	(1,4)	(2,1)	(2,3)	(2,2)	(2,2)	(2,2)
4^{th}	(1,3)	(1,3)	(1,4)	(1,3)	(1,3)					

Table 12: Median ranking of criteria

group 1 median ranking	group 2 median ranking
$(1,1) \succsim (1,2) \succsim (1,4) \succsim (1,3)$	$(2,3) \succsim (2,1) \succsim (2,2)$

- In a third stage, the KEMIRA algorithm is implemented with the parameters indicated in relations (18), (19), (20). The results with different execution times associated are shown in Table 13 and Table 14.

$$max_iter = 10000 \quad (18)$$

$$\alpha_i = p_i\% max_{k=1}^{15} W_i(a^k); i \in \{1,2,3\}, p_i \in \{10,20,30,40,50,60,70,75,80\} \quad (19)$$

$$p_1 = p_2 = p_3 \quad (20)$$

Table 13: KEMIRA G-I results

p_i	Criteria weights							Best varieties	Time
	(1,1)	(1,2)	(1,3)	(1,4)	(2,1)	(2,2)	(2,3)		
	$w_{1,1}$	$w_{1,2}$	$w_{1,3}$	$w_{1,4}$	$w_{2,1}$	$w_{2,2}$	$w_{2,3}$		
10;20;30;40;50	0.5	0.25	0.23	0.03	0.5	0.35	0.15	$\{a^1, a^6, a^7\}$	14.001
60;70;80	0.33	0.3	0.21	0.16	0.39	0.39	0.21	$\{a^7\}$	14.29

- Looking Table 13, KEMIRA G-I method select a^7 as the best location site.
- Looking Table 14, for $p_i = 80$: $B^{d_1} = B^{d_2} = B^{d_3} = B^{d_4} = B^{d_5} = \{a^7\}$. So the alternative a^7 is selected as the best location site by KEMIRA G-II method.

4.2.2. Resolution by Lon-Zo et MACASP methods

Since we have taken this case study from the literature[6], we have considered the weights obtained with the KEMIRA G-II method and assumed that the two groups of criteria are of equal importance. The weights thus determined are presented in table 15.

Lon-Zo and MACASP methods also select a^7 as the best location site as showed in Table 16 and Table 17 respectively.

5. Discussion

In the first case study, we saw that the new methods and the classic methods proposed different results. The Nafi (a^4) is the best variety for the highest possible threshold ac-

Table 14: KEMIRA G-II results

DMs	p_i	Criteria weights (1,1) $w_{1,1}$	(1,2) $w_{1,2}$	(1,3) $w_{1,3}$	(1,4) $w_{1,4}$	(2,1) $w_{2,1}$	(2,2) $w_{2,2}$	(2,3) $w_{2,3}$	Best varieties	Time
d_1	10;20	0.34	0.34	0.26	0.06	0.44	0.28	0.28	$\{a^1, a^2, a^3, a^4, a^6, a^7\}$	14.54
	30	0.33	0.30	0.24	0.12	0.35	0.33	0.32	$\{a^1, a^2, a^4, a^6, a^7\}$	14.26
	40	0.27	0.26	0.25	0.22	0.42	0.39	0.17	$\{a^1, a^4, a^6, a^7\}$	14.04
	50	0.33	0.32	0.18	0.17	0.46	0.43	0.11	$\{a^1, a^6, a^7\}$	14.10
	60;70	0.41	0.37	0.13	0.09	0.55	0.29	0.17	$\{a^1, a^7\}$	14.45
	80	0.39	0.38	0.12	0.11	0.69	0.18	0.13	$\{a^7\}$	13.92
d_2	10;20	0.36	0.3	0.19	0.15	0.64	0.22	0.13	$\{a^1, a^2, a^3, a^4, a^6, a^7\}$	13.64
	30;40	0.38	0.27	0.18	0.15	0.37	0.34	0.27	$\{a^1, a^2, a^4, a^6, a^7\}$	13.71
	50	0.37	0.33	0.25	0.05	0.54	0.34	0.11	$\{a^1, a^6, a^7\}$	14.22
	60;70	0.38	0.3	0.22	0.11	0.46	0.42	0.12	$\{a^1, a^7\}$	14.03
	80	0.46	0.26	0.17	0.11	0.47	0.39	0.14	$\{a^7\}$	14.20
d_3	10;20;30;40;50	0.56	0.27	0.11	0.06	0.43	0.33	0.24	$\{a^1, a^6, a^7\}$	14.69
	60;70;80	0.45	0.35	0.16	0.04	0.5	0.36	0.14	$\{a^7\}$	14.07
d_4	10;20;30;40;50	0.5	0.25	0.23	0.03	0.5	0.35	0.15	$\{a^1, a^6, a^7\}$	14.001
	60;70;80	0.33	0.3	0.21	0.16	0.39	0.39	0.21	$\{a^7\}$	14.29
d_5	10;20;30;40;50	0.56	0.27	0.11	0.06	0.43	0.33	0.24	$\{a^1, a^6, a^7\}$	14.69
	60;70;80	0.45	0.35	0.16	0.04	0.5	0.36	0.14	$\{a^7\}$	14.07

Table 15: Criteria weights

Criteria	g_1 $w_1^{d_i}$	g_2 $w_2^{d_i}$	g_3 $w_3^{d_i}$	g_4 $w_4^{d_i}$	g_5 $w_5^{d_i}$	g_6 $w_6^{d_i}$	g_7 $w_7^{d_i}$
d_1	0.195	0.06	0.055	0.19	0.065	0.345	0.09
d_2	0.23	0.085	0.055	0.13	0.235	0.195	0.07
d_3	0.225	0.175	0.08	0.02	0.18	0.07	0.25
d_4	0.165	0.15	0.08	0.105	0.195	0.11	0.195
d_5	0.225	0.175	0.02	0.08	0.25	0.07	0.18

cording to the KEMIRA G-I and KEMIRA G-II methods, *i.e.*, this variety outperforms the highest possible threshold on all the criteria.

Unlike the Lon-Zo and MACASP methods, the a^1 variety is considered the best. This can be explained by the fact that this variety performs very well on some criteria and very poorly on others without said criteria being homogeneous. However, such compensation are generally allowed only when the criteria are all homogeneous, *i.e.* when natural compensation are allowed between criteria. This is not the case, for example, when considering in group 1, the criterion (1,4): disease resistance and in group 2, the criterion (2,2): seed color. These two criteria are said to be heterogeneous. The KEMIRA G-I and KEMIRA G-II methods are designed to avoid such compensation between heterogeneous criteria, unlike the Lon-Zo and MACASP methods where this is not the case

Table 16: Lon-Zo results

	$g^{d_1}(a^k) = \sum_{j=1}^{j=7} w_j^{d_1}.g_j^{d_1}(a^k)$	$g^{d_2}(a^k) = \sum_{j=1}^{j=7} w_j^{d_2}.g_j^{d_2}(a^k)$	$g^{d_3}(a^k) = \sum_{j=1}^{j=7} w_j^{d_3}.g_j^{d_3}(a^k)$	$g^{d_4}(a^k) = \sum_{j=1}^{j=7} w_j^{d_4}.g_j^{d_4}(a^k)$	$g^{d_5}(a^k) = \sum_{j=1}^{j=7} w_j^{d_5}.g_j^{d_5}(a^k)$	$g(a^k) = \frac{5}{\sum_{l=1}^{5} \frac{1}{g^{d_l}(a^k)}}$	Time(s)
a^1	0.5562	0.6055	0.6202	0.5935	0.6343	0.6007	
a^2	0.3447	0.4006	0.3248	0.3707	0.4147	0.3680	
a^3	0.4358	0.4300	0.406	0.4106	0.3848	0.4126	
a^4	0.2017	0.3859	0.4770	0.4460	0.4814	0.3580	0.0039987
a^5	0.3148	0.2700	0.3257	0.3669	0.3168	0.3158	
a^6	0.5168	0.5126	0.6875	0.6247	0.6373	0.5875	
a^7	0.8994	0.7118	0.7612	0.7353	0.7124	0.7538	

Table 17: MACASP results

Variétés	$g(a^k) = \frac{\sum_{j=1}^{j=12} \sum_{l=1}^{l=5} w_j^{d_l} . g_j^{d_l}(a^k)}{5}$	Time(s)
a^1	0.6019	
a^2	0.3711	
a^3	0.4134	
a^4	0.3984	0.0030000
a^5	0.3188	
a^6	0.5958	
a^7	0.7580	

In the second case study, all four methods selected a^7 as the best site. All criteria can be considered in a single group. In fact, the criteria in both groups are all distance-related, except for one criterion which refers to the number of people (and therefore has no unit). Consequently, all the criteria of the two groups can naturally compensate for each other, *i.e.*, the criteria can all be considered as homogeneous.

Based on the partial or incomplete information given by the decision makers (we have only asked to the DMs to rank the alternatives from best to the worst in each group as showed in Table 3 and Table 11), KEMIRA G-I and KEMIRA G-II methods have proposed an elicitation of criteria weights as showed in Table 13 and Table 14. This is an advantage in a decision-making process, as the process of weighting criteria is generally tedious for decision-makers. Of course, applying the Lon-Zo and MACASP methods assumes that you have previously determined the weights of all the criteria.

With regard to the execution times of the KEMIRA G-I and KEMIRA G-II methods on the one hand, and Lon-Zo and MACASP on the other, we note that the latter two are faster. This is also an advantage to use Lon-Zo or MACAP methods in case where all the criteria can be considered as homogenous.

The relatively long runtimes of the KEMIRA G-I and KEMIRA G-II methods can be explained by the fact that, due to incomplete information on criteria weights, more iterations are needed to stabilize the corresponding algorithms, as the search space for suitable weights that allow an alternative to be selected as the best is too large.

6. Conclusion

By solving two multi-criteria choice problems, we were able to demonstrate the effectiveness of the two new methods, KEMIRA G-I and KEMIRA G-II, when the criteria are heterogeneous. However, when the criteria can be considered as homogeneous, the Lon-Zo and MACASP methods seem to be better suited, given their speed. Another advantage of the two new methods, KEMIRA G-I and KEMIRA G-II, is the elicitation of criteria weights based on incomplete information, which is not the case with Lon-Zo and MACASP methods. In our future work, we intend to implement a user friendly computer system that integrates these four decision group methods to better handle multi-criteria group problems. On the other, we want to see how we can reinforce the process of eliciting the weights of the criteria proposed by the new KEMIRA G-I and KEMIRA G-II methods by adding information on the intensity between criteria in their respective algorithms.

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