



Scientific Data Visualization with Shape Preserving C^1 Rational Cubic Interpolation

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Abstract. This paper deals with the shape preserving C^1 rational cubic interpolation. The developed rational cubic interpolating function has only one free parameter. The approximation order of rational cubic function is investigated and range of optimal error constant is determined. Moreover, positive, constrained and monotone data preserving schemes are developed.

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1. Introduction

The development of interpolating and approximating techniques for shape control and shape preservation is a germane area of research in Computer Graphics and Data Visualization environment. The data are classified as positive, constrained, monotone and convex according to their shapes. Stability of radioactive substance and chemical reactions, solvability of solute in solvent, population statistics [3], resistance offered by an electric circuit, probability distribution [3] are a few examples of entities which are always positive. Monotonicity is applied in the specification of Digital to Analog Converters (DACs), Analog to Digital Converters (ADCs) and sensors. These devices are used in control system applications where non-monotonicity is unacceptable [9]. Erythrocyte sedimentation rate (E.S.R.) in cancer patients [9], uric acid level in patients suffering from gout [9], approximation of couples and quasi couples in statistics [2], data generated from stress of a material [9], rate of dissemination of drug in blood [2] are a few examples of entities which are always monotone.

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In the past quite a few authors have worked in the area of shape preserving interpolation. The positivity preserving scheme, by Butt and Brodlie [3], preserved the shape of data by interval subdivision technique. In the interval, where the piecewise cubic interpolant lost the positive shape of data, the authors inserted an additional knot (increase in number of data points) in such a way that original shape of the data was preserved. Fahr and Kallay [5] used a monotone rational B-spline of degree one to preserve the shape of monotone data. Goodman et al [7] developed rational cubic interpolating schemes to preserve the shape of data lying on the either side of straight line. The first scheme scaled weights by some scale factors and the second scheme adopted the method of insertion of a new interpolation point. Goodman [8] surveyed the shape preserving interpolating schemes for planar data. Hussain and Sarfraz used four parameter family of rational cubic function to preserve the shape of positive and monotone data in [10] and [11] respectively. Lamberti and Manni [12] used cubic Hermite in parametric form to preserve the shape of data. The step length was used as shape preserving parameters. The first order derivatives at the knots were estimated imposing second derivative continuity at the knots, which resulted in tridiagonal system of equations. Sarfraz et al [13] addressed the problems of positive and monotone curve data interpolation using rational cubic function with two parameters. Sarfraz et al [14] developed a GC^1 cubic function with one free parameter to preserve the positive, monotone and convex data. Schmidt and Hess [15] developed sufficient conditions on derivatives at the knots to assure positivity of interpolating cubic polynomial.

This paper presents shape preserving interpolating schemes for positive, constrained and monotone data. A one parameter family of rational cubic function has been developed to preserve the shape of the data. In different intervals, the free parameter adopts different value. That is, the interpolant has same structure but different functions in different intervals. The presented schemes are neither dependent on data nor on derivatives. Unlike the methodologies in [3, 7,12], the scheme developed in this paper does not constrain interval length. The developed scheme is equally applicable whether the data is with derivatives or the derivative is estimated by derivative estimation techniques [10-11]. The schemes presented in this paper use one parameter family of rational function to preserve the shape of data thus computationally less expensive than [10-11, 13]. The order of continuity achieved is C^1 , whereas, in [14] it was GC^1 .

The remainder of the paper is organized as follows. In Section 2, the C^1 rational cubic interpolant with one free parameter is developed and Section 2.1 discusses its approximation properties. The visualization problems of positive, constrained, and monotone data interpolation are discussed in Sections 3, 4 and 5 respectively. Section 6 concludes the paper.

2. Rational Cubic Function

In this Section, a C^1 rational cubic function with one parameter and linear denominator has been developed. Let $\{(x_i, f_i), i = 0, 1, 2, \dots, n\}$ be a given set of data points defined over the interval $[a, b]$, where $a = x_0 < x_1 < x_2 < \dots < x_n = b$. The C^1 piecewise rational cubic function with one parameter α_i is defined over each subinterval $I_i = [x_i, x_{i+1}]$, $i =$

0, 1, 2, ..., n - 1 as:

$$S(x) \equiv S_i(x) = \frac{p_i(\theta)}{q_i(\theta)}, \tag{1}$$

where

$$\begin{aligned} p_i(\theta) &= \sum_{i=0}^3 (1 - \theta)^{3-i} \theta A_i, \\ A_0 &= (\alpha_i + 1) f_i, \\ A_1 &= (2\alpha_i + 3) f_i + h_i d_i, \\ A_2 &= (\alpha_i + 3) f_{i+1} - h_i d_{i+1}, \\ A_3 &= f_{i+1}, \\ q_i(\theta) &= 1 + \alpha_i(1 - \theta), \\ h_i &= x_{i+1} - x_i, \theta = \frac{(x - x_i)}{h_i}, \\ S(x_i) &= f_i, S(x_{i+1}) = f_{i+1}, S^{(1)}(x_i) = d_i, S^{(1)}(x_{i+1}) = d_{i+1}. \end{aligned} \tag{2}$$

$S^{(1)}(x)$ denotes the derivative with respect to x and d_i denotes derivative values estimated or given. It is noted that when $\alpha_i = 0$, the piecewise rational cubic function (1) reduces to cubic Hermite spline. In this paper, the parameter α_i can assume any positive real value.

2.1. Error Estimation of Interpolation

In this Section, the error of interpolation is estimated when the function being interpolated is $f(x) \in C^2[x_0, x_n]$. The interpolation scheme developed in Section 2 is local, which allows investigating the error in an arbitrary subinterval $I_i = [x_i, x_{i+1}]$ without loss of generality. Using Peano Kernel Theorem [16] the error of interpolation in each subinterval $I_i = [x_i, x_{i+1}]$ is:

$$R[f] = f(x) - S_i(x) = \frac{1}{2} \int_{x_i}^{x_{i+1}} f^{(2)}(\tau) R_x[(x - \tau)_+] d\tau. \tag{3}$$

It is assumed that the function being interpolated is $f(x) \in C^2[x_0, x_n]$. The absolute error in $I_i = [x_i, x_{i+1}]$ is:

$$|f(x) - S_i(x)| \leq \frac{1}{2} \|f^{(2)}(\tau)\| \int_{x_i}^{x_{i+1}} |R_x[(x - \tau)_+]| d\tau, \tag{4}$$

where

$$R_x[(x - \tau)_+] = \left\{ \begin{array}{ll} u(\tau, x), & x_i < \tau < x, \\ v(\tau, x), & x < \tau < x_{i+1}. \end{array} \right\}, \tag{5}$$

The integral involved in (4) is expressed as:

$$\int_{x_i}^{x_{i+1}} |R_x[(x - \tau)_+]| d\tau = \int_{x_i}^x |u(\tau, x)| d\tau + \int_x^{x_{i+1}} |v(\tau, x)| d\tau. \tag{6}$$

For the C^1 rational cubic function (1), $u(\tau, x)$ and $v(\tau, x)$ have the values

$$u(\tau, x) = (x - \tau) - \frac{\theta^2}{q_i(\theta)} [(1 - \theta)\{(\alpha_i + 3)(x_{i+1} - \tau) - h_i\} + \theta(x_{i+1} - \tau)], \quad (7)$$

$$v(\tau, x) = -\frac{\theta^2}{q_i(\theta)} [(1 - \theta)\{(\alpha_i + 3)(x_{i+1} - \tau) - h_i\} + \theta(x_{i+1} - \tau)]. \quad (8)$$

The roots of $u(x, x)$ and $v(x, x)$ in $[0, 1]$ are $\theta = 0$ and $\theta = 1$.

The root of $u(\tau, x) = 0$ is $\tau_1 = x - \frac{h_i \theta^2 (\alpha_i + 2)}{(\alpha_i + 1) + \theta(\alpha_i + 2)}$.

The root of $v(\tau, x) = 0$ is $\tau_2 = x_{i+1} - \frac{h_i(1 - \theta)}{1 + (\alpha_i + 2)(1 - \theta)}$.

The above discussion leads to the following manipulation:

$$\begin{aligned} |f(x) - S_i(x)| &\leq \frac{1}{2} \|f^{(2)}(\tau)\| h_i^2 \omega(\alpha_i, \theta), \\ \omega(\alpha_i, \theta) &= \int_{x_i}^x |u(\tau, x)| d\tau + \int_x^{x_{i+1}} |v(\tau, x)| d\tau \\ &= \int_{x_i}^{\tau_1} u(\tau, x) d\tau - \int_{\tau_1}^x u(\tau, x) d\tau - \int_x^{\tau_2} v(\tau, x) d\tau + \int_{\tau_2}^{x_{i+1}} v(\tau, x) d\tau \\ &= \frac{\theta^2 \{-\theta^2(\alpha_i + 2)^2 q_i(\theta) + (\alpha_i + 1)^2 (1 + \theta)^2 ((\alpha_i + 3) - \theta(\alpha_i + 2))\}}{q_i(\theta) \{1 + \alpha_i + \theta(\alpha_i + 2)\}^2} \\ &\quad + \frac{2\theta^2(1 - \theta)^2}{q_i(\theta)} - \frac{2\theta^4(1 - \theta)(\alpha_i + 2)}{q_i(\theta) \{1 + \alpha_i + \theta(\alpha_i + 2)\}} + \frac{\theta^2(1 - \theta)^2}{q_i(\theta) \{1 + (\alpha_i + 2)(1 - \theta)\}}. \end{aligned}$$

The above discussion is summarized as follows:

Theorem 1. The error of C^1 rational cubic function (1), for $f(x) \in C^2[x_0, x_n]$, in each subinterval $I_i = [x_i, x_{i+1}]$ is

$$|f(x) - S_i(x)| \leq \frac{1}{2} \|f^{(2)}(\tau)\| h_i^2 c_i, \quad (9)$$

where c_i is the maximum value of $\omega(\alpha_i, \theta)$ for $0 \leq \theta \leq 1$ and $\alpha_i \geq 0$.

Theorem 2. For any given positive parameter α_i , the error optimal constant c_i in Theorem 1 are bounded with $0 \leq c_i \leq 0.2685$.

3. Positivity Preserving Interpolation

This section provides sufficient conditions on parameters for positive interpolation of curve data. Let $\{(x_i, f_i), i = 0, 1, 2, \dots, n\}$ be the positive data defined over the interval $[a, b]$. The necessary condition for the positivity of data is

$$f_i > 0, i = 0, 1, 2, \dots, n - 1. \quad (10)$$

The piecewise rational cubic function (1) preserves positivity if

$$S_i(x) > 0, \quad i = 0, 1, 2, \dots, n - 1.$$

Now, $S_i(x) > 0$ if

$$p_i(\theta) > 0 \quad \text{and} \quad q_i(\theta).$$

But, $q_i(\theta)$ if

$$\alpha_i > 0.$$

Using the result developed by Schmidt and Hess in [15], the cubic polynomial $p_i(\theta) > 0$ if

$$(p'_i(0), p'_i(1)) \in R_1UR_2,$$

where

$$R_1 = \left\{ (a, b) : a > \frac{-3f_i}{h_i}, \quad b < \frac{3f_{i+1}}{h_i} \right\},$$

$$R_2 = \{ (a, b) : 36f_i f_{i+1} (a^2 + b^2 + ab - 3\Delta_i(a + b) + 3\Delta_i^2) + 4h_i(a^3 f_{i+1} - b^3 f_i) + 3(a f_{i+1} - b f_i)(2h_i ab - 3a f_{i+1} + 3b f_i) - h_i^2 a^2 b^2 \geq 0 \}.$$

For the rational cubic function (1), we have

$$p'_i(0) = \frac{-\alpha_i f_i + h_i d_i}{h_i} \quad \text{and} \quad p'_i(1) = \frac{-\alpha_i f_{i+1} + h_i d_{i+1}}{h_i}.$$

$(p'_i(0), p'_i(1)) \in R_1$ if

$$\frac{-\alpha_i f_i + h_i d_i}{h_i} > \frac{-3f_i}{h_i}, \tag{11}$$

and

$$\frac{-\alpha_i f_{i+1} + h_i d_{i+1}}{h_i} < \frac{3f_{i+1}}{h_i}. \tag{12}$$

The inequality (11) leads to the following relation:

$$\alpha_i < \frac{h_i d_i}{f_i} + 3. \tag{13}$$

The inequality (12) imposes the following restriction on the free parameter α_i :

$$\alpha_i > \frac{h_i d_{i+1}}{f_{i+1}} - 3. \tag{14}$$

Further $(p'_i(0), p'_i(1)) \in R_2$ if

$$\begin{aligned} \phi(\alpha_i) = & 36f_i f_{i+1} [\phi_1^2(\alpha_i) + \phi_2^2(\alpha_i) + \phi_1(\alpha_i)\phi_2(\alpha_i) - 3\Delta_i(\phi_1(\alpha_i) + \phi_2(\alpha_i)) + 3\Delta_i^2] \\ & + 3[f_{i+1}\phi_1(\alpha_i) - f_i\phi_2(\alpha_i)][2h_i\phi_1(\alpha_i)\phi_2(\alpha_i) - 3f_{i+1}\phi_1(\alpha_i) + 3f_i\phi_2(\alpha_i)] \\ & + 4h_i[f_{i+1}\phi_1^3(\alpha_i) - f_i\phi_2^3(\alpha_i)] - h_i^2\phi_1^2(\alpha_i)\phi_2^2(\alpha_i) > 0, \end{aligned} \tag{15}$$

with $\phi_1(\alpha_i) = p'_i(0)$ and $\phi_2(\alpha_i) = p'_i(1)$. The C^1 rational cubic function is positive if either (13)-(14) or (15) are satisfied, but (13)-(14) involves less computation hence more suitable for practical applications. This discussion can be summarized as:

Theorem 3. The piecewise C^1 rational cubic interpolant $S(x)$, defined over the interval $[a, b]$, in (1), is positive if in each subinterval $I_i = [x_i, x_{i+1}]$ the following sufficient conditions are satisfied:

$$Con_1 < \alpha_i < Con_2,$$

where

$$Con_1 = \text{Max} \left\{ 0, \frac{h_i d_{i+1}}{f_{i+1}} - 3 \right\} \text{ and } Con_2 = \frac{h_i d_i}{f_i} + 3.$$

The above constraints can be rearranged as:

$$Con_1 + k_i = \alpha_i = Con_2 - l_i,$$

where $k_i > 0$ and $l_i > 0$.

Thus we have the following algorithm for the manipulation of positive curve design.

Algorithm 1

Step 1. Enter the $(n + 1)$ positive data points $\{(x_i, f_i) : i = 0, 1, 2, \dots, n\}$.

Step 2. Estimate the first order derivatives $d_i, i = 0, 1, 2, \dots, n$ at knots. (Note: The step 2 is only applicable if data is not provided with derivatives).

Step 3. Calculate the value of free parameter α_i using Theorem 3.

Step 4. Substitute the values of $f_i, d_i, i = 0, 1, 2, \dots, n$ and $\alpha_i, i = 0, 1, 2, \dots, n - 1$ in rational cubic function (1) to obtain positive curve through positive data.

3.1. Demonstration

Example 1. A positive data set in Table 1 is taken from [6]. The Figure 1 is produced from

Table 1: A positive data taken from [6].

x	0.5	0.6	0.7	0.8	0.9	1.0	1.1
f	0.4804	0.5669	0.7262	0.1	7985	0.8658	0.9281

the data set in Table 1 using the cubic Hermite spline interpolation technique which loses the positive shape of data. The positive curve in Figure 2 is produced by using positive curve data interpolation scheme developed in Section 3.

Example 2. The positive data set in Table 2 is taken from [6]. The Figure 3 is produced from the

Table 2: Another positive data taken from [6].

x	0.0	0.05	0.10	0.15	0.20
f	0.302	0.278	0.10	0.268	0.298

data set in Table 2 using the cubic Hermite spline interpolation technique which loses the positive shape of data. Positive curve is produced in Figure 4 using positive curve data interpolation scheme developed in Section 3.

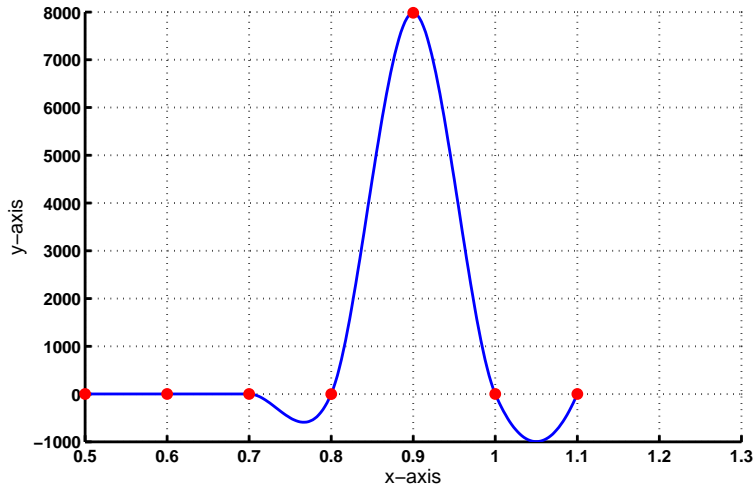


Figure 1: Cubic Hermite spline.

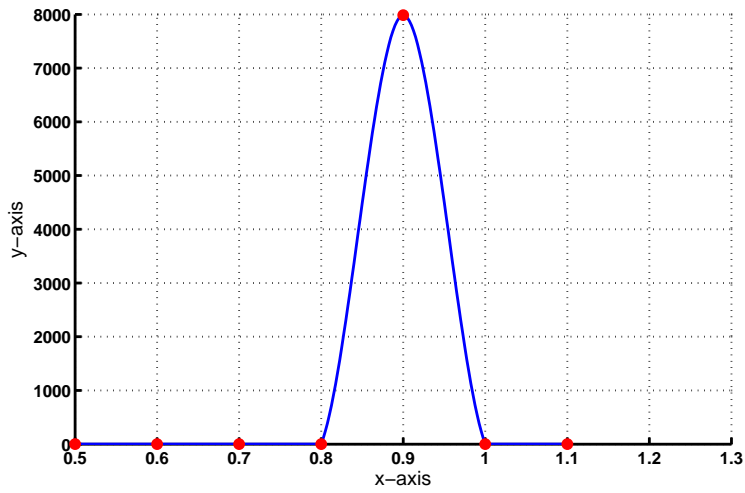


Figure 2: Positive C^1 rational cubic function.

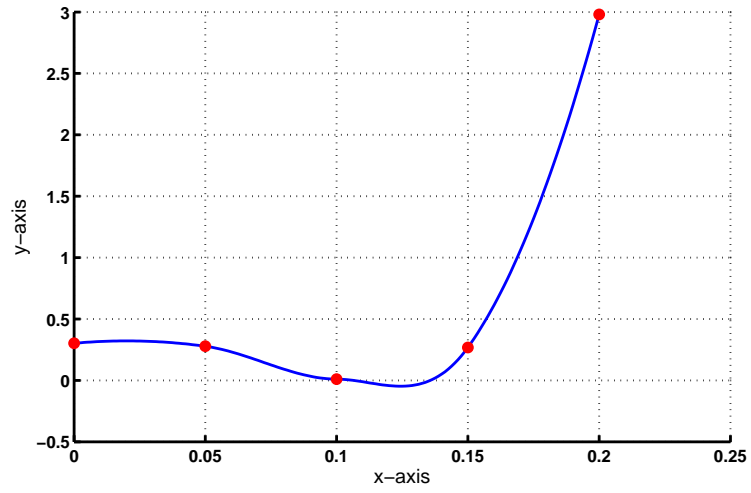


Figure 3: Cubic Hermite spline.

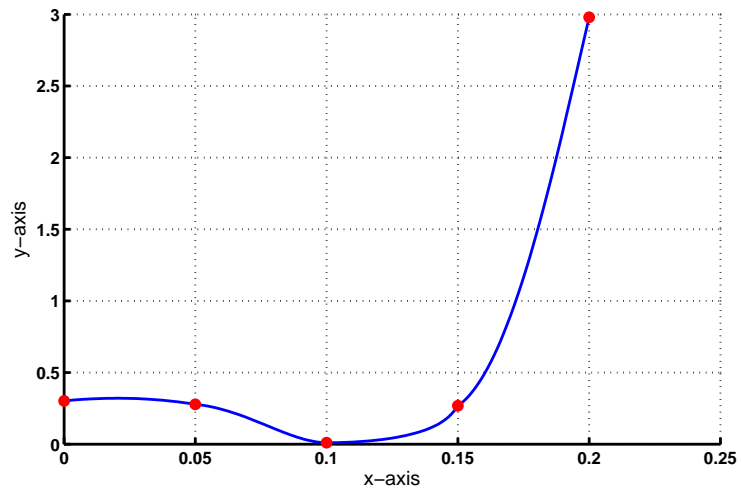


Figure 4: Positive C^1 rational cubic function.

4. Constrained Data Interpolation

Let $\{(x_i, f_i), i = 0, 1, 2, \dots, n\}$ be the given set of data points lying above the straight line $y = mx + c$ i.e.

$$f_i > mx_i + c, \quad \forall \quad i = 0, 1, 2, \dots, n. \tag{16}$$

The curve will lie above the straight line if the C^1 rational cubic function (1) satisfies the following condition:

$$S(x) > mx + c, \quad \forall \quad x \in [x_0, x_n].$$

For each subinterval $I_i = [x_i, x_{i+1}]$, the above relation in parametric form is expressed as

$$S_i(x) > a_i(1 - \theta) + b_i\theta, \tag{17}$$

where $\theta = \frac{x-x_i}{h_i}$ and $a_i(1-\theta)+b_i\theta$ is the parametric equation of straight line with $a_i = mx_i + c$ and $b_i = mx_{i+1} + c$. The rearrangement of (17) leads to the following relation:

$$U_i(\theta) = \sum_{i=0}^3 (1 - \theta)^{3-i} \theta^i B_i, \tag{18}$$

where

$$\begin{aligned} B_0 &= (\alpha_i + 1)(f_i - a_i), \\ B_1 &= \alpha_i(2f_i - a_i - b_i) + 3f_i - 2a_i - b_i + h_i d_i, \\ B_2 &= (\alpha_i + 2)(f_{i+1} - b_i) + f_{i+1} - h_i d_{i+1} - a_i, \\ B_3 &= f_{i+1} - b_i. \end{aligned}$$

Now, $U_i(\theta) > 0$ if $B_i > 0, i = 0, 1, 2, 3$. It is straightforward to know that $B_0 > 0$ and $B_3 > 0$ are true from the necessary condition defined in (16) and $\alpha_i > 0$. $B_1 > 0$ provides the following constraints on α_i :

If $2f_i - a_i - b_i > 0$ then $\alpha_i > \frac{-f_i + b_i - h_i d_i}{2f_i - a_i - b_i}$. Similarly, if $2f_i - a_i - b_i < 0$ then $\alpha_i < \frac{-f_i + b_i - h_i d_i}{2f_i - a_i - b_i}$. One can also note that $B_2 > 0$ if $\alpha_i > \frac{-f_{i+1} + h_i d_{i+1} + a_i}{f_{i+1} - b_i}$.

The above discussion can be summarized as:

Theorem 4. *The piecewise C^1 rational cubic interpolant $S(x)$, defined over the interval $[a, b]$, in (1), preserves the shape of data that lies above the straight line if in each subinterval $I_i = [x_i, x_{i+1}]$ the following sufficient conditions are satisfied:*

$$\begin{aligned} Con_3 &< \alpha_i < Con_4, \\ Con_3 &= \text{Max} \left\{ 0, \frac{-f_i + b_i - h_i d_i}{2f_i - a_i - b_i}, \frac{-f_{i+1} + h_i d_{i+1} + a_i}{f_{i+1} - b_i} \right\}, \\ Con_4 &= \frac{-f_i + b_i - h_i d_i}{2f_i - a_i - b_i}. \end{aligned}$$

The above constraints can be rearranged as:

$$Con_3 + m_i = \alpha_i = Con_4 - n_i,$$

where $m_i > 0$ and $n_i > 0$.

Thus we have the following algorithm for the manipulation of curve design.

Algorithm 2

Step 1. Enter the $(n + 1)$ positive data points $\{(x_i, f_i) : i = 0, 1, 2, \dots, n\}$.

Step 2. Estimate the first order derivatives $d_i, i = 0, 1, 2, \dots, n$ at knots. (Note: The step 2 is only applicable if data is not provided with derivatives).

Step 3. Calculate the value of free parameter α_i using Theorem 4.

Step 4. Substitute the values of $f_i, d_i, i = 0, 1, 2, \dots, n$ and $\alpha_i, i = 0, 1, 2, \dots, n - 1$ in rational cubic function (1) to obtain the curve lying above the straight line.

4.1. Demonstration

Example 3. The data set in Table 3 is lying above the straight line $y = \frac{x}{2} + 1$ and is taken from [1] with slight modification. The Figure 5 is produced from the data set in Table 3 using cubic

Table 3:

x	0	2	3	5	6	8	11	12	14	15
f	6.5	6.5	6.5	6.5	6.5	6.5	16	50	60	85

Hermite spline. It is clear from the Figure 5 that some part of the curve is lying below the line $y = \frac{x}{2} + 1$. This flaw is recovered nicely in Figure 6 using constrained data interpolation scheme developed in Section 4. It is clear from Figure 6 shape of the data is recovered.

Example 4. The data set [10] in Table 4 is lying above the straight line $y = \frac{x}{2} + 1$. Figure 7

Table 4: The data set lying above the straight line $y = \frac{x}{2} + 1$.

x	2	3	7	8	10	13	14
f	12	4.5	6.5	10	6.3	12	18

is produced from the data set in Table 4 using cubic Hermite spline. It is clear from Figure 7 that cubic Hermite spline loses the shape of data. Figure 8 is produced from the same data set using constrained curve data interpolation scheme developed in Section 4. It is clear from the Figure 8 that curve is lying above the straight line $y = \frac{x}{2} + 1$.

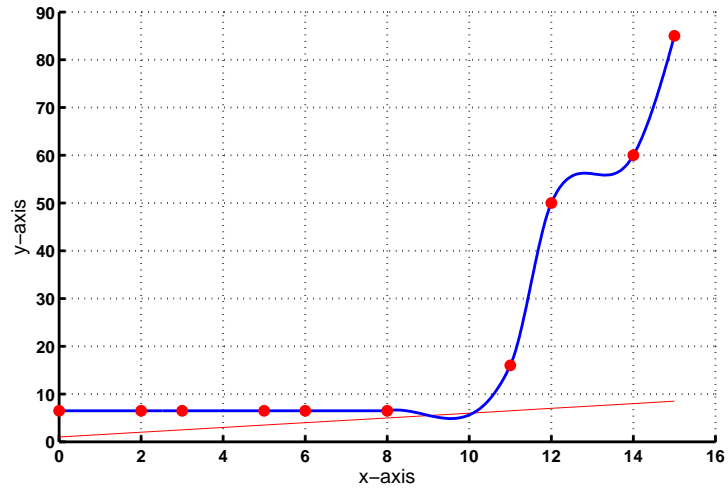


Figure 5: Cubic Hermite spline.

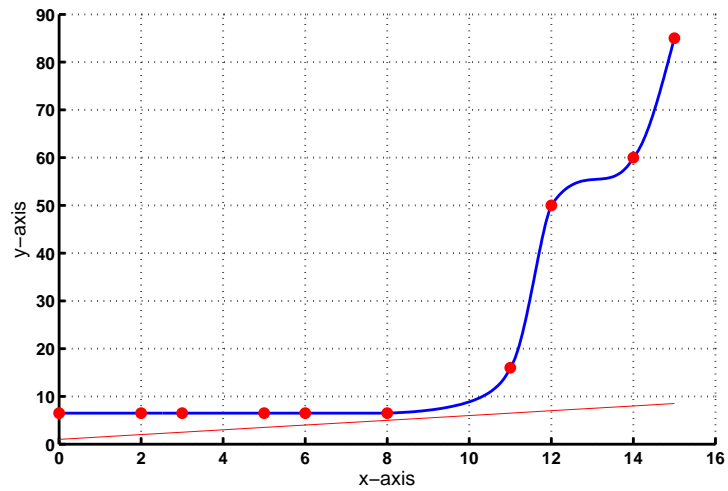


Figure 6: Constrained C^1 rational cubic function.

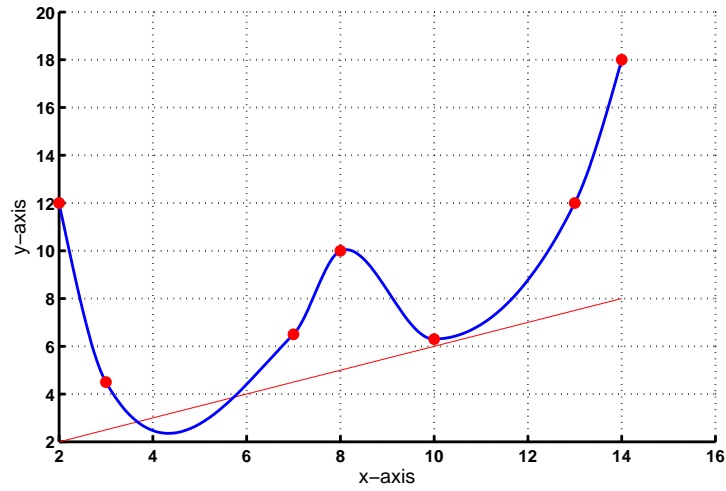


Figure 7: Cubic Hermite spline.

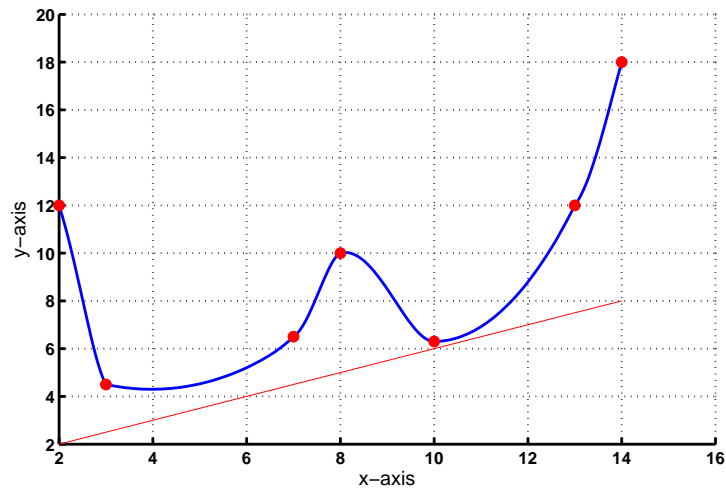


Figure 8: Constrained C^1 rational cubic function.

5. Monotonicity Preserving Interpolation

Let $\{(x_i, f_i), i = 0, 1, 2, \dots, n\}$ be the monotone data defined over the interval $[a, b]$ such that

$$\Delta_i = \frac{f_{i+1} - f_i}{h_i} >, d_i > 0, i = 0, 1, 2, \dots, n - 1. \tag{19}$$

The piecewise rational cubic function (1) preserves monotonicity if

$$S^{(1)}(x) > 0, i = 0, 1, 2, \dots, n - 1,$$

where

$$S_i^{(1)}(x) = \frac{\sum_{i=0}^3 (1 - \theta)^{3-i} \theta^i C_i}{(q_i(\theta))^2}, \tag{20}$$

$$\begin{aligned} C_0 &= (\alpha_i + 1)d_i, \\ C_1 &= (\alpha_i + 1)\{(2\alpha_i + 6)\Delta_i - 2d_{i+1} - d_i\}, \\ C_2 &= (4\alpha_i + 6)\Delta_i - d_{i+1} - 2d_i, \\ C_3 &= d_{i+1}. \end{aligned}$$

From (20), $S_i^{(1)}(x)$ if $C_i > 0, i = 0, 1, 2, 3$. We know that $C_0 > 0$ and $C_3 > 0$ are always true from the necessary condition of monotonicity defined in (19) and $\alpha_i > 0$. One can see that $C_1 > 0$ if

$$\alpha_i > \text{Max} \left\{ 0, \frac{2d_{i+1} + d_i}{2\Delta_i} \right\}.$$

Similarly, $C_2 > 0$ if

$$\alpha_i > \text{Max} \left\{ 0, \frac{d_{i+1} + 2d_i}{4\Delta_i} \right\}.$$

The above can be summarized as:

Theorem 5. *The piecewise C^1 rational cubic interpolant $S(x)$, defined over the interval $[a, b]$, in (1), is monotone if in each subinterval $I_i = [x_i, x_{i+1}]$ the following sufficient conditions are satisfied:*

$$\alpha_i > \text{Max} \left\{ 0, \frac{2d_{i+1} + d_i}{2\Delta_i}, \frac{d_{i+1} + 2d_i}{4\Delta_i} \right\}.$$

The above constraints can be rearranged as:

$$\alpha_i = m_i + \text{Max} \left\{ 0, \frac{2d_{i+1} + d_i}{2\Delta_i}, \frac{d_{i+1} + 2d_i}{4\Delta_i} \right\}, m_i > 0.$$

Thus we have the following algorithm for the manipulation of curve design.

Algorithm 3

Step 1. Enter the $(n + 1)$ points $\{(x_i, f_i) : i = 0, 1, 2, \dots, n\}$.

Step 2. Estimate the first order derivatives $d_i, i = 0, 1, 2, \dots, n$ at knots. (Note: The step 2 is only applicable if data is not provided with derivatives).

Step 3. Calculate the value of free parameter α_i using Theorem 5.

Step 4.

If $\Delta_i > 0$ then

$d_i = 0$

$S(x) \equiv S_i(x) = f_i$

else

$S(x) \equiv S_i(x) = \frac{p_i(\theta)}{q_i(\theta)}$

end

5.1. Demonstration

Table 5: A monotone data set.

x	4.0	6.0	7.0
f	3.9	4.2	5.7

Example 5. A monotone data set is taken in Table 5. Non-monotone curve in Figure 9 is produced from the monotone data set taken in Table 5 using cubic Hermite spline. The monotone curve (from the same data set) is produced in Figure 10 using monotonicity preserving scheme developed in Section 5.

Table 6: A monotone data set.

x	1710	2650	2760
f	500	1360	2940

Example 6. A monotone data set is taken in Table 6. The Figure 11 is produced from the monotone data set taken in Table 6 using cubic Hermite spline which loses the monotone shape of data. Monotone curve in Figure 12 is produced using the monotonicity preserving scheme developed in Section 5.

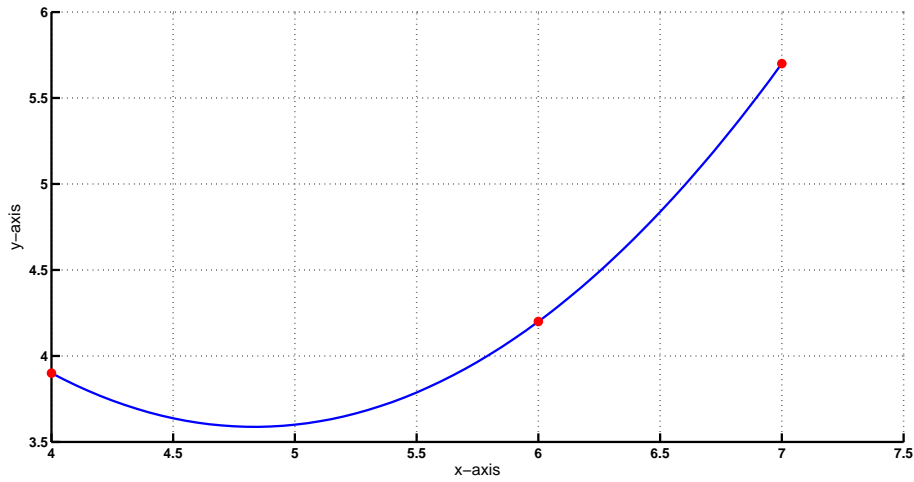


Figure 9: Cubic Hermite spline.

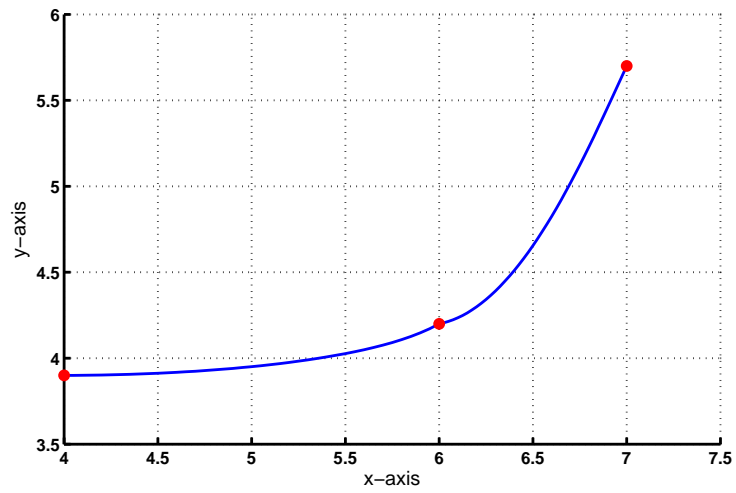


Figure 10: Monotone C^1 rational cubic function.

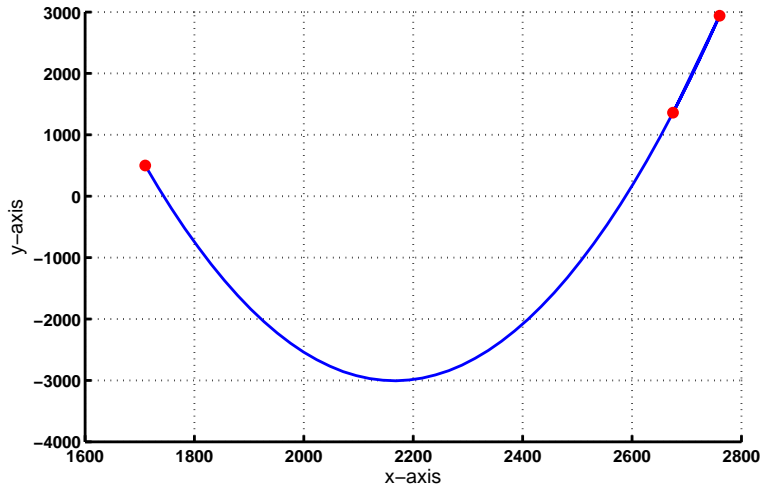


Figure 11: Cubic Hermite spline.

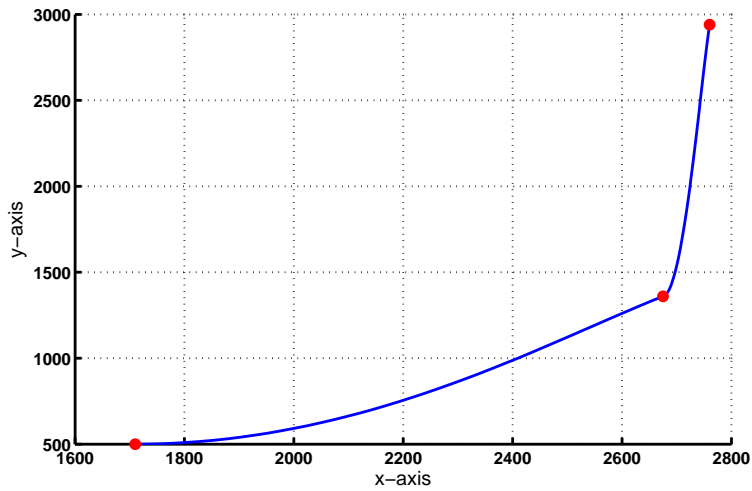


Figure 12: Monotone C^1 rational cubic function.

6. Conclusion

The work in this paper is concerned to the development of shape preserving interpolating schemes. The developed schemes have the following advantageous features over the existing schemes. These are equally worthy for data with and without derivatives, whereas, authors in [15] had to constrain derivatives at knots to preserve the shape of positive data. The developed schemes are based on automated selection of free parameters thus do not require the modification of data. But, the authors in [3, 7, 12] constrained the interval length to preserve the shape of data. In [13] and [10-11], rational functions with two and four family of free parameters were used to preserve the shape of data hence computationally expensive than shape preserving schemes developed in this paper. The order of continuity attained in [14] was GC^1 , whereas, in this paper it is C^1 .

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