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Employing the Sadik Residual Power Series Method to Analyze a System of Nonlinear Caputo Time-Fractional Partial Differential Equations

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Abstract. The present study proposes an approximate analytical solution to a nonlinear system of time-fractional partial differential equations. The Sadik residual power series method, which integrates the two-part Sadik integral transform with the residual power series technique, is employed to solve the fractional differential equation in the Caputo sense. Nonlinear problems with known and unknown solutions are examined to demonstrate the capacity of the technique. Numerical simulations and 3D visualizations are conducted for various values of the fractional order to further understand the solution's behavior. Additionally, the results are validated against exact solutions or existing methodologies to ensure their reliability and accuracy. A key advantage of the proposed method is its ability to generate results without the need for Adomian polynomials, perturbation techniques, discretization, or linearization, enabling a more efficient.

2020 Mathematics Subject Classifications: 26A33, 35R11, 35A22

Key Words and Phrases: Residual Power Series, Sadik transform, Nonlinear System, Caputo fractional derivatives, Approximate solution, method

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1. Introduction

A fractional differential equation is a generalization of an integer-order differential equation that appears in various scientific frameworks [18]. The study of this sort of equation has gained increasing interest among scholars because it can describe real-world physical phenomena in greater detail and accuracy than integer differential equations offer, especially in the physical processes involving memory and heritability properties. Analyzing non-linear fractional differential equations has become an essential challenge for a century. Scientists have attempted to devise and enhance novel approaches, whether analytical or numerical, for conquering such circumstances. Among these intriguing methods are the Adomian decomposition method [15], [24], [30], homotopy-perturbation method [1], the homotopy analysis method [16], the differential transform method [12], the Chebyshev

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Wavelets method [19]. Although the procedures described above are reliable and verifiable, they come with practical constraints and require a long time to generate results. As a result, establishing a strategy for effectively addressing issues, reducing determining time, and making it advantageous is challenging for researchers.

Sadik integral transformation is one of the most vital tools used to address the issues dealing with fractional calculus in science and engineering [25], [28], [26]. The transform is considered to generalize other important integral transformations since this transformation can be reduced to others by assigning the parameter. Due to their straightforward use, the Sadik transformation has continued to gain popularity among academics since it was first introduced in 2018 [27]. The advancement of the transform in solving fractional differential equations is found in much literature and references therein, such as Alabdala et al. [2] used the Sadik transform to solve the Caputo delayed fractional differential equation; Derakhshan [9] employed the Sadik transform to obtain the solution for the equal width fractional differential equation; Pue-on [20] utilized the Sadik decomposition method to generate the solution for a system of nonlinear fractional Volterra Integro differential equations of convolution type; Pue-on [21], [22] also obtained the solution of the space and time fractional telegraph equation and well-known fractional partial differential equation by double Sadik transform.

The residual power series approach is a highly effective semi-analytic procedure for handling scientific challenges. The aforementioned strategy is based on the generalized Taylor series expansion. The concept is to view the solution as a fractional power series with unknown coefficients and establish a residual function. A traditional residual power series method and an improved version have been widely utilized in literature to address a system of fractional differential equations, both linear and non-linear. Recently, some of these developments in non-linear system can be observed in [3], [5], [13], [14] and [23].

The inspiration for this work was to devise an analytical method that leveraged two widely recognized mathematical tools, the Sadik integral transform and the residual power series technique, to solve a nonlinear system of fractional order differential equations. The primary objective of the current study is to deploy the Sadik residual power series approach to solve the initial value problem for a system of nonlinear Caputo fractional partial differential equations with the form

$$D_t^{\gamma} u_i(\mathbf{x},t) = N_i(u_j(\mathbf{x},t),\mathbf{x},t) + f_i(\mathbf{x},t),\ i = 1,\ldots,n \tag{1}$$

$$u_i(\mathbf{x},0) = g_i(\mathbf{x}). \tag{2}$$

$0 < \gamma \leqslant 1$. Here $D_t^{\gamma}(\cdot)$ denotes the Caputo partial fractional derivative of order γ, N_i represents the general nonlinear differential operator and f_i is the source term. This system is highly significant in the fields of science and technology.

The structure of the manuscript is designed as follows: Section 2 presents fundamental definitions and theorems of fractional calculus, the Sadik transform, and the residual power series method. These theoretical foundations support the methodology employed in this study. Section 3 describes the implementation of the Sadik residual power series method to nonlinear systems of fractional partial differential equations. This section also comprises illustrative examples that serve to demonstrate and validate the proposed technique. Section 4 shows a comprehensive discussion of the findings and presents the conclusions from this research.

2. Basic Definitions and Theorems of Fractional Calculus, Sadik Integral Transform and Residual Power Series Method

This section reviews the basic notions and essential properties of fractional calculus. Furthermore, the definitions and theorems of the Sadik integral transform and residual power series technique are discussed.

2.1. Fractional Calculus

Definition 1. *[18] The fractional integral operator of order $\gamma(\gamma \geqslant 0)$ of $\phi(t)$ of Riemann-Liouville type is defined as*

$$I^{\gamma}\phi(t) = \begin{cases} \frac{1}{\Gamma(\gamma)} \displaystyle\int_0^t \frac{\phi(\tau)}{(t-\tau)^{1-\gamma}} d\tau, \ \gamma > 0, \ t > 0, \\ \phi(t), \ \gamma = 0 \end{cases}$$

Definition 2. *The Caputo fractional derivative operator D^{γ} of order γ, $(n-1 < \gamma \leqslant n, n \in \mathbb{N})$ is defined in the following form,*

$$D^{\gamma}\phi(t) = \frac{1}{\Gamma(n-\gamma)} \int_0^t (t-\tau)^{-\gamma+n-1} \phi^{(n)}(\tau) d\tau, \tag{3}$$

$\gamma > 0$, $t > 0$, *where the function $\phi(t)$ has absolutely continuous derivatives up to order $n-1$.*

Moreover, the following basic properties can be proved

1.) $D^{\gamma}C = 0$, C is a constant

2.) For $p \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$ and the ceiling function $\lceil \gamma \rceil$ refers to the smallest integer greater than or equal to γ

$$D^{\gamma}x^p = \begin{cases} 0, & \text{for } p < \lceil \gamma \rceil \\ \dfrac{\Gamma(p+1)}{\Gamma(p+1-\gamma)} x^{p-\gamma}, & \text{for } p \geqslant \lceil \gamma \rceil \end{cases}$$

3.) $D^{\gamma}I^{\gamma}\phi(t) = \phi(t)$,

4.) $I^{\gamma}D^{\gamma}\phi(t) = \phi(t) - \displaystyle\sum_{k=1}^{n-1} \frac{\phi^{(k)}(0)}{k!} t^k$.

Definition 3. *[7] The fractional derivative of order $\gamma > 0$ in the Caputo sense is stated as*

$$D_t^{\gamma} u(x,t) = \frac{\partial^{\gamma} u(x,t)}{\partial t^{\gamma}} = \begin{cases} \frac{1}{\Gamma(n-\gamma)} \int_0^t (t-\tau)^{n-\gamma-1} \frac{\partial^n u(x,\tau)}{\partial \tau^n} d\tau, & n-1 < \gamma < n \\ \frac{\partial^n u(x,t)}{\partial t^n}, & \gamma = n \in \mathbb{N}. \end{cases}$$

Definition 4. *[17] The Mittag-Leffler function $E_p(z)$ with $p > 0$ is defined as*

$$E_p(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(pn+1)}.$$

2.2. The Sadik Transform

Definition 5. *[27] If $f(t)$ is piecewise continuous function on the interval $0 \leqslant t \leqslant A$ for any $A > 0$ and $|f(t)| \leqslant Ke^{Bt}$ when $t \geqslant M$, for any real constant B and some positive constant K and M. Then Sadik transform of $f(t)$ is defined by*

$$\mathcal{S}\left[f(t)\right] = \frac{1}{v^{\beta}} \int_0^{\infty} f(t) e^{-tv^{\alpha}} dt = F(v^{\alpha}, \beta) \tag{4}$$

where v is complex variable, α is any non zero real number, and β is any real number. Here $\mathcal{S}$ is called the Sadik transform operator.

Remark 1. *The Sadik transform can be evolved to the other intrgral transform by assigning the parameter α and β as follows : $\alpha = 1, \beta = 0$ (Laplace Transform), $\alpha = 1, \beta = 1$ (Aboodh Transform), $\alpha = 1, \beta = -1$ (Laplace–Carson transform), $\alpha = -1, \beta = 0$ (Kamal TransformX, $\alpha = -1, \beta = 1$ (Sumudu Transform), $\alpha = -1, \beta = -1$ (Elzaki Transform), $\alpha = -2, \beta = 1$ (Tarig Transform).*

Remark 2. *The basic property of Sadik transform and its transform of elemetary functions are shown in [27].*

Theorem 1. *[25] Let $F(v^{\alpha}, \beta)$ denote the Sadik transform of $f(t)$ and $f'(t), f''(t), f'''(t), \ldots,$ $f^{(n-1)}(t)$ are continuous on $[0, \infty.)$ Then the Sadik transform of $f^{(n)}(t)$ is*

$$\mathcal{S}\left[f^{(n)}(t)\right] = v^{n\alpha} F(v^{\alpha}, \beta) - \sum_{k=0}^{n-1} v^{k\alpha-\beta} f^{(n-1-k)}(0). \tag{5}$$

Theorem 2. *[25] Let $n-1 < \gamma < n, (n = [\gamma]+1)$ and $f(t), f'(t), \ldots, f^{(n-1)}(t)$ are continuous on $[0, \infty)$ and of exponential order, while $D^{\gamma} f(t)$ is piecewise continuous on $[0, \infty)$. Then Sadik transform of Caputo fractional derivative of order γ of function f is given by*

$$\mathcal{S}[D^{\gamma} f(t)] = v^{\gamma\alpha} F(v^{\alpha}, \beta) - \sum_{k=0}^{n-1} v^{(\gamma-n+k)\alpha-\beta} f^{(n-1-k)}(0^{+}).$$

2.3. The Residual Power Series Method

Definition 6. *[10, 11] If $n \in \mathbb{N}$ where $n-1 < \gamma \leqslant n$. A power series expansion of the form*

$$\sum_{n=0}^{\infty} a_n(t-t_0)^{n\gamma} = a_0 + a_1(t-t_0)^{\gamma} + a_2(t-t_0)^{2\gamma} + \ldots, t \geqslant t_0$$

is called fractional power series about t_0.

Theorem 3. *[10, 11] Suppose that $u(t)$ has a fractional power series representation at $t = t_0$ of the form*

$$u(t) = \sum_{n=0}^{\infty} a_n(t-t_0)^{n\gamma}, \quad 0 \leqslant m-1 < \gamma \leqslant m,\ t_0 \leqslant t < t_0 + R.$$

If $u(t) \in C[t_0, t_0+R)$ and $D^{n\gamma}u(t) \in C(t_0, t_0+R)$, $n = 0, 1, 2, \ldots,$ then $a_n = \frac{D^{n\gamma}u(t)}{\Gamma(n\gamma+1)}$.

Definition 7. *[10, 11] A power series of the form*

$$\sum_{n=0}^{\infty} f_n(x)(t-t_0)^{n\gamma} = f_0(x) + f_1(x)(t-t_0)^{\gamma} + f_2(x)(t-t_0)^{2\gamma} + \ldots$$

is called multiple fractional power series about $t = t_0$, where t is a variable and f_m's are functions of x called the coefficients of the series.

Theorem 4. *[10, 11] Suppose that $u(x,t)$ has a multiple fractional power series representation at $t = t_0$ of the form*

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = \sum_{n=0}^{\infty} f_n(x)(t-t_0)^{n\gamma},$$

$0 \leqslant m-1 < \gamma \leqslant m$, $x \in I$, $t_0 \leqslant t < t_0 + R$. If $D_t^{n\gamma}u(x,t)$, $n = 0, 1, 2, \ldots$ are continuous on $I \times (t_0, t_0+R)$, then

$$f_n(x) = \frac{D_t^{n\gamma}u(x,t)}{\Gamma(n\gamma+1)}, n = 0, 1, 2, \ldots.$$

Here $D_t^{n\gamma}(\cdot) = \frac{\partial^{n\gamma}}{\partial t^{n\gamma}}(\cdot) = \frac{\partial^{\gamma}}{\partial t^{\gamma}}(\frac{\partial^{\gamma}}{\partial t^{\gamma}}(\ldots(\frac{\partial^{\gamma}}{\partial t^{\gamma}}(\cdot))))\,(n-times)$, and $R = \min_{c \in I} R_c$, in which R_c is the radius of convergence of the fractional power series $\sum_{k=0}^{\infty} f_n(c)(t-t_0)^{n\gamma}$.

According to the convergence of the classic residual power series method, there is a real number $0 < \lambda < 1$, such that $||u_{n+1}(x,t)|| \leqslant \lambda ||u_n(x,t)||$, $t \in (t_0, t_0+R)$.

3. Main Results

This section explains the Sadik residual power series method and presents revealed examples that support its suggested concept.

3.1. Implementation of the Sadik residual power series method in a system of time-fractional PDEs

Consider the nonlinear system of time-fractional partial differential equation (1)

$$D_t^{\gamma} u_i(\mathbf{x},t) = N_i(u_j(\mathbf{x},t),\mathbf{x},t) + f_i(\mathbf{x},t),\ i = 1,\dots,n$$

with initial conditions $u_i(\mathbf{x},0) = g_i(\mathbf{x})$. Applying the Sadik transform and along with the initial condition yields

$$v^{\gamma\alpha}U_i(\mathbf{x},v^{\alpha},\beta) - v^{(\gamma-1)\alpha-\beta}g_i(\mathbf{x}) = \mathcal{S}\Big[N_i(u_j(\mathbf{x},t),\mathbf{x},t)\Big] + F_i(\mathrm{x},v^{\alpha},\beta).$$

Rearranging the equation obtained

$$U_i(\mathbf{x},v^{\alpha},\beta) = \frac{1}{v^{\alpha+\beta}}g_i(\mathbf{x}) + \frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[N_i(u_j(\mathbf{x},t),\mathbf{x},t)\Big] + \frac{1}{v^{\gamma\alpha}}F_i(\mathrm{x},v^{\alpha},\beta).$$

Performing the inverse Sadik transform results in

$$u_i(\mathbf{x},t) = g_i(\mathbf{x}) + \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[N_i(u_j(\mathbf{x},t),\mathbf{x},t)\Big]\Big] + \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}F_i(\mathrm{x},v^{\alpha},\beta)\Big].$$

Assume the solution to (1) is provided within the infinite series

$$u_i(\mathbf{x},t) = \sum_{n=0}^{\infty} a_{i,n}(\mathbf{x})\frac{t^{n\gamma}}{\Gamma(n\gamma+1)}. \tag{6}$$

According to the residual power series strategy, we define the residual function as follows

$$\mathrm{Res}_i(\mathbf{x},t) = u_i(\mathbf{x},t) - g_i(\mathbf{x}) - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[N_i(u_j(\mathbf{x},t),\mathbf{x},t)\Big]\Big] - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}F_i(\mathrm{x},v^{\alpha},\beta)\Big],$$

and the kth truncated series of (6) is stated as

$$u_k(\mathbf{x},t) = \sum_{n=0}^{k} a_{i,n}(\mathbf{x})\frac{t^{n\gamma}}{\Gamma(n\gamma+1)}.$$

Since the solution satisfies the initial condition, $a_{i,0}(\mathbf{x}) = g_i(\mathbf{x})$. Hence, the k- order approximated solution turns into

$$u_{i,k}(\mathbf{x},t) = g_i(\mathbf{x}) + \sum_{n=1}^{k} a_{i,n}(\mathbf{x})\frac{t^{n\gamma}}{\Gamma(n\gamma+1)}. \tag{7}$$

Furthermore, the k- residual function is expressed as

$$\mathrm{Res}_{i,k}(\mathbf{x},t) = u_{i,k}(\mathbf{x},t) - g_i(\mathbf{x}) - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[N_i(u_{j,k}(\mathbf{x},t),\mathbf{x},t)\Big]\Big] - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}F_i(\mathrm{x},v^{\alpha},\beta)\Big].$$

For the calculation of the coefficients $a_{i,n}(\mathbf{x}),\ n = 1,\ 2,\ 3,\ \ldots$ in equation (7), construct the relation for recurrence

$$\begin{cases} u_{i,0}(\mathbf{x}, t) = g_i(\mathbf{x}) \\ \mathrm{Res}_{i,k}(\mathbf{x}, t) = u_{i,k}(\mathbf{x}, t) - g_i(\mathbf{x}) - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[N_i(u_{j,k}(\mathbf{x}, t), \mathbf{x}, t)\Big]\Big] - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}F_i(\mathrm{x}, v^{\alpha}, \beta)\Big] \end{cases} \tag{8}$$

and and afterward follow the condition [29]

$$t^{-k\alpha} \cdot \mathrm{Res}_{i,k}(\mathbf{x}, t)\Big|_{t=0} = 0. \tag{9}$$

Convergence criteria for Sadik residual power series method

Theorem 5. *[8] Let the Banach space $B \equiv C(X \times [0, T])$ is defined on $X \times [0, T]$, where $X = [a_1, b_1] \times [a_2, b_2] \times \ldots \times [a_n, b_n]$, then Eq. (6) as $u(\boldsymbol{x}, t) = \sum_{n=0}^{k} u_k(\boldsymbol{x}, t)$ is convergent series, if $u_0 \in B$ is bounded and $\forall u_k \in B, \|u_{k+1}\| \leqslant \lambda\|u_k\|$, where $0 < \lambda < 1$ and $\|u\| = \sup_{\boldsymbol{x}\in X, t\in[0,T]} |u(\boldsymbol{x}, t)|$.*

Theorem 6. *[8] The maximum absolute error for (6) is defined by*

$$\|u(\boldsymbol{x}, t) - \sum_{n=0}^{k} u_n(\boldsymbol{x}, t)\| \leqslant \frac{\lambda^{k+1}}{1-\lambda}\|u_0(\boldsymbol{x}, t)\|.$$

Remark 3. *The procedure for proving the aforementioned theorems is the same as that used in the reference, which will not be examined further here.*

3.2. Illustrative Examples

Example 1. *[4] Consider the nonlinear system of fractional partial differential equation*

$$\begin{aligned} D_t^{\gamma} u(x, t) + w^2(x, t)u_x^2(x, t) + u(x, t) = 1, \\ D_t^{\gamma} w(x, t) + u^2(x, t)w_x^2(x, t) - w(x, t) = 1, \end{aligned}$$

with initial conditions

$$u(x, 0) = e^{x},\ w(x, 0) = e^{-x}.$$

The exact solution to the problem when $\gamma = 1$ is $u(x, t) = e^{x-t},\ w(x, t) = e^{-x+t}$.

Here $N_1(u, w) = -w^2u_x^2 - u, N_2(u, w) = -u^2w_x^2 + w, f_1(x, t) = f_2(x, t) = 1, g_1(x) = e^x, g_2(x) = e^{-x}$. The $k^{th}-$ truncated term series solution for this problem is

$$u_k(x, t) = \sum_{n=0}^{k} a_n(x)\frac{t^{n\gamma}}{\Gamma(n\gamma + 1)},$$

$$w_k(x,t) = \sum_{n=0}^{k} b_n(x) \frac{t^{n\gamma}}{\Gamma(n\gamma+1)}.$$

The recurrence relation to the above issue is

$$\begin{aligned}
&u_0(x,t) = e^x,\ w_0(x,t) = e^{-x},\\
&\mathrm{Res}_{1,k}(x,t) = u_k(x,t) - e^x - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[-w_{k-1}^2(u_{k-1})_x^2 - u_{k-1}\Big]\Big] - \frac{t^\gamma}{\Gamma(\gamma+1)},\\
&\mathrm{Res}_{2,k}(x,t) = w_k(x,t) - e^{-x} + \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[(u_{k-1})^2(w_{k-1})_x^2 - w_{k-1}\Big]\Big] - \frac{t^\gamma}{\Gamma(\gamma+1)}, k \geqslant 1.
\end{aligned}$$

For $k = 1$. The first-order approximate solution is assumed

$$u_1(x,t) = e^x + a_1(x)\frac{t^\gamma}{\Gamma(\gamma+1)}, \quad w_1(x,t) = e^{-x} + b_1(x)\frac{t^\gamma}{\Gamma(\gamma+1)}.$$

Substituting into the above iteration yields

$$\begin{aligned}
\mathrm{Res}_{1,1}(x,t) &= u_1(x,t) - e^x - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[-w_0^2(u_0)_x^2 - u_0\Big]\Big] - \frac{t^\gamma}{\Gamma(\gamma+1)},\\
&= a_1(x)\frac{t^\gamma}{\Gamma(\gamma+1)} + e^x\frac{t^\gamma}{\Gamma(\gamma+1)},\\
\mathrm{Res}_{2,1}(x,t) &= w_1(x,t) - e^{-x} + \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[(u_0)^2(w_0)_x^2 - w_0\Big]\Big] - \frac{t^\gamma}{\Gamma(\gamma+1)},\\
&= b_1(x)\frac{t^\gamma}{\Gamma(\gamma+1)} - e^x\frac{t^\gamma}{\Gamma(\gamma+1)}.
\end{aligned}$$

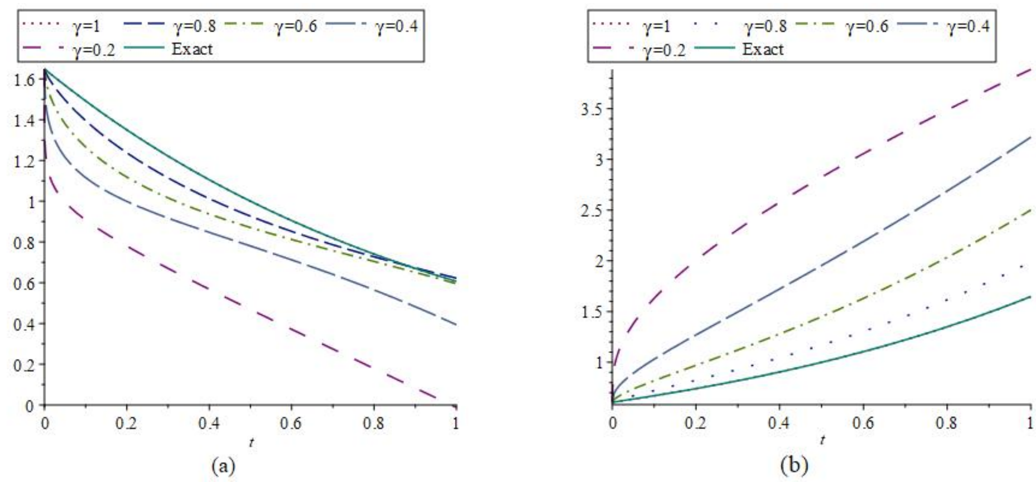


Figure 1: Subplot (a) displays the approximate solution $u_5(x,t)$, and subplot (b) presents the approximate solution $w_5(x,t)$ for Example 1 with different γ values at $x = 0.5$.

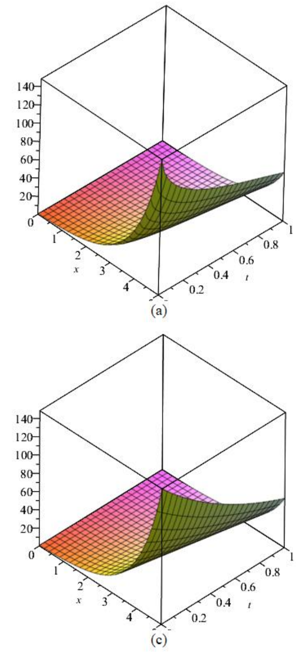


Figure 2: The graphics represent the approximate solution $u_5(x,t)$ with different γ values: (a) $\gamma = 0.6$, (b) $\gamma = 0.8$, (c) $\gamma = 1$, and (d) the exact solution for Example 1.

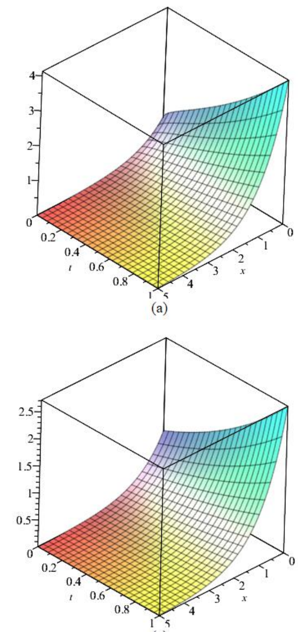


Figure 3: The graphics represent the approximate solution $w_5(x,t)$ with different γ values: (a) $\gamma = 0.6$, (b) $\gamma = 0.8$, (c) $\gamma = 1$, and (d) the exact solution for Example 1.

Using the condition (9) with $k = 1$, $t^{-\gamma}\text{Res}_{1,1}(x,t)\big|_{t=0} = 0$, $t^{-\gamma}\text{Res}_{2,1}(x,t)\big|_{t=0} = 0$, produces the coefficients that are $a_1(x) = -e^{-x}$ and $b_1(x) = e^x$. As a result, the first-order truncated series solution is

$$u_1(x,t) = e^x - e^x \frac{t^\gamma}{\Gamma(\gamma+1)}, \quad w_1(x,t) = e^{-x} + e^{-x} \frac{t^\gamma}{\Gamma(\gamma+1)}.$$

Table 1: Comparison of the approximate solution $u_5(x,t)$ for $\gamma = 1$ and error analysis in Example 1 using different t, x values.

t	x	$u_5(x,t)$	Exact Soln.	Abs.Error (SRPSM)	Abs.Error (LRPSM) [4]
0.1	0.2	1.105170916	1.105170918	2×10^{-9}	2×10^{-9}
	0.4	1.349858806	1.349858808	2×10^{-9}	2×10^{-9}
	0.6	1.648721268	1.648721271	3×10^{-9}	3×10^{-9}
	0.8	2.013752704	2.013752707	3×10^{-9}	3×10^{-9}
	1.0	2.459603107	2.459603111	4×10^{-9}	4×10^{-9}
0.25	0.2	0.9512290245	0.9512294245	4×10^{-7}	4×10^{-7}
	0.4	1.161833755	1.161834243	4.88×10^{-7}	4.88×10^{-7}
	0.6	1.419066952	1.419067549	5.97×10^{-7}	5.97×10^{-7}
	0.8	1.733252289	1.733253018	7.29×10^{-7}	7.29×10^{-7}
	1.0	2.116999126	2.117000017	8.90×10^{-7}	8.90×10^{-7}

Table 2: Comparison of the approximate solution $w_5(x,t)$ for $\gamma = 1$ and error analysis in Example 1 using different x, t values.

t	x	$w_5(x,t)$	Exact Soln.	Abs.Error (SRPSM)	Abs.Error (LRPSM) [4]
0.1	0.2	0.9048374172	0.9048374180	8×10^{-10}	8×10^{-10}
	0.4	0.7408182199	0.7408182207	8×10^{-10}	8×10^{-10}
	0.6	0.6065306591	0.6065306597	6×10^{-10}	6×10^{-10}
	0.8	0.4965853033	0.4965853038	5×10^{-10}	5×10^{-10}
	1.0	0.4065696594	0.4065696597	3×10^{-10}	3×10^{-10}
0.25	0.2	1.051270808	1.051271096	2.88×10^{-7}	2.88×10^{-7}
	0.4	0.8607077406	0.8607079764	2.358×10^{-7}	2.358×10^{-7}
	0.6	0.7046878967	0.7046880897	1.93×10^{-7}	1.93×10^{-7}
	0.8	0.5769496523	0.5769498104	1.581×10^{-7}	1.581×10^{-7}
	1.0	0.4723664234	0.4723665527	1.293×10^{-7}	1.293×10^{-7}

For $k \geqslant 2$. By employing the offered procedure and condition (9), the coefficients of the series solution are

$$a_k(x) = (-1)^k e^x, \; b_k(x) = e^{-x}.$$

Hence, the k-truncated series solution is

$$u_k(x,t) = \sum_{n=0}^{k} (-1)^n e^x \frac{t^{n\gamma}}{\Gamma(n\gamma+1)} = e^x \sum_{n=0}^{k} \frac{(-t^\gamma)^n}{\Gamma(n\gamma+1)},$$

$$w_k(x,t) = \sum_{n=0}^{k} e^{-x} \frac{t^{n\gamma}}{\Gamma(n\gamma+1)} = e^{-x} \sum_{n=0}^{k} \frac{t^{n\gamma}}{\Gamma(n\gamma+1)}.$$

It is important to note that when $k \to \infty$, the approximate solution converges on the exact solution

$$u(x,t) = e^{x} \sum_{n=0}^{\infty} \frac{(-t^{\gamma})^n}{\Gamma(n\gamma+1)} = e^{x} E(-t^{\gamma}),$$
$$w(x,t) = e^{-x} \sum_{n=0}^{k} \frac{t^{n\gamma}}{\Gamma(n\gamma+1)} = e^{-x} E(t^{\gamma}).$$

If $\gamma = 1$, the solutions is the exact solution of first-order nonlinear PDEs system

$$u(x,t) = e^{x-t},\ w(x,t) = e^{-x+t}.$$

The approximate solution for Example 1 are shown in Figure 1. Figures 2 and 3 compare the visualizations to the approximate and exact solutions.

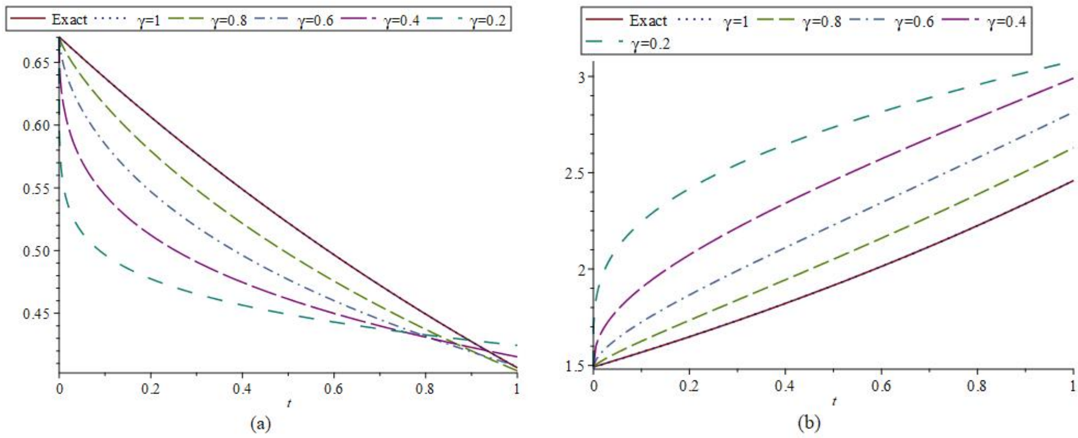


Figure 4: subplot (a) depicts the approximate solution $u_5(x,y,t)$ and subplot (b) presents the approximate solution $w_5(x,y,t)$ for Example 2 at different γ values for $x = y = 0.2$.

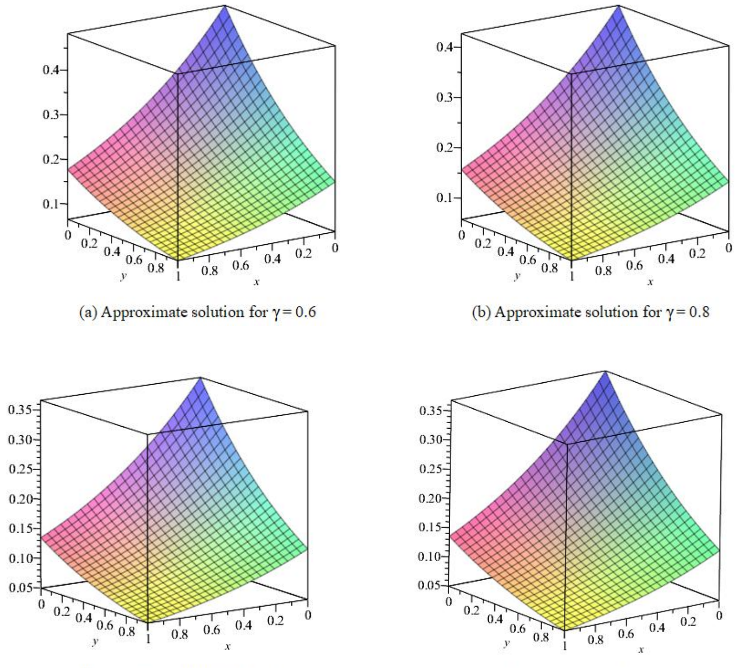
Example 2. *[6] Consider the Fractional Reaction-Diffusion Brusselator System*

$$D_t^{\gamma} u(x,y,t) = -2u(x,y,t) + u^2(x,y,t)w(x,y,t) + \frac{1}{4}\big[u_{xx}(x,y,t) + u_{yy}(x,y,t)\big],$$
$$D_t^{\gamma} w(x,y,t) = u(x,y,t) - u^2(x,y,t)w(x,y,t) + \frac{1}{4}[w_{xx}(x,y,t) + w_{yy}(x,y,t)],$$

subject to the initial conditions

$$u(x,y,0) = e^{-x-y},\ w(x,y,0) = e^{x+y}.$$

The exact solution when $\gamma = 1$ *is* $u(x,y,t) = e^{-x-y-\frac{1}{2}t},\ w(x,y,t) = e^{x+y+\frac{1}{2}t}$.



(a) Approximate solution for γ = 0.6

(b) Approximate solution for γ = 0.8

(c) Approximate solution for γ = 1

(d) Exact solution for γ = 1

Figure 5: The graphics represent the approximate solution $u_5(x,y,t)$ when $t=1, 0 \leqslant x, y \leqslant 1$, with different γ values: (a) $\gamma = 0.6$, (b) $\gamma = 0.8$, (c) $\gamma = 1$, and (d) the exact solution for Example 2.

Note that $N_1(u,w) = -2u + u^2w + \frac{1}{4}(u_{xx}+u_{yy}), N_2(u,w) = u - u^2w_x^2 + \frac{1}{4}(w_{xx} + w_{yy}), f_1(x,t) = f_2(x,t) = 0, g_1(x,y) = e^{-x-y}, g_2(x,y) = e^{x+y}$. The $k^{th}-$ truncated term series solution to the problem is

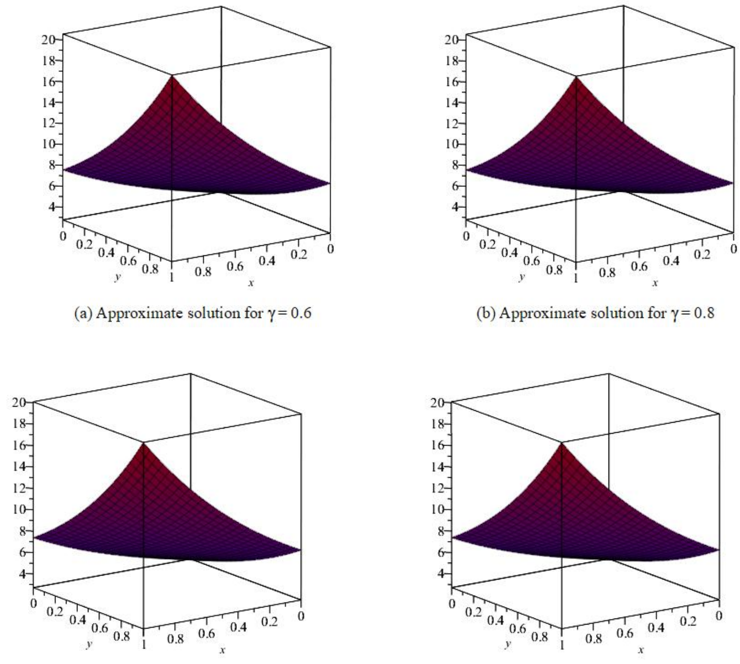
$$u_k(x,y,t) = \sum_{n=0}^{k} a_n(x,y)\frac{t^{n\gamma}}{\Gamma(n\gamma+1)} \quad \text{and} \quad w_k(x,y,t) = \sum_{n=0}^{k} b_n(x,y)\frac{t^{n\gamma}}{\Gamma(n\gamma+1)}.$$

The recursive relation for this problem is

$$\begin{aligned} &u_0(x,y,t) = e^{-x-y},\ w_0(x,y,t) = e^{x+y},\\ &\text{Res}_{1,k}(x,y,t) = u_k(x,y,t) - e^{-x-y} + 2\mathcal{S}^{-1}\Big[\frac{1}{\upsilon^{\gamma\alpha}}\mathcal{S}\Big[u_{k-1}\Big]\Big] - \mathcal{S}^{-1}\Big[\frac{1}{\upsilon^{\gamma\alpha}}\mathcal{S}\Big[u_{k-1}^2 w_{k-1}\Big]\Big]\\ &\qquad - \frac{1}{4}\mathcal{S}^{-1}\Big[\frac{1}{\upsilon^{\gamma\alpha}}\mathcal{S}\Big[(u_{k-1})_{xx} + (u_{k-1})_{yy}\Big]\Big],\ k \geqslant 1\\ &\text{Res}_{2,k}(x,y,t) = w_k(x,y,t) - e^{x+y} - \mathcal{S}^{-1}\Big[\frac{1}{\upsilon^{\gamma\alpha}}\mathcal{S}\Big[u_{k-1}\Big]\Big] + \mathcal{S}^{-1}\Big[\frac{1}{\upsilon^{\gamma\alpha}}\mathcal{S}\Big[(u_{k-1})^2 w_{k-1}\Big]\Big]\\ &\qquad - \frac{1}{4}\mathcal{S}^{-1}\Big[\frac{1}{\upsilon^{\gamma\alpha}}\mathcal{S}\Big[(w_{k-1})_{xx} + (w_{k-1})_{yy}\Big]\Big],\ k \geqslant 1. \end{aligned}$$

Proceed to continue the recurrence to obtain the coefficients,

$$a_1(x,y) = -\frac{1}{2}e^{-x-y},\ a_2(x,y) = \frac{1}{4}e^{-x-y},\ \ldots, a_k(x,y) = \frac{1}{(-2)^k}e^{-x-y}$$



(a) Approximate solution for γ = 0.6 (b) Approximate solution for γ = 0.8

(c) Approximate solution for γ = 1 (d) Exact Solution for γ = 1

Figure 6: The graphics represent the approximate solution $w_5(x,y,t)$ when $t=1, 0\leqslant x,y\leqslant 1$, with different γ values: (a) $\gamma=0.6$, (b) $\gamma=0.8$, (c) $\gamma=1$, and (d) the exact solution for Example 2.

$$b_1(x,y)=\frac{1}{2}e^{x+y}, b_2(x,y)=\frac{1}{4}e^{x+y}, \ldots, b_k(x,y)=\frac{1}{2^k}e^{x+y}.$$

Hence, the k-order approximate series solution is

$$u_k(x,y,t)=e^{-x-y}\Big[1-\frac{1}{2}\frac{t^{\gamma}}{\Gamma(\gamma+1)}+\frac{1}{4}\frac{t^{2\gamma}}{\Gamma(2\gamma+1)}+\ldots+\frac{1}{(-2)^k}\frac{t^{k\gamma}}{\Gamma(k\gamma+1)}\Big]$$

$$w_k(x,y,t)=e^{x+y}\Big[1+\frac{1}{2}\frac{t^{\gamma}}{\Gamma(\gamma+1)}+\frac{1}{4}\frac{t^{2\gamma}}{\Gamma(2\gamma+1)}+\ldots+\frac{1}{2^k}\frac{t^{k\gamma}}{\Gamma(k\gamma+1)}\Big].$$

When $k\to\infty$, the approximate solution converges to

$$u(x,y,t)=e^{-x-y}\sum_{n=0}^{\infty}\frac{(-\frac{t^{\gamma}}{2})^n}{\Gamma(n\gamma+1)}=e^{-x-y}E(\frac{-t^{\gamma}}{2})$$

$$w(x,y,t)=e^{x+y}\sum_{n=0}^{\infty}\frac{(\frac{t^{\gamma}}{2})^n}{\Gamma(n\gamma+1)}=e^{x+y}E(\frac{t^{\gamma}}{2}).$$

If $\gamma=1$, the result is reduced to the solution of the first-order equation

$$u(x,y,t)=e^{-x-y-\frac{1}{2}t} \quad \text{and} \quad w(x,y,t)=e^{x+y+\frac{1}{2}t}.$$

The approximate solution for Example 2 are shown in Figure 4. Figures 5 and 6 compare the visualizations to the approximate and exact solutions.

Table 3: Comparison of the approximate solution $u_5(x, y, t)$ for $x = y = 0.2$ in Example 2 using different γ values.

t	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1$	Exact Soln.
0.2	0.477488627	0.512326854	0.547063183	0.579385607	0.606530658	0.606530659
0.4	0.456444828	0.474696013	0.496237125	0.521635809	0.548811578	0.548811636
0.6	0.443011054	0.449925766	0.460073110	0.475641940	0.496584653	0.496585303
0.8	0.432811651	0.430960053	0.431579433	0.437159809	0.449325358	0.449328964
1.0	0.424420175	0.415283298	0.407895963	0.404110210	0.406556090	0.406569659

Table 4: Comparison of the approximate solution $w_5(x, y, t)$for $x = y = 0.2$ in Example 2 using different γ values.

t	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1$	Exact soln.
0.2	2.417846010	2.074372749	1.865586354	1.734071972	1.648721268	1.648721271
0.4	2.646725351	2.342078614	2.111643502	1.944446048	1.822118665	1.822118800
0.6	2.816085119	2.572130993	2.344840870	2.160190740	2.013751129	2.013752707
0.8	2.956794558	2.785737340	2.577721089	2.387602322	2.225531932	2.225540928
1.0	3.079989436	2.990722585	2.815404246	2.630080531	2.459568271	2.459603111

Table 5: Comparison of the absolute error for $u_2(x, y, t)$ when $x = y = 0.2$ and $x = y = 0.5$ in Example 2 using different t values.

(x,y)	t	$u_2(x, y, t)$	Exact Soln.	Abs. Error (SRPSM)	Abs. Error (CRPS)[6]
$(0.2, 0.2)$	0.2	0.6066396416	0.6065306597	1.089819×10^{-4}	1.0898×10^{-4}
	0.4	0.5496624377	0.5488116361	8.508016×10^{-4}	8.5080×10^{-4}
	0.6	0.4993884343	0.4965853038	2.8031305×10^{-3}	2.8031×10^{-3}
	0.8	0.4558176313	0.4493289641	6.4886672×10^{-3}	6.4886×10^{-3}
$(0.5, 0.5)$	0.2	0.3329308943	0.3328710837	5.98106×10^{-5}	5.9810×10^{-5}
	0.4	0.3016611418	0.3011942119	4.669299×10^{-4}	4.6692×10^{-4}
	0.6	0.2740701836	0.2725317930	1.5383906×10^{-3}	1.5383×10^{-3}
	0.8	0.2501580200	0.2465969639	3.5610561×10^{-3}	3.5610×10^{-3}

Table 6: Comparison of the absolute error for $w_2(x, y, t)$ when $x = y = 0.2$ and $x = y = 0.5$ in Example 2 using different t values.

(x, y)	t	$w_2(x, y, t)$	Exact Soln.	Abs. Error (SRPSM)	Abs. Error (CRPS)[6]
$(0.2, 0.2)$	0.2	1.648466291	1.648721271	2.54980×10^{-4}	2.5497×10^{-4}
	0.4	1.820026132	1.822118800	2.092668×10^{-3}	2.0926×10^{-3}
	0.6	2.006504218	2.013752707	7.248489×10^{-3}	7.2484×10^{-3}
	0.8	2.207900553	2.225540928	1.7640375×10^{-2}	1.7640×10^{-2}
$(0.5, 0.5)$	0.2	3.003701420	3.004166024	4.64604×10^{-4}	4.6460×10^{-4}
	0.4	3.316303831	3.320116923	3.813092×10^{-3}	3.8130×10^{-3}
	0.6	3.656089058	3.669296668	1.3207610×10^{-2}	1.3207×10^{-3}
	0.8	4.023057105	4.055199967	3.2142862×10^{-2}	3.2142×10^{-2}

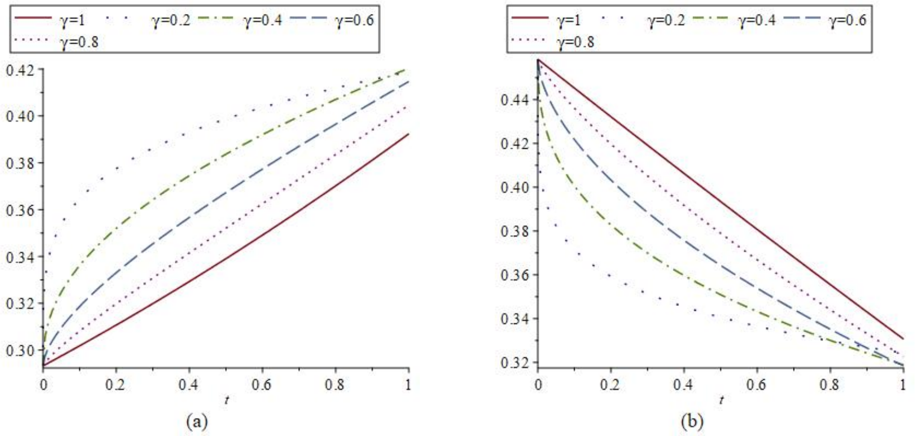


Figure 7: subplot (a) depicts the approximate solution $u_2(x,t)$ and subplot (b) presents the approximate solution $w_2(x,t)$ for Example 3 at different γ values for $x = 0.5$.

Example 3. *Consider the coupled system of reaction-diffusion equations*

$$D_t^{\gamma}u(x,t) = u(x,t) - u^2(x,t) - u(x,t)w(x,t) + u_{xx}(x,t),$$
$$D_t^{\gamma}w(x,t) = w_{xx}(x,t) - u(x,t)w(x,t),$$

subject to the initial conditions

$$u(x,0) = \frac{e^{px}}{(1+e^{0.5px})^2},\ w(x,0) = \frac{1}{1+e^{0.5px}}$$

where p is constant.

Here $N_1(u,w) = u - u^2 - uw + u_x$, $N_2(u,w) = w_{xx} - uw$, $f_1(x,t) = f_2(x,t) = 0$, $g_1(x) = \frac{e^{px}}{(1+e^{0.5px})^2}$, $g_2(x) = \frac{1}{1+e^{0.5px}}$. The $k^{th}-$ truncated term series solution to the problem is

$$u_k(x,t) = \sum_{n=0}^{k} a_n(x)\frac{t^{n\gamma}}{\Gamma(n\gamma+1)} \quad \text{and} \quad w_k(x,t) = \sum_{n=0}^{k} b_n(x)\frac{t^{n\gamma}}{\Gamma(n\gamma+1)}.$$

The iteration for this problem is

$$u_0(x,t) = \frac{e^{px}}{(1+e^{0.5px})^2},\ w_0(x,t) = \frac{1}{1+e^{0.5px}},$$

$$\text{Res}_{1,k}(x,t) = u_k(x,t) - \frac{e^{px}}{(1+e^{0.5px})^2} - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[u_{k-1}\Big]\Big] + \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[u_{k-1}^2\Big]\Big]$$
$$+ \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[u_{k-1}w_{k-1}\Big]\Big] - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[(u_{k-1})_{xx}\Big]\Big],\ k \geqslant 1$$

$$\text{Res}_{2,k}(x,t) = w_k(x,t) - \frac{1}{1+e^{0.5px}} - \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[(w_{k-1})_{xx}\Big]\Big] + \mathcal{S}^{-1}\Big[\frac{1}{v^{\gamma\alpha}}\mathcal{S}\Big[u_{k-1}w_{k-1}\Big]\Big]\ k \geqslant 1.$$

After a few iterations, the coefficients are obtained:

$$a_1(x) = \frac{e^{px}}{(1+e^{0.5px})^4}\big[e^{0.5px} - 0.5p^2e^{0.5px} + p^2\big],$$

$$b_1(x) = \frac{0.25}{(1+e^{0.5px})^3}\big[p^2e^{px} - 4e^{px} - p^2e^{0.5px}\big],$$

and

$$a_2(x) = \frac{1}{8(1+e^{0.5px})^6}\big[-32p^2e^{2px} + 16e^{2px} + 28p^2e^{1.5px} - 33p^4e^{1.5px} + 4p^2e^{2.5px} + 18p^4e^{2px} - p^4e^{2.5px} + 8p^4e^{px}\big],$$

$$b_2(x) = \frac{1}{16(1+e^{0.5px})^5}\big[p^4e^{2px} + 16e^{2px} - 8p^2e^{2px} + 11p^4e^{px} - 11p^4e^{1.5px} + 40p^2e^{1.5px} - p^4e^{0.5px} - 32p^2e^{px} - 16e^{1.5px}\big],$$

Hence, the 2-nd order approximate solution is

Table 7: Comparison of the approximate solution $u_2(x,t)$ in Example 3 using different γ values.

t	x	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1$
	0.2	0.3346368216	0.3075765318	0.2909673743	0.2809869555	0.2750395108
	0.4	0.3545182352	0.3265194339	0.3093155911	0.2989687897	0.2927994540
0.1	0.6	0.3747087628	0.3458387005	0.3280801409	0.3173903381	0.3110127944
	0.8	0.3951398996	0.3654736576	0.3472053559	0.3361990391	0.3296288598
	1.0	0.4157415419	0.3853612031	0.3666326584	0.3553391449	0.3485936321

Table 8: Comparison of the approximate solution $w_2(x,t)$ in Example 3 using different γ values.

t	x	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1$
	0.2	0.3968241630	0.4264368572	0.4475212727	0.4616009993	0.4705466352
	0.4	0.3798767149	0.4091653431	0.4303163383	0.4445647827	0.4536616511
0.1	0.6	0.3632965320	0.3921407562	0.4132861957	0.4276608484	0.4368839467
	0.8	0.3471168287	0.3754019837	0.3964705401	0.4109280813	0.4202514134
	1.0	0.3313660940	0.3589848110	0.3799068930	0.3944037158	0.4038005919

$$u_2(x,t) = \frac{e^{px}}{(1+e^{0.5px})^2} + \frac{e^{px}(e^{0.5px} - 0.5p^2e^{0.5px} + p^2)}{(1+e^{0.5px})^4}\cdot\frac{t^\gamma}{\Gamma(\gamma+1)} + \frac{(18p^4e^{2px} - 32p^2e^{2px} + 16e^{2px} + 28p^2e^{1.5px} - 33p^4e^{1.5px} + 4p^2e^{2.5px} - p^4e^{2.5px} + 8p^4e^{px})}{8(1+e^{0.5px})^6} \times \frac{t^{2\gamma}}{\Gamma(2\gamma+1)},$$

$$w_2(x,t) = \frac{1}{1+e^{0.5px}} + \frac{0.25(p^2e^{px} - 4e^{px} - p^2e^{0.5px})}{(1+e^{0.5px})^3}\cdot\frac{t^\gamma}{\Gamma(\gamma+1)} + \frac{(p^4e^{2px} + 16e^{2px} - 8p^2e^{2px} + 11p^4e^{px} - 11p^4e^{1.5px} + 40p^2e^{1.5px} - p^4e^{0.5px} - 32p^2e^{px} - 16e^{1.5px})}{16(1+e^{0.5px})^5} \times \frac{t^{2\gamma}}{\Gamma(2\gamma+1)}.$$

The approximate solution for Example 3 where $p = 2/3$ are shown in Figure 7. Figures 8 and 9 provide the visualizations to the approximate solutions.

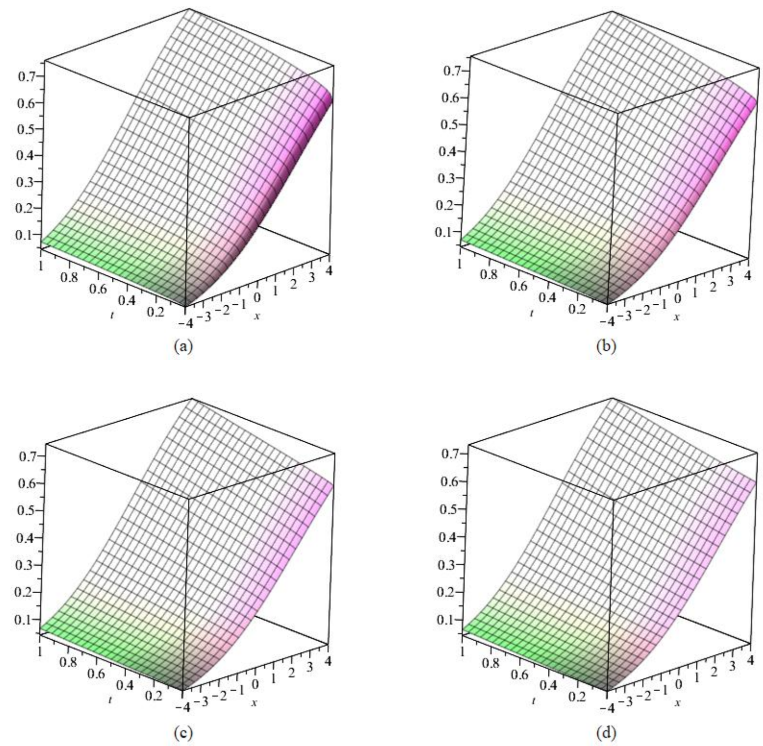


Figure 8: The graphics represent the approximate solution $u_2(x,t)$ with different γ values: (a) $\gamma = 0.4$, (b) $\gamma = 0.6$, (c) $\gamma = 0.8$, and (d) $\gamma = 1.0$ for Example 3.

4. Discussion

Based on the findings in Example 1, the suggested method provides a solution in the form of an infinite series that converges to exact solutions. Figure 1 compares the approximate solutions at different γ values with the exact solutions when $x = 0.5$. Figures 2 and 3 compare the visualizations to the approximate and exact solutions. These graphs show how the solution shapes will approach the precise solution as gamma approaches 1. Tables 1 and 2 deliver a comparison of numerical solutions derived with accurate solutions. Furthermore, an error analysis of the Sadik residual power series method (SRPSM) and the Laplace residual power series method (LRPSM) is carried out, demonstrating that both performed similarly.

In Example 2, the approximate solution in the form of an infinite convergent series is found. Figure 4. provides the outcome at different values of γ when $x = y = 0.2$ and the 3D

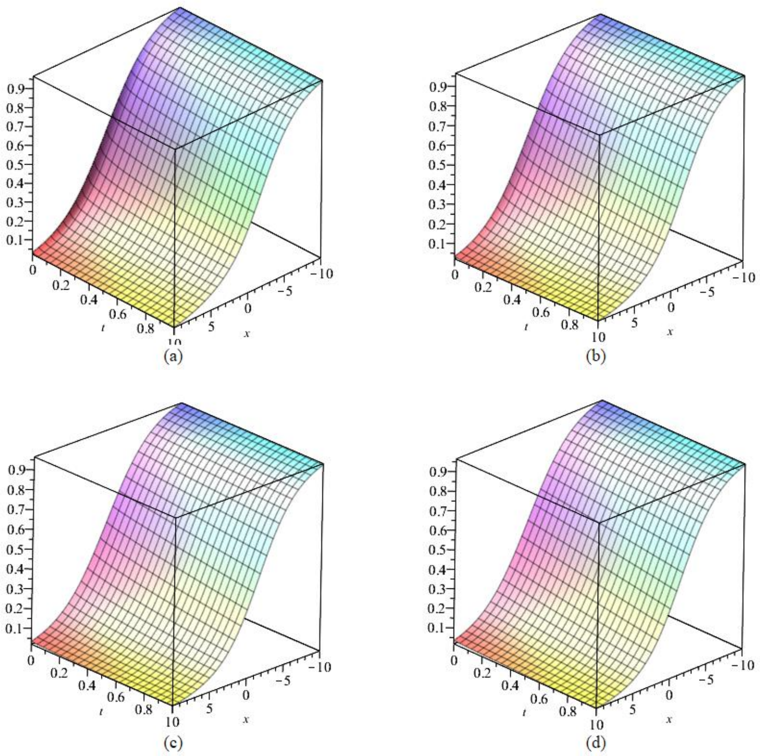


Figure 9: The graphics represent the approximate solution $w_2(x,t)$ with different γ values: (a) $\gamma = 0.4$, (b) $\gamma = 0.6$, (c) $\gamma = 0.8$, and (d) $\gamma = 1.0$ for Example 3.

graphic of solutions are displayed in Figures 5 and 6. Moreover, the numerical simulation when $x = y = 0.2$ at different values of γ is shown in Tables 3 and 4. The Sadik residual power series method and conformable residual power series method (CRPS) [6] errors are analyzed at the corresponding points shown in Tables 5 and 6. This demonstrates that both have equivalent potential.

In Example 3, the graph of approximate solutions at different values of γ when $x = 0.5$ are indicated in Figure 7 and the 3D graphics of approximate solutions are manifested in Figures 8 and 9. In addition, Tables 7 and 8 present the numerical solution at different values of γ.

5. Conclusion

This investigation successfully addressed the system of nonlinear fractional partial differential equations by using the Sadik residual power series method. The employed methodology offers a solution to the nonlinear problem without necessitating the utilization of Adomian polynomials, linearization techniques, or perturbation processes. The ease of use of the technique due to condition (9) is an advantage compared to traditional power series approaches, which involve differentiation to obtain the series solution coefficients. Therefore, the recommended approach requires less time for computation. The obtained results are expressed as a power series that converges to the closed form via Mittag-Leffler functions, as shown in Example 1 and Example 2, while in Example 3, the outcome is written in the form of a truncated power series. Numerical simulation results and 3D graphics confirm that the method is an efficient, reliable, and accurate method for solving non-linear problems.

Based on the information above, the Sadik residual power series approach is an efficient, dependable, and practical method for solving nonlinear fractional order partial differential equation systems. It is suitable for tackling other types of non-linear fractional calculus issues.

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