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Computing and Simulating Fractal Dimensions to Advance Sustainable Cities: The Road Networks of Buraydah, Saudi Arabia

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Abstract. Urban road networks possess fractal characteristics due to ordered division and structural layout of the road networks which contain inherent self-similarity and scale invariance. We investigate urban planning of road networks in the Buraydah City, Al Qassim, Saudi Arabia using techniques from fractal geometry via fractal dimension (a non-integer number which measures geometric complexity of an object) calculations. The main objective is to study the fractal structure of the road network of the Buraydah city. To accomplish this goal we provide simulation results and compare various kinds of fractal dimensions including box dimension (D_{box}), capacity dimension (D_{cap}), information dimension (D_{inf}), correlation dimension (D_{cor}), and probability dimension (D_{prob}) for the entire road network of the Buraydah city. The study reveals that better planning and optimization of road networks for urban sustainability can be obtained using these fractal indicators. The computational experiments are done using QGIS software and open source fractal dimension programs to understand fractal spatial patterns in the road network.

2020 Mathematics Subject Classifications: 28A80, 65D18, 68U05

Key Words and Phrases: Fractal dimension, Road network, Sustainable cities, Urban planning, Urban sustainability, Simulation

1. Introduction

Ever since the industrial revolution, cities have grown enormously and nowadays more than 50% of the world's total population resides in major cities and urban areas. Many urban planners, economists, politicians, and environmentalists deplore that urban growth appears difficult to control and it has become a source of increasing environmental burden for sustainable development. The continuous growth and evolution of a city lead to substantial changes in the existing road networks and other structures over a period of

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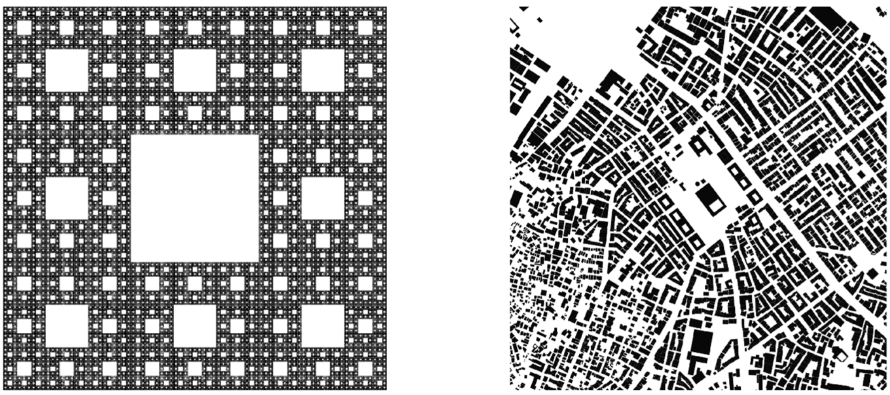
time and the size of road networks also grows with continuous expansion of roads. The planning and systematic structure of urban road networks have become an integral and important part of urbanization which provides good connectivity, attracts industrialization, globalization and helps seamlessly in creating sustainable cities. The structural features of urban road networks affect overall functions and services in a city or urban area. Therefore, an optimal planning of urban road networks is extremely important from multiple perspectives including smoother transportation and sustainable development of any city.

Researchers have observed that almost all cities contain fractal rules in some or the other form while occupying the available free space as they evolve over time. The space-filling processes demonstrate how new developments, access and inclination to services scales up over the time. Fractal techniques have been implemented as a tool to characterize and interpret urban areas in urban simulation models [6, 14, 45] to characterize and interpret urban areas.

The ordered division and structural planning (layout) of urban road networks contain self-similarity and scale invariance therefore urban road networks possess fractal characteristics (see Figure 1 below for an example). The urban planning models based on fractal



(a) (b)

Figure 1: Fractal patterns. (a) Sierpinski carpet, a well-known fractal used as a basis for comparison. (b) Sierpinski type fractal patterns in Istanbul city [21].

geometry can

(i) simulate urban growth and development to improve understanding of underlying processes to a large extent.

(ii) suggest various schemes and patterns for effective urban planning.

In this article we study, analyze and quantify structural characteristics of road networks in the Buraydah City, Al Qassim, Saudi Arabia using fractal geometry and fractal dimension calculations. The prime objective is to provide precise characterization of urban road networks via fractal indicators. The study suggests feasibility of using fractal indicators in process of design, construction and evaluation of urban road network layouts in an optimal manner for better urban traffic control and sustainability.

In the rapidly urbanizing world, major cities are a big attraction for companies, business, jobs, higher studies, living standards, schooling etc. and a clear understanding of how cities run and how urban life is a subject of utmost importance today.

Several studies including recent developments have established that the fractal geometry is a powerful tool for urban form [3, 7, 8, 13, 14, 42], fractal cities [4–6, 10], urban modelling [9, 12, 17], urban pattern design [22], road networks [2, 18, 25, 26, 37], fractal patterns in transit networks [38, 39, 41, 44], complex networks [36, 43], urban planning [10, 16, 21] and socioeconomic functions [29, 30, 34] wherein computation of fractal dimension is the key tool for characterization in most of the works. The fractal dimension along with other techniques of fractal geometry can provide very reliable, accurate and effective solutions to the design, planning, development and evaluation of urban road and traffic flow networks which are not possible using the classical Euclidean geometry. A fractal structure occupies available space by imitating its structure at ever increasing thinners scale.

The term "fractal" was coined by Mandelbrot who used it to delineate the natural geometry, such as leaves of tree, the profile of cloud, the silhouette of a tree covered with ice, the surface of cauliflower, the shape of coastlines etc. which are jagged, irregular, self-similar and fragmented in multiple scales at finer levels [31].

The urban road network optimization is typically done using 2 approaches:

(i) Traditional approach, which is based on finding various indicators such as road network density, connectivity etc. for optimizing road network structures via traditional data analysis methods.

(ii) Fractal geometry approach, which examines the spatial structure features as well as measures the coverage degree; access depth of the road network to represent the spatial topological relationship of the internal structure of the road network.

In various road network layout designs, expressway, arterial, sub-arterial network, and branch network are relevant with the overall layout. This is a kind of scale invariance and fractal geometry fits naturally into this to assess internal structural characteristics of urban road networks. The fractal dimension of planer shapes lies between 1 and 2 [20]. Several studies have been conducted on urban road network structure optimization and the fractal dimensions of urban road networks [2, 11, 14, 18, 26, 29, 30, 37, 43]. The urban road traffic networks suffer difficulties in terms of jagged spatial distribution, obstructive network layout, and poor traffic accessibility. Over the last two decades, accurate analysis, description and characterization of urban road network structures has become a topic of great importance for precise decision-making and guidance towards the design and layout of urban sustainability. In order to improve fractal dimension calculation, we require high-resolution remote sensing and big traffic data which contain more realistic features and information (e.g., the complexity and nonlinearity) of road networks under consideration.

Euclidean geometry and standard linear algebra have limited capabilities in handling with complex and nonlinear road networks. Fractal geometry provides appropriate framework and tools to study and simulate complex (irregular) shapes such as urban road networks. A number of studies illustrates that cities exhibit fractal postulates both in terms of filling the space available to them and in terms of statistical self-similar patterns that they create (at-least at some scales) [18, 25–27, 44]. These space-filling properties and fractal patterns contain the size distribution of the built environment, and road networks in the city.

In this article, we simulate and compare 5 kinds of fractal dimensions (box, capacity, information, correlation and probability dimensions) using two open source software for the road network in the Buraydah city. The findings in this research elaborates on comparison among each type of computed fractal dimension and some insights are provided on which type of computed dimension may have the capability to measure the space filling properties more accurately.

2. Preliminaries

Cities are complex systems that behave like living organisms in some or the other ways and possesses chaotic characteristics. The laws of fractals and chaos theory apply straight forward to the evolution of cities. The study of urban patterns allows us to benefit from the experiments of past cultures to shape our own future with as much awareness of the consequences of our actions as possible.

Urban road networks face difficulties in terms of uneven spatial distribution, obstructive network layouts, and poor traffic accessibility. Scientific and objective description of the characteristics of an urban road network is a subject of major practical significance from the prospective of preparing guidelines and making informed quantitative and scientific decisions for the planning and layout as well as for the purpose of analyzing the development characteristics of spatial structures in urban road networks for urban sustainability.

To study and precisely calculate the fractal dimension, high-resolution remote sensing images are extracted containing information on more realistic features such as complexity and nonlinearity present in the road networks at finer scales which contributes to the development of the network structure. A conventional geographic urban road network model is typically constructed on Euclidean geometry and techniques from linear algebra. It is well known that the Euclidean geometry based approach combined with linear algebra methods can provide accurate solutions to linear problems but their applicability in dealing with irregularities and nonlinear systems is very limited. To overcome this, modern computational tools including fractal geometry is introduced to apply iterative law of road network structures. Furthermore, fractal dimension can usually only be applied to depict the relevant features of a single level of road network in a given region [32, 33]. Hence, it is an absolute necessity to develop an integrated analytic procedure that may combine the relationship among diverse types of roads and calculate the fractal dimension of the road network. These new tools are required to effectively unfold the fractal characteristics at all levels in the road network for decision making and planning of urban sustainability cities.

Fractal Dimension (FD) denoted by D is usually a non-integer positive number which measures geometric complexity (irregularity) of an object in the underlying space of fractals. It provides intimation regarding an object's geometrical structure at diverse scales. This permits to take over spatial object by estimating how fast it grows or reduces with varying scales. Thus, FD is a statistical number measuring the space filling efficiency. Various types of fractal dimensions have been developed in the literature to characterize specific features of fractals such as Minkowski dimension, similarity dimension, box dimension, divider (ruler) dimension, capacity dimension, mass dimension, information dimension, correlation dimension and so on. Most of these dimensions are special cases of a more general fractal dimension namely the Hausdorff dimension [19].

In the context of urban planning and road networks, these dimensions play an important role since they can characterize and estimate the distribution density and network complexity. These provide a means to determine the coverage depth, connectivity degree, and coverage degree of urban traffic [12, 18]. For road networks, the box-counting and mass dimensions are main tools to explore the fractal properties [2]. Deng et. al [18] recently, studied the structural features of the urban road network by means of five various kinds of fractal dimension calculations in several districts of the Harbin city in China. These are based on various district patters including population, district built-up areas, building number as well as built-up area, road number and lengths.

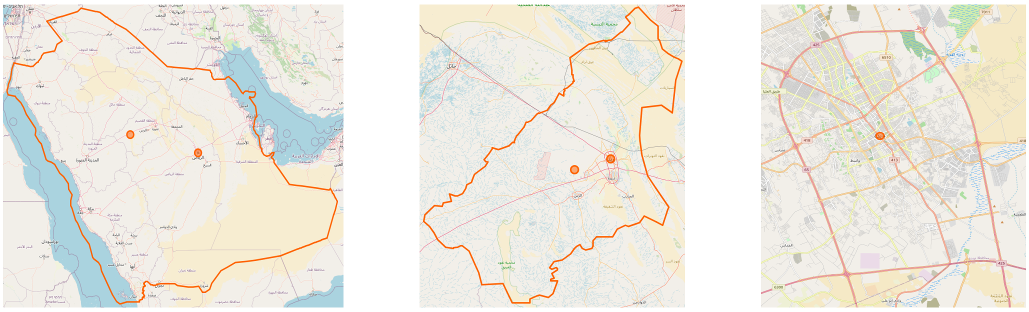
3. Methodology

In this work, we study and analyze fractal characteristics of the road networks in the Buraydah City, Al Qassim, Saudi Arabia through fractal geometry and fractal dimension simulations.

3.1. Study Areas

Buraydah is the capital and largest city as well as the commercial and administrative center of Al-Qassim region in north-central Saudi Arabia in the heart of the Arabian Peninsula. Buraydah is equidistant from the Red Sea (in the west) and Persian Gulf (in the east) among longitudes $43°42'E$ to $43°90'E$ and latitudes $26°10'N$ to $26°45'N$ at an elevation of approximately 650m above the sea level. The city of Buraydah is an important agricultural center that plays a significant role in the economy of Saudi Arabia. Figure 2 provides the administrative maps of Saudi Arabia, Al Qassim province and Buraydah city.

The Buraydah city has giant landscapes for the mass production of vegetables, wheat, and dates and serves as the food basket for the Saudi Arabia. The city also hosts the world's biggest dates festival. It consists of 70 districts with large business centers and recreational services which attracts high rate of population migration from other cities and villages of Al Qassim to share these services. As of 2024 data, Buraydah's metropolitan city has a total built-up area and population of 1291 square kilometer and 677,647 as per General Authorities for Statistics (available at *https://www.stats.gov.sa/en*) respectively with an average annual population change rate of 1.68%.



(a) (b) (c)

Figure 2: Maps of (a) Saudi Arabia, (b) Al Qassim province and (c) Buraydah city (Source: OpenStreetMap, https://www.openstreetmap.org/).

3.2. Data Collection

The necessary data for the road network analysis and calculation of fractal dimensions consisting of the road network map is obtained for the entire city of Buraydah.

The Buraydah city map consisting of features such as roads, buildings, places, highways etc. is taken from, Open Street Map (OSM), which is an open source geographic database, in the shape file as displayed in Figure 3. The road network for the entire Buraydah city is then obtained from the shape files using in-built tools in the QGIS software which is a standard open source Geographic Information System (GIS) software.

Most of the research in urban network analysis makes use of fractal dimension for analysis and interpretations. Based on the available studies, fractal analysis for urban road networks can be categorized in 3 classes:

(i) Revealing fractal properties of the road network [24].

(ii) Using fractal properties of road networks to find patterns in urban development [15, 28].

(iii) Computing fractal dimension for a road network or subnetwork [30].

3.3. Fractal Dimension

The underlying rule of a fractal measurement is to choose a basic unit with a uniform scale size ϵ (for example, side of regular polygon such as a square or an equilateral triangle, or the radius of a circle) for spatial measurement of an object, and the result of the measurement is $M(\epsilon)$. Modifying the measurement scale ϵ will change $M(\epsilon)$ accordingly. An object is said to possess fractal characteristics if ϵ and $M(\epsilon)$ satisfy

$$M(\lambda\epsilon) \propto \lambda^{\pm\alpha} M(\epsilon), \tag{1}$$

within a specific range. Here, λ is the scale ratio and α is a function of fractal dimension. Usually, $\alpha = D$, where D is the fractal dimension. In fractal geometry, the law of degree invariance states that D does not change with change in the scale size ϵ.

3.3.1. Box dimension

The box dimension or box-counting dimension is denoted by D_{box}. It is also called grid dimension because a grid of boxes (usually squares) is imposed on the object under consideration to calculate the box dimension. The minimum number of boxes required to cover the object at a chosen side length ϵ is counted and denoted by $N(\epsilon)$. The logarithm of $N(\epsilon)$ is plotted on the vertical axis versus the logarithm of ϵ on the horizontal axis for a range of values of ϵ. If the object is a fractal, then the plot adheres a straight line with a negative slope which is $-D_{\text{box}}$. For simulating D_{box} we use Fractal Dimension Estimator software which automatically selects the side length ϵ of the box based on shape and size. The software then selects the RGB threshold to convert the image into binary data and its automatic extraction. It also selects the "scaling window" of the box-counting algorithm with inputs as the minimum to maximum box size ϵ. Once compiled it generates the required log-log graph whose slope represents the D_{box}.

Figure 3: Road network in the Buraydah city as of 2024 (extracted using QGIS).

3.3.2. Capacity dimension

The capacity dimension is denoted by D_{cap}. To determine the capacity dimension of an object, it is assumed that the number of elements covering an object (or data set) is inversely proportional to ϵ^D, where ϵ is the scale of covering cells and D is the (Euclidean) dimension of the object. Thus, for an arbitrary object, we have

$$N(\epsilon) \propto \epsilon^{-D} \tag{2}$$

where ϵ is the side of the covering element and $N(\epsilon)$ is the number of elements of size ϵ desired to cover the object. If object is scaled, it fits into a single element (say square) with side $\epsilon = 1$, then $N(1) = 1$. This gives,

$$D_{\text{cap}} = \lim_{\epsilon \to 0} \frac{\log(N(\epsilon))}{\log(1/\epsilon)}. \tag{3}$$

In other words, we can measure the amount of space a subset of the two-dimensional space covers by examining how efficiently the set can be covered by elements of various sizes.

The D_{cap} is calculated by drawing logarithm of $N(\epsilon)$ on the vertical axis against the logarithm of $\frac{1}{\epsilon}$ on the horizontal axis for different length scales ϵ. If the relationship possesses fractal characteristics, then the plot is a straight line with a positive slope D_{cap}. Thus, we see that the capacity dimension is essentially similar to the box dimension with the difference that in the box dimension minimum number of boxes is counted while in the capacity dimension total number of elements which covers the given object are counted. The FDim software is used to compute D_{cap} for the road network under consideration. The input is the road network map in .bmp format and the software automatically selects size of the covering element and then calculate the capacity dimension using the log-log plot.

3.3.3. Information dimension

The information dimension is denoted by D_{inf} and it is essentially a weighted variant of box dimension in which weights (probabilities) are assigned to the boxes in such a way that the boxes containing more number of points gets higher weight (probability) as compared to the boxes containing lesser number of points. To calculate D_{inf}, an information function or information entropy $I(\epsilon)$ is calculated using the formula

$$I(\epsilon) = -\sum_{i=1}^{N} P_i(\epsilon) \log(P_i(\epsilon)), \tag{4}$$

where $P_i(\epsilon)$ is the natural measure, or the probability that element i is populated and normalized such that $\sum_{i=1}^{N} P_i(\epsilon) = 1$. The information dimension is now defined by

$$D_{\text{inf}} = -\lim_{\epsilon \to 0} \frac{I(\epsilon)}{\log \epsilon} = -\lim_{\epsilon \to 0} \frac{\sum_{i=1}^{N} P_i(\epsilon) \log(P_i(\epsilon))}{\log \epsilon}. \tag{5}$$

D_{inf} is calculated for various length scales ϵ by drawing logarithm of information entropy $I(\epsilon)$ versus logarithm of side length ϵ. If the object is indeed fractal, a plot of $I(\epsilon)$ versus ϵ on a $\log - \log$ scale will follow a straight line with a negative slope which is equal to $-D_{\text{inf}}$. The FDim software does this computation automatically once the object is given as input in BMP file format by selecting the corresponding algorithm.

It follows from the formula (3.3) and (3.5) that the correlation dimension coincides with the information dimension if $P_i(\epsilon)$ is independent of i, i.e., every element is equally likely to be visited and in that case $\sum_{i=1}^{N} P_i(\epsilon) = NP_i(\epsilon) = 1$, so that $P_i(\epsilon) = \frac{1}{N}$.

3.3.4. Correlation dimension

The correlation dimension is denoted by D_{cor}. It is a measure of the dimensionality of the space occupied by a set of random points. It has the advantage of being calculated quickly and easily as follows:

Consider a set $S = \{X_1, X_2, \cdots, X_N\}$ containing N random points on an object. Let $|X_i - X_j|$ denotes the distance between each pair of points (X_i, X_j). Define a correlation function $C(\epsilon)$ by

$$C(\epsilon) = \frac{M}{N^2}, \tag{6}$$

where M is the total number of pairs of points having distance between them less than ϵ, i.e., $|X_i - X_j| < \epsilon$. Then, the function $C(\epsilon)$ can be written more conveniently in the form

$$C(\epsilon) = \lim_{n\to\infty} \frac{1}{N^2} \sum_{j=1}^{N} \sum_{i=j+1}^{N} \theta(\epsilon - |X_i - X_j|), \tag{7}$$

where θ is the Heaviside function, defined by

$$\theta(\epsilon - |X_i - X_j|) = \begin{cases} 1, & |X_i - X_j| \le \epsilon \\ 0, & |X_i - X_j| > \epsilon. \end{cases} \tag{8}$$

As the number of points tends to ∞ and the distance between them tends to 0, the correlation function follow a power law similar to the capacity dimension and takes the form

$$C(\epsilon) \propto \epsilon^D. \tag{9}$$

Thus, D_{cor} can be calculated using the formula

$$D_{\text{cor}} = \lim_{\epsilon\to 0} \frac{\log(C(\epsilon))}{\log \epsilon} = \lim_{N\to\infty} \lim_{\epsilon\to 0} \frac{\log\left(M/N^2\right)}{\log \epsilon}. \tag{10}$$

If the number of points is evenly distributed and sufficiently large, the slope of the log-log graph between $C(\epsilon)$ and ϵ will yield D_{cor} for small ϵ. This is accomplished using the FDim software automatically in our simulations. Indeed for higher-dimensional objects, the number M will grow rapidly since there will be more ways for points to be close to each other. Moreover, the inequality

$$D_{\text{cor}} \le D_{\text{inf}} \le D_{\text{cap}}. \tag{11}$$

holds.

3.3.5. Probability dimension

The probability dimension is denoted by D_p [23, 40] and it can be regarded as a variant of the capacity dimension. Let $P(m, \epsilon)$ be the probability that m points (pixels) lie in a box of side length ϵ centered over any pixel in the image (object). Then for each ϵ, we have

$$\sum_{i=1}^{N} P(m, \epsilon) = 1$$

where, N is the maximum number of possible pixels within the box. Now let M denotes the total number of pixels in the image. If the image is covered with boxes of side length ϵ, the number of boxes with m pixels inside the box is approximately $\frac{M}{m}P(m, \epsilon)$. Therefore, the expected total number of boxes needed to overlay the entire image is

$$\langle N(\epsilon)\rangle = \sum_{m=1}^{N} \frac{M}{m} P(m, \epsilon) = M \sum_{m=1}^{N} \frac{P(m, \epsilon)}{m}. \tag{12}$$

If we take, $N(\epsilon) = \sum_{m=1}^{N} \frac{P(m,\epsilon)}{m}$ then as in capacity dimension,

$$N(\epsilon) \propto \epsilon^{-D_{\text{prob}}},$$

where D_{prob} is the probability dimension. Hence, we get the formula

$$D_{\text{prob}} = \lim_{\epsilon \to 0} \frac{-\log(N(\epsilon))}{\log \epsilon}. \tag{13}$$

4. Simulation, Comparison and Discussion

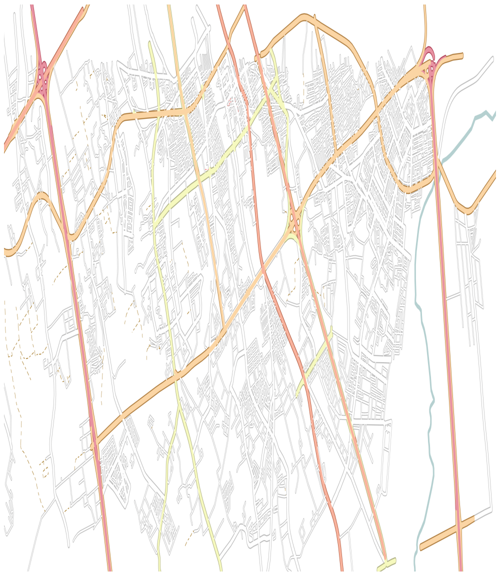
Several software tools are available to estimate fractal dimensions which include Fractalyse, Fraclab, FracLac, TruSoftBenoit, etc. Nowadays, many common GIS software are pre-loaded with a fractal dimension calculation module such as ArcGIS, SpaDiS, Exeter GS, Geostat Office, etc.

4.1. Data Processing & Software/Codes

We have used two open source software namely the Fractal Dimension Estimator software [1] and FDim software [35] which are image analysis programs and can be applied to compute various types of fractal dimensions described in Section 3.

Fractal Dimension Estimator is a tool to measure the fractal dimension (FD) of a 2D image by means of the box-counting method and it supports BMP, JPG, PNG, TIFF, GIF file formats. FDim is a GUI program to compute the fractal dimension of a gray scale image. It supports the capacity, information, correlation, and probability dimension algorithms. FDim software requires an image in BMP format (.bmp) as input. Gray scale images are provided as inputs in both the programs. Measurements scale are automatically

(a)



(b)

(c)

Figure 4: Road maps showing details of network (a) segment-I, (b) segment-II and (c) segment-III.

chosen for a range of sizes by both the software. The scale sizes for all methods range from 2 to around 128 pixels. For simulation purposes, Buraydah city maps were processed and extracted in the BMP format in a uniform resolution 500 dpi and in various sizes (width, height) as inputs to measure fractal dimensions. In order to compute fractal dimensions and compare simulation results, we divided the entire Buraydah city road network of Figure 3 into three segments as shown in Figure 4 (extracted and processed via QGIS). Various types of fractal dimensions are simulated for these segments as well as for the entire road network of the city of Buraydah.

4.2. Fractal Characteristics of the Road Networks

The simulation results for the entire road network in the Buraydah city are provided in the Table 1. It is clear from the table that the fractal dimension values lie between 1 and 2. It indicates that the case study is fractal with regard to all types of fractal dimensions under consideration.

Table 1: Fractal dimension calculations and corresponding correlation coefficients

Network	Box dimension		Capacity dimension		Information dimension		Correlation dimension		Probability dimension	
	D_{box}	R^2	D_{cap}	R^2	D_{inf}	R^2	D_{cor}	R^2	D_{prob}	R^2
Segment-I	1.133	0.9977	1.902	0.9998	1.645	0.9994	1.463	0.9998	1.320	1.0000
Segment-II	1.169	0.9989	1.808	0.9999	1.791	0.9999	1.658	0.9999	1.820	0.9991
Segment-III	1.239	0.9981	1.739	0.9999	1.679	0.9999	1.561	0.9999	1.670	0.9992
Full Network	1.906	0.9813	1.990	0.9995	1.904	0.9981	1.876	0.9997	1.495	0.9971

Also, seeing a straight line plots for each type of fractal dimension using FD Estimator and FDim software, the following are noted: (i) For box counting method, $\log N(\epsilon)$ vs $\log \epsilon$ gets a negative slope equal to $-D_{\text{box}}$. (ii) For the capacity method, $\log N(\epsilon)$ vs $\log \epsilon$ gives a negative slope equal to $-D_{\text{cap}}$. (iii) For the information method, $\log I(\epsilon)$ vs $\log \epsilon$ provides a negative slope equal to $-D_{\text{inf}}$. (iv) For correlation method, $\log C(\epsilon)$ vs $\log \epsilon$ gets a positive slope equal to D_{cor}. (v) For the probability method, $\log N(\epsilon)$ vs $\log \epsilon$ gives a negative slope that equal $-D_{\text{prob}}$.

All methods have strong correlation among the values on the vertical and horizontal axes. This is verified from the correlation coefficients given in the Table 1. Figure 5 displays $\log - \log$ plots for the D_{box} which are obtained using the FD Estimator software.

There is no general rule to compare fractal dimension values for each segment but we observe that for the same region D_{box}, D_{cap} and D_{inf}, values have a high linear positive correlation due to the similarity among these methods. Further, they have higher values than D_{cor} and D_{prob} which gives weak linear positive correlation. Also D_{prob} represents distinct behavior since sometimes it has a bigger value than D_{box} and D_{inf}; and sometimes it has a smaller value.

Referring to the correlation data as well as $\log - \log$ plot behaviors: D_{box}, D_{cap}, D_{inf}, and D_{cor} may describe the fractal feature of roads network better than D_{prob} since D_{prob} values exhibit random behavior as compared to other FDs. It is also clear form the FD values in Table 1 that the fractal dimensions satisfy the relation given in (3.11).

4.3. Comparison of Buraydah's fractal dimensions

On comparing the minimum, maximum and average values of each fractal dimension (see Table 2) with fractal dimensions of the entire Buraydah city we noticed that D_{box}, D_{cap}, D_{inf} and D_{cor} for Buraydah are higher than the corresponding maximum values from all segments. However, D_{prob} for the Buraydah city is less than the maximum D_{prob} from

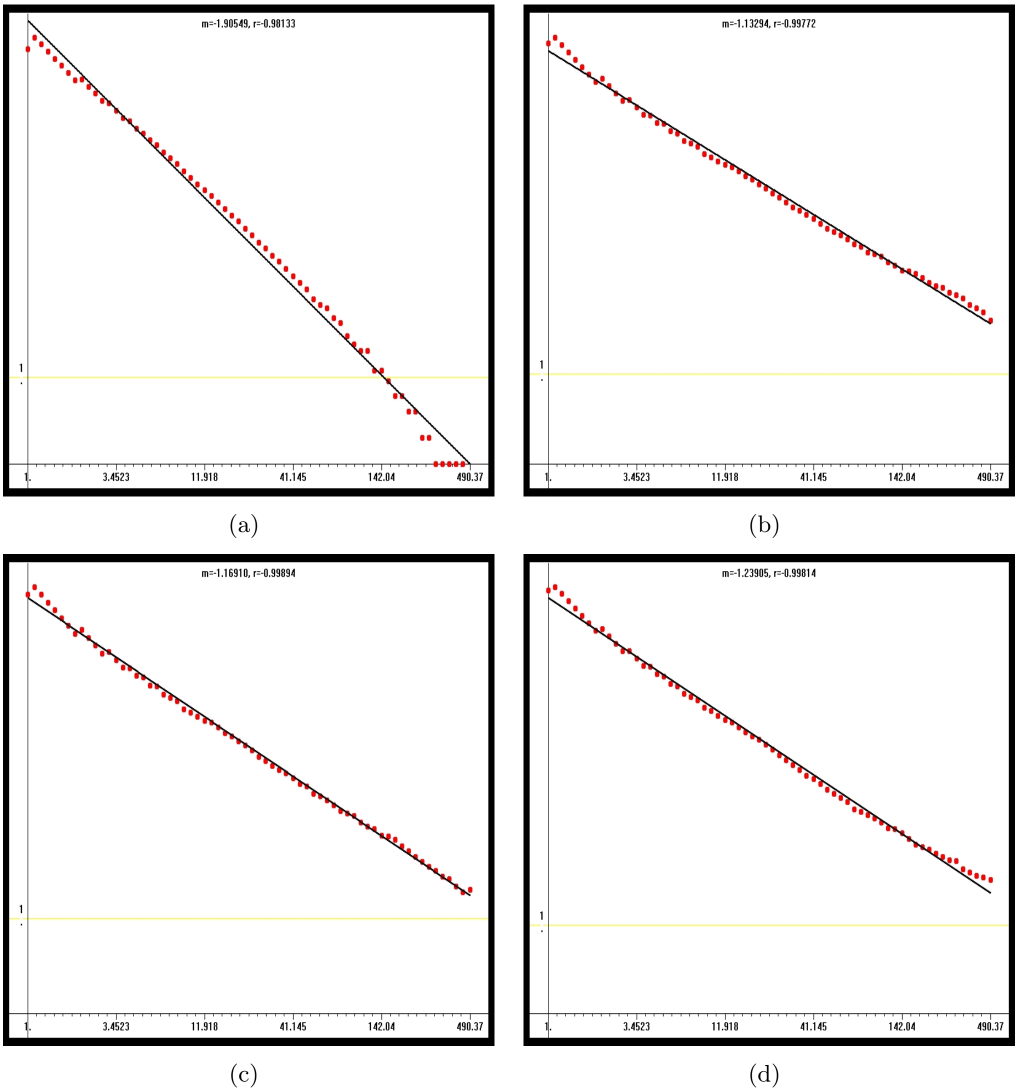


Figure 5: log − log plots of D_{box} for (a) Buraydah city, (b) segment-I, (c) segment-II and (d) segment-III from FD Estimator software

all segments. Moreover, the minimum value of D_{prob} from all segments is smaller than the D_{prob} for Buraydah city.

Table 2: Minimum, maximum and average values of fractal dimensions for various segments of the city.

Fractal Dimension	Minimum	Maximum	Average
D_{box}	1.133	1.239	1.180
D_{cap}	1.739	1.902	1.816
D_{inf}	1.645	1.791	1.705
D_{cor}	1.463	1.658	1.561
D_{prob}	1.320	1.820	1.603

In terms of measuring the complexity of the road network, D_{box}, D_{cap}, D_{inf} and D_{cor} seems to be the best indicators to characterize the fractal nature of the road network. When compares the whole network with the segments, then the entire Buraydah city road network has higher D_{box}, D_{cap}, D_{inf} and D_{cor} values than those of segments.

4.4. Discussions

Simulation results from fractal dimension calculations clearly indicated that the computation of D_{box}, D_{cap}, D_{inf} and D_{cor} fractal dimensions is capable of revealing the fractal properties of the road networks under our case study. As stated above and known from the available literature, fractal dimensions individually cannot describe the fractal features of any road network. To predict better urban planning, data from district patterns (population, areas, buildings etc.) could be included in the analysis. Outcomes from statistical analysis among all kinds of fractal dimensions show that D_{cap}, and D_{inf} values agree closely as we can see from Table 1 and they contribute a similar calculation method (box occupation) too. Moreover, the roads or segments have almost the similar patterns but may not have the same fractal dimensions. Calculation of dimension is also impacted by many factors such as image resolution, distribution of pattern in the network and so on.

5. Conclusions

Any urban city may be regarded as a very complex, integrated structure and main essential objective of urban planning is sustainable urban design and form. The study of urban road network structures is necessary for planning, design and management. The analysis and evaluation of urban road networks using fractal geometry and fractal dimension framework describe its complexity to a more desired and practical level. The tools from fractal geometry can provide a vast and in-depth quantitative research in this subject which will lead to richer systematic conclusions. Fractal dimension is an important spatial analysis measurement of fractal geometry which has been applied to many various fields including science, engineering, medicine and others. To calculate fractal dimension accurately, good analytical and computational tools are required.

This work expresses the structural characteristics of the urban road network. It describes the significance of fractal geometry to characterize the road network in the Buraydah city by calculating 5 kinds of fractal dimensions namely the box dimension D_{box}, capacity dimension D_{cap}, information dimension D_{inf}, correlation dimension D_{cor} and the probability dimension D_{prob}. Open source software FD Estimator and FDim were employed to simulate fractal dimension. The desired images have been proceeded through the QGIS software in various file formats. Simulation results prove that the strongest positive correlation is 0.9999 among D_{box}, D_{cap} and D_{inf} and this is due to the theoretical similarity of these methods. A given segment of the road network has more than one fractal dimension therefore, it follows that the fractal dimension itself is not enough to explore regarding the spatial distribution in the space and the complexity level of the city road network.

In summary, the computed results reveal that D_{box}, D_{cap} and D_{inf} provide good signs of the fractal nature of Buraydah's road networks compared to other by types of dimensions. In the upcoming works, we plan to consider the effect of district patterns (e.g., districts' built-up areas, buildings, built-up area, road number and lengths) on the fractal dimension as well as its application for creating sustainable cities.

Data Availability

No any underlying data used in this paper.

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